April 14, 1936. C. E. LANE 2,037,171

WAVE FILTER

Filed June 23, 1933

FIG. 1

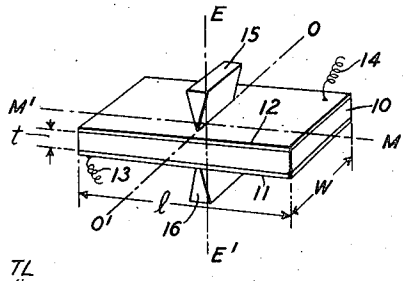
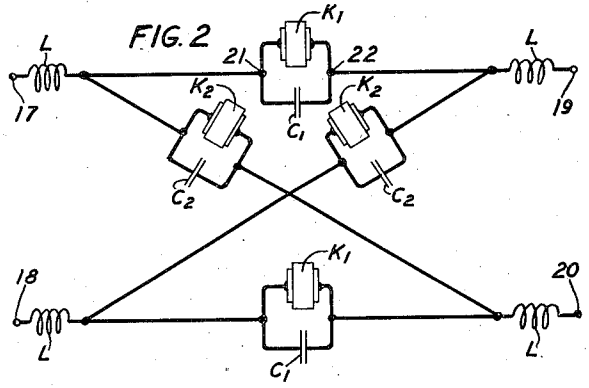


FIG. 2



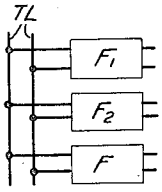


FIG. 10

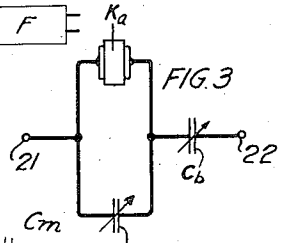


FIG. 3

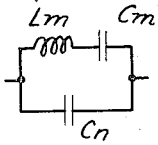


FIG. 9

FIG. 5

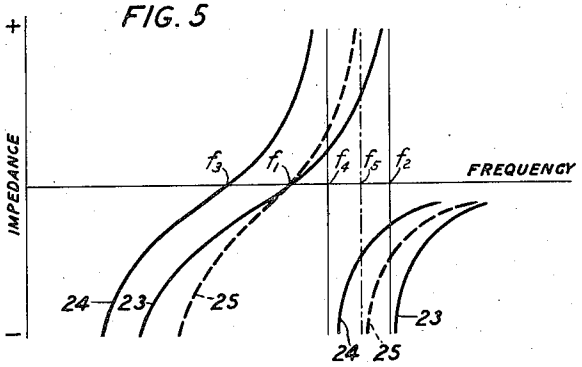


FIG. 4

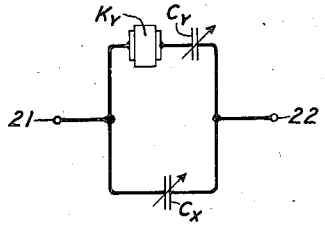


FIG. 6

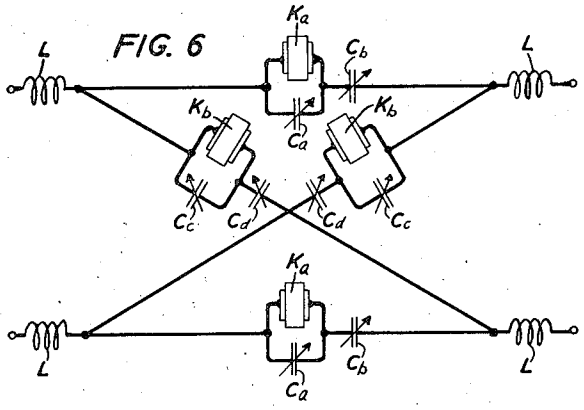
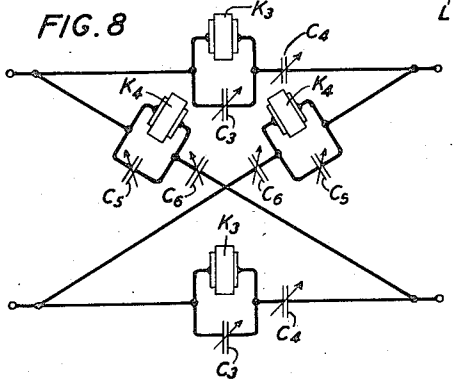


