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Zaverucha

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(54) **VERIFYING IMPLICIT CERTIFICATES AND DIGITAL SIGNATURES**

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H04L 9/30	(2006.01)
H04L 9/32	(2006.01)

(52) **U.S. Cl.**
CPC ***H04L 9/3066*** (2013.01); ***H04L 9/3242*** (2013.01); *H04L 9/3247* (2013.01)
USPC **713/156**

(58) **Field of Classification Search**
CPC H04L 9/3242; H04L 9/3066; H04L 9/3247
USPC 713/156
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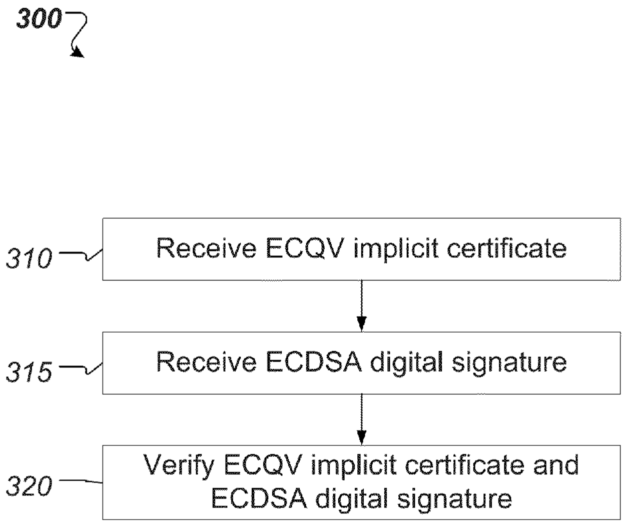
(57) **ABSTRACT**

Methods, systems, and computer programs for verifying a digital signature are disclosed. The verifier accesses an implicit certificate and a digital signature provided by the signer. The implicit certificate includes a first elliptic curve point representing a public key reconstruction value of the signer. The verifier accesses a second elliptic curve point representing a pre-computed multiple of the certificate authority's public key. The verifier uses the first elliptic curve point and the second elliptic curve point to verify the digital signature. The verifier may also use a third elliptic curve point representing a pre-computed multiple of a generator point. Verifying the digital signature may provide verification that the implicit certificate is valid.

17 Claims, 5 Drawing Sheets


300
310
Receive ECQV implicit certificate
315
Receive ECDSA digital signature
320
Verify ECQV implicit certificate and ECDSA digital signature

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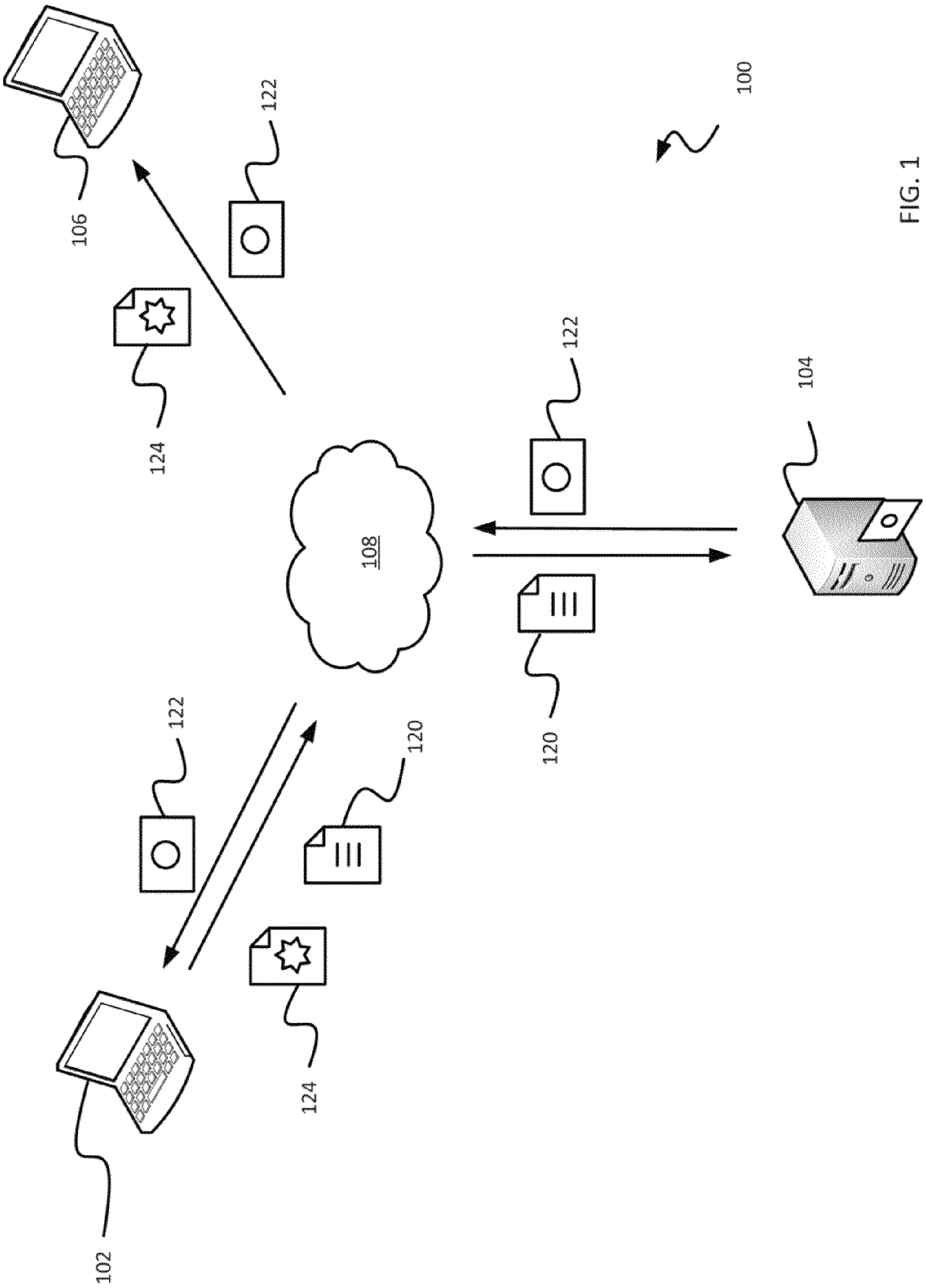

FIG. 1

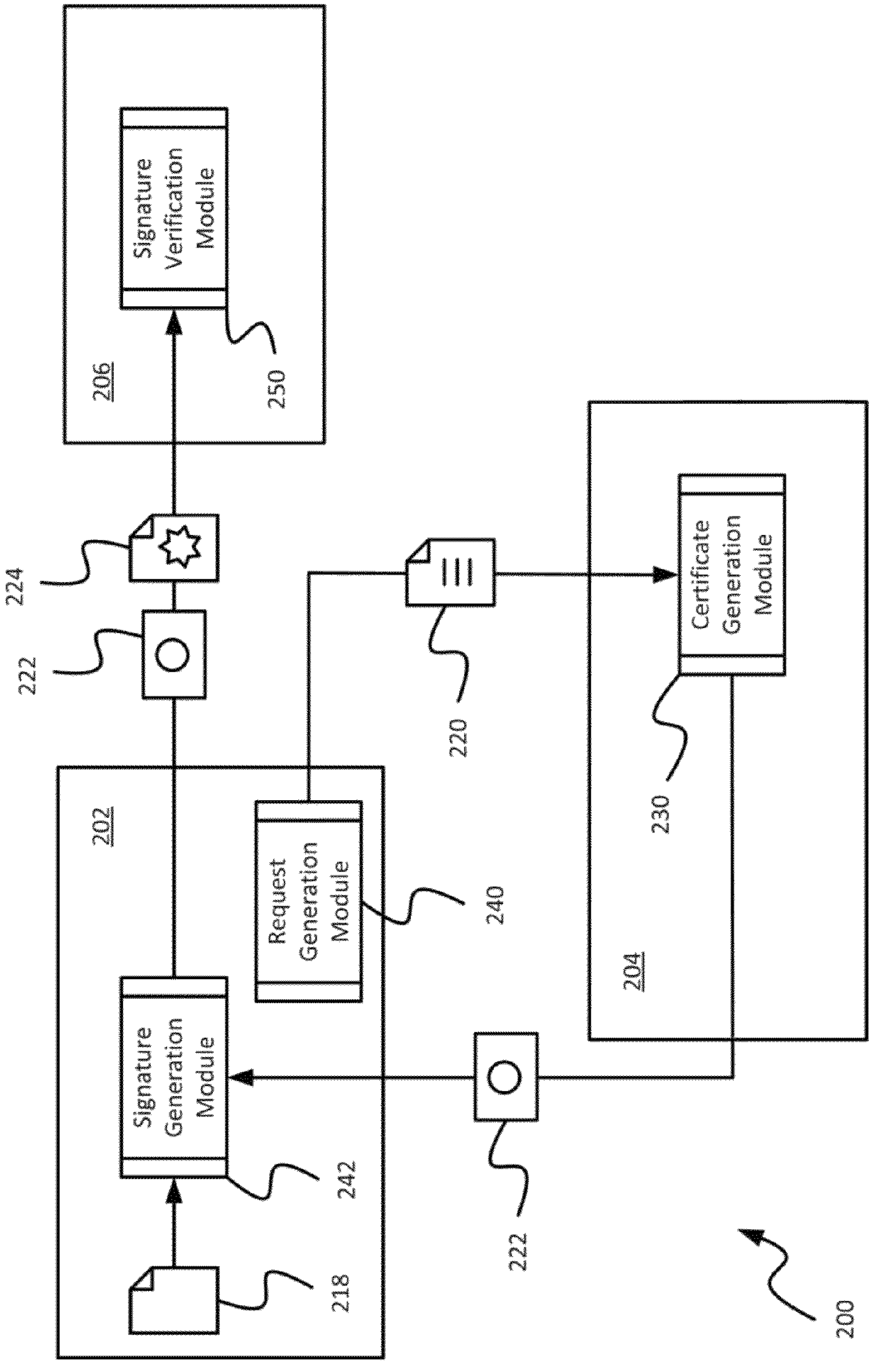
206
Signature Verification Module
250
224
222
220
202
Request Generation Module
240
Certificate Generation Module
230
204
Signature Generation Module
242
218
222
200

