A circularly housed transducer transmitting and/or receiving array of square transducer elements is so shaded as to produce patterns equivalent to a transducer composed of a superpositioned set of concentric disk elements of consecutively smaller radii from the largest disk to produce stepwise shading thereby truncating a square array to maximize the radiating area and minimize the beamwidth with low side lobes.

6 Claims, 5 Drawing Figures
Fig. 4

Fig. 5

45dB CHEBYSHEV SHADING (PRIOR ART)
MODIFIED 45dB SHADING

ANGLE OF INCIDENCE (DEGREES)
RELATIVE PRESSURE LEVEL (dB)
ADAPTING CIRCULAR SHADING TO A TRUNCATED ARRAY OF SQUARE ELEMENTS

BACKGROUND OF THE INVENTION

This invention relates to transducer arrays for transmitting and/or receiving acoustical or electromagnetic signals and more particularly to the adaptation of a square array of uniform, equispaced, square elements by the truncation of the corner elements into concentric circular groupings to fit within a circular baffle to maintain maximum radiation area with little increase in beamwidth and with low side lobes.

Torpedo-borne transducer arrays are normally designed with uniform, equispaced, square elements and are mounted into a circular baffle at the head of the torpedo. In order to maximize the radiating area and minimize the beamwidth, the corner elements are eliminated. This truncation of the array invalidates the strict application of the second product theorem which is used to extend Dolph-Chebyshev shading from a line array to a two-dimensional array. Indeed, when Dolph-Chebyshev shading is applied, the first on-axis side lobe level is raised and a troublesome off-axis side lobe appears. The only success achieved was to lower the first on-axis side lobe to the design level while leaving the off-axis lobe at a higher value (5 to 7 dB higher). Since the housing into which the transducer array is mounted is circular, it seems natural to try a circular shading scheme. The major problem is obviously a geometric one since the array consists of square, equispaced elements.

SUMMARY OF THE INVENTION

In the present invention a uniform, equispaced, square element array is truncated to fit within the circular baffle in which it is to be used and its radiation pattern is compared to that of a circular disk of comparable size. The radiation pattern of consecutively smaller disks which are subdivided into square elements or portions of square elements are superpositioned on the first disk concentrically with predetermined dimensions to achieve the nearest to optimum circular shading. This method successfully adapts circular shading to a truncated square array and eliminates the problem of a high off-axis side lobe with very little increase in beamwidth over a Dolph-Chebyshev design. Accordingly, it is a general object of this invention to provide a circular shaded array for transmitting and/or receiving acoustical or electromagnetic signals by the use of a circularly shaded truncated, uniform, equispaced square element array to maximize the radiating area and minimize the beamwidth to the greatest advantage while keeping side lobes lower than −45 dB.

BRIEF DESCRIPTION OF THE DRAWINGS

These and other objects and the attendant advantages, features, and uses of the invention will become more apparent to those skilled in the art as a more detailed description proceeds when considered along with the accompanying drawings in which:

FIG. 1 is a diagrammatic view showing a partial torpedo nose section and showing therein the transducer array of the present invention with associated electronic devices;

FIG. 2 is a diagram depicting concentric superpositioned disks which would provide shading equivalent to shading provided by the array shown in FIG. 1 of the drawings;

FIG. 3 is a view taken on line 3–3 of FIG. 1 showing a 9×9 array of transducer elements and showing, in broken lines, the concentric disks depicted in FIG. 2 of the drawings;

FIG. 4 is a view similar to FIG. 3 of the drawings only showing an array with an even number of transducer elements, that is, a 10×10 array; and

FIG. 5 is a graph comparing shading produced by Chebyshev shading and shading produced by the present invention.

DESCRIPTION OF THE PREFERRED EMBODIMENT

Referring first to FIG. 1 of the drawings, there is illustrated an enclosure for an array which, by way of example, might be a torpedo housing 11 having a window 12 of rubber or other suitable material, as well understood by those skilled in the art of torpedo design. A circular baffle 13 is positioned behind window 12 and attached thereto are a plurality of transducer elements mounted in an array 14. An electronic assembly 15 is provided and has means for transmitting and receiving acoustical or electromagnetic signals. A plurality of shading resistances 16 are provided between the electronic assembly 15 and array 14 and each of these shading resistances provide a coefficient of radiation value for the transducers in array 14.

