May 15, 1934. W. CAUER 1,958,742

ARTIFICIAL NETWORK

Filed Dec. 1, 1930 3 Sheets-Sheet 1

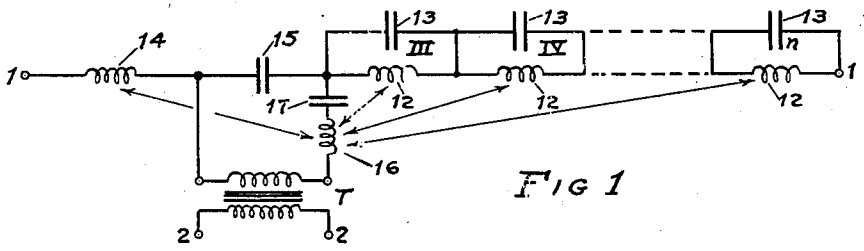


FIG 1

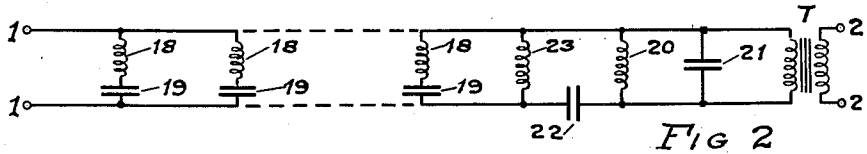


FIG 2

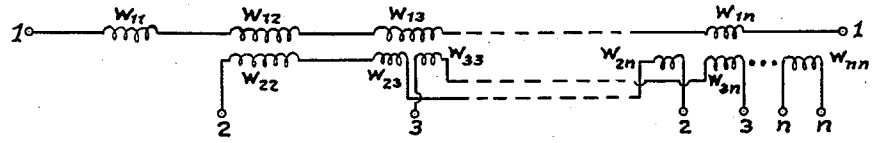


FIG 3

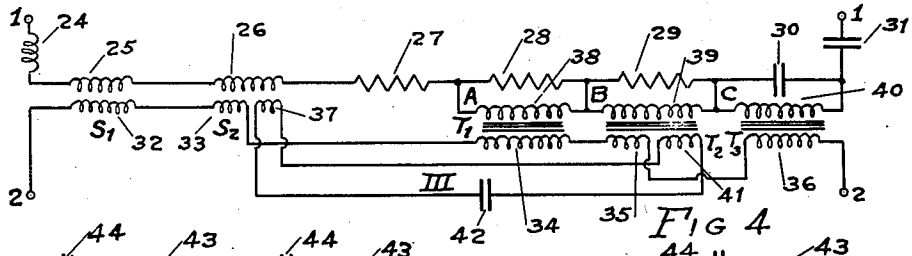


FIG 4

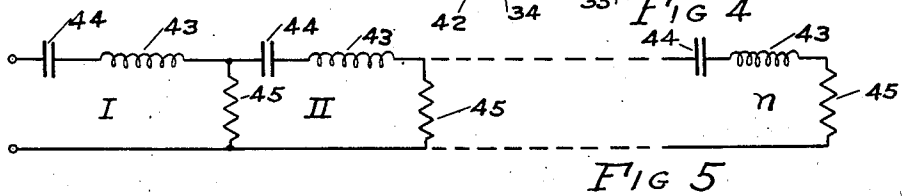


FIG 5

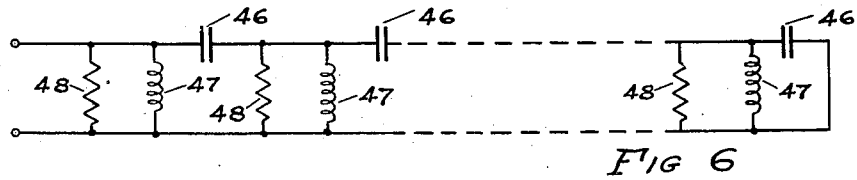


FIG 6

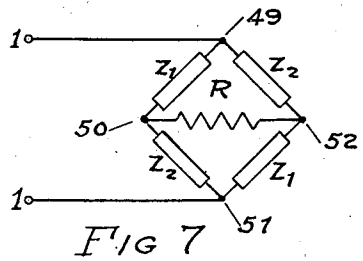


FIG 7

INVENTOR
WILHELM CAUER
BY David Rines
ATTORNEY

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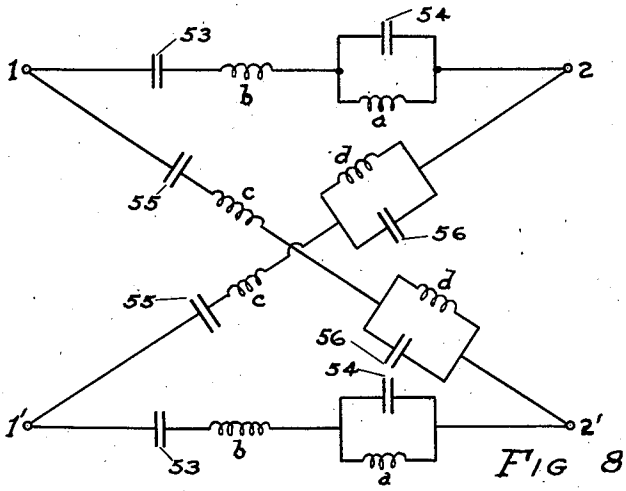


FIG 8

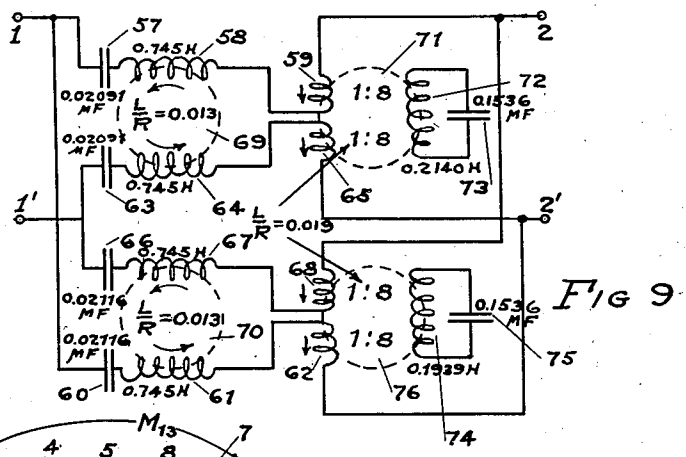


FIG 9

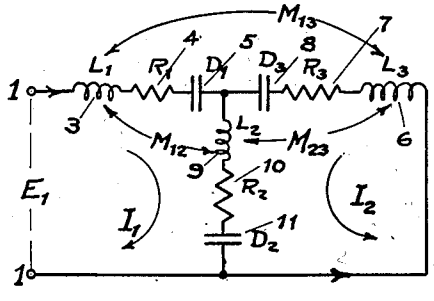


FIG 10

INVENTOR
WILHELM CAUER
BY David Rines
ATTORNEY

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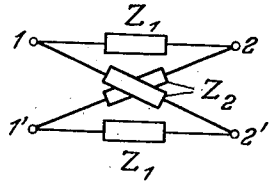