FIG. 8



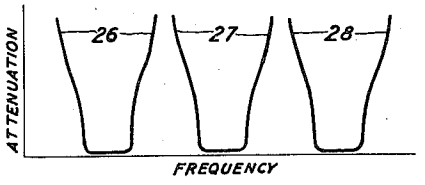


FIG. 7

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UNITED STATES PATENT OFFICE

2,037,171

WAVE FILTER

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Application June 23, 1933, Serial No. 677,244

20 Claims. (Cl. 178—44)

This invention relates to wave transmission networks and more particularly to frequency selective networks, such as wave filters, which employ piezoelectric crystals as impedance elements.

An object of the invention is to reduce the cost of manufacturing wave filters employing piezoelectric crystals.

Another object is to permit the use of thinner piezoelectric crystals in such filters.

Another object is to extend the frequency range and the impedance range over which a series of wave filters employing piezoelectric crystals may be made to function successfully.

A further object is to vary the characteristic impedance of such a filter over a wide range without materially changing the attenuation characteristic.

A feature of the invention is the combination of a piezoelectric crystal with a capacitance connected in shunt therewith and a second capacitance connected in series, the combination being the electrical equivalent of a second thicker piezoelectric crystal shunted by a capacitance.

Another feature is a wave filter employing a plurality of piezoelectric crystals all of which have the same thickness.

Another feature is a series of such filters designed to operate over different frequency ranges, all of the component piezoelectric crystals being of the same thickness.

Still another feature of the invention is a band filter comprising piezoelectric crystals, the characteristic impedance of which may be varied over a wide range, without material change in the width or location of the band, by means of variable condensers only.

The well known property of low energy dissipation in piezoelectric crystals such as quartz crystals makes them highly suitable for use as impedance elements in frequency selective networks. Such crystal elements may, for example, be combined with simple inductances and capacitances to form the branches of a lattice type wave filter which will provide a wide transmission band, while at the same time the advantages arising from the low energy dissipation in the crystal are maintained. The length of the crystal required in such a filter depends primarily upon the location of the frequency band to be transmitted, and in order that the crystal will vibrate in the proper mode the thickness of the crystal must be kept small relative to its length. But, in general, the higher the characteristic impedance of the filter is made, the thicker the required crystal becomes. It follows, therefore, that if a filter designed to pass a band of a certain width centered at a given frequency is to have a high impedance then conflicting requirements are placed upon the component crystal, namely, that its thickness shall be small in order for the crystal to vibrate in the desired mode, and that its thickness shall be large in order to provide a high impedance filter. In accordance with the present invention this difficulty is obviated by the use of a crystal which is thin enough to assure the proper mode of vibration but the impedance of which is stepped up by the addition of a capacitance in series therewith and the adjustment of the condenser which normally shunts the crystal. In this way it is possible to build a filter having a higher characteristic impedance, or one which will operate successfully at a higher frequency than would otherwise be possible.

As an adaptation of the invention there is presented a lattice type filter section employing a piezoelectric crystal in each branch in which all of the crystals are of the same thickness. Also, as an extension of this method of construction it is shown how a series of band-pass filters may be built having transmission bands spaced at intervals along the frequency spectrum but employing crystals of uniform thickness for all of the filters. In this way the cost of manufacturing the filters is materially reduced because it is more economical to grind a number of crystals all to the same thickness than it is to grind each crystal to a different thickness.

The fact that the impedance of a crystal element may be stepped up by the addition of a capacitance in series therewith is also utilized in the provision of a variable impedance filter comprising only piezoelectric crystals and variable capacitances. The characteristic impedance of the filter may be continuously varied over a wide range simply by varying the component capacitances. The location of the transmission band and its width remain practically unchanged. In this way a very useful type of filter is provided, since its impedance may readily be adjusted to match the load impedances between which it is to operate, and thereby eliminate reflection effects.