Referring now to FIG. 2 of the drawings, the transducers forming array 14 are arranged, and the value of the shading resistances 16 are selected such that array 14 will produce a radiation pattern which is equivalent to the radiation pattern produced by concentric stacks 17, 18, 19, 20, and 21 of transducer arrays, hereinafter referred to as disks. The amplitude of the superpositioned disks can be represented by adding their respective amplitudes and this is illustrated by stacking the disks. The smallest of the disks, which is disk 17, is stacked on the next smallest disk 18 and succeeding disks are stacked on increasingly larger diameter disks. The radii of these disks are designated a_1 through a_n and the thicknesses of the various disks are designated A_1 through A_n, representing the weighting coefficients which determines the annulus shading coefficients of the array 14 illustrated in FIG. 3 of the drawings.

Referring more particularly to FIG. 3, there is illustrated a 9×9 flat array 14 which consists of a normally uniform, equispaced, square array of elements truncated at the corner elements so as to be mounted on circular baffle 13 in housing 11. Each square element in the array 14 is considered to have dimensions (x), and 14 represents the placement of elements in an equispaced truncated square array. Such arrays as array 14 are made of crystal or ceramic transducer elements, well known to those skilled in the art and more fully discussed in the text, Underwater Acoustics Handbook — II, published by the Pennsylvania State University Press (1965) in Chapter 10. The circles in FIG. 3 represent the radii of the disks 17 through 21, illustrated in FIG. 2, and these circles have radii which conveniently enclose elements on the horizontal and vertical rows forming as many circles as there are rows in half of the array. By extensive geometrical manipulation the percentage of the area of each square, which lies within a given circle, can be determined. The reasoning by which the squares can be divided between
3,760,345

different annular rings which have different source strengths, is based on the fact that when an individual source measures a small fraction of a wavelength, the shape of its radiating face is unimportant and only the volume velocity that it pumps is of concern. As is shown in FIG. 3, 13 elements, composed of array squares within the confines of the circles and designated (1) through (13) provide all the possible combinations the five rings or annuli. Each circle has a radius that conveniently encloses elements on the horizontal and vertical rows of the array 14 forming as many circles as there are rows in half an array. The percentages of these elements within each annulus is shown in FIG. 3 and the radius of each $a_i$ through $a_{14}$ illustrates the contribution for the element shading from each of the five annular disks that can be made from a 9x9 transducer array. For the purpose of invention these radii have been found to be

\[ a_1 = 0.564(x), \quad a_2 = 3.54(x), \]

\[ a_3 = 1.58(x), \quad a_4 = 4.53(x), \]

\[ a_5 = 2.55(x) \]

where $(x)$ is a dimension of each square element in array 14.

Under radiation theory it can be shown that the acoustic far field pressure due to radiation from a flat circular disk mounted in an infinite baffle is proportional to

\[ P \propto \pi \alpha^2 (2 J_1(\alpha \sin \theta)/\alpha \sin \theta) \]

where

- $\alpha$ = radius of the disk
- $k$ = wave number
- $\theta$ = angle of incidence measured from the normal to the disk
- $J_1$ = Bessel function of the first order.

Such a circular disk shading theory has been more fully discussed in Chapter 7 of the Text, Fundamentals of Acoustics, published by John Wiley & Sons, Inc. (1967).

Since finite sized elements are being used, the shading will be stepwise rather than continuous. This shading is achieved simply by superpositioning disks 11 through 14 of smaller radii and concentrically on larger or originally used circular disk 15 under the circular shading theory. By this building up process any degree of shading can be realized. The total radiation pattern will then be the sum of the radiation from each disk. Hence, the far field pressure is

\[ P = 2A_1(\pi \alpha_1^2) J_1(\alpha_1 \sin \theta)/\alpha_1 \sin \theta + A_2(\pi \alpha_2^2) J_1(\alpha_2 \sin \theta)/\alpha_2 \sin \theta + \ldots + A_n(\pi \alpha_n^2) J_1(\alpha_n \sin \theta)/\alpha_n \sin \theta ] . \]

Where

- $A_n$ is the weighting coefficient of the nth disk,
- $\alpha_n$ is the radius of the nth disk.

Dividing by the area of the smallest disk 11 and dropping the constants we have

\[ P = A_1 J_1(\alpha_1 \sin \theta)/k \alpha_1 \sin \theta + A_2 J_1(\alpha_2 \sin \theta)/k \alpha_2 \sin \theta + \ldots + A_n J_1(\alpha_n \sin \theta)/k \alpha_n \sin \theta \]

It is one of the purposes of this invention to assemble these five concentric disks 11 through 15 with the radii found and shaded such that no side lobe will have a higher level than $-45$ dB. In order to approximate the desired (not unknown) shading, the initial attempt used $45$ dB Dolph-Chebyshev shading even though its application is restricted to line arrays or extension of line arrays to non-truncated square or rectangular two dimensional arrays. The basic circular shading theory developed by Dolph-Chebyshev (Tchebyscheff) is fully discussed in the above text “Underwater Acoustic Handbook — II,” Chapter 13 and more particularly in Section 13.1.2 of this text.