FIG. 2

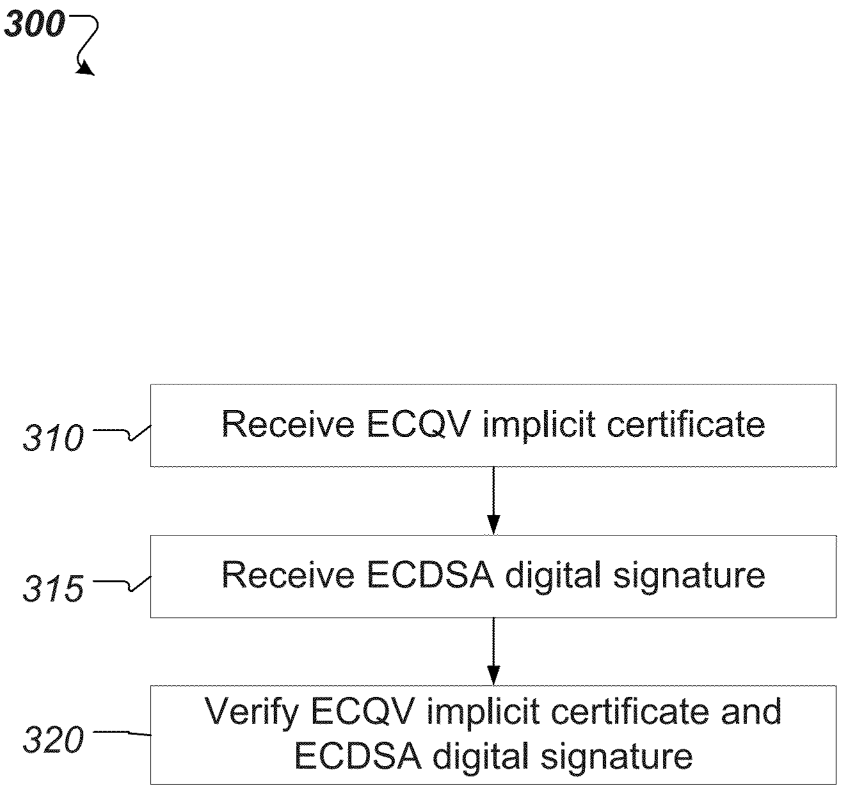
300
310
Receive ECQV implicit certificate
315
Receive ECDSA digital signature
320
Verify ECQV implicit certificate and ECDSA digital signature

FIG. 3

$$
\begin{array}{lcl}
\underline{\qquad\qquad U \qquad\qquad} & & \underline{\qquad\qquad CA \qquad\qquad} \\
k_U \in_R [1..n-1] & & \\
R_U := k_U \cdot G & & \\
 & \xrightarrow{U,\, R_U} & \\
 & & k \in_R [1, \ldots, n-1];\ P = R_U + kG \\
 & & Cert_U := Encode(P, I, *) \\
 & & e := H(Cert_U) \\
 & & r := ek + d_{CA} \pmod{n} \\
 & \xleftarrow{r,\, Cert_U} & \\
e = H(Cert_U) & & \\
d_U := ek_U + r \pmod{n} & & \\
B := eP + C & &
\end{array}
$$

FIG. 3A

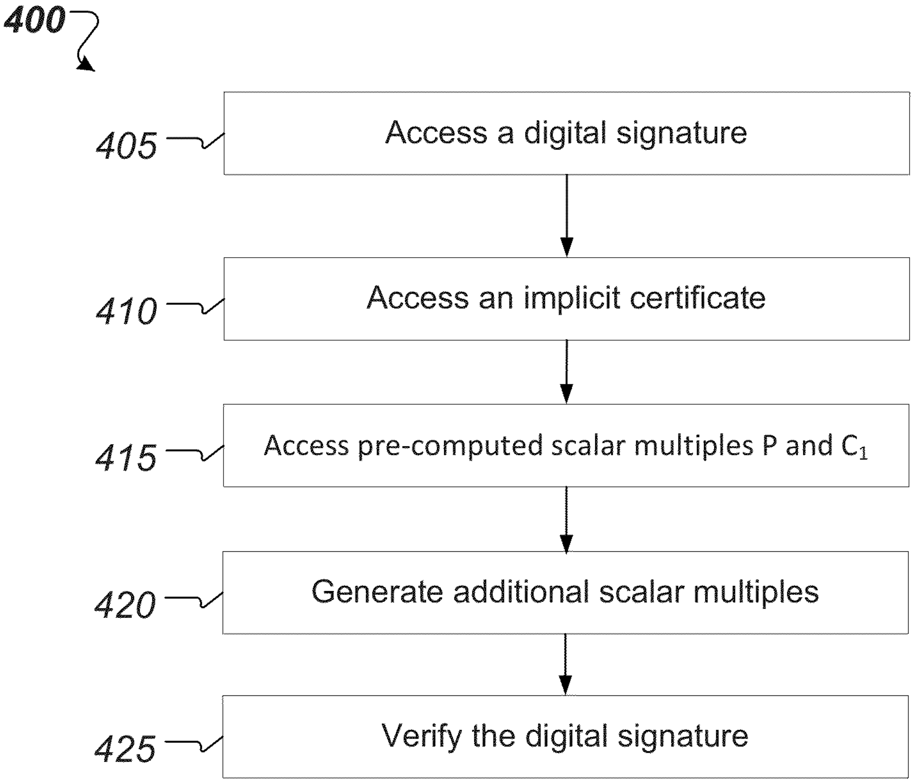
400
405
Access a digital signature
410
Access an implicit certificate
415
Access pre-computed scalar multiples P and C_1
420
Generate additional scalar multiples
425
Verify the digital signature

FIG. 4

VERIFYING IMPLICIT CERTIFICATES AND DIGITAL SIGNATURES

BACKGROUND

This specification relates to verifying signatures in a cryptography system. Cryptography systems enable secure communication over public channels. For example, digital signature schemes can be implemented in a public key cryptography system. In some cryptography systems, users verify the authenticity of other users' digital signatures based on certificates issued by a trusted third party.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** is a schematic diagram of an example data communication system.

FIG. **2** is a schematic diagram of an example cryptography system.

FIG. **3** is a flow chart showing an example verification process.

FIG. **3**A is a diagram example of certificate issuance protocol.

FIG. **4** is a flow chart showing an example process for verifying combined signatures.

Like reference numbers and designations in the various drawings indicate like elements.