Fig. 11

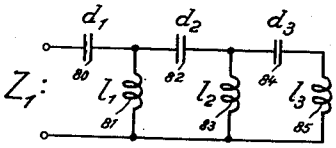


Fig. 12

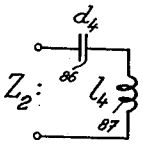


Fig. 13

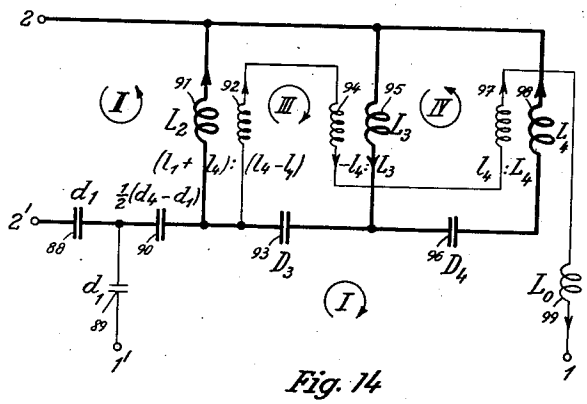


Fig. 14

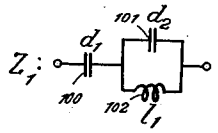


Fig. 15

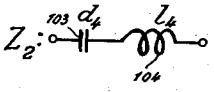


Fig. 16

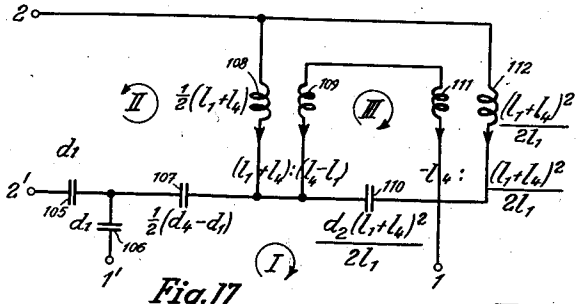


Fig. 17

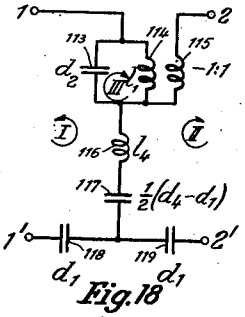


Fig. 18

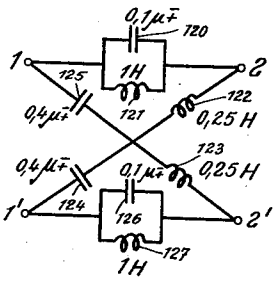


Fig. 19

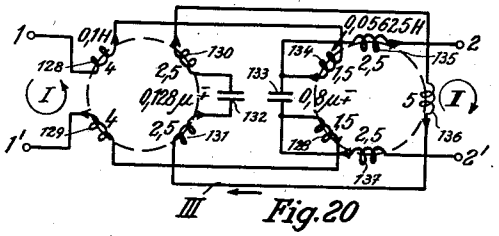


Fig. 20

INVENTOR
WILHELM CAUER
BY David Rines
ATTORNEY

Patented May 15, 1934 1,958,742

UNITED STATES PATENT OFFICE

1,958,742

ARTIFICIAL NETWORK

Wilhelm Cauer, Gottingen, Germany

Application December 1, 1930, Serial No. 499,233
In Germany June 8, 1928

21 Claims. (Cl. 178—44)

My invention relates to artificial networks which depend essentially on frequency, such networks, for example, as filters, distortion and other correction lines and circuits, simulating circuits and the like, and also to methods of designing the same. In particular, the invention, though not restricted thereto, relates to artificial networks having mutual inductances.

An artificial network, as the term is used in the art, is a network designed to attain, or approximate to, predetermined frequency characteristics. For example, a network which imitates the impedance of a given long line or a combination of lines is an artificial network because it is designed to approximate to the frequency characteristics of the long line or the combination of lines. An attenuation-correction circuit or a phase-correction circuit is an artificial network because it produces the desired attenuation or phase correction as a function of frequency.

In the case of a filter, let it be assumed, for example, that it is desired to have zero attenuation in the transmission ranges and infinite attenuation in the attenuating ranges. It is impossible, usually, to attain the ideal conditions exactly, but a filter that will attain or approximate to the desired frequency characteristics is an artificial network.

An object of the invention is to provide a new and improved method of designing circuits equivalent to other circuits.

A further object is to improve upon lines and circuits of the above-described character with the end in view of improving their efficiency and field of utility and reducing their cost of manufacture.

Another object is to provide a new and improved coupling of inductances in the said circuits and, in particular, tight coupling.

Other and further objects will be explained hereinafter and pointed out in the appended claims, it being understood that I intend to claim all the novelty that my invention may possess.

The invention will now be explained in connection with the accompanying drawings, in which Fig. 1 is a diagrammatic view of a canonical form of a general, four-terminal network without resistances; Fig. 2 is a similar view of a network that is reciprocal or inverse to the network of Fig. 1, the coils being mutually inductively related; Fig. 3 is a similar view of a network all the mutual inductances of which are tightly coupled; Fig. 4 is a diagram of a canonical form of a four-terminal network having inductance, capacity and resistance; Fig. 5 is a diagrammatic view of a canonical form of a two-terminal network, all the coils being mutually inductively related; Fig. 6 is a view of a network reciprocal or inverse to Fig. 5; Fig. 7 is a view of a two-terminal network having impedances with mutual inductances; Fig. 8 is a diagrammatic view of a wave-filter circuit; Fig. 9 is a corresponding view of a network which constitutes an improvement over the network of Fig. 8; Fig. 10 is a diagram for aiding the understanding of the invention; Fig. 11 is a diagrammatic view of a reactive four-terminal network; Figs. 12 and 13 are similar views of two-terminal portions that may be embodied in Fig. 11; Fig. 14 is a view of a network equivalent to Fig. 11 and having mutual inductance; Figs. 15 and 16 are views of two-terminal portions that may be embodied in Fig. 11 alternatively to the portions illustrated in Figs. 12 and 13, respectively; Figs. 17 and 18 are views of networks, with mutual inductance, each equivalent to the network of Fig. 11 when the latter embodies the two-terminal portions illustrated in Figs. 15 and 16, Fig. 17 being a special case of Fig. 14; and Figs. 19 and 20 are views of two equivalent phase-compensating networks, Fig. 20 affording also a numerical example illustrative of the invention.

Some of the terminology hereinafter employed will first be explained in connection with Fig. 10, showing a general, two-terminal network containing two meshes, the terminals being indicated at 1, 1. The invention, of course, is general, and is not restricted either to two-terminal networks or to networks having two meshes, but this simple case will serve for illustrative purposes.

The first mesh of the network of Fig. 10 is shown provided with series-arranged inductance 3, resistance 4, and capacitance 5. The inductance 3 has a value L_1, the resistance 4 has a value R_1 and the capacitance 5 has an inverse or reciprocal value D_1.

In the second mesh, similarly, there is inductance 6, resistance 7 and capacitance 8, denoted by L_3, R_3 and D_3, which symbols represent also the values of the inductance and the resistance and the inverse or reciprocal value of the capacitance in the said second mesh.

A branch between the two meshes is illustrated as having inductance 9, resistance 10 and capacitance 11, and having, respectively, the values L_2, R_2 and the reciprocal of D_2.

The value of the mutual inductance between the inductance L_1 and L_2 will be represented by M_{12}; that between the inductance L_2 and L_3 by

M_{23}; and that between the inductance L_1 aud L_3 by M_{13}.

The current flowing in the first mesh will be denoted by I_1 and that in the second mesh by I_2. The voltage across the two terminals 1, 1 will be referred to as E_1. The driving-point impedance z, then, will be

$$z=\frac{E_1}{I_1}$$

It will be useful to use also the following symbols, which relate to meshes as the L_1, L_2, . . . relate to branches:

$$\begin{aligned}
L_{11}&=L_1+L_2+2M_{12}\\
L_{22}&=L_2+L_3+2M_{23}\\
L_{12}&=L_2+M_{12}+M_{13}+M_{23}\\
R_{11}&=R_1+R_2\\
R_{22}&=R_2+R_3\\
R_{12}&=R_2\\
D_{11}&=D_1+D_2\\
D_{22}&=D_2+D_3\\
D_{12}&=D_2
\end{aligned}$$

Similar symbols will be used for more complicated artificial networks having a larger number of meshes. The complex currents flowing in the successive meshes

$$\text{I, II, III, . . . } n$$

will be denoted by

$$I_1, I_2, I_3, \ldots I_n,$$

respectively. The inductances and the mutual inductances between the meshes s and t will be denoted by L_{st}. Thus, the symbol for the inductances and the mutual inductances between the first and second meshes will be represented by L_{12}. The corresponding resistance will be denoted by R_{st} and the corresponding reciprocal or inverse capacity by D_{st}. The pulsatance will be represented, as usual, by ω, where

$$\omega=2\pi f$$

f, being the frequency of the electromotive force in the first mesh. The following customary abbreviation will also be used:

$$\lambda=j\omega$$

where

$$j=\sqrt{-1}.$$

A very brief outline of the theory of equivalence will now be given. For a fuller discussion, reference may be had to my paper, entitled "Eine Klasse von Funktionen, die die Sheltjesschen Kettenbruche als Sonderfall enthalt", in Jahresbericht der deutschen Mathematikervereinigung, 1929.