The nature of the invention will be more fully understood from the following detailed description and by reference to the accompanying drawing, of which:

Fig. 1 shows a form of piezoelectric crystal suitable for use in wave filters;

Fig. 2 represents schematically a type of filter to which the invention is applicable;

Figs. 3 and 4 show combinations of crystal element, series capacitance and shunt capacitance, embodying the invention;

Fig. 5 illustrates the type of impedance characteristics obtainable with the combinations shown in Figs. 3 and 4;

Fig. 6 shows the embodiment of the invention in a lattice type section;

Fig. 7 represents the type of transmission characteristics obtainable from a series of band filter sections embodying the invention, in which crystals of uniform thickness are employed;

Fig. 8 illustrates schematically a variable impedance band filter embodying the invention;

Fig. 9 represents the equivalent electrical circuit of a piezoelectric crystal to which reference is made in explaining the invention; and

Fig. 10 shows a series of filters embodying the invention.

A form of piezoelectric crystal suitable for use in wave filters for frequencies up to about 500 kilocycles per second is shown in Fig. 1, in which **10** represents a rectangular crystal, preferably of quartz, having its length l parallel to the mechanical axis MM', its width w parallel to the optical axis OO' and its thickness t in the direction of the electrical axis EE'. Electrodes **11** and **12** are applied to the large faces of the crystal, that is, to the surfaces perpendicular to the electrical axis, preferably by the electrical deposition of a layer of silver or other metal to secure an intimate contact over the whole surface. Leads **13** and **14** are connected to the electrodes by soldering with soft solder or by other appropriate means. Crystals of this type when subjected to an alternating potential difference between the electrodes vibrate mainly by expansion and contraction in the direction along the mechanical axis. They should, therefore, be supported between points or knife-edge clamps such as **15** and **16**, located near the center of the crystal along the optical axis.

A type of wave filter to which the invention is applicable is shown in Fig. 2. The filter comprises a symmetrical lattice each line branch of which is constituted by a piezoelectric crystal K_1 in parallel with a capacitance C_1, and each diagonal branch of which consists of a crystal K_2 shunted by a capacitance C_2, the structure being completed by the four equal inductances L connected between the terminals of the lattice section and the input terminals **17**, **18** and the output terminals **19**, **20** of the filter. It is well known that the impedance elements of such a filter may be proportioned to produce a band-pass characteristic or other desired attenuation characteristic. For a detailed explanation of how this may be done reference is made to the copending applications of W. P. Mason, Serial No. 489,268, filed October 17, 1930 and Serial No. 653,622, filed January 26, 1933.

For frequencies up to and well above the first resonance the electrical impedance of a parallel combination of crystal and capacitance such as is shown between terminals **21** and **22** of Fig. 2, is of a simple character corresponding to that of an electrical circuit comprising a capacitance shunted by an arm consisting of an inductance and capacitance in series. The impedance of such a combination, which is practically wholly reactive, has a frequency characteristic of the type shown by the solid line curve **23** of Fig. 5. At low frequencies the reactance is negative or capacitative, with the magnitude diminishing with increasing frequency to zero at a resonance frequency f_1. Above this frequency the reactance becomes positive and rises rapidly to an infinite value at an anti-resonance frequency f_2, above which it again becomes negative and decreases in magnitude with increasing frequency. The principal resonance frequency of the combination is determined primarily by the length l of the crystal. Other resonances, representing changes in the mode of vibration of the crystal, are practically eliminated so long as the length of the crystal is greater than its width and its thickness is relatively small. But if the characteristic impedance of the filter is to be high the crystal must be made relatively thick. This latter requirement directly conflicts with the requirement that the crystal should be kept relatively thin for vibration in the proper mode. It follows, then, that if the resonance f_1 is to be placed at a certain frequency, and the filter is to have a comparatively high characteristic impedance, it will be impossible by ordinary methods to build a filter which will function satisfactorily.

In accordance with the invention, this difficulty is avoided by replacing the combination shown between terminals **21** and **22** of Fig. 2 by the combination of elements shown in Fig. 3 which comprises a thinner crystal K_a shunted by a variable capacitance C_a, and in series with the combination a second variable capacitance C_b. By the proper choice of the series capacitance C_b and by the adjustment of the shunt capacitance C_a the impedance of the combination shown in Fig. 3 may be made exactly the same as the impedance of the crystal K_1 shunted by the capacitance C_1, and therefore the former combination may be substituted for the latter in the filter of Fig. 2 without affecting its impedance or transmission characteristics.