DETAILED DESCRIPTION

Techniques for verifying a digital signature and an implicit certificate are presented in this disclosure. In many instances, an Elliptic Curve Digital Signature Algorithm (ECDSA) signature may be sent with an implicit certificate such as an Elliptic Curve Qu-Vanstone (ECQV) certificate. The sum of individual computation cost for verifying the ECDSA signatures and the ECQV certificate may be significant, and a joint operation for simultaneous verification for the both may require less total time or computation resources. The disclosed verification techniques provide an example of such a joint operation. Moreover, the verification techniques disclosed here can be used with other types of digital signature schemes and digital certificate schemes, such as, for example, other types of Elliptic Curve Cryptography (ECC)-based systems.

FIG. **1** is a schematic diagram of an example data communication system **100**. The data communication system **100** includes a certificate authority server **104**, two terminals **102**, **106**, and a data network **108**. The data communication system **100** can include additional, fewer, or different components. For example, the data communication system **100** may include additional storage devices, additional servers (including additional certificate authority servers), additional terminals, and other features not shown in the figure.

The certificate authority server **104** and the terminals **102**, **106** can communicate with each other and with other components of the data communication system **100** over the data network **108**. In the example shown in FIG. **1**, the terminal **102** can send a certificate request **120** to the certificate authority server **104**, and the certificate authority can respond by sending an implicit certificate **122** to the terminal **102**. The terminal **102** can send a signed message **124** to the terminal **106**, and the terminal **106** can verify the authenticity of the signed message **124** using the implicit certificate **122** from the certificate server authority **104**. The data communication system **100** can support additional or different types of communication. In some implementations, the terminals **102**, **106** can also exchange encrypted messages and other types of information with each other, with the certificate authority server **104**, and with other components of the data communication system **100**.

The certificate authority server **104** is a computing system that can perform operations of a certificate authority in a cryptography system. The certificate authority server **104** is generally operable to receive, transmit, process, and store information associated with the cryptography system. Although FIG. **1** shows a single certificate authority server **104**, a certificate authority can be implemented using multiple certificate authority servers **104**, including server clusters, as well as additional or different types of computing devices other than servers.

The certificate authority server **104** and the terminals **102**, **106** can be implemented as computing devices that generally include a data processing apparatus, a data storage medium, and a data communication interface. The example certificate authority server **104** shown in FIG. **1** can include a processor, a memory, an input/output controller, and other components. The memory can include, for example, a random access memory (RAM), a storage device (e.g., a writable read-only memory (ROM), etc.), a hard disk, or another type of storage medium. A computing device can be preprogrammed or it can be programmed (and reprogrammed) by loading a program from another source (e.g., from a CD-ROM, from another computer device through a data network, or in another manner). The input/output controller can be coupled to input/output devices (e.g., a monitor, a keyboard, etc.) and to the data network **108**. The input/output devices receive and transmit data in analog or digital form over communication links such as a serial link, wireless link (e.g., infrared, radio frequency, etc.), parallel link, or another type of link.

The memory of a computing device can store instructions (e.g., computer code) associated with computer applications, programs and computer program modules, and other resources. For example, the memory can store instructions associated with computer program modules shown in FIG. **2**. The memory can also store application data and data objects that can be interpreted by applications, programs, modules, or virtual machines. The memory can store additional information, for example, files and instruction associated with an operating system, device drivers, archival data, or other types of information.

The processor of a computing device can execute instructions to generate output data based on data inputs. For example, the processor can run applications and programs by executing or interpreting the software, scripts, functions, executables, and other types of computer program modules. For example, the processor may perform one or more of the operations shown in FIGS. **3** and **4**. The input data received by the processor and the output data generated by the processor can be stored in a computer-readable medium, such as the memory or a storage device.

The data network **108** can include any type of data communication network. For example, the data network **108** can include a wireless or wired network, a cellular network, a telecommunications network, an enterprise network, an application-specific public network, a Local Area Network (LAN), a Wide Area Network (WAN), a private network, a public network (such as the Internet), a WiFi network, a network that includes a satellite link, or another type of data communication network. The data network **108** can include a tiered structure defined by firewalls or similar features that implement various levels of security.

The terminals **102**, **106** can communicate over the data network **108** based on communication schemes specified by the cryptography system. The terminals **102**, **106** are generally operable to receive, transmit, process, and store information. Although FIG. **1** shows two terminals **102**, **106**, a data communication system **100** may include any number of terminals. The data communication system **100** can include groups or subgroups of terminals that can communicate with each other, but not necessarily with the terminals in other groups or subgroups. In some implementations, each group of terminals can access a certificate authority server and a database of implicit certificates that have been issued by the certificate authority server. The data communication system **100** can include terminals of disparate types, having different types of hardware and software configurations, and in a variety of different locations. In some cases, multiple devices or subsystems can be identified together as a single terminal.

The terminals **102**, **106** can be computing devices that include a memory, a data processor, and an input/output controller. A terminal can include user interface devices, for example, a monitor, touchscreen, mouse, or keyboard. The terminals **102**, **106** interface with the data network **108**. The memory of a terminal can store messages and information associated with the cryptography system. For example, a terminal may store public and private key data, digital certificate data, and other types of information. The memory of the terminal can store instructions (e.g., computer code) associated with computer applications, programs and computer program modules, and other resources. For example, the terminals can store instructions associated with the computer program modules of the terminal modules **202**, **206** shown in FIG. **2**.

Terminals can include handheld devices such as smart phones, personal digital assistants (PDAs), portable media players, laptops, notebooks, tablets, and others. Terminals can include work stations, mainframes, non-portable computing systems, devices installed in structures, vehicles, and other types of installations. Terminals can include embedded communication devices. For example, the terminals can include messaging devices that are embedded in smart energy meters of a smart energy system. Other types of terminals may also be used.

In one aspect of operation, the terminal **102** sends the certificate request **120** to the certificate authority server **104**, and the certificate authority server **104** generates the implicit certificate **122** for the terminal **102**. The implicit certificate **122** associates a particular public key value with a particular user entity (e.g., the terminal **102**, a user associated with the user terminal **102**, a module implemented at the terminal **102**, etc.). The terminal **102** receives the implicit certificate **122** from the certificate authority server **104**. When the terminal **102** has a message to send to the terminal **106**, the terminal **102** generates a digital signature for the message based on the implicit certificate **122**. The digital signature can be combined with the message to form the signed message **124**, which the terminal **102** sends to the terminal **106**. In some implementations, the digital signature and the message are sent separately. The terminal **106** receives the signed message **124**, obtains the implicit certificate **122**, and verifies the digital signature based on the implicit certificate **122**. Implicit certificates can also be used in other types of schemes, for example, encryption schemes.

An implicit certificate scheme implemented by the data communication system **100** allows the terminals **102**, **106** to communicate with each other in a secure manner, even when communications on the data network **108** are observable by malicious users. The implicit certificate **122** binds a user entity associated with the terminal **102** to a particular public key value that can be used to verify digital signatures generated by the terminal **102**. The terminal **106** can obtain the implicit certificate **122** to verify that the digital signature was generated by the user entity associated with the terminal **102**, and not by an impostor. The terminal **106** can also verify that the implicit certificate **122** was generated by a trusted third party at the certificate authority server **104**. In this manner, the implicit certificate **122** serves as confirmation by the trusted third party that the signed message **124** was signed by the user entity associated with the terminal **102** and not by an impostor.

The example implicit certificate **122** shown in FIG. **1** includes neither an explicit representation of the public key nor an explicit representation of the certificate authority's digital signature. Thus, in some implementations, the implicit certificate **122** is more compact than some other types of digital certificates. In some cases, the implicit certificate **122** includes a digital signature of the certificate authority that allows user entities, for example a user entity associated with the terminal **106**, to verify that the implicit certificate **122** was generated by the trusted certificate authority. The certificate authority can, in some cases, require the user entity to prove knowledge of the user entity's private key. In some cases, the implicit certificate **122** includes an explicit representation of the user's public key.

Instead of explicitly representing the public key of the terminal **102**, the example implicit certificate **122** in FIG. **1** includes public key reconstruction data that can be combined with other information (e.g., the certificate authority's public key, etc.) to generate the public key of the user entity associated with the terminal **102**. The example implicit certificate **122** is constructed such that successful verification of a digital signature generated by the terminal **102** serves as confirmation that the terminal **102** is in possession of the private key. Thus, according to some implicit certificate schemes, binding of a user entity to its public key and the user entity's knowledge of its private key can be verified in unison during key usage.