Referring more particularly to FIG. 4 of the drawings, there is shown that the resulting sound pressure level using Dolph-Chebyshev shading has unequal side lobes and a higher first minor lobe at $90^\circ$ Theta than the design goal of $-45$ dB indicating, as expected, that this shading does not apply to circular shading. The radiation from each of the five disks 17 through 21 illustrated in FIG. 2 represent the contribution to the far field radiation pattern of the two curves shown in FIG. 4. By judiciously modifying the disk shading coefficients a radiation pattern with side lobe levels below $45$ dB is achieved. It should be mentioned at this point that this analysis of circular shading includes an area or the finite size of the transducer. The Dolph-Chebyshev shading technique applies to point sources, hence the directivity of an individual element must be added to the computed pattern. For those interested in how the modification proceeded note that the two highest side lobes are negative and that radiation from disk 20 has a predominate influence on the first side lobe. The coefficient $A_n$ was lowered by an amount equal to that required to reduce the first side lobe of $\Theta = 30^\circ$ to a level of $-45$ dB. In order to equalize the remaining side lobes an amount was added to the coefficient $A_1$ of the central small disk 17 to achieve a desirable pattern. These are by no means optimized shading coefficients but are only one set of numbers which achieves that $-45$ dB shading being sought.

Referring more particularly to TABLE I there is listed the 13 elements (1) through (13) which have different shading coefficients and the percentage of the area of that element which lies in each of the five annuli.

**TABLE I**

<table>
<thead>
<tr>
<th>Transducer element number</th>
<th>Percent of element area within an annulus DISK 17 DISK 18 DISK 19 DISK 20 DISK 21</th>
<th>Shading coefficients</th>
<th>Normalized</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>.912</td>
<td>.088</td>
<td>.984</td>
</tr>
<tr>
<td>2</td>
<td>.622</td>
<td>.078</td>
<td>.608</td>
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<tr>
<td>3</td>
<td>.005</td>
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</tr>
<tr>
<td>15</td>
<td>.005</td>
<td>.005</td>
<td>.005</td>
</tr>
</tbody>
</table>
The following relationship exists between the annulus shading coefficients, \( C_n \), and the concentric disk coefficients, \( A_i \), as derived for the shaded disks.

1. \( C_0 = A_1 + A_2 + A_3 + A_4 + A_5 = 1.000 \)
2. \( C_1 = A_1 + A_2 + A_3 + A_4 = 0.868 \)
3. \( C_2 = A_1 + A_2 + A_3 = 0.594 \)
4. \( C_3 = A_1 + A_2 = 0.293 \)
5. \( C_4 = A_1 = 0.0984 \)

The portion of the square element which lies beyond the largest annulus (disk 21) will be given an amplitude of zero, hence, that need not be of concern.

The annuli shading coefficients listed in Table I are the sums of the modified 45 dB shading coefficients as derived for the shaded disks. The element shading coefficient is derived for each element by summing the products of the annulus shading coefficient times the percentage of the corresponding area and then normalizing to the coefficient of the central element (1) of Fig. 3. It is considered that the shading technique which has best control over the side lobe levels and produce a narrow 3 dB beamwidth is the circular shading scheme, the object being, as well understood by those skilled in the art, to obtain the narrowest beamwidth and the maximum radiating area with the greatest suppression of side lobes. It should be emphasized however that the circular shading coefficients derived here are probably not quite optimum. The major drawback of the circular shading is this present inability to design for exact minor lobe levels; however, the present method is not difficult since computer programs are available to the computations.

Referring now to Fig. 4 of the drawings, there is shown a transducer array 22 comprised of an even number of transducer elements. For purposes of illustration, a 10X10 array of elements is shown with the four central elements 23 through 26 being treated as a single element for purposes of determining the radii \( A_1 \) through \( A_5 \).

While the above element percentages, annuli diameters, and annulus shading coefficients derived may not be optimum, they present hereinabove as described, shading for a transducer array for a torpedo nose or usage in other devices providing a superior narrow beam and maximum radiating area with lower side lobe to those of pure Dolph-Chebyshev shading on a truncated square array. Accordingly, shading truncated square arrays by adapting the radiation characteristics of circular arrays, as shown and described herein with a more or lesser number of circular concentric disks, may be used for different purposes, and it is to be understood that I desire to be limited in the spirit of my invention only by the scope of the appended claims.