A general passive network of the character heretofore described, with n independent meshes, is determined, in the steady state, by the following equations:

$$\begin{aligned}
&A_{11}\,I_1+A_{12}\,I_2+A_{13}\,I_3+\ldots+A_{1n}\,I_n=E_1\\
&A_{21}\,I_1+A_{22}\,I_2+A_{23}\,I_3+\ldots+A_{2n}\,I_n=0.\\
&\ldots\ldots\ldots\\
&\ldots\ldots\ldots\\
&A_{n1}\,I_1+A_{n2}\,I_2+A_{n3}\,I_3+\ldots+A_{nn}\,I_n=0,
\end{aligned}$$

where

$$A_{st}=A_{ts}=\lambda L_{st}+R_{st}+\lambda^{-1}D_{st}$$

Let

$$a=\begin{vmatrix}A_{11} & A_{12} & A_{13} & \ldots & A_{1n}\\ A_{21} & A_{22} & A_{23} & \ldots & A_{2n}\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ A_{n1} & A_{n2} & A_{n3} & \ldots & A_{nn}\end{vmatrix}$$

and

$$a_{11}=\begin{vmatrix}A_{22} & A_{23} & \ldots & A_{2n}\\ A_{32} & A_{33} & \ldots & A_{3n}\\ \ldots & \ldots & \ldots & \ldots\\ \ldots & \ldots & \ldots & \ldots\\ A_{n2} & A_{n3} & \ldots & A_{nn}\end{vmatrix}$$

Then the driving-point impedance for the first mesh is

$$z=\frac{E_1}{I_1}=\frac{a}{a_{11}}.$$

Two networks are said to be equivalent when they have the same exterior efficiency as functions of frequency, irrespective of what might be the case interiorly.

A circuit is a canonical circuit for a certain kind of network if it can be made equivalent to all circuits of this kind by suitable choice of the size of its elements.

More especially, two-terminal networks are equivalent if they have the same driving-point-impedance functions of frequency.

For getting equivalent circuits, it is desired to transform the quadratic form

$$\sum_{s,\ t=1}^{n} A_{st}x_s x_t,$$

where x_s and x_t are any variables, by the affine, real transformation:

$$\begin{aligned}
x_1&=x_1'\\
x_2&=\alpha_{21}x_1'+\alpha_{22}x_2'+\alpha_{23}x_3'+\ldots+\alpha_{2n}x_n'\\
x_3&=\alpha_{31}x_1'+\alpha_{32}x_2'+\alpha_{33}x_3'+\ldots+\alpha_{3n}x_n'\\
&\ldots\ldots\\
&\ldots\ldots\\
x_n&=\alpha_{n1}x_1'+\alpha_{n2}x_2'+\alpha_{n3}x_3'+\ldots+\alpha_{nn}x_n'
\end{aligned}$$

where x_s' is likewise any variable, and where α_{st} is an arbitrary, real constant. Then

$$\sum_{s,\ t=1}^{n} A_{st}x_s x_t=\sum_{s,\ t=1}^{n} A'_{st}x'_s x'_t,$$

and

$$\sum_{s,\ t=2}^{n} A_{st}x_s x_t=\sum_{s,\ t=2}^{n} A'_{st}x'_s x'_t.$$

The new coefficients A'_{st}, where

$$A'_{st}=\lambda L'_{st}+R'_{st}+\lambda^{-1}D'_{st}$$

thus represent a new network, having new inductances L'_{st}, new resistances R'_{st} and new reciprocal capacities D'_{st}. The new coefficients may be calculated by known algebraic methods such, for example, as the multiplication of matrices.

$$a'=\begin{vmatrix}A'_{11} & A'_{12} & A'_{13} & \ldots & A'_{1n}\\ A'_{21} & A'_{22} & A'_{23} & \ldots & A'_{2n}\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ A'_{n1} & A'_{n2} & A'_{n3} & \ldots & A'_{nn}\end{vmatrix}=a\alpha^2,$$

where

$$\alpha=\begin{vmatrix}\alpha_{22} & \alpha_{23} & \ldots & \alpha_{2n}\\ \alpha_{32} & \alpha_{33} & \ldots & \alpha_{3n}\\ \ldots & \ldots & \ldots & \ldots\\ \alpha_{n2} & \alpha_{n3} & & \alpha_{nn}\end{vmatrix}\neq 0;$$

and similarly,

$$a'_{11}=\begin{vmatrix}A'_{22} & A'_{23} & \ldots & A'_{2n}\\ A'_{32} & A'_{33} & \ldots & A'_{3n}\\ \ldots & \ldots & \ldots & \ldots\\ A'_{n2} & A'_{n3} & & A'_{nn}\end{vmatrix}=a_{11}\alpha^2,$$

whence it follows that

$$\frac{a'}{a'_{11}}=\frac{a}{a_{11}}=z$$

The new network represented by the coefficients A'_{st}, therefore, has the same driving-point impedance z in the first mesh thereof. The new network is therefore one equivalent of the original network.

An important fact to be noticed is that, in the new network, as in the original network, the quadratic form

$$\sum_{s,\ t=1}^{n} L_{st}x_s x_t,$$

representing the energy of the coils;

$$\sum_{s,\ t=1}^{n} R_{st}x_s x_t,$$

representing Joule's heat; and

$$\sum_{s,\ t=1}^{n} D_{st}x_s x_t,$$

representing the energy of the capacities, satisfy the necessary conditions that they be positive definite.

It follows that the above group of linear transformations fundamentally determines the theory of equivalence of driving-point impedances. The theory includes, for example, as a special case, the discussion of the driving-point impedances of two-mesh circuits considered by Foster in the Bell System Technical Journal, 1924, and by me in Archiv fur Elektrotechnik, 1926. As will be clear by reference to my paper entitled, "Untersuchungen über ein Problem, das drei positiv definite quadratische Formen mit Streckenkomplexen in Beziehung setzt", in Mathematische Annalen, Band 105, Heft 1, 1931, indeed, the complete theory is somewhat more complicated. This is particularly because the above-given necessary conditions for the resistances and capacities are not sufficient, for there is no such thing possible physically as a mutual resistance and a mutual capacity, with the result that the transformed network is not always immediately capable of physical realization in the precise form determined by the linear transformation. This exception, however, is more apparent than real, because it is possible, as is also explained in my said paper, to produce an actual, physical network in all cases,—even the cases mentioned of mutual resistances and capacities,—by suitable further modification and interpretation. With this explanation, therefore, it will be understood that the theory is perfectly general. The very fact that there are no physical mutual resistances and capacities, on the other hand, emphasizes the importance of those cases involving the use of mutual inductances. There are other complications also, particularly in the case where all three kinds of impedances, namely, inductance, capacitance and resistance, are found in the network.

It will be obvious that to every network there correspond a large number of equivalent networks. Some of these have special properties that make them more advantageous for use in particular cases than others. For example, if some of the coefficients are zero, the corresponding network will have a fewer number of elements, such as inductances, capacitances, or resistances. In particular, it is possible to choose a network equivalent to some other network and that shall have the minimum number of elements. This result may be obtained by the same linear-transformation method, as before described, and is similar to the well-known method for reducing two quadratic forms, by means of a general linear transformation, to a sum of squares. As an illustration, let the following linear transformation be applied to a four-terminal network in which I_1 is the current at the input terminals 1, 1 and I_2 at the output terminals 2, 2. This network is to be perfectly general except that it is to have no resistances:

$$\begin{aligned}
x_1&=x_1'\\
x_2&=x_2'\\
x_3&=\alpha_{31}x_1'+\alpha_{32}x_2'+\alpha_{33}x_3'+\ \ldots\ +\alpha_{3n}x_n'\\
x_4&=\alpha_{41}x_1'+\alpha_{42}x_2'+\alpha_{43}x_3'+\ \ldots\ +\alpha_{4n}x_n'\\
&\ldots\ldots\\
&\ldots\ldots\\
x_n&=\alpha_{n1}x_1'+\alpha_{n2}x_2'+\alpha_{n3}x_3'+\ \ldots\ +\alpha_{nn}x_n'
\end{aligned}$$

Any given pair of matrices (L_{st}), (D_{st}) may be reduced, by the same transformation, to the forms

$$\begin{bmatrix}
L_{11} & L_{12} & L_{33} & L_{44} & \ldots & L_{nn}\\
L_{21} & L_{22} & L_{23} & L_{24} & \ldots & L_{2n}\\
L_{33} & L_{32} & L_{33} & 0 & \ldots & 0\\
L_{44} & L_{42} & 0 & L_{44} & \ldots & 0\\
\ldots & & & & & \\
\ldots & & & & & \\
L_{nn} & L_{n2} & 0 & 0 & \ldots & L_{nn}
\end{bmatrix}$$

and

$$\begin{bmatrix}
D_{11} & D_{12} & 0 & \ldots & 0\\
D_{21} & D_{22} & 0 & \ldots & 0\\
0 & 0 & D_3 & \ldots & 0\\
\ldots & & & & \\
\ldots & & & & \\
0 & 0 & 0 & \ldots & D_n
\end{bmatrix}$$

respectively. Then the canonical form of circuit illustrated in Fig. 1 will be obtained (if greater details are desired, they will be found in my said paper in Mathematische Annalen, particularly commencing with page 121). This circuit comprises a number of parallel-connected coils and condensers 12, 13 arranged in series with a coil 14 and a condenser 15, the last-named condenser 15 having a shunt-arranged coil 16 and condenser 17. An ideal transformer T is connected in this shunt circuit. By "ideal" is meant a tightly-coupled transformer having great inductance. The arrows connect coils that are mutually inductively related, and only the coils so indicated by the arrows are so related.