FIG. **2** is a schematic diagram of an example cryptography system **200** that implements an implicit certificate scheme. The cryptography system **200** includes terminal modules **202**, **206**, and a certificate authority module **204**. The cryptography system **200** can include additional or different components. The terminal modules **202**, **206** can each be computer program modules implemented by one or more terminals. For example, the terminal module **202** can be implemented by the terminal **102** of FIG. **1**, and the terminal module **206** can be implemented by the terminal **106** of FIG. **1**. The certificate authority module **204** can be a computer program module implemented by one or more certificate authority servers. For example, the certificate authority module **204** can be implemented by certificate authority server **104** of FIG. **1**.

The terminal modules **202**, **206**, the certificate authority module **204**, and the certificate database **236** can be implemented by additional or different types of hardware systems. For example, the certificate authority module **204**, or in some instances individual modules, data, or other aspects of the certificate authority module **204** can be offloaded to non-certificate authority devices. In some instances, for example in a peer-to-peer computing environment, server functionality can be distributed among client devices. As another example, terminal modules, or in some instances individual modules, data, or other aspects of a terminal module, can be provided on a server device, such as a certificate authority server or another type of server.

The terminal modules **202**, **206** and the certificate authority module **204** can communicate with each other, for example, over a data network or another type of communication link. In some implementations, the terminal modules **202**, **206** and the certificate authority module **204** can communicate with each other by messages transmitted over the data network **108** of FIG. **1**. In the example shown in FIG. **2**, the terminal module **202** can send a certificate request **220** to the certificate authority module **204**. The certificate authority module **204** can receive the certificate request **220** from the terminal module **202** and send an implicit certificate **222** to the terminal module **202** in response to the certificate request **220**. The certificate authority module **204** can also send the terminal module **202** private key contribution data. The private key contribution data can be sent to the terminal module **202** together with or separate from the implicit certificate **222**. The certificate authority module **204** can also publish the implicit certificate **222**, for example, to a certificate database. The terminal module **202** can receive the implicit certificate **222** from the certificate authority module **204** and send a signed message **224** to the terminal module **206**. The terminal module **206** can receive the signed message **224** and the implicit certificate **222** from the terminal module **202**. The terminal module **206** can verify the digital signature and the implicit certificate **222**, for example, using the techniques shown in FIGS. **3** and **4**. The cryptography system **200** can support additional or different types of communications.

The cryptography system **200** utilizes an implicit certificate and digital signature scheme that allows the terminal modules to verify the authenticity of messages received from other terminal modules. According to the implicit certificate scheme, implicit certificates issued by the certificate authority bind each user entity to a particular public key value. The ECQV implicit certificate scheme, as well as others, may be implemented using a group of points on an elliptic curve, a multiplicative group of a finite field, or other groups where the discrete logarithm problem may be hard.

Some of the example operations and capabilities of the cryptography system **200** shown in FIG. **2** are described with respect to the ECQV implicit certificate scheme. In some implementations, the ECQV implicit certificate scheme can function as a general purpose digital signature scheme for applications within computer and communications systems. Some implementations of the ECQV implicit certificate scheme are well suited for application environments where resources, such as bandwidth, computing power, and storage are limited. In those cases, ECQV implicit certificates may provide a more efficient alternative to some other types of certificates. Some implementations of the ECQV implicit certificate scheme are well suited for other types of application environments, for example, with superior resources. Examples of elliptic curve-based digital signatures schemes include ECDSA (Elliptic Curve Digital Signature Algorithm), ECPVS (Elliptic Curve Pintsov Vanstone Signatures), and ECNR (Elliptic Curve Nyberg Rueppel).

In an elliptic curve cryptography (ECC) scheme, information is encoded in elliptic curve points in an elliptic curve group. An elliptic curve group can be described in terms of a solution to an equation over a finite field, for example, a prime finite field or a characteristic-two finite field. Each point in the elliptic curve group is a pair of field elements corresponding to a solution to an elliptic curve equation. The elliptic curve group also includes an identity element. As a particular example, let $\mathbb{F}_p$ represent a prime finite field where p is an odd prime number, and let a, b $\in \mathbb{F}_p$ satisfy $4.a^3+27.b^2\neq 0$ (mod p). The elliptic curve group E ($\mathbb{F}_p$) over $\mathbb{F}_p$, which is defined by the parameters a, b $\in \mathbb{F}_p$ includes the set of points M=(x,y) for x,y $\in \mathbb{F}_p$ that represent a solution to the equation $y^2 \equiv x^3+a.x+b$ (mod p), together with a point $\mathcal{O}$ that is the identity element of the elliptic curve group E ($\mathbb{F}_p$). The identity element $\mathcal{O}$ is sometimes referred to as the point at infinity.

In an ECC scheme, elliptic curve domain parameters over $\mathbb{F}_p$ can be identified by a sextuple T=(p,a,b,G,n,h). The integer p specifies the finite field $\mathbb{F}_p$. Field elements a, b $\in \mathbb{F}_p$ specify an elliptic curve E($\mathbb{F}_p$) over $\mathbb{F}_p$ as discussed above. The elliptic curve point G=(x_G,y_G) on E($\mathbb{F}_p$) is a base point generator. The integer n specifies the order of the base point generator G, having the property nG=$\mathcal{O}$. The cofactor h is equal to #E($\mathbb{F}_p$)/n, which is the number of points on the elliptic curve E($\mathbb{F}_p$) divided by the order of the base point generator G. Elliptic curve domain parameters may alternatively be identified over other types of finite fields. For example, elliptic curve domain parameters over the characteristic two field $\mathbb{F}_{2^m}$ can be identified by a septuple T=(m,f(x),a,b,G,n,h), where m is an integer specifying the finite field $\mathbb{F}_{2^m}$ and f(x) is an irreducible binary polynomial of degree m specifying the representation of $\mathbb{F}_{2^m}$ In some implementations, the elliptic curve domain parameters can be generated, validated, and utilized by the terminal modules **202**, **206** or by the certificate authority module **204** in the cryptography system **200**. In some implementations, the elliptic curve domain parameters can be shared among the modules in the cryptography system **200**.

In an ECC scheme, an elliptic curve key pair (d,Q) can be generated based on valid elliptic curve domain parameters, for example, T=(p,a,b,G,n,h) or T=(m,f(x),a,b,G,n,h). The key pair may be generated by selecting a random integer d in the interval [1,n−1], computing Q=dG, and outputting the key pair (d,Q). The random integer d may be selected or obtained by a random number generator. In some implementations, the elliptic curve key pairs can be generated, validated, and processed by the terminal modules **202**, **206** or by the certificate authority module **204** in the cryptography system **200**.

The terminal module **202** includes a signature generation module **242**, a request generation module **240**, and other possibly other modules. The request generation module **240** can generate a certificate request **220**. The certificate request **220** can include an identification U of a user entity. The certificate request **220** can include an elliptic curve point R_U. The certificate request **220** can include additional or different information. The identification value U can be a unique identifier for a particular user entity, a particular device, or both. The request generation module **240** can generate the elliptic curve point R_U by selecting a random number k_U and computing R_U=k_UG. For example, the terminal module **202** may have a random number generator module that generates random numbers. The request generation module **240** can perform a validity check to ensure that the values k_U and R_U correspond to a valid key pair. The requester can convert the elliptic curve point R_U, the identification value U, and any other information to be included in the certificate request **220** to an appropriate data format (e.g., an octet string).

The signature generation module **242** can use the implicit certificate **222** to generate a digital signature for a message **218**. An example technique for generating a digital signature based on an elliptic curve key pair is provided by the Elliptic Curve Digital Signature Algorithm (ECDSA). The message **218** can include any type of electronic document, data file, data object, or other form of information. In some cases, the message **218** is an e-mail message, an electronic document, or an electronic data file that can be edited and rendered by appropriate software applications. In some cases, the message **218** is a data message or a combination of data messages used in signaling applications among hardware components. For example, the message **218** can include status information from a smart energy meter in a smart energy infrastructure.

