July 28, 1964 W. J. SILBERKUHL ETAL 3,142,136

HYPERBOLOIDAL SHELL FOR ROOF VAULTS AND THE LIKE

Filed Jan. 25, 1962 3 Sheets-Sheet 1

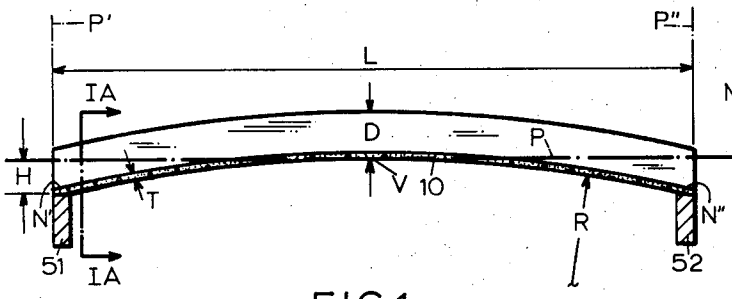


FIG.1

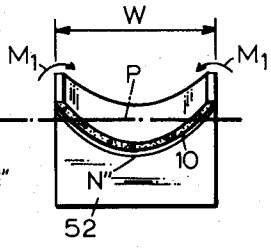


FIG.1A

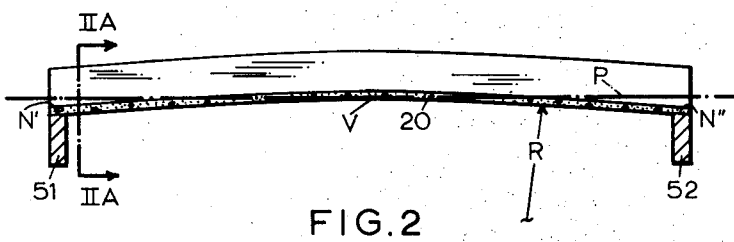


FIG.2

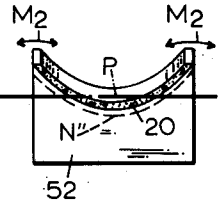


FIG. 2A

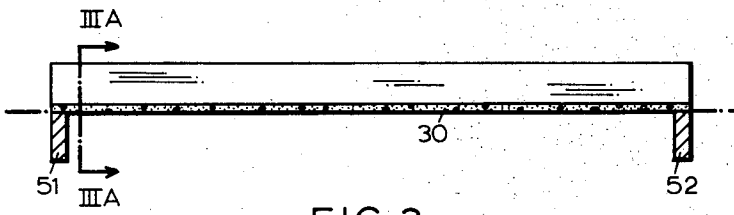


FIG.3

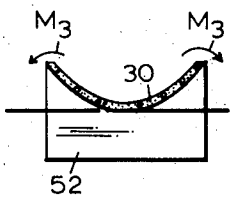


FIG. 3A

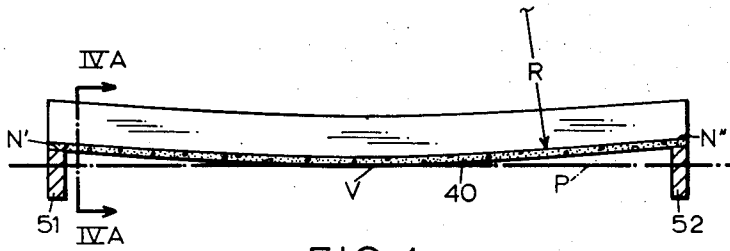


FIG.4

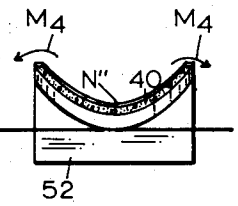


FIG. 4A

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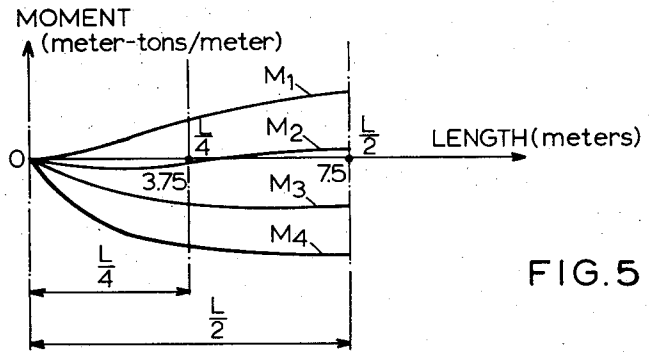