The numerical values of the transformed elements L_{st}, D_{st} of the new network obtained from the original network, may be calculated also by other methods, and even without knowledge of the reducing transformation. The following is one method that may be employed (and it will be understood that the same method is applicable also to the treatment of certain other canonical forms):

Let

$$a_{12}=a_{21}$$

represent the minor of the determinant a that is complementary to the element A_{12}; let a_{22} represent the corresponding minor complementary to the element A_{22}; and let a_{1122} represent the minor which is the algebraic complement of

$$\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}$$

The first step is to construct a two-terminal circuit for the primary open-circuit impedance

$$\frac{a_{22}}{a_{1122}};$$

that is, the impedance obtained across the ter-

minals 1, 1 in Fig. 1, for example, when the terminals 2, 2 are open. In the circuit of this Fig. 1, this may be effected by resolution into partial fractions. In order not to obtain physically unrealizable mutual capacities, I shall assume that the mesh 2 is not capacitatively connected with the meshes III, IV . . . n. As for the quantities.

$$D_{21}, L_{21}, L_{23}, L_{24} \ldots L_{2n}$$

they may be determined uniquely by means of a system of linear equations, derived by comparison of the coefficients of the various powers of λ^2 in $\lambda^{n-1}a_{12}$. Finally, L_{22} and D_{22} may be uniquely calculated from a_{11}. The network constants so obtained will always correspond to physically realizable circuits.

The only case that might at all be considered troublesome is when

$$D_{11} < |D_{12}|$$

or

$$D_{22} < |D_{12}|$$

In that event, however, a transformer,—such as the transformer T of Fig. 1,—may be connected either to the input or the output terminals of the system. The necessity for the use of a transformer does not arise in the case of symmetrical four-terminal networks. Instead of the transformer, an additional capacity may, in general, be required.

Several important results flow from a study of this general theory, which results are true for all networks, whether two-terminal, four-terminal, etc.

First, in many cases, the economy of the network system is greatly improved if the network contains at least three mutually inductively related coils, no two of which are directly galvanically connected together. For example, in Fig. 3, the coils w_{13}, w_{23}, w_{33}, as will be explained hereinafter, are no two of them galvanically connected together, and they are mutually inductively related.

Secondly, it is possible to reduce any network with any frequency characteristics to equivalent canonical forms.

Thirdly, among these canonical forms, for any network, it is always possible to find some that have all their mutual inductances tightly coupled.

Fourthly, corresponding to every two-terminal or four-terminal network (and also other networks with corresponding definitions) having mutual inductance, it is always possible to design a reciprocal or inverse network. The possibility of constructing reciprocal four-terminal networks may be useful also in such cases where two reciprocal networks are not found in a larger network. For a four-terminal network, the definition of a reciprocal or inverse network is as follows:

Let y_{1k} be the impedance across the terminals 1, 1 when the terminals 2, 2 are short circuited, and y_{10} be the impedance when the terminals 2, 2 are open circuited. Secondly, let y_{2k} and y_{20} be the short-circuit and the open-circuit impedances at the terminals 2, 2. Let z_{1k}, z_{10} and z_{2k}, z_{20} be the corresponding impedances for the second network. Then the second network is reciprocal or inverse to the first if

$$y_{1k}z_{10}=y_{10}z_{1k}=y_{2k}z_{20}=y_{20}z_{2k}=R^2$$

where R is a constant.

Similar definitions hold for networks having any other number of terminals. Thus, in the case of the reciprocal or the inverse of a two-terminal network, the product of the impedances which are known as reciprocal to each other is a positive constant, the square of a resistance. The general definition of the reciprocal of a $2n$-terminal network may be found in my paper in the Mathematische Annalen, Vol. 105 (1931) and in my paper "New Theory and Design of Wave-Filters", Physics 1932, especially Appendix II, at the top of page 265. As indicated on this page 265, the reciprocal of a $2n$-terminal network is a network whose characteristic matrix is the reciprocal of the characteristic matrix of the original network, except for a positive, constant factor.

It is, of course, a physical impossibility to enumerate and describe herein all the numerous equivalent two-terminal or four-terminal network circuits. Only a few examples can here be given.

Fig. 1, for example, as already stated, represents a canonical, four-terminal network having negligible resistances, and is equivalent to any other four-terminal network without resistances with n independent meshes. It may have many uses, such as for wave-filter purposes. As a filter, it is, in general, equivalent to any unsymmetrical wave filter having n independent meshes, the circuit elements being suitably chosen and including mutual inductances. The $3n$ elements of the circuit all have independent influence on the frequency characteristics. The transformer T should have very great inductance and negligible stray, as before described.

The four-terminal network of Fig. 2 is reciprocal or inverse to that of Fig. 1. The coils 18, 20 and 23 are connected by mutual inductances. The transformer T is of the same character as that of Fig. 1. The reciprocal or inverse circuit of Fig. 2 is obtained from that of Fig. 1 in the the following manner. The canonical circuit of Fig. 1 is first replaced by an imaginary equivalent network in which no coils are connected by mutual inductances, but the capacities are connected by mutual capacities; for instance, in the canonical circuit of Fig. 1, by interchanging the role of the capacities and the inductances. That is, in the imaginary network, the coils 12, 14, 16, which are mutually related, but no two of which are directly connected together galvanically, are replaced by interlinked capacities, and the capacities 13, 15, 17 are replaced by non-interlinked inductances. The imaginary circuit thus produced is then replaced, in a known manner, by another arrangement, branch for branch,—branches in parallel, like each branch 12 and the corresponding branch 13 of Fig. 1, being replaced by branches in series, like each branch 19 and the corresponding branch 18 of Fig. 2, and vice versa. The inductances thus become replaced by proportional capacities. The quadratic form

$$\sum_{s,t=1}^{n} D_{st}x_s x_t$$

which represents the interlinked capacities becomes replaced by the reciprocal or inverse quadratic form which is also positive definite, and the coefficients thereof represent therefore inductances and mutual inductances that are always physically realizable. In the reciprocal or inverse circuit of Fig. 2, the inductance 23 corresponds to the imaginary capacity corresponding

to the inductance 14 of Fig. 1. The capacity 21 of Fig. 2, similarly, corresponds to the capacity 17 of Fig. 1; the capacity 22 of Fig. 2 to the capacity 15 of Fig. 1; and the inductance 20 of Fig. 2 to the inductance 16 of Fig. 1.

In the circuit arrangement of Fig. 3, there are n series of coils each series being connected between two terminals 1, 1; 2, 2; 3, 3 . . . n, n, respectively. The first series of coils is indicated at

$$w_{11}, w_{12}, w_{13} \ldots w_{1n};$$

the second series at

$$w_{22}, w_{23} \ldots w_{2n};$$

the third series at

$$w_{33}, w_{34} \ldots w_{3n},$$

etc.

The last series consists of a single coil w_{nn}. The coils of the second series are tightly connected with each coil of the first series except one; the coils of the third series are tightly connected with each coil of the second series except one, and with all those of the first series except two; and so on. The last series, which consists of only one coil w_{nn}, is tightly connected with one coil of each of the other series.