The signature generation module **242** can generate the digital signature using the private key of the terminal module **202** and the implicit certificate **222**. The signature generation module can generate the private key of the terminal module **202** based on private key contribution data r, the implicit certificate **222**, and the random value k_U that was used to generate the certificate request **220**. The digital signature generated by the signature generation module **242** can be appended to, combined with, or otherwise associated with the message **218** to create the signed message **224**. The digital signature can be sent separately from the message **218**. The terminal module **202** can send the implicit certificate **222** to the terminal module **206** along with the signed message **224**.

The terminal module **206** includes a signature verification module **250** and possibly other modules. The signature verification module **250** can verify the digital signature associated with the signed message **224**. The terminal module **206** can include a data input module that obtains or provides the input values used by the signature verification module **250**. Example techniques for verifying a digital signature are provided in FIGS. **3** and **4**. Additional or different techniques may be used. The signed message **224** includes a digital signature purportedly generated by a user entity associated with an identification value U. The signature verification module **250** can receive the implicit certificate **222** from the terminal module **206** or retrieve the implicit certificate **222** associated with the identification value U from another source. The signature verification module **250** can verify the authenticity of the digital signature and the validity of implicit certificate **222**.

The certificate authority module **204** includes a certificate generation module **230**, and possibly other modules. The certificate generation module **230** can perform one or more operations for issuing the implicit certificate **222** for use in the cryptography system **200**. For example, the certificate generation module **230** may be configured to perform one or more of the operations presented in FIG. **3**A, or the certificate generation module **230** may be configured to issued implicit certificates in a different manner.

The certificate generation module **230** generates the implicit certificate **222** based on the information in the certificate request **220**. For example, the certificate generation module **230** can select a random value k and generate public key reconstruction data P_U by computing $P_U=R_U+kG$, where R_U is the elliptic curve point generated by the request generation module **240** and included in the certificate request **220**. The certificate authority module **204** may have a random number generator module that generates random numbers. The certificate generation module **230** can encode the public key reconstruction data P_U, and sometimes other information, in an implicit certificate Cert_U. The implicit certificate Cert_U can be generated by a certificate encoding scheme, for example, a fixed-length field scheme, a minimal ASN.1 encoding scheme, or an X.509-compliant ASN.1 encoding scheme.

FIG. **3** is a flow chart showing an example verification process **300**. The process **300** can be implemented by a terminal of a cryptography system. For example, the process **300** can be implemented by the terminal **106** shown in FIG. **1**, the terminal module **206** shown in FIG. **2**, or by another type of system or module. The example process **300** shown in FIG. **3** can be implemented using additional, fewer, or different operations, which can be performed in the order shown or in a different order. In some implementations, one or more of the operations can be repeated or iterated, for example, until a terminating condition is reached. For purposes of illustration, the operations of the example process **300** are described below as implemented by a terminal of an elliptic curve cryptography system. The example process **300** can also be in other types of cryptography systems. Moreover, one or more of the operations of the example process **300** can be implemented by another type of entity in the cryptography such as a certificate authority.

In some implementations, the process **300** may be used to simultaneously verify an ECQV implicit certificate and an ECDSA digital certificate that is based on the implicit certificate. The ECQV implicit certificate scheme may be described with respect to operations performed by three entities with the following representations—a Certificate Authority CA, a certificate requester U, and a certificate processer V. FIG. **3**A shows an example protocol executed by the certificate requester U and the certificate authority CA when the certificate authority CA issues the implicit certificate for the requester U. As shown in FIG. **3**A, certificate issuance can include a two pass protocol between the requester U and the certificate authority CA, where the first flow is the certificate request from U, and the second flow is the response from CA, containing the certificate.

In the present discussion of FIGS. **3** and **3**A, the following notation is used. G represents a base point operable to generate the group of elliptic curve points used to implement ECQV. The group $\mathbb{G}$ may be of prime order n. C represents the CA's public key, which is a point in the group $\mathbb{G}$ The ECQV implicit certificate can be represented as (P,I), where P represents an elliptic curve point in $\mathbb{G}$ and I is a bit string that may contain information about the requester U. A hash value e=H (P,I) represents the hash of the certificate (P,I). The choice of the hash function H can be fixed, for example, as part of the domain parameters. B represents the requester's public key, according to the equation B=C+eP. The value b represents the requester's secret key, and b satisfies the relationship of B=bG.

In some implementations, the certificate authority CA or another entity establishes the elliptic curve domain parameters, a hash function, the certificate encoding format, and all parties have selected a random number generator. The certificate authority CA can generate a key pair. All parties can receive authentic copies of CA's public key and domain parameters. The elliptic curve domain parameters, which can be generated by the certificate authority or another entity, include the field size q, the elliptic curve coefficients a and b, the base point generator G, the order n of the base point generator, the cofactor h (where hn is the number of points on the elliptic curve), and others. In some instances, the elliptic curve domain parameters include a seed value for selecting random values. In cases where the field is a characteristic two finite field (i.e., $q=2^m$), the elliptic curve domain parameters include an indication of the basis (e.g., the reduction polynomial). The hash function H can be a hash function that has been approved for the specified security level in the cryptography system. In some implementations, one or more conventional hash functions in the SHA-2 family can be used (e.g., SHA-256, SHA-512). Additional or different hash functions may be used.

After the requester U receives the implicit certificate from the certificate authority, the requester U can use the implicit certificate. For example, the requester U may use the implicit certificate to generate an ECDSA digital signature. In some implementations, an ECDSA digital signature can be generated based on the following example technique. Let Q=xG, where G represents the base point. Q represents the public key and x represents the private key. The hash function $H': \{0, 1\}^* \rightarrow Z_n$ may be the same as the function H used with ECQV, or may be different. The ECDSA signing process may utilize as input a message m and a private key x, and produce as output a signature (r,s). The signing procedure can be summarized as follows: (1.) Choose $k \in_R \mathbb{Z}_n$; (2.) Compute T=kG; (3.) Let r=f(T) where f: $\mathbb{G} \rightarrow \mathbb{Z}_n$; (4.) If r=0 then go to step (1.); (5.) Compute h=H'(m); (6.) Compute s=(h+rx)/k (mod n); (7.) If s=0 then go to step (1.); and (8.) return (r,s). Additional or different digital signature techniques may be used.

The requester U may then send the digital signature and the implicit certificate to a verifier V. The requester U may send the implicit certificate with the digital signature, the requester U may send the implicit certificate and the digital signature separately, or the verifier may obtain the implicit certificate from another source. As shown in FIG. **3**, an ECQV implicit certificate is received by a verifier V, and at **315**, an ECDSA digital signature is received by the verifier V.

At **320**, the ECQV implicit certificate and the ECDSA digital signature are verified by the verifier V. A combined fast verification technique can be used for verifying an ECDSA signed message where the signer has an ECQV implicit certificate. For example, the technique can combine the two processes of recovering the signer's ECQV public key and ECDSA. The technique may access inputs that include an ECDSA signed message m, the digital signature (r,s), and an ECQV certificate (P,I). The technique may produce outputs that include reporting "accept" or "reject".

In some aspects, the verification process can be summarized as follows: (1.) Reject if r,s∈{1, . . . q−1}; (2.) Reject if P∈$\mathbb{G}$; (3.) Invert the function ƒ to find R∈$\mathbb{G}$ where ƒ(R)=r from the ECDSA signing operation, under an assumption that only one such R exists; (4.) Compute h=H'(m), e=H (P,I); (5.) Write (re)/s as u/v (mod n) such that u, v are small integers; (6.) If the equation

$$\left(\frac{hv}{s}\right)G + \left(\frac{vr}{s}\right)C + uP - R = \mathcal{O} \tag{1}$$

is satisfied, where $\mathcal{O}$ is the identity element, then output "accept", else output "reject".

The process of inverting the function ƒ can be efficiently conducted, for example, where the co-factor of the elliptic curve group is small. When the co-factor is one, there can be a single inverse for each r, and when the cofactor is greater than one, but still small, with a few "hint bits", the inversion may be made efficient. In some implementations, (R, s) may be sent as the signature instead of (r=ƒ(R), s). Operation (5.) in the verification process outlined above can use the extended Euclidean algorithm to express x(mod n) as a fraction u/v (mod n) such that u,v≤$\sqrt{n}$.