FIG.5

FIG.7

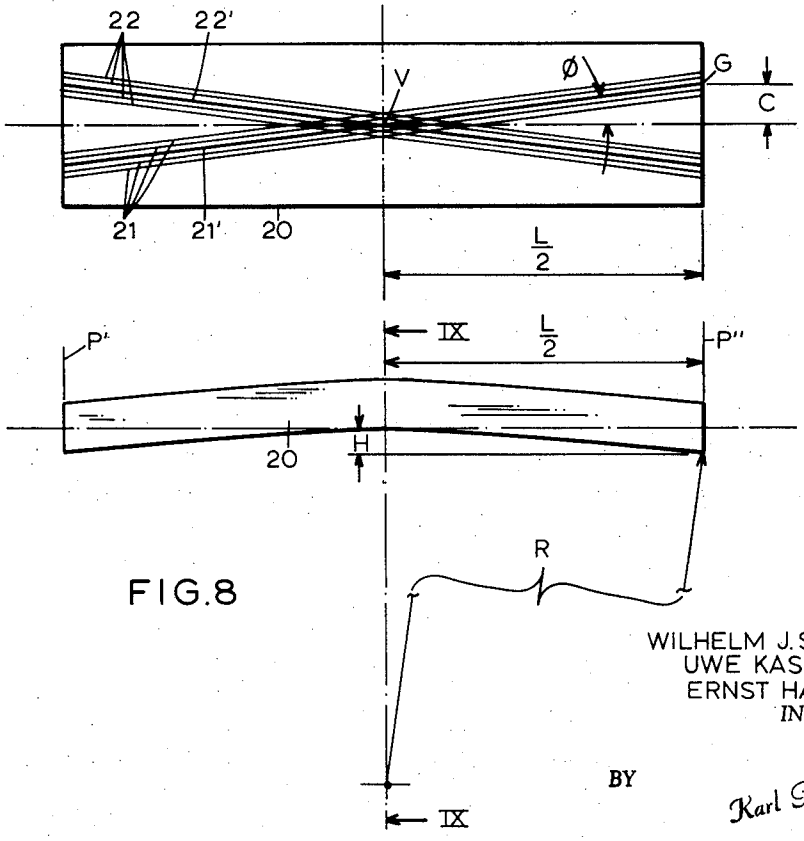


FIG.8

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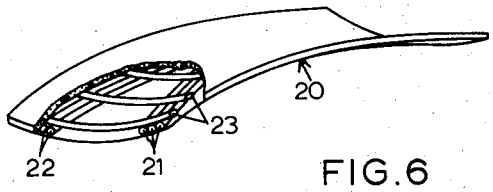


FIG.6

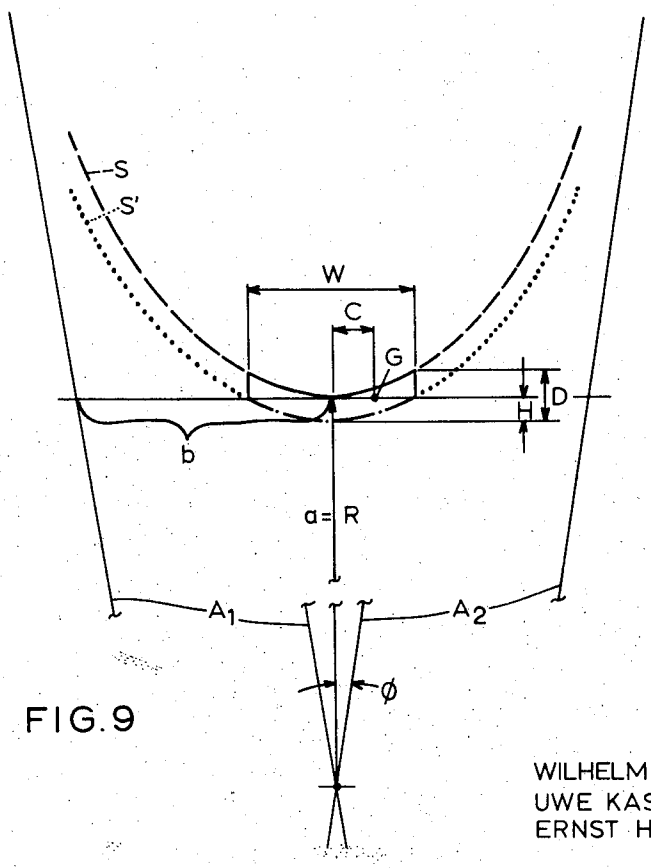


FIG.9

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United States Patent Office 3,142,136 Patented July 28, 1964