The network is, in general, equivalent to n coils arbitrarily interlinked, with inductances L_{st}, and is therefore a canonical form for n arbitrarily interlinked coils. The couplings are all ideal and tight, with w_{st} turns. The inductances of the primary coils are

$$\begin{aligned} L_1 &= Lw^2_{11} \\ L_2 &= Lw^2_{12} \\ &\cdots\cdots \\ &\cdots\cdots \\ L_n &= Lw^2_{1n} \end{aligned}$$

For $n=3$, the following relations hold good between L, w_{st} and L_{st}:

$$w^2_{11} = \frac{\begin{vmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{vmatrix}}{L \cdot \begin{vmatrix} L_{22} & L_{23} \\ L_{32} & L_{33} \end{vmatrix}}$$

$$w^2_{12} = \frac{\begin{vmatrix} L_{12} & L_{13} \\ L_{32} & L_{33} \end{vmatrix}^2}{L \cdot \begin{vmatrix} L_{22} & L_{23} \\ L_{32} & L_{33} \end{vmatrix} \cdot L_{33}},$$

$$w^2_{13} = L\frac{L^2_{13}}{L_{33}},$$

$$\frac{w_{12}}{w_{22}} = \frac{\begin{vmatrix} L_{12} & L_{13} \\ L_{32} & L_{33} \end{vmatrix}}{\begin{vmatrix} L_{22} & L_{23} \\ L_{32} & L_{33} \end{vmatrix}},$$

and

$$w_{13} : w_{23} : w_{33} = L_{13} : L_{23} : L_{33}.$$

A more complicated case, including resistances as well as inductances and capacitances, is illustrated by the canonical, four-terminal network of Fig. 4 and is equivalent to every four-terminal network constituted of three independent meshes, The diagram is restricted to only three independent meshes, for the sake of simplicity only, in order not to complicate the drawing, the same principle being applicable irrespective of the number of meshes. Between the terminals 1, 1, there are connected coils 24, 25, 26, resistances 27, 28, 29 and condensers 30, 31. Between the terminals 2, 2, there are connected coils 32, 33, 34, 35, 36. The coils 25, 32 form a tightly coupled transformer S_1. The coils 26, 33 and a coil 37 form a tightly coupled transformer S_2. Coils 38, 39 and 40 are connected in shunt across the resistances 28 and 29 and the condenser 30, respectively, in the meshes A, B and C. If the meshes A, B and C are omitted, the circuit will have exactly three independent meshes. The coils 34 and 38 constitute a tightly coupled transformer T_1. The coils 35 and 39 and a coil 41 constitute a tightly coupled transformer T_2. The coils 36 and 40 constitute a tightly coupled transformer T_3. The transformers T_1, T_2 and T_3 are ideal transformers.

The coils 37 and 41 are connected in a circuit III in series with a condenser 42. This forms a canonical circuit with tight coupling. It is an advantage to have a circuit with tight coupling because it is more easily realizable in practice with given values of the elements. The necessity of taking the transformers T_1, T_2 and T_3 arises from the fact that no mutual capacities or resistances exist. The importance lies, not so much in having the transformers T_1, T_2 and T_3 ideal, as in having mutual inductances. This is true in general, and is not restricted to the particular circuit shown.

In the most general case, three independent meshes will involve 18 elements. If we count several coils on a single core, as one coil, and if we use the circuit arrangement of Fig. 4, then the 18 elements will reduce to 12. These 12 are as follows:

(1) Coil 24;
(2) The transformer S_1;
(3) The transformer S_2;
(4), (5), (6) The resistances 27, 28, 29;
(7), (8), (9) The transformers T_1, T_2, T_3;
(10) The condenser 42;
(11) The condenser 30; and
(12) The condenser 31.

It will be noted that, in the transformer S_2, as in the case of the coils w_{13}, w_{23} and w_{33} of Fig. 3, there are three coils, not immediately galvanically connected together, but all tightly coupled together. The same is true of transformer T_2.

In Fig. 5, there is shown a canonical two-terminal network, the coils 43 having inductance, and the network having also mutual inductance, capacities and resistances. By our definition, this circuit is equivalent to all other two-terminal circuits with n independent meshes containing inductances, including mutual inductance, capacities and resistances. Each mesh has a coil 43 and a condenser 44 in series and a resistance 45 in shunt. The coils 43 are all interlinked by mutual inductances.

The network of Fig. 6 is reciprocal or inverse to that of Fig. 5, the analogous relations being similar, but not exact, to that existing between Figs. 1 and 2, and the mutual inductances of the reciprocal or inverse circuit being calculated in the same manner as before described.

To obtain the imaginary circuit equivalent to the circuit of Fig. 5, I replace the capacities 44 by resistances, the inductances 43 by imaginary capacities which are interlinked, and the resistances 45 by non-interlinked inductances.

The said imaginary circuit may be obtained by using the principle of linear transformation for the given circuit. The coils 47 are mutually interlinked, the L_{st}'s being determined by the quadratic form which is reciprocal or inverse to the quadratic form of the D_{st}'s of the imaginary circuit equivalent to the circuit of Fig. 5. This imaginary circuit will correspond to the recip-

rocal or inverse circuit. Fig. 6, in the same manner as described above in connection with Figs. 1 and 2. This circuit of Fig. 6 contains condenser 46 in series and inductances 47 and resistances 48 in parallel.

In Fig. 7, one of the networks Z_1 has two terminals 49 and 50 and the other has two terminals 51 and 52. The networks Z_2 have each the two terminals 49 and 52, and 50 and 51, respectively. The two-terminal networks Z_1 and Z_2 contain mutual inductance, satisfying the relation

$$Z_1 Z_2 = R^2$$

where R is a constant, equal to the value of the resistance across the terminals 50 and 52. The practical value of this arrangement is that, given any impedance Z_1, it is possible to design a second impedance Z_2 so as to have a constant resistance across the terminals 50, 52.

Fig. 8 illustrates a lattice network having four branches. The branches 1, 2 and the branches 1', 2' are equal to each other, as also are the branches 1, 2' and 1', 2. The branches 1, 2 and 1' 2' each contains a condenser 53, an inductance *b* and an inductance *a* in parallel with a condenser 54. The branches 1, 2' and 1', 2 similarly each have a condenser 55, an inductance *c* and an inductance *d* in parallel with a condenser 56. The coils *a*, *a* are mounted on a single core. The same is true of the coils *b*, *b*, as well as of the coils *c*, *c* and the coils *d*, *d*. This lattice-type network is useful as a wave-filter circuit if the elements *a*, *b*, *c*, *d*, 53, 54, 55 and 56 are suitably chosen.

Fig. 9 illustrates a balanced four-terminal network circuit comprising a multiple-inductance wave filter embodying the present invention. A balanced network, as is well understood in the art, involves symmetry along the horizontal line. The network of Fig. 9 can be made equivalent to the filter of Fig. 8 by suitable choice of dimensions of the elements and which contains a smaller number of capacities and also a more suitable choice of dimensions of the capacities than is true of the circuit of Fig. 8. This circuit comprises four branches, as in Fig. 8. The first branch contains a condenser 57, and two coils 58, 59 in series between the terminals 1, 2. The second branch, between the terminals 1, 2', contains a condenser 60 and two coils 61 and 62 in series. The third branch contains a condenser 63 and two coils 64, 65 between the terminals 1', 2'. The fourth branch contains a condenser 66 and two coils 67 and 68 between the terminals 1', 2.

The coils 58 and 64 are tightly coupled and mounted upon a single core 69. The same is true of the coils 61 and 67, upon a single core 70. The coils 59 and 65 are similarly mounted upon a single core 71 with a third coil 72, which latter is shunted by a condenser 73. It will be noted that the coils 59, 65 and 72 are none of them directly galvanically connected, but are mutually tightly coupled. A similar construction obtains for the coils 62 and 68 and a third coil 74, which latter is shunted by a condenser 75. The coils 62, 68 and 74 are tightly coupled and are all mounted upon a single core 76 similarly to the coils 59, 65 and 72.

The six condensers 57, 60, 63, 66, 73 and 75 are smaller in number than the eight condensers 53, 54, 55 and 56 of Fig. 8. A further advantage of the circuit of Fig. 8 is to avoid too great capacities such as would be necessary, in practice, with the circuit of Fig. 8. The dimensions of the elements indicated in Fig. 9 were calculated beforehand for a given attenuation requirement and were actually checked experimentally.

The networks disclosed, for example, in Figs. 9 and 20, being symmetrical about a median, horizontal line, are balanced with respect to the ground potential. Thus, in Fig. 20, the middle point of the right-hand coil 5 may be connected with the ground. To avoid confusion, it may be stated that the expression "symmetrical four-terminal network", as used in this specification, is not synonymous with "balanced network", as it assumes that the efficiency of the four-terminal network is unaffected by an interchange of the input and output terminals. A network may, of course, be both balanced and symmetrical, as is the case with the networks illustrated in Figs. 9 and 20.