After R is computed from r, the ECDSA verification equation can check whether the following equation is satisfied:

$$R = \left(\frac{h}{s}\right)G + \left(\frac{r}{s}\right)B$$

where B is the public key. Since B=C+eP, it is mathematically equivalent to check

$$R = \left(\frac{h}{s}\right)G + \left(\frac{r}{s}\right)C + \left(\frac{re}{s}\right)P. \tag{2}$$

Further, (re)/s can be substituted with u/v and the equation becomes:

$$R = \left(\frac{h}{s}\right)G + \left(\frac{r}{s}\right)C + \left(\frac{u}{v}\right)P$$

As such, this is mathematically equivalent to the verification equation:

$$\left(\frac{hv}{s}\right)G + \left(\frac{vr}{s}\right)C + uP - R = \mathcal{O}$$

The efficiency of the combined verification process can be higher than the sum of calculation in each component. The resulting verification equation can be computed by computing the sum of scalar multiples (sometimes called a "multi scalar multiplication" or MSM). Two of the scalar multiples (base G and C) may have full sized exponents and fixed bases. For example, assuming there is a small number of CAs, pre-processing can be conducted to pre-compute and store multiples of G and C. The other two bases (P and R) may vary with each signer/signature, but here the scalars are half-sized, on account of the choice of u and v.

The verification equation can be rewritten as

$$\lambda G + \gamma C + uP - vP = \mathcal{O} \tag{3}$$

where λ=hv/s and γ=vr/s. Let t=[$\log_2$(n+1)]. Then

$$\lambda = \lambda_0 + \lambda_1 2^{\lceil t/2 \rceil}$$

$$\gamma = \gamma_0 + \gamma_1 2^{\lceil t/2 \rceil}$$

for integers λ_0, λ_1, γ_0, γ_1 that are about t/2 bits long in some instances. The notation ⌈/2⌉ denotes the ceiling of t/2, which refers to the smallest integer not less than t/2. If two points are pre-computed, such as, for example, by computing $G_1 = 2^{\lceil t/2 \rceil}G$ and $C_1 = ^{\lceil t/2 \rceil}C$, the verification criterion can be written:

$$\lambda_0 G + \lambda_1 G_1 + \gamma_0 C + \gamma_1 C_1 + uP - vR = \mathcal{O} \tag{4}$$

Any suitable technique may be used to implement the scalar multiplications. The form of the verification criterion in Equation (4) uses six scalar multiplications with half-length scalars, compared to three scalar multiplications with full-length scalars in Equation (2). In some implementations, each "half-length" scalar is less than or equal to ⌈t/2⌉ bits. For example, in some instances, each of the scalars λ_0, λ_1, γ_0, γ_1, u, v in Equation (4) can be less than or equal to ⌈t/2⌉ bits. The cost comparison between the Equations (2) and (4) can be reduced to comparing an MSM with three terms and full-length (t-bit) scalars to another MSM with six terms and half-length (t/2-bit) scalars.

Some example algorithms for evaluating the speedup (e.g., the Strauss Algorithm) use (1+p/$\log_2$ $\log_2$ B) $\log_2$ B group operations, where p is the number of terms in the MSM and the scalars are less than B. Using the Strauss estimate for comparing Equation (2) to Equation (4), when t=384, it is found that Equation (4) generates a 33 percent speedup over Equation (2).

A second estimate can be given, assuming the MSM is implemented using a conventional w-NAF, to facilitate calculation. In Equation (2) two of the three terms are known. In Equation (4) four of the six terms are known in advance. The tables of points used by the NAF algorithm may be pre-computed for the known points. Each table has $(2^w-(-1)^w)/3$ entries, and takes one addition per entry to create. The tables for the unknown points can be computed on-the-fly. The scalars are w-NAF re-encoded, reducing the overall number of additions by about 1/(w+1).

Let A be the number of elliptic curve additions and D be the number of doublings required. In this implementation, Equation (2) uses

$$[3t/(w+1)+(2^w-(-1)^3)/3]A+[t-1]D$$

additions and doublings, while Equation (4) uses

$$[6t/2(w+1)+2(2^w-(-1)^2)/3]A+[(t-1)/2]D$$

additions and doublings. By subtracting these two costs, we can estimate the speedup of Equation 4 to be

$$[(t-1)/2]D-[(2^w-(-1)^w)/3]A.$$

We now look at two specific choices of w, namely w=4, 5. When w=4, Equation (2) uses

$$[3t/5+5]A+[t-1]D$$

and Equation (4) uses

$$[3t/5+10]A+[(t-1)/2]D.$$

Using the estimates, A=10.4M and D=7.2M, where M is the cost of a field multiplication, it can be found that when w=4, and t=384 the estimated speedup is 25.5 percent. When w=5, a similar calculation estimates the speedup to be 26.1 percent. Based on these estimates, combining ECQV and ECDSA verification as in Equation (4) may provide faster verification in some instances.

FIG. **4** is a flow chart showing an example verification process **400**. The verification process **400** can be implemented by any suitable hardware, for example, by a data processing apparatus, a computing device or a system of computing devices. In some instances, the process **400** can be implemented by the terminal **106** shown in FIG. **1**, by the terminal module **206** shown in FIG. **2**, or by another type of system or module. The example verification process **400** shown in FIG. **4** can be implemented using additional, fewer, or different operations, which can be performed in the order shown or in a different order. In some implementations, one or more of the operations can be repeated or iterated, for example, until a terminating condition is reached. For purposes of illustration, the operations of the example verification process **400** are described below as implemented by a terminal of an elliptic curve cryptography system. For example, the verification process may be used to verify an ECDSA digital signature and an ECQV implicit certificate. The example process **400** can also be used in different types of cryptography systems. Moreover, one or more of the operations of the example process **400** can be implemented by another type of entity in the cryptography such as a certificate authority.

At **405**, a digital signature is accessed. For example, the digital signature can be an ECDSA digital signature or another type of ECC-based digital signature. The digital signature may be associated with an entity. For example, the entity can be a user, a computing device, a user account, or another type of entity. The digital signature can be accessed from a local memory, from a remote device, or another source. In some instances, the digital signature is received from the entity.

At **410**, an implicit certificate is accessed. For example, the implicit certificate can be an ECQV implicit certificate or another type of ECC-based implicit certificate. The implicit certificate may be issued by a certificate authority and associated with the entity. The implicit certificate can be accessed from a local memory, from a remote device, or another source. In some instances, the digital signature is received from the entity. The implicit certificate may include an elliptic curve point P representing a public key reconstruction value of the entity.

At **415**, a second elliptic curve point C_1 is accessed. The second elliptic curve point C_1 represents a pre-computed multiple of a public key C of the certificate authority. In some instances, the pre-computed multiple of the certificate authority's public key C can be used to verify the digital signature faster. For example, performing an elliptic curve point multiplication can be time-consuming. By calculating and storing certain pre-computed multiples (e.g., prior to receiving a digital signature to be verified), the time required to verify the digital signature may be reduced. The pre-computed multiples can values that are likely to be used to verify digital signatures. For example, the pre-computed multiples can include one or more pre-computed multiples of the certificate authority's public key, one or more pre-computed multiples of the generator point, or any suitable combination of these and other values. Such values can be "pre-computed" by computing them before a particular digital signature is to be verified. The pre-computed multiples can be stored and made accessible to be used when a digital signature is to be verified.

At **420**, additional scalar multiples are generated based on a number of half-length scalars and a plurality of elliptic curve points. Each of the scalar multiples may be generated by multiplying one of the half-length scalars by one of the elliptic curve points. The elliptic curve points that are used to produce the scalar multiples may include P, C_1 and G_1, of which G_1 represents a pre-computed multiple of a generator point G. At **425**, the digital signature is verified using the first elliptic curve point P and the second elliptic curve point C_1. For example, the digital signature can be verified based on the scalar multiples generated at **420**, which may include a scalar multiple of the first elliptic curve point P, a scalar multiple of the second elliptic curve point C_1, a scalar multiple of the third elliptic curve point G_1, or a combination of these and other scalar multiples. In some implementations, the digital signature may be verified at **420** using the example Equation 4 provided above, or by another suitable technique.

Subject matter and operations described in this specification can be implemented in digital electronic circuitry, or in computer software, firmware, or hardware, including the structures disclosed in this specification and their structural equivalents, or in combinations of one or more of them. Some of the subject matter described in this specification can be implemented as one or more computer programs, i.e., one or more modules of computer program instructions, encoded on non-transitory computer storage medium for execution by, or to control the operation of, data processing apparatus. Alternatively or in addition, the program instructions can be encoded for transmission to suitable receiver apparatus for execution by a data processing apparatus. A computer storage medium can be, or be included in, a computer-readable storage device, a computer-readable storage substrate, a random or serial access memory array or device, or a combination of one or more of them. The computer storage medium can be, or be included in, one or more separate physical components or media (e.g., multiple cards, disks, or other storage devices).