3,142,136

HYPERBOLOIDAL SHELL FOR ROOF VAULTS AND THE LIKE

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Filed Jan. 25, 1962, Ser. No. 168,700
Claims priority, application Germany Mar. 3, 1958
2 Claims. (Cl. 50—52)

Our present invention relates to a concrete shell adapted to be used as a precast construction element for roof vaults and similar structures. This application is a continuation-in-part of our copending application Serial No. 796,818, filed March 3, 1959, and now abandoned.

In pending application Serial No. 715,750, filed February 17, 1958, and now abandoned, by Wilhelm J. Silberkuhl (one of the present applicants), there has been disclosed a saddle-shaped precast shell in the form of a one-sheet hyperboloid which is prestressed, in whole or in part, with the aid of tensionable reinforcements imbedded in the concrete layer along a pair of straight-line generatrices, It is known that a conoidal body such as a one-sheet hyperboloid of rotation has such generatrices because is can be produced by the displacement of two intersecting straight lines, symmetrically disposed with respect to a given plane passing through their intersection, along a conic section located within that plane; in a hyperboloid of rotation these straight lines are parallel to the asymptotes of the hyperbola which constitutes an axial section of the resulting body parallel to their own plane. A specific proposal set forth in that application is to dimension the shell of generally rectangular horizontal outline in such manner that the straight-line generatrices pass diagonally from one corner of the rectangle to another, the central vertex of the shell thus lying substantially on the level of the upper edges of the piers by which the shell is supported along the minor sides of the rectangle.

Further research has established, however, that this proposal does not represent the optimum solution from the viewpoint of minimizing the transverse bending moments which develop, even under dead load, along the major edges of the rectangle and tend to deform the shell unless absorbed by marginal reinforcements, particularly in the case of shells that are dimensioned only thick enough to sustain the shear stresses and longitudinal bending moments due to dead and live loads. More particularly, as disclosed in our above-identified copending application, these transverse bending moments tend to be of substantial magnitudes and positive sign (i.e. inwardly directed) if the vertex is at the level indicated heretofore, the sign changing from positive to negative as the shell flattens out to approach the shape of a cylinder segment. With a cylindrical configuration the vertex rests, of course, on the level of the nadir of the cross-section of the shell above the supporting piers, hence the optimum (i.e. substantial disappearance of the transverse marginal moments) occurs with an elevation of the vertex between the two levels referred to, i.e. between the highest and lowest points of the terminal cross-sections in the planes of the supports. As a first approximation we indicated in our prior application that this vertex, and therefore the two straight-line generatrices passing through it, may lie in a horizontal plane passing substantially through the center of gravity of the projection of the shell upon a transverse plane (such as either plane of support).

Our present invention has for its object the establishment of convenient rules for determining the optimum dimensioning of a generally hyperboloidal shell from the points of view outlined above.

At the outset it should be noted that the type of shell here invisaged has the outline of an elongated rectangle, the term "elongated" being here understood as implying a

longitudinal span which is a multiple of the transverse span and preferably lies between about five and ten times the latter although these values are not to be regarded as sharp limits of the operative range. While the shell thickness is not critical, a preferred value is about 5 to 12 cm. The longitudinal curvature, which is upwardly convex, has a radius which is large in comparison with the length of the shell; its arch, therefore, extends over only a few degrees and may be regarded as an arc of either a circle or some other conic section osculating that circle so that there exists practically very little difference between a hyperboloid of rotation and, for example, a parabolic hyperboloid. Thus, for the sake of simplicity, the following discussion will be confined to a one-sheet hyperboloid of rotation without any intention of excluding related conoidal bodies with straight-line generatrices.

While the cross-sectional curve of the shell in any transverse plane is ideally a hyperbola, it should be remembered that the imaginary axis thereof coincides with the axis of the longitudinal arch so that, in view of the large radius mentioned above, the relatively short section of curve becomes almost indistinguishable from a parabolic segment and may in practice be treated as such.