It would be a practical impossibility to illustrate all the many uses of my invention. Filters have already been mentioned, and a suggestion has also been made in connection with Fig. 7. The invention finds use also in simulating or correction or compensating circuits, as well as in other relations. According to present practice, particularly in the more complicated cases, the known methods of correction are not always successful. It is possible to attain success by using mutual inductances in accordance with my invention.

It will be unnecessary to recapitulate all the advantages of this invention. Mention has been made, for example, of the desirability of having several coils on a single core. It is thus possible to have the same number of arbitrary constants in the circuit, though using fewer elements and occupying less space. The counting of several coils on a single core as one element has already been referred to.

Again, if a given network is to be reduced to an equivalent network more suitable for practical design,—as, for example, having less elements,—it is often not the best way to apply the principle of linear transformation directly to the circuit. In all cases where the given circuit has not the least number of independent meshes, we replace, first, the network by any equivalent network which has the least number of elements. For instance, if we have a bridge-type, four terminal network,—as the circuit, Fig. 7, input terminals 1, 1 and output terminals 50, 52, with the resistance R removed,—and arbitrary z_1 and z_2 without resistances, then a first equivalent network without superfluous meshes may be obtained in the following way: We use the relations

$$\frac{z_1 + z_2}{2} = \frac{a_{22}}{a_{1122}}$$

and

$$\frac{z_1 - z_2}{2} = \frac{a_{12}}{a_{1122}}.$$

As the network is symmetrical,

$$a_{11} = a_{22}.$$

The procedure from this point on for obtaining a first network without superfluous meshes is the same as before described.

As a further example, if we have several bridge-type, four-terminal networks of the form shown in Fig. 7, (with the terminals 1, 1, 50, 52 and the resistance R being removed), in series the impedances Z_1, Z_2 having the same product, though not necessarily constant, we can reduce this circuit to a single, bridge-type circuit according to the procedure on page 279 of my paper entitled

"Vierpole" in Elektrische Nachrichtentechnik, 1929.

The invention will now be further explained in connection with Figs. 11 to 20.

Let it be assumed that it is desired to obtain a more economical, balanced network equivalent to the symmetrical, four-terminal, reactive network illustrated in Fig. 11, having four branches, as in Fig. 8, with the respective impedance Z_1 and Z_2, as indicated in the said Fig. 11. The impedances Z_1 and Z_2 are each a two-terminal portion, as represented in Figs. 12 and 13, respectively. The two-terminal portion Z_1, has series capacitances 80, 82 and 84 and shunt inductances 81, 83 and 85. The two-terminal portion Z_2 has a series capacitance 86 and a shunt inductance 87. Let the reciprocal values of the capacitances 80, 82, 84 and 86 be represented by d_1, d_2, d_3 and d_4, respectively. Let the values of the inductances be represented by l_1, l_2, l_3 and l_4, respectively.

Following out the procedure outlined above, the first step is to construct a two-terminal circuit for the secondary open-circuit impedance, that is, the impedance obtained across the terminals 2, 2' of Fig. 11, the terminals 1, 1' being disregarded, as though they were open, with the modification, however, that the secondary open-circuit impedance will be taken now instead of the primary open circuit impedance:

$$\frac{a_{11}}{a_{1122}}=\frac{Z_1+Z_2}{2}.$$

Instead of resolving

$$\frac{a_{11}}{a_{1122}}$$

in partial fractions, which would result in a special case of the circuit illustrated in Fig. 1, it is preferred here to develop in a Stieltjes continued fraction as explained for example, in my paper entitled, "Verwirklichung von Wechselstromwiderstanden vorgeschriebener Frequenzabhängigkeit", in Archiv für Electrotechnik, Heft 4, Band XVII, 1920. The two-terminal network shown by heavy lines in Fig. 14, and having the two terminals 2, 2', is thus obtained. The remaining circuit elements of the final network are next calculated, as also described above. The first four-terminal network thus obtained, as an equivalent to the four-terminal network illustrated in Fig. 11, will be that illustrated in Fig. 14.

To obtain the said heavy-line, two-terminal network of Fig. 14, bearing in mind that Z_1 and Z_2 are as illustrated in Figs. 12 and 13, the following values are first written down:

$$L_2=\frac{1}{2}(I_1+I_4)$$

$$L_3=\frac{(I_1+I_4)(I_1I_2+I_2I_4+I_4I_1)}{2I^2_1}$$

$$L_4=\frac{(I_1I_2+I_2I_4+I_4I_1)[I_1I_4(I_2+I_3)+I_2I_3(I_1+I_4)]}{2I^2_1I^2_2}$$

$$D_3=\frac{(I_1+I_4)^2d_2}{2I^2_1}$$

$$D_4=\frac{(I_1I_2+I_2I_4+I_4I_1)^2d_3}{2I^2_1I^2_2}$$

To simplify the notation, the following are introduced also:

$$D_2=\frac{1}{2}(d_1+d_4)$$

$$L_0=\frac{2I_1I_2I_3I_4}{I_1I_4(I_2+I_3)+I_2I_3(I_1+I_4)}.$$

If the meshes II, III and IV are oriented as in Fig. 14, and assuming arbitrarily, but premissibly, that mesh II is not capacitatively connected with the meshes III and IV, so as to have

$$D_{13}=D_{31}=D_{14}=D_{41}=0,$$

the matrices (L_{st}) and (D_{st}) of the final network, with four meshes, will have the following values:

$$(L_{st})=\begin{vmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_2 & L_2 & 0 \\ L_{31} & L_2 & L_2+L_3 & L_3 \\ L_{41} & 0 & L_3 & L_3+L_4 \end{vmatrix}$$

$$(D_{st})=\begin{vmatrix} D_{11} & D_{12} & 0 & 0 \\ D_{21} & D_2 & 0 & 0 \\ 0 & 0 & D_3 & 0 \\ 0 & 0 & 0 & D_4 \end{vmatrix}$$

To determine D_{12}, L_{12}, L_{13}, L_{14}, the coefficients of the various powers of $\lambda^2=x$, in λ^3a_{12} may be compared.

$$\lambda^2a_{1122}=\begin{vmatrix} (L_2+L_3)x+D_3 & L_3x \\ L_3x & (L_3+L_4)x+D_4 \end{vmatrix}=$$

$$K\{(I_2I_3+I_3I_1+I_1I_2)x^2+[(I_1+I_2)d_3+(I_2+I_3)d_2]x+d_2d_3\}$$

where

$$K=\frac{(I_1+I_4)^2(I_1I_2+I_2I_4+I_4I_1)^2}{4I^4_1I^2_2}$$

Since, from the network of Fig. 11,

$$\frac{a_{12}}{a_{1122}}=\frac{Z_1-Z_2}{2}$$

it follows that

$$\lambda^3a_{12}=\frac{K}{2}(e_0x^3+e_1x^2+e_2x+e_3)$$

where

$$\begin{aligned} e_0&=I_1I_2I_3-(I_2I_3+I_3I_1+I_1I_2)I_4 \\ e_1&=I_1I_2(d_1+d_2+d_3)+I_1I_3(d_1+d_2)+I_2I_3d_1 \\ &\quad -\{[(I_1+I_2)d_3+(I_2+I_3)d_2]I_4+(I_2I_3+I_3I_1+I_1I_2)d_4\} \\ e_2&=I_1(d_1+d_2)d_3+I_2(d_2+d_3)d_1+I_3d_1d_2-\{I_4d_2d_3+ \\ &\quad [(I_1+I_2)d_3+(I_2+I_3)d_2]d_4\} \\ e_3&=d_1d_2d_3-d_2d_3d_4 \end{aligned}$$

On the other hand, according to the definition of a_{12} given in the general treatment above,

$$\lambda^3a_{12}=-\begin{vmatrix} L_{12}x+D_{12} & L_{13}x & L_{14}x \\ L_2x & (L_2+L_3)x+D_3 & L_3x \\ 0 & L_3x & (L_3+L_4)x+D_4 \end{vmatrix}$$

$$\begin{aligned} &=[-L_{12}(L_2L_3+L_3L_4+L_4L_2)+L_{13}L_2(L_3+L_4)- \\ &\quad L_{14}L_2L_3]x^3 \\ &-\{D_{12}(L_2L_3+L_3L_4+L_4L_2)+L_{12}[(L_2+L_3)D_4+ \\ &\quad (L_3+L_4)D_3]-L_{13}L_2D_4\}x^2 \\ &-\{D_{12}[(L_2+L_3)D_4+(L_3+L_4)D_3]+L_{12}D_3D_4\}x \\ &-D_{12}D_3D_4. \end{aligned}$$