A method which may be followed in choosing the required values for the capacitances C_a and C_b will now be explained by reference to the impedance curves shown in Fig. 5, in which curve **23** represents the reactance of the crystal K_1 in parallel with the capacitance C_1, f_1 being the resonance frequency and f_2 the frequency of anti-resonance of the combination. If the crystal K_1 is replaced by a thinner and slightly longer crystal K_a, having the same ratio of width to length as the replaced crystal, the shunt condenser C_1 remaining unchanged, the new combination will have a somewhat lower resonance frequency f_3 and a lower frequency of anti-resonance f_4, as indicated by the full line curve **24** of Fig. 5. Now, if a variable capacitance C_b be added in series with the combination, the resonance frequency f_3 will be raised, and by properly adjusting the value of C_b this frequency may be made to coincide with the frequency f_1. At the same time, the frequency of anti-resonance will be raised from f_4 to some new frequency f_5, as indicated by the dotted curve **25** of Fig. 5. Now, in order to make the curve **25** coincide at all points with the curve **23** it is only necessary to increase the value of the shunt capacitance C_1 until the frequency of anti-resonance f_5 is raised to the frequency f_2. For this purpose it is convenient to replace C_1 by the variable capacitance C_a and adjust the latter to the required value. Changing the value of C_a affects only the anti-resonance frequency f_5, the resonance frequency f_1 remaining undisturbed. Once the required values for C_a and C_b have been found, the variable condensers may be replaced by fixed ones, if

desired. Also, the proper values for the capacitances C_a and C_b may be determined by direct computation from equations 5 and 4 presented below, if that method is preferred, and in that case no variable condensers are required.

It will now be explained how the required length of the replacing crystal K_a may be evaluated in terms of the physical dimensions of the replaced crystal K_1, the thickness of the replacing crystal K_a and the magnitude of the capacitance C_1 expressed in micro-microfarads. For a crystal of the type shown in Fig. 1 the equivalent electrical circuit, as shown in Fig. 9, may be represented by a capacitance C_n shunted by a branch consisting of an inductance L_m and a second capacitance C_m. The capacitance C_n is the simple electrostatic capacitance between the electrodes of the crystal. The values of the inductance L_m and the capacitance C_m depend not only upon the dimensions of the crystal but also upon its piezoelectric and elastic constants. For quartz crystals of the type shown in Fig. 1 the values of the elements of the equivalent electrical circuit are given in terms of the crystal dimensions by the following equations:

$$L_m=\frac{118It}{w} \text{ henries} \tag{1}$$

$$C_m=\frac{0.00289wI}{t} \text{ micro-microfarads} \tag{2}$$

$$C_n=\frac{0.405wI}{t} \text{ micro-microfarads} \tag{3}$$

where l is the length, t the thickness, and w the width of the crystal in centimeters.

If the equivalent reactance elements of the crystal K_1 are designated L_m, C_m and C_n as in Fig. 9, the value of the series capacitance C_b of Fig. 3 may be expressed as

$$C_b=\frac{C_n+C_1}{1-\sqrt{\frac{t_2}{t_1}}} \tag{4}$$

and the value of the shunting capacitance C_a is given by the equation

$$C_a=\frac{C_1+(C_m+C_n)\left(1-\sqrt{\frac{t_1}{t_2}}\right)}{C_1+C_m+C_n+(C_n-C_m)\sqrt{\frac{t_1}{t_2}}} \tag{5}$$

where t_1 is the thickness of the crystal K_1 and t_2 is the thickness of the replacing crystal K_a. The corresponding equivalent reactance elements L_m' and C_m' of the replacing crystal K_a may be found from the expressions

$$L_m'=L_m\frac{t_2}{t_1} \tag{6}$$

and

$$C_m'=\frac{C_m(C_1+C_n)\frac{t_1}{t_2}}{C_1+C_m+C_n-C_m\sqrt{\frac{t_1}{t_2}}} \tag{7}$$

The length l_1 of the crystal K_1 is given by the equation

$$l_1=2\pi k\sqrt{L_mC_m} \tag{8}$$

where k is a factor relating the length of the crystal to its resonant frequency, and in any given crystal depends upon the ratio of the width of the crystal to its length. Since, as stated above, the crystals K_1 and K_a both have the same ratio of width to length, the length l_2 of the crystal K_a may be expressed as

$$l_2=2\pi k\sqrt{L_m'C_m'} \tag{9}$$

Substituting in Equation (9) the values of L_m' and C_m' given by Equations (6) and (7) gives

$$l_2=2\pi k\sqrt{\frac{(C_n+C_1)C_mL_m}{C_m+C_n+C_1-C_m\sqrt{\frac{t_1}{t_2}}}} \tag{10}$$