Let us consider a shell having a longitudinal span of length L, a transverse width W, a radius of longitudinal curvature R, and a depth D as measured in a transverse plane from the level of its elevated edges to the bottom of its trough (neglecting the shell thickness). Let, furthermore,

$$\frac{L}{2R}=K$$

and let H denote the elevation of the midpoint or vertex of the shell above the nadir of its supported edges. From the geometry of the structure it can be shown that

$$H=R(\sqrt{1+K^2}-1)\approx\frac{RK^2}{2} \tag{1}$$

whence, with good approximation,

$$H=\frac{KL}{4} \tag{2}$$

We have found, in accordance with this invention, that optimum or near-optimum conditions with regard to transverse moments can be realized if, substantially,

$$0.015L\geqslant H\geqslant 0.005L \tag{3}$$

i.e. if

$$0.06\geqslant K\geqslant 0.03 \tag{4}$$

From both a structural and an esthetic viewpoint it is desirable to observe certain well-defined relationships between the aforestated parameters and the other shell dimensions identified above. It is preferred that the depth D be related to the width W substantially as

$$\frac{W}{3}\geqslant D\geqslant\frac{W}{5} \tag{5}$$

and that the elevation H have a maximum value approximately equal to half the depth D of the shell, i.e. that substantially

$$0<H\leqslant\frac{D}{2} \tag{6}$$

We have found that observance of these limitations contributes further to the stability of the structure through the suppression of undesirable edge forces; inequality (6), moreover, denotes an arrangement whereby the straight-line generatrices passing through the vertex emerge from the shell well within the outline of its terminal sections so that a considerable number of prestressing rods or wires may be accommodated within the concrete body along two families of such generatrices.

The foregoing features will become more apparent from the following detailed description given with reference to the drawing in which:

FIGS. 1, 2, 3 and 4 illustrate in longitudinal sectional elevation four types of span with substantially hyperbolical transverse curvature;

FIGS. 1A, 2A, 3A and 4A show respective cross-sections taken on the lines IA—IA, IIA—IIA, IIIA—IIIA and IVA—IVA of FIGS. 1–4;

FIG. 5 is a set of graphs relating to the structures of FIGS. 1–4 and showing the transverse moments developed therein;

FIG. 6 is a perspective view of a shell (parts broken away) as shown in FIGS. 2 and 2A;

FIG. 7 diagrammatically illustrates the plan of the shell shown in FIG. 6;

FIG. 8 is a diagrammatic side view of the shell shown in FIGS. 6 and 7; and

FIG. 9 is a diagrammatic cross-sectional view of the same shell, taken on the line IX—IX of FIG. 8.

FIGS. 1, 2, 3 and 4 show four types of concrete shells **10**, **20**, **30** and **40**, of rectangular horizontal outline and generally hyperbolical transverse curvature as illustrated in FIGS. 1A–4A, which are supported along the minor sides of the rectangle by a pair of piers invariably designated **51**, **52**. The four shells are all of the same length L, width W, depth D and thickness T whose values are given, by way of example, as follows:

L	m—	15
W	m—	2.4
D	m—	0.6
T	cm—	5

The longitudinal radius of curvature R differs progressively from shell **10** to shell **40**; while the proportions have been distorted in the drawing for convenience of illustration, it is assumed that this radius has the following values:

R(**10**)	m—	90
R(**20**)	m—	180
R(**30**)		∞
R(**40**)	m—	—180

Thus, shell **10** is strongly upwardly cambered in its longitudinal plane; shell **20** is also upwardly convex but is less strongly curved in its longitudinal dimension; shell **30** has no longitudinal curvature and represents, therefore, a hyperbolic cylinder; and shell **40** is upwardly concave.