A comparison of the coefficients of x^0 yields:

$$-D_{12}D_3D_4=\frac{K}{2}e_3=\frac{K}{2}d_2d_3(d_1-d_4)$$

from which it follows that

$$D_{12}=D_{21}=\frac{d_4-d_1}{2}$$

A similar comparison of the coefficients of x^1, x^2, x^3 in λ^3a_{12} gives similarly

$$L_{12}=L_{27}=\frac{I_4-I_1}{2}$$

$$L_{13}=L_{31}=-\frac{(I_1+I_4)}{2}=-L_2$$

$$L_{14}=L_{41}=0.$$

To calculate the remaining unknowns, L_{11} and

D_{11}, a similar comparison is made of the coefficients of the different powers of x in $\lambda^3 a_{22}$, taking

$$\frac{a_{22}}{a_{1122}}$$

from the original network. In the particular case of Fig. 11, the original network happens to be symmetrical, so that

$$a_{11}=a_{22}$$

from which it follows that

$$\begin{vmatrix} L_{11}x+D_{11} & L_{13}x & L_{14}x \\ L_{31}x & (L_2+L_3)x+D_3 & L_3x \\ L_{41}x & L_3x & (L_3+L_4)x+D_4 \end{vmatrix} = \begin{vmatrix} L_2x+D_2 & L_2x & 0 \\ L_2x & (L_2+L_3)x+D_3 & L_3x \\ 0 & L_3x & (L_3+L_4)x+D_4 \end{vmatrix}$$

A comparison of the coefficient of x^0 on both sides yields

$$D_{11}=D_2,$$

and a similar comparison of the coefficients of x^1 yields, finally,

$$L_{11}=L_2.$$

The resulting matrices (L_{st}) and (D_{st}) of the electrict constants of the different meshes are now

$$(L_{st})=\begin{vmatrix} L_2 & \frac{1}{2}(l_4-l_1) & -L_2 & 0 \\ \frac{1}{2}(l_4-l_1) & L_2 & L_2 & 0 \\ -L_2 & L_2 & L_2+L_3 & L_3 \\ 0 & 0 & L_3 & L_3+L_4 \end{vmatrix}$$

$$(D_{st})=\begin{vmatrix} D_2 & \frac{1}{2}(d_4-d_1) & 0 & 0 \\ \frac{1}{2}(d_4-d_1) & D_2 & 0 & 0 \\ 0 & 0 & D_3 & 0 \\ 0 & 0 & 0 & D_4 \end{vmatrix}$$

Inspection of Fig. 14 demonstrates that the network illustrated therein fulfills the conditions required by these electric constants, with tight couplings. Thus, the mutual inductance between the meshes I and III is

$$\frac{l_4-l_1}{l_1+l_4}L_2-\frac{l_4}{L_3}L_3=-L_2$$

and the self-inductance of mesh I is

$$\frac{(l_4-l_1)^2}{(l_1+l_4)^2}L_2+\frac{l_4^2}{L_3^2}L_3+\frac{l_4^2}{L_4^2}L_4+L_0=L_2$$

as it should be. This equivalent network is more economical than the network illustrated in Fig. 11. It contains mutual inductances in tightly coupled form, as described above. The coils 91 and 92 are tightly coupled, the turn-ratio being

$$(l_1+l_4) : (l_4-l_1).$$

The coils 94 and 95 are tightly coupled, with a turn-ratio

$$-l_4 : L_3.$$

The turn-ratio of the tightly coupled coils 97 and 98 is

$$l_4 : L_4.$$

The turn-ratio may be regarded as positive when the inductances of a pair of coupled coils are additive when connected in series in the same direction.

In the special case when

$$l_2=0$$

the connections of Fig. 12 reduce to those of Fig. 15. The condensers 80 and 82 of Fig. 12 become the condensers 100 and 101 of Fig. 15, with the same reciprocal values d_1, d_2 respectively. The inductance 81 of Fig. 12 becomes the inductance 102 of Fig. 15, with the same value l_1.

In this special case, however, no change occurs in the connections of Fig. 13 which may, however, be represented as in Fig. 16, the condensers 86 being there illustrated at 103 and the inductance 87 at 104.

The network of Fig. 14 thus becomes reduced to the special case illustrated in Fig. 17.

As a check upon the work, and to give an illustration of the method of matrix multiplication referred to in the general treatment above, the equivalence of the networks illustrated in Figs. 11 and 17, for the special case where

$$l_2=0,$$

will be demonstrated independently.

The network of Fig. 11, where Z_1 and Z_2 are now as illustrated in Figs. 15 and 16, is first replaced by its equivalent network, illustrated in Fig. 18, which is a modification of a special case of the network of Fig. 1, where $n=3$.

Let the oriented mesh I in Fig. 18 be so chosen that it shall contain the capacity 113.

The electric constants (L_{st}) and (D_{st}) of the network Fig. 18 are given by the following matrices:

$$(L_{st})=\begin{vmatrix} \frac{I_4}{2} & \frac{I_4}{2} & 0 \\ \frac{I_4}{2} & \frac{I_1+I_4}{2} & \frac{I_1}{2} \\ 0 & \frac{I_1}{2} & \frac{I_1}{2} \end{vmatrix},$$

$$(D_{st})=\begin{vmatrix} \frac{d_1+d_2+d_4}{2} & \frac{d_4-d_1}{2} & \frac{d_2}{2} \\ \frac{d_4-d_1}{2} & \frac{d_1+d_4}{2} & 0 \\ \frac{d_2}{2} & 0 & \frac{d_2}{2} \end{vmatrix}$$

Equivalent circuits, with new constants L'_{st} and D'_{st}, may be obtained by means of the relations

$$\sum_{s,\,t=1}^{3} L_{st}x_s x_t = \sum_{s,\,t=1}^{3} L'_{st}x'_s x'_t$$

$$\sum_{s,\,t=1}^{3} D_{st}x_s x_t = \sum_{s,\,t=1}^{3} D'_{st}x'_s x'_t$$

by the transformation which is obtained from the general transformation discussed in an earlier part of this specification, letting

$$n=3,$$

and writing

$$\begin{aligned} \alpha_{31}&=\alpha \\ \alpha_{32}&=\beta \\ \alpha_{33}&=\gamma \\ x_1&=x_1' \\ x_2&=x_2' \\ x_3&=\alpha x_1'+\beta x_2',+\gamma x_3'. \end{aligned}$$

It results that the (L'_{st}) and (D'_{st}) may be calculated by the following matrix multiplication:

$$(L'_{st})=\begin{vmatrix}1&0&\alpha\\0&1&\beta\\0&0&\gamma\end{vmatrix}\begin{vmatrix}L_{11}&L_{12}&L_{13}\\L_{21}&L_{22}&L_{23}\\L_{31}&L_{32}&L_{33}\end{vmatrix}\begin{vmatrix}1&0&0\\0&1&0\\\alpha&\beta&\gamma\end{vmatrix}$$

$$=\begin{vmatrix}L_{11}+2\alpha L_{13}+\alpha^2L_{33} & L_{12}+\alpha L_{23}+\beta L_{13}+\alpha\beta L_{33} & \gamma L_{13}+\alpha\gamma L_{33}\\ L_{12}+\beta L_{13}+\alpha L_{23}+\alpha\beta L_{33} & L_{22}+2\beta L_{23}+\beta^2L_{33} & \gamma L_{23}+\beta\gamma L_{33}\\ \gamma L_{13}+\alpha\gamma L_{33} & \gamma L_{23}+\beta\gamma L_{33} & \gamma^2L_{33}\end{vmatrix}$$

An analogous matrix may be obtained for (D'_{st}). α, β, γ may be determined by the assumptions

$$L'_{22}=L'_{23},$$
$$D'_{13}=0,$$
$$D'_{23}=0.$$

which conditions are satisfied in the network Fig. 17. It follows that

$$\alpha=-1,\ \beta=0,\ \gamma=\frac{l_1+l_4}{l_1}.$$

Putting in these values and the above values of Fig. 18 for the (L_{st}) and (D_{st}), it results that:

$$(L'_{st})=\begin{vmatrix}\frac{l_1+l_4}{2} & \frac{l_4-l_1}{2} & -\frac{(l_1+l_4)}{2}\\ \frac{l_4-l_1}{2} & \frac{l_1+l_4}{2} & \frac{l_1+l_4}{2}\\ -\frac{(l_1+l_4)}{2} & \frac{l_1+l_4}{2} & \frac{(l_1+l_4)^2}{2l_1}\end{vmatrix},$$

$$(D'_{st})=\begin{vmatrix}\frac{d_1+d_4}{2} & \frac{d_4-d_1}{2} & 0\\ \frac{d_4-d_1}{2} & \frac{d_1+d_4}{2} & 0\\ 0 & 0 & \frac{(l_1+l_4)^2d_2}{2l_1^2}\end{vmatrix}$$

These circuit elements are found in the network of Fig. 17, which is thus equivalent to that of Fig. 18 which, in turn, is equivalent to that of Fig. 11. The second calculation has independently thus yielded the same results as the first calculation.