The operations described in this specification can be implemented as operations performed by a data processing apparatus on data stored on one or more computer-readable storage devices or received from other sources. The term "data processing apparatus" encompasses all kinds of apparatus, devices, and machines for processing data, including by way of example a programmable processor, a computer, a system on a chip, or multiple ones, or combinations, of the foregoing.

The apparatus can include special purpose logic circuitry, e.g., an FPGA (field programmable gate array) or an ASIC (application-specific integrated circuit). The apparatus can also include, in addition to hardware, code that creates an execution environment for the computer program in question, e.g., code that constitutes processor firmware, a protocol stack, a database management system, an operating system, a cross-platform runtime environment, a virtual machine, or a combination of one or more of them. The apparatus and execution environment can realize various different computing model infrastructures, such as web services, distributed computing and grid computing infrastructures.

A computer program (also known as a program, software, software application, script, or code) can be written in any form of programming language, including compiled or interpreted languages, declarative or procedural languages, and it can be deployed in any form, including as a stand-alone program or as a module, component, subroutine, object, or other unit suitable for use in a computing environment. A computer program may, but need not, correspond to a file in a file system. A program can be stored in a portion of a file that holds other programs or data (e.g., one or more scripts stored in a markup language document), in a single file dedicated to the program in question, or in multiple coordinated files (e.g., files that store one or more modules, sub-programs, or portions of code). A computer program can be deployed to be executed on one computing device or on multiple computers that are located at one site or distributed across multiple sites and interconnected by a communication network.

The processes and logic flows described in this specification can be performed by one or more programmable processors executing one or more computer programs to perform actions by operating on input data and generating output. The processes and logic flows can also be performed by, and apparatus can also be implemented as, special purpose logic circuitry, e.g., an FPGA (field programmable gate array) or an ASIC (application-specific integrated circuit).

Processors suitable for the execution of a computer program include, by way of example, both general and special purpose microprocessors, and any one or more processors of any kind of digital computing device. Generally, a processor will receive instructions and data from a read-only memory or a random access memory or both. The essential elements of a computing device are a processor for performing actions in accordance with instructions and one or more memory devices for storing instructions and data. Generally, a computing device will also include, or be operatively coupled to receive data from or transfer data to, or both, one or more storage devices for storing data. However, a computing device need not have such devices. Moreover, a computer can be embedded in another device, e.g., a mobile telephone, a personal digital assistant (PDA), a mobile audio or video player, a game console, a Global Positioning System (GPS) receiver, or a portable storage device (e.g., a universal serial bus (USB) flash drive), to name just a few. Devices suitable for storing computer program instructions and data include all forms of non-volatile memory, media and memory devices, including by way of example semiconductor memory devices, e.g., EPROM, EEPROM, and flash memory devices; magnetic disks, e.g., internal hard disks or removable disks; magneto-optical disks; and CD-ROM and DVD-ROM disks. The processor and the memory can be supplemented by, or incorporated in, special purpose logic circuitry.

To provide for interaction with a user, subject matter described in this specification can be implemented on a computer having a display device, e.g., an LCD (liquid crystal display) screen for displaying information to the user and a keyboard and a pointing device, e.g., touch screen, stylus, mouse, etc. by which the user can provide input to the computer. Other kinds of devices can be used to provide for interaction with a user as well; for example, feedback provided to the user can be any form of sensory feedback, e.g., visual feedback, auditory feedback, or tactile feedback; and input from the user can be received in any form, including acoustic, speech, or tactile input. In addition, a computing device can interact with a user by sending documents to and receiving documents from a device that is used by the user; for example, by sending web pages to a web browser on a user's client device in response to requests received from the web browser.

Some of the subject matter described in this specification can be implemented in a computing system that includes a back-end component, e.g., as a data server, or that includes a middleware component, e.g., an application server, or that includes a front-end component, e.g., a client computing device having a graphical user interface or a Web browser through which a user can interact with an implementation of the subject matter described in this specification, or any combination of one or more such back-end, middleware, or front-end components. The components of the system can be interconnected by any form or medium of digital data communication, e.g., a data network.

The computing system can include clients and servers. A client and server are generally remote from each other and typically interact through a data network. The relationship of client and server arises by virtue of computer programs running on the respective computers and having a client-server relationship to each other. In some implementations, a server transmits data to a client device. Data generated at the client device can be received from the client device at the server.

While this specification contains many implementation details, these should not be construed as limitations on the scope of what may be claimed, but rather as descriptions of features specific to some implementations. Certain features that are described in this specification in the context of separate implementations can also be implemented in combination in a single implementation. Conversely, various features that are described in the context of a single implementation can also be implemented in multiple implementations separately or in any suitable subcombination. Moreover, although features may be described above as acting in certain combinations and even initially claimed as such, one or more features from a claimed combination can in some cases be excised from the combination, and the claimed combination may be directed to a subcombination or variation of a sub combination.

Similarly, while operations are depicted in the drawings in a particular order, this should not be understood as requiring that such operations be performed in the particular order shown or in sequential order, or that all illustrated operations be performed, to achieve desirable results. In certain circumstances, multitasking and parallel processing may be advantageous. Moreover, the separation of various system components in the implementations described above should not be understood as requiring such separation in all implementations, and it should be understood that the described program components and systems can generally be integrated together in a single software product or packaged into multiple software products.

In a general aspect, a technique for verifying digital signatures is disclosed. The technique may take an implicit certificate and a digital signature as input. The technique may determine and generate a positive output if the certificate is valid and the signature on the message is valid.

In some aspects, a digital signature associated with an entity is accessed. An implicit certificate issued by a certificate authority and associated with the entity is accessed. The implicit certificate may include a first elliptic curve point P representing a public key reconstruction value of the entity. A second elliptic curve point C_1 representing a pre-computed multiple of a public key C of the certificate authority is accessed. The first elliptic curve point P and the second elliptic curve point C_1 are used to verify the digital signature by data processing apparatus.

Implementations of these and other aspects can include one or more of the following features. Verifying the digital signature provides verification that the implicit certificate is valid. A third elliptic curve point G_1 representing a pre-computed multiple of a generator point G is accessed and the third elliptic curve point G_1 is accessed with the first elliptic curve point P and the second elliptic curve point C_1 to verify the digital signature.

Additionally or alternatively, implementations of these and other aspects can include one or more of the following features. Verifying the digital signature can include generating a plurality of scalar multiples based on a plurality of half-length scalars and a plurality of elliptic curve points. Each of the scalar multiples is generated by multiplying one of the plurality of half-length scalars by one of the plurality of elliptic curve points. The plurality of elliptic curve points includes the first elliptic curve point P, the second elliptic curve point C_1, and the third elliptic curve point G_1. Verifying the digital signature can also include combining the plurality of scalar multiples according to a digital signature verification equation.

Additionally or alternatively, implementations of these and other aspects can include one or more of the following features. The digital signature includes a first signature element r and a second signature element s. The generator point G is a generator of an elliptic curve group of order n. Based on the first signature element r, a fourth elliptic curve point R can be obtained. A first integer u and a second integer n are then computed based on the digital signature.

Additionally or alternatively, implementations of these and other aspects can include one or more of the following features. The first integer u and the second integer v are each less than $\sqrt{n}$. A third integer λ_0 and a fourth integer λ_1 are then computed such that $\lambda=\lambda_0+\lambda_1 2^{\lceil t/2 \rceil}$, where $t=\lceil \log_2(n+1) \rceil$, $\lambda=hv/s$, and h is a hash of a message m associated with the digital signature (r,s). A fifth integer γ_0 and a sixth integer γ_1 can then be computed such that $\gamma=\gamma_0+\gamma_1 2^{\lceil t/2 \rceil}$, where $\gamma=vr/s$. Verifying the digital signature includes verifying $\lambda_0 G+\lambda_0 G_1+\gamma_0 C+\gamma_1 C_1+uP-vR=\mathcal{O}$, where $\mathcal{O}$ represents an identity element of the elliptic curve group.