Squaring Equations (8) and (10) and dividing the latter by the former gives

$$\frac{l_2^2}{l_1^2}=\frac{C_n+C_1}{C_m+C_n+C_1-C_m\sqrt{\frac{t_1}{t_2}}} \tag{11}$$

Substituting in Equation (11) the values of C_m and C_n given by Equations (2) and (3) gives

$$l_2^2=\frac{l_1^2\left(\frac{0.405w_1l_1}{t_1}+C_1\right)}{\frac{0.40789w_1l_1}{t_1}+C_1-\frac{0.00289w_1l_1}{\sqrt{t_1t_2}}} \tag{12}$$

from which the length l_2 of the replacing crystal K_a may be determined in terms of the capacitance C_1 in micro-microfarads, the length l_1, width w_1, and thickness t_1 of the crystal K_1 and the thickness t_2 of the crystal K_a, all dimensions being expressed in centimeters.

If the elements K_a, C_a and C_b have been proportioned as explained above then the combination shown in Fig. 3 may be substituted for the crystal K_1 and capacitance C_1 appearing between the terminals **21** and **22** indicated in Fig. 2. A similar combination comprising K_a, C_a and C_b may be substituted for the other line branch of the lattice of Fig. 2. The same procedure may be followed in working out a combination comprising a crystal K_b and capacitance C_c, C_d which may be substituted for each diagonal branch of the lattice. The resulting filter, with the new branches of the lattice in place, is shown schematically in Fig. 6. The inductances L are left unchanged.

As a concrete illustration giving the physical dimensions of the crystal element and the magnitudes of the associated capacitances required in each line branch of a wave filter embodying the invention the following data are presented. In a band-pass filter of the configuration shown in Fig. 2 having a characteristic impedance of 1100 ohms, a mid-band frequency of 98280 cycles per second and a theoretical band width of 3800 cycles per second the crystal K_1 was found to have the following dimensions in millimeters:

t_1= 2.767
l_1=26.724
w_1=12.908

The shunting capacitance C_1 had a magnitude of 52.139 micro-microfarads. In accordance with the invention, this branch of the filter may be replaced, as explained above, by an impedance branch of the form shown in Fig. 3 which employs a crystal element having a more convenient thickness. The dimensions in millimeters of the replacing crystals K_a may, for example, be as follows:

t_2= 0.883
l_2=26.731
w_2=12.911

It will be noted that, in the illustration, the thickness t_2 is less than a third of the thickness t_1 of the replaced crystal K_1. Any other ratio of t_2 to t_1 less than unity could, of course, be used. For a crystal having the dimensions given above for K_a the magnitudes of the associated capac-

itances in micro-microfarads are found to be as follows:

$C_a = 86.442$
$C_b = 131.380$

The combination shown in Fig. 3 may be replaced, if desired, by the equivalent circuit shown in Fig. 4, comprising a variable capacitance C_x shunted by an arm consisting of the crystal K_y in series with a second variable capacitance C_y. The impedance of the crystal K_y will be the same at all frequencies as that of the crystal K_a, multiplied by a constant. The necessary relationships existing between the values of the reactances in the equivalent circuits are well known and are given, for example, in "Transmission Circuits for Telephonic Communication", by K. S. Johnson, page 267, Equations (1), (2) and (3).

As an adaptation of the invention the crystals K_b in the diagonal branches of Fig. 6 may be chosen of the same thickness as the crystals K_a in the line branches.

As a further extension of the invention, a series of band-pass filters may be built having transmission bands placed at different frequencies, such for example, as those shown by curves 26, 27 and 28 of Fig. 7, but employing crystals of uniform thickness for all of the filters. Fig. 10 shows a series of three such filters F_1, F_2 and F_3 connected in parallel to a transmission line TL. Each of the filters may have the configuration shown in Fig. 6, for example. This construction conduces to economical manufacture of the filters inasmuch as grinding a number of crystals to the same thickness by a machine lapping process is less expensive than grinding each crystal to a different thickness, which latter method generally requires tedious hand work.