I claim:

1. A shaded transducer system comprising:
   a. an annular baffle a symmetrical array of uniform equispaced square transducer elements truncated to fit onto said annular baffle;
   b. a multiplicity of shading resistances, each of said resistances providing a coefficient of radiation value;
   c. an electronic assembly having means to energize said elements of said array and having at least transmitter capability;
   d. connecting means connecting each of said elements to said electronic assembly through at least one of said multiplicity of shading resistances; and
   e. said elements, said resistances, said coefficient of radiation values, and said electronic assembly so correlated that said array will produce a radiation pattern which is equivalent to the radiation pattern which would be produced by a concentric stack of decreasing radii circular transducer elements, with the largest radius circular element positioned adjacent said annular baffle.

2. A transducer system as set forth in claim 1 wherein:
   said concentric stack of circular transducer elements includes a circular element of smallest diameter which has a radius which encloses as completely as possible the most central of said square transducer elements and has the same area as the area of the most central of said square transducer elements, and a circular element of largest diameter which has a radius which encloses as completely as possible all of said square transducer elements which lie along the row and the column of said square transducer elements which include said most central of said square transducer elements.

3. A transducer system as set forth in claim 2 wherein:
   said concentric stack of circular transducer elements have radii of about 0.564(\( x \)), 1.58(\( x \)), 2.55(\( x \)), 3.54(\( x \)), and 4.53(\( x \)), where \( x \) is a dimension of said square element, from the smallest circular element to the largest circular element, respectively; and
   have annular shading coefficient of radiation values of 0.132(\( x \)) for the smallest circular element and about 0.274(\( x \)), 0.301(\( x \)), 0.195(\( x \)), and 0.0984(\( x \)) respectively for the remaining circular elements whereby said array produces a first side lobe level lower than \(-45 \) dB.

4. A shaded transducer system comprising:
   an annular baffle a symmetrical array of uniform equispaced square transducer elements truncated to fit onto said annular baffle;
   a multiplicity of shading resistances, each of said resistances providing a coefficient of radiation value;
   an electronic assembly having means to detect signals from said elements of said array and having at least receiver capability;
   connecting means connecting each of said elements to said electronic assembly through at least one of said multiplicity of shading resistances; and
   said elements, said resistances, said coefficient of radiation values, and said electronic assembly so correlated that said array will produce a receiving pattern which is equivalent to the receiving pattern which would be produced by a concentric stack of decreasing radii circular transducer elements, with the largest radius circular element positioned adjacent said annular baffle.

5. A transducer system as set forth in claim 4 wherein:
   said concentric stack of circular transducer elements includes a circular element of smallest diameter which has a radius which encloses as completely as possible the most central of said square transducer elements and has the same area as the area of the most central of said square transducer elements, and a circular element of largest diameter which has a radius which encloses as completely as possible all of said square transducer elements which lie along the row and the column of said square trans-
7 ducer elements which include said most central of said square transducer elements.

6. A transducer system as set forth in claim 5 wherein:

said concentric stack of circular transducer elements have radii of about 0.564(x), 1.58(x), 2.55(x), 3.54(x), and 4.53(x), where (x) is a dimension of said square element from the smallest circular element to the largest circular element, respectively;

and have annular shading coefficient of radiation values of 0.132(x) for the smallest circular element and about 0.274(x), 0.301(x), 0.195(x), and 0.0984(x) respectively for the remaining circular elements whereby said array produces a first side lobe level lower than −45 dB.

* * * * *
CERTIFICATE OF CORRECTION
UNDER RULE 322

Patent No. 3,760,345 Dated September 18, 1973

Inventor(s) WILLIAM J. HUGHES

It is certified that error appears in the above-identified patent and that said Letters Patent are hereby corrected as shown below:

Column 4, line 5, that portion of the formula reading

\[ P = A_1 \frac{J_1(ka_1 \sin \theta)}{kd_1 \sin \theta} \]

should read

\[ P = A_1 \frac{J_1(ka_1 \sin \theta)}{ka_1 \sin \theta} \]

Column 4, line 26 change "90°" to read 30°

Signed and sealed this 26th day of February 1974.

(SEAL)

Attest:

EDWARD M. FLETCHER, JR.
Attesting Officer

C. MARSHALL DANN
Commissioner of Patents
UNITED STATES PATENT OFFICE
CERTIFICATE OF CORRECTION
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