Additionally or alternatively, implementations of these and other aspects can include one or more of the following features. Prior to accessing the digital signature associated with the entity, the fast verification method may further include computing the second elliptic curve point $C_1=2^{\lceil t/2 \rceil}C$ and computing the third elliptic curve point $G_1=2^{\lceil t/2 \rceil}G$. The second elliptic curve point C_1 and the third elliptic curve point G_1 may be stored for subsequent access.

Thus, implementations of the subject matter have been described. Other implementations are within the scope of the following claims. In some cases, the actions recited in the claims can be performed in a different order and still achieve desirable results. In addition, the processes depicted in the accompanying figures do not necessarily require the particular order shown, or sequential order, to achieve desirable results. In certain implementations, multitasking and parallel processing may be advantageous.

What is claimed is:

1. A computer implemented method for verifying a digital signature, the method comprising:

accessing a digital signature associated with an entity, wherein the digital signature includes a first signature element r and a second signature element s;

accessing a message m associated with the digital signature (r,s);

accessing an implicit certificate issued by a certificate authority and associated with the entity, the implicit certificate including a first elliptic curve point P representing a public key reconstruction value of the entity;

accessing a second elliptic curve point C_1 representing a pre-computed multiple of a public key C of the certificate authority;

accessing a third elliptic curve point G_1 representing a pre-computed multiple of a generator point G, the generator point G representing a generator of an elliptic curve group of order n;

obtaining a fourth elliptic curve point R based on the first signature element r;

computing a first integer u and a second integer v based on the digital signature, wherein the first integer u and the second integer v are each less than $\sqrt{n}$;

computing a hash h of the message m;

computing a third integer λ_0 and a fourth integer λ_1 such that $\lambda=\lambda_0+\lambda_1 2^{\lceil t/2 \rceil}$, where $t=\lceil \log_2(n+1) \rceil$, $\lambda=hv/s$;

computing a fifth integer γ_0 and a sixth integer γ_1 such that $\gamma=\gamma_0+\gamma_1 2^{\lceil t/2 \rceil}$, where $\gamma=vr/s$;

verifying that $\lambda_0 G+\lambda_1 G_1+\gamma_0 C+\gamma_1 C_1+uP-vR=\mathcal{O}$ to verify the digital signature by data processing apparatus, where $\mathcal{O}$ represents an identity element of the elliptic curve group.

2. The method of claim **1**, comprising:

generating a plurality of scalar multiples based on a plurality of half-length scalars and a plurality of elliptic curve points, wherein each of the scalar multiples is generated by multiplying one of the plurality of half-length scalars by one of the plurality of elliptic curve points, and the plurality of elliptic curve points includes the first elliptic curve point P and the second elliptic curve point C_1,

wherein verifying the digital signature includes combining the plurality of scalar multiples according to a digital signature verification equation.

3. The method of claim **2**, wherein each of the half-length scalars has a bit length less than or equal to $\lceil t/2 \rceil$.

4. The method of claim **1**, wherein verifying the digital signature provides verification that the implicit certificate is valid.

5. The method of claim **1**, further comprising, prior to accessing the digital signature associated with the entity:

computing the second elliptic curve point $C_1=2^{\lceil t/2 \rceil}C$;

computing the third elliptic curve point $G_1=2^{\lceil t/2 \rceil}G$; and

storing the second elliptic curve point C_1 and the third elliptic curve point G_1 for subsequent access.

6. A computing device comprising:

one or more hardware processors configured to:

access a digital signature associated with an entity, wherein the digital signature includes a first signature element r and a second signature element s;

access a message m associated with the digital signature (r,s);

access an implicit certificate issued by a certificate authority and associated with the entity, the implicit certificate including a first elliptic curve point P representing a public key reconstruction value of the entity;

access a second elliptic curve point C_1 representing a pre-computed multiple of a public key C of the certificate authority;

access a third elliptic curve point G_1 representing a pre-computed multiple of a generator point G, the generator point G representing a generator of an elliptic curve group of order n;

obtain a fourth elliptic curve point R based on the first signature element r;

compute a first integer u and a second integer v based on the digital signature, wherein the first integer u and the second integer v are each less than $\sqrt{n}$;

compute a hash h of the message m;

compute a third integer λ_0 and a fourth integer λ_1 such that $\lambda=\lambda_0+\lambda_1 2^{\lceil t/2 \rceil}$, where $t=\lceil \log_2(n+1) \rceil$, $\lambda=hv/s$;

compute a fifth integer γ_0 and a sixth integer γ_1 such that $\gamma=\gamma_0+\gamma_1 2^{\lceil t/2 \rceil}$, where $\gamma=vr/s$;

verify that $\lambda_0 G+\lambda_1 G_1+\gamma_0 C+\gamma_1 C_1+uP-vR=\mathcal{O}$ to verify the digital signature, where $\mathcal{O}$ represents an identity element of the elliptic curve group.

7. The computing device of claim **6**, further comprising one or more memory components, wherein the one or more processors are configured to access the second elliptic curve point from the one or more memory components.

8. The computing device of claim **6**, further comprising one or more communication interface components, wherein the implicit certificate and the digital signature are received by the one or more communication interface components from another computing device.

9. The computing device of claim **6**, wherein the one or more processors are configured to:

generate a plurality of scalar multiples based on a plurality of half-length scalars and a plurality of elliptic curve points, wherein each of the scalar multiples is generated by multiplying one of the plurality of half-length scalars by one of the plurality of elliptic curve points, and the plurality of elliptic curve points includes the first elliptic curve point P and the second elliptic curve point C_1,

wherein verifying the digital signature includes combining the plurality of scalar multiples according to a digital signature verification equation.

10. The computing device of claim **9**, wherein each of the half-length scalars has a bit length less than or equal to t/2.

11. The computing device of claim **6**, wherein verifying the digital signature provides verification that the implicit certificate is valid.

12. A non-transitory computer-readable medium storing instructions that are operable when executed by data processing apparatus to perform operations for verifying a digital signature, the operations comprising:

accessing a digital signature associated with an entity, wherein the digital signature includes a first signature element r and a second signature element s;

accessing a message m associated with the digital signature (r,s);

accessing an implicit certificate issued by a certificate authority and associated with the entity, the implicit certificate including a first elliptic curve point P representing a public key reconstruction value of the entity;

accessing a second elliptic curve point C_1 representing a pre-computed multiple of a public key C of the certificate authority;

accessing a third elliptic curve point G_1 representing a pre-computed multiple of a generator point G, the generator point G representing a generator of an elliptic curve group of order n;

obtaining a fourth elliptic curve point R based on the first signature element r;

computing a first integer u and a second integer v based on the digital signature, wherein the first integer u and the second integer v are each less than $\sqrt{n}$;

computing a hash h of the message m; computing a third integer λ_0 and a fourth integer λ_1 such that $\lambda=\lambda_0+\lambda_1 2^{\lceil t/2 \rceil}$, where $t=\lceil \log_2(n+1) \rceil$, $\lambda=hv/s$;

computing a fifth integer γ_0 and a sixth integer γ_1 such that $\gamma=\gamma_0+\gamma_1 2^{\lceil t/2 \rceil}$, where $\gamma=vr/s$;

verifying that $\lambda_0 G+\lambda_1 G_1+\gamma_0 C+\gamma_1 C_1+uP-vR=\mathcal{O}$ to verify the digital signature, where $\mathcal{O}$ represents an identity element of the elliptic curve group.

13. The computer-readable medium of claim **12**, the operations comprising:

generating a plurality of scalar multiples based on a plurality of half-length scalars and a plurality of elliptic curve points, wherein each of the scalar multiples is generated by multiplying one of the plurality of half-length scalars by one of the plurality of elliptic curve points, and the plurality of elliptic curve points includes the first elliptic curve point and the second elliptic curve point,

wherein verifying the digital signature includes combining the plurality of scalar multiples according to a digital signature verification equation.

14. The computer-readable medium of claim **13**, wherein each of the half-length scalars has a bit length less than or equal to t/2.

15. The computer-readable medium of claim **12**, wherein verifying the digital signature provides verification that the implicit certificate is valid.

16. The computer-readable medium of claim **12**, the operations further comprising, prior to accessing the digital signature associated with the entity:

computing the second elliptic curve point; and

storing the second elliptic curve point for subsequent access.

17. The method of claim **1**, wherein verifying the digital signature includes combining the second elliptic curve point C_1 with a first half-length scalar, and combining the public key C of the certificate authority with a second half-length scalar, and the first and second half-length scalars are computed based on the digital signature.

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