Another embodiment of the invention is illustrated diagrammatically in Fig. 8, which shows a lattice type band-pass filter the characteristic impedance of which may be varied over a wide range of values simply by the manipulation of variable capacitances. It is well known how to construct a band filter of the lattice type in which each line branch and each lattice branch consists of a piezoelectric crystal element shunted by a capacitance. For a more detailed description of the design of such a filter, reference is made to the aforementioned Mason applications. In accordance with the present invention each of the shunting capacitances is made variable, and a second capacitance is inserted in series with each branch so that its impedance level may be raised or lowered without appreciably changing the frequencies of resonance and anti-resonance, as explained above in connection with Fig. 3. Thus, each line branch of the filter shown in Fig. 8 consists of a crystal K_3 shunted by a variable capacitance C_3, the combination being connected in series with a second variable capacitance C_4; and each lattice branch is made up of a variable capacitance C_6 in series with the parallel combination comprising the crystal K_4 and the variable capacitance C_5. In order to increase the impedance of the filter all of the variable capacitances, C_3, C_4, C_5 and C_6, are decreased in value; in order to decrease the filter impedance, these capacitances are increased in magnitude. The width of band transmitted by the filter will remain fixed, regardless of the impedance setting, but as the impedance is raised the mid-band frequency may increase slightly. This latter effect, however, is unimportant in most instances.

As an example illustrating the actual values of capacitance required in a variable impedance filter of the kind described above, the following data are presented. With no series capacitances in the branches, the filter has a characteristic impedance of 10,000 ohms and passes a 20 cycle band centered at 50 kilocycles per second. With the addition of series capacitance the impedance may be varied continuously from 10,000 ohms to nearly 1,000,000 ohms, the latter figure representing the condition where the capacitance in shunt with the crystal has been reduced to zero. The following table gives the values required, in micro-microfarads for the capacitances C_3 and C_4 of the line branches when the characteristic impedance Z_0 of the filter is set, respectively, at 10,000, 20,000 and 939,700 ohms.

Z_0	C_3	C_4
10,000	281.0	∞
20,000	189.6	535.3
939,700	0	3.7

The magnitudes of the capacitances C_5 and C_6 associated with the lattice branches will vary approximately in the same ratio as the corresponding capacitances in the line branches, given in the table. The crystals K_3 and K_4, of course, remain unaltered. When the impedance of the filter is increased from 10,000 to 20,000 ohms, the center of the transmission band is moved up in frequency approximately eight cycles, but this shift is of little importance when it is considered that the band is centered at 50,000 cycles. The width of the band, 20 cycles, remains unchanged as the impedance of the filter is varied.

What is claimed is:

1. In a wave transmission network of the lattice type having a pair of input terminals and a pair of output terminals, a plurality of pairs of impedance branches connected between said input terminals and said output terminals, one of said impedance branches comprising a piezoelectric crystal as an impedance element, a capacitance shunting said crystal and a second capacitance connected effectively in series with said crystal, said impedance branch being the electrical equivalent of a different thicker crystal shunted by a capacitance.

2. A wave transmission network of the lattice type comprising a plurality of pairs of impedance branches connected between a pair of input terminals and a pair of output terminals, one of said impedance branches comprising in combination a capacitance shunted by an impedance arm comprising a second capacitance and a piezoelectric crystal element connected in series relation.

3. A wave filter comprising a plurality of pairs of impedance branches connected between a pair of input terminals and a pair of output terminals to form a lattice network, one of said impedance branches comprising a capacitance in series with a combination comprising a second capacitance and a piezoelectric crystal element connected in parallel.

4. A wave filter of the lattice type having a plurality of pairs of impedance branches connected between a pair of input terminals and a pair of output terminals, one of said impedance branches comprising a variable capacitance connected in series with a combination comprising a second variable capacitance in shunt with a piezoelectric crystal, whereby, by the adjustment of said variable capacitances, the impedance of said branch may be varied without displacing its critical frequencies.