The network of Fig. 19 is a special case of the network of Fig. 11, for the following special values:

$$d_1=l_2=0$$
$$d_2^{-1}=0.1\mu F$$
$$d_4^{-1}=0.4\mu F$$
$$l_1=1H$$
$$l_4=0.25H.$$

This network represents a known form of phase-compensating network.

The network of Fig. 20 is an equivalent network of the same type as that of Fig. 19, with mutual inductances all in the form of tight couplings. For example, the coils 128, 130 and 131 are mutually inductively related in this manner.

The numerical values annexed to the various parts in Fig. 20 make it easy to read from the diagram that the circuit elements are determined by the matrices:

$$(L'_{st})=\begin{vmatrix}0.625&-0.375&-0.625\\-0.375&0.625&0.625\\-0.625&0.625&0.78125\end{vmatrix},$$

$$(D'_{st})=\begin{vmatrix}1.25\cdot10^6&1.25\cdot10^6&0\\1.25\cdot10^6&1.25\cdot10^6&0\\0&0&7.8125\cdot10^6\end{vmatrix},$$

which are the same as the former matrices (L'_{st}), (D'_{st}) for the above numerical values.

It is desirable to have some convention for determining the sign of a mutual inductance between two oriented meshes, each containing an oriented coil.

The mutual inductance L_{12} between meshes I and II may be assumed positive if the orientation of the coils has the same sense as the orientation of the mesh containing it, and if the turn-ratio is positive. Each change in there orientations, or in the sign of the turn-ratio, involves the introduction of one minus sign for L_{12}. For instance, if both meshes have the opposite orientation to that of the corresponding coils contained therein, and if the turn-ratio is negative, three negative signs are involved, so that, therefore, L_{12} is negative in this case.

The numbers 4, 2.5 etc. on the coils 128, 130, etc. are proportional to the turn number.

Therefore, the turn-ratio between the coils 128 and 130, for instance, is 4: 2.5. It follows that the inductivity of coil 130 is

$$0.1\times\left(\frac{2.5}{4}\right)^2=0.0390625 \text{ henry.}$$

The self-inductivity of mesh III is

$$L'_{33}=4\times0.0390625+\left(\frac{5}{1.5}\right)^2\times0.05625=$$
$$0.78125 \text{ henry.}$$

As an example of mutual inductance, let L'_{12} be calculated. According to the above explanations,

$$L'_{12}=-4\times0.05625\times\frac{2.5}{1.5}=-0.375.$$

Though the networks of Figs. 19 and 20 are equivalent, and thus attain, or approximate to, the same, predetermined, frequency characteristics, no two of the three mutually inductively related coils 128, 134 and 136 of the network of Fig. 20 are directly connected together galvanically. No such arrangement is disclosed in Fig. 19. The employment of this mutual inductance, as illustrated, for example, in Fig. 20, thus enables one to attain equally good, or better, results, but with greater economy, involving the employment of a fewer number of circuit elements. This is not true of the network of Fig. 19. One glance suffices to show how much more economical is the network of Fig. 20.

The network of Fig. 20 is not quite a special case of that of Fig. 17, as it is somewhat changed, so as to be balanced.

It will be understood that the invention is not restricted to the exact embodiments herein illustrated and described, but that modifications may be made by persons skilled in the art, and all such are considered to fall within the scope of this invention.

What I claim is:

1. An artificial network attaining or approximating predetermined frequency characteristics and having one or more passive network sections, one of the sections having three or more mutually inductively related coils no two of which are directly galvanically connected together.

2. An artificial network attaining or approximating predetermined frequency characteristics and having one or more passive network sections, one of the sections having three or more mutually inductively related coils tightly coupled together no two of which are directly galvanically connected together.

3. An electric system having four two-terminal portions containing mutual inductance and connected together at four vertices, the oppositely

disposed portions being substantially alike, the product of the unlike portions being substantially equal to the square of a resistance across two of the vertices.

4. An artificial network having one or more passive network sections and provided with two terminals only, one of the sections having three or more mutually inductively related coils no two of which are directly galvanically connected together, and means connecting the coils with the terminals.

5. An artificial network having one or more passive network sections including a two-terminal network portion provided with three or more mutually inductively related coils no two of which are directly galvanically connected together, and means connecting the coils with the terminals.

6. An artificial network having one or more passive network sections and provided with two terminals only, one of the sections having three or more mutually inductively related coils tightly coupled together, no two of which are directly galvanically connected together, and means connecting the coils with the terminals.

7. An artificial network having one or more passive network sections including a two-terminal network portion provided with three or more mutually inductively related coils tightly coupled together, no two of which are directly galvanically connected together, and means connecting the coils with the terminals.

8. An artificial network having four terminals only and provided with one or more passive network sections, one of the sections having three or more mutually inductively related coils no two of which are directly galvanically connected together, and means connecting the coils with the terminals.

9. An artificial network having one or more passive network sections including a four-terminal network portion, provided with three or more mutually inductively related coils no two of which are directly galvanically connected together, and means connecting the coils with the terminals.

10. An artificial network having four terminals only and provided with one or more passive network sections, one of the sections having three or more mutually inductively related coils tightly coupled together, no two of which are directly galvanically connected together, and means connecting the coils with the terminals.

11. An artificial network having one or more passive network sections including a four-terminal network portion provided with three or more mutually inductively related coils tightly coupled together, no two of which are directly galvanically connected together, and means connecting the coils with the terminals.

12. An artificial network attaining or approximating predetermined frequency characteristics and having one or more passive network sections, one of the sections having three or more mutually inductively related coils no two of which are directly galvanically connected together, the network being balanced with respect to the ground potential.

13. An artificial network attaining or approximating predetermined frequency characteristics and having one or more passive network sections, one of the sections having three or more mutually inductively related coils tightly coupled together no two of which are directly galvanically connected together, the network being balanced with respect to the ground potential.

14. An artificial network having four terminals only and provided with one or more passive network sections, one of the sections having three or more mutually inductively related coils no two of which are directly galvanically connected together, and means connecting the coils with the terminals, the network being balanced with respect to the ground potential.

15. An artificial network having one or more passive network sections including a four-terminal network portion provided with three or more mutually inductively related coils no two of which are directly galvanically connected together, and means connecting the coils with the terminals, the network being balanced with respect to the ground potential.

16. An artificial network having four terminals only and provided with one or more passive network sections, one of the sections having three or more mutually inductively related coils tightly coupled together, no two of which are directly galvanically connected together, and means connecting the coils with the terminals, the network being balanced with respect to the ground potential.

17. An artificial network having one or more passive network sections including a four-terminal network portion provided with three or more mutually inductively related coils tightly coupled together, and means connecting the coils with the terminals, the network being balanced with respect to the ground potential.

18. An artificial network having two network portions each provided with mutual inductances therein, the characteristic matrix of the impedance forming one of said network portions being the reciprocal of the characteristic matrix of the impedances forming said other network portion, multiplied by a positive, constant factor.

19. An artificial network having two network portions each provided with mutual inductances therein and each having a number of pairs of terminals, the characteristic matrix of the impedances forming one of said network portions being the reciprocal of the characteristic matrix of the impedances forming said other network portion, multiplied by a positive, constant factor.

20. An artificial network having two network portions each provided with mutual inductances therein tightly coupled together, the characteristic matrix of the impedances forming one of said network portions being the reciprocal of the characteristic matrix of the impedances forming said other network portion, multiplied by a positive, constant factor.

21. An artificial network having two network portions each provided with mutual inductances therein tightly coupled together and each having a number of pairs of terminals, the characteristic matrix of the impedances forming one of said network portions being the reciprocal of the characteristic matrix of the impedances forming said other network portion, multiplied by a positive, constant factor.

WILHELM CAUER.