5. A broad band wave filter having a plurality

of pairs of impedance branches connected between a pair of input terminals and a pair of output terminals to form a lattice network, one of said impedance branches comprising two capacitances connected in series, one of said capacitances being shunted by a piezoelectric crystal element, and said impedance branch being the electrical equivalent of a different thicker crystal shunted by a capacitance.

6. A wave filter comprising a plurality of impedance branches connected between a pair of input terminals and a pair of output terminals, each of said branches including two capacitances connected in series, one of said capacitances being shunted by a piezoelectric crystal element, and all of said piezoelectric crystal elements being of the same thickness.

7. A broad band wave filter comprising a plurality of impedance branches arranged to form a lattice network, each of said branches including a piezoelectric crystal impedance in combination with two separate capacitances, and all of said piezoelectric crystals being of the same thickness.

8. A broad band wave filter comprising a plurality of impedance branches arranged to form a lattice network, each of said branches including a piezoelectric crystal impedance, a capacitance in shunt thereto and a second capacitance in series therewith, and all of said crystals being of the same thickness.

9. A broad band wave filter comprising a plurality of impedance branches arranged to form a lattice network, each of said branches including two capacitances and a piezoelectric crystal element in shunt with one of said capacitances, all of said crystals being of the same thickness.

10. A broad band wave filter comprising a plurality of impedance branches arranged to form a lattice network between a pair of input terminals and a pair of output terminals, each of said branches including as impedance elements a piezoelectric crystal and two separate capacitances, one of said capacitances being connected effectively in series with said crystal and the other of said capacitances being connected effectively in shunt with said crystal, and all of said crystals being of the same thickness.

11. A broad band wave filter comprising two pairs of similar impedance branches arranged to form a symmetrical lattice network between a pair of input terminals and a pair of output terminals, each of said branches including as an impedance element a piezoelectric crystal, all of said crystals being of the same thickness, the crystals included in one of said pairs of branches having a frequency characteristic different from that of the crystals in the other pair and proportioned with respect thereto to provide a single transmission band.

12. A lattice-type wave filter comprising a plurality of pairs of impedance branches connected between a pair of input terminals and a pair of output terminals, each of said branches including as impedance elements a piezoelectric crystal and two variable capacitances, all of said crystals being of the same thickness, and the crystals in one of said pairs of branches having a frequency characteristic different from that of the crystals in another pair of said branches and being proportioned with respect thereto to provide a single transmission band.

13. A plurality of wave transmission networks having transmission bands centered at different frequencies, each of said networks comprising a piezoelectric crystal as an impedance element, and all of said crystals being of the same thickness.

14. A plurality of wave filters having diverse transmission characteristics, each of said filters comprising a plurality of impedance branches connected between a pair of input terminals and a pair of output terminals, each of said branches including a piezoelectric crystal as an impedance element, and all said crystals being of the same thickness.

15. A plurality of wave filters having different attenuation characteristics, each of said filters comprising two pairs of similar impedance branches disposed between a pair of input terminals and a pair of output terminals to form a symmetrical lattice network, each of said branches including a piezoelectric crystal as an impedance element, and all of said crystals being of the same thickness.

16. A plurality of wave filters having transmission bands centered at different frequencies, each of said filters comprising a plurality of impedance branches arranged to form a lattice network, each of said branches including a piezoelectric crystal in combination with two capacitances, all of said crystals being of the same thickness.

17. A plurality of wave filters having transmission bands centered at different frequencies, each of said filters comprising a plurality of impedance branches arranged to form a lattice network, each of said branches including a piezoelectric crystal, and all of said crystals being of the same thickness.

18. A variable impedance wave filter comprising a plurality of impedance branches arranged to form a lattice network, each of said branches including a piezoelectric crystal element and a plurality of variable capacitances cooperating therewith for varying the impedance of said branches without materially changing the transmission characteristics of said filter.

19. A variable impedance wave filter comprising a plurality of impedance branches arranged to form a lattice network, each of said branches including a piezoelectric crystal impedance in combination with two variable capacitances, whereby the characteristic impedance of said filter may be varied without materially altering its attenuation characteristic.

20. A variable impedance wave filter comprising a plurality of impedance branches arranged to form a lattice network, each of said branches including a piezoelectric crystal element, a variable capacitance in shunt thereto, and a second variable capacitance in series therewith, whereby the characteristic impedance of said filter may be varied while the width and location of its transmission bands remain unaltered.

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