A horizontal plane P passes through the vertex V of each shell and intersects the terminal transverse planes P′, P″ at an elevation H from the level of the nadirs N′, N″ of the shell in these latter planes. A comparison of the four pairs of figures shows that

$$H(10)>\frac{D}{2}$$

$$H(20)<\frac{D}{2}$$

$$H(30)=0$$

$$H(40)<0$$

Reference is now being made to FIG. 5 for a representation of the transverse bending moments M_1, M_2, M_3, M_4 which occur along the longitudinal edges of the several shells and have been designated by arrows in FIGS. 1A–4A. These moments are due to (*a*) the dead weight of the cross-section of the shell, (*b*) the shear stresses caused by the dead weight of the shell, and (*c*) the longitudinal stresses resulting from the arching of the shell (except in the case of shell **30**). We have found that, under certain conditions, the contribution of factor (*c*) can be made to balance the contributions of factors (*a*) and (*b*) as averaged over the length of the span. As will be apparent of the several graphs of FIG. 5, the

moments M_1 of shell **10** are positive (i.e. inwardly directed) from the vicinity of the supported section at O to the midpoint

$$\frac{L}{2}$$

of the span; the moments M_3 of cylindrical shell **30** are wholly negative, i.e. outwardly directed, as are the moments M_4 of shell **40** (which are of a magnitude substantially double that of moments M_3).

In contradistinction thereto, the moments M_2 of shell **20** are of considerably reduced magnitude and are slightly positive at the midpoint

$$\frac{L}{2}$$

while being slightly negative near the supported ends, their magnitude dropping to zero near the quarter-length point

$$\frac{L}{4}$$

of the structure. The average value of these moments is therefore substantially zero.

FIG. 8 shows the relationship between the parameters R and L as expressed by Equation 1. Upon a substitution of the given numerical values for R and L in Equations 1 and 2 the following values are obtained for K and H:

$$K(10)=\frac{1}{12} \qquad H(10)=\frac{L}{48}=32 \text{ cm.} \approx 0.02L$$

$$K(20)=\frac{1}{24} \qquad H(20)=\frac{L}{96}=16 \text{ cm.} \approx 0.01L$$

$$K(30)=0 \qquad H(30)=0$$

$$K(40)=-\frac{1}{24} \qquad H(40)=-\frac{L}{96}=-16 \text{ cm.} \approx -0.01L$$

If R remains constant and L is varied, the magnitudes of K and H also change. Thus, if L(**20**) were reduced from 15 to 10 m. (i.e. slightly more than 4 W), K(**20**) would be $\frac{1}{36}$ and H(**20**) would be

$$\frac{L}{144}$$

or, roughly, 0.7% of L; if L(**20**) were increased to, say, 20 m. (i.e. slightly more than 8 W), K(**20**) would be $\frac{1}{18}$ and H(**20**) would be

$$\frac{L}{72}$$

or 1.4% of L. While under these circumstances the absolute value of H(**20**) will be 28 cm. and will therefore be close to half the depth D which constitutes the altitude of the outmost cross-section of the shell, it should be noted that even longer spans could be constructed without change in the parameters R, W and D, though in such extreme cases the value of H might be somewhat greater than

$$\frac{D}{2}$$

it should be remembered that, particularly with long spans, the vertex V of the shell tends to settle below its design level even when the structure is not loaded. Moreover, since it is contemplated to use the structure with different degrees of prestress depending upon the expected loading conditions, some departures from the theoretical optimum illustrated in graph M_2 of FIG. 5 are permissible.

FIG. 6 shows the shell **20** with the prestressing rods **21**, **22** imbedded under tension along straight-line generatrices thereof; unprestressed reinforcements **23** of approximately hyperbolic or parabolic shape extend in transverse planes within the shell and overlie the prestressing elements **21** and **22**. As will be seen from FIG.

7, in which the elements **21** and **22** have been illustrated diagrammatically, each of these elements extends from one of the minor sides of the rectangle to a symmetrically located point along the opposite minor side; the lines **21'** and **22'** pass through the center of the span in the region of the vertex V and are intercepted by these minor sides at a location G whose distance from the longitudinal center line has been designated C.

As previously explained, the straight-line generatrices **21''**, **22''** are parallel to the asymptotes of a cross-sectional curve of the hyperboloid of rotation, taken in a plane parallel to these generatrices, this curve being of course identical in shape with the hyperbolic sections S, S' which have been illustrated in FIG. 9 and are located, respectively, at the midsection of the span and at an end section nearly coincident with plane P'. The curve S is a hyperbola with asymptotes A_1, A_2, a real half-axis $a=R$, and an imaginary half-axis b; from the equality of the angles ϕ in FIGS. 7 and 9 it will be apparent that

$$b:R=C:\frac{L}{2} \tag{7}$$

Since both the point $G(R+H, C)$ and a corner point

$$Q\left(R+D, \frac{W}{2}\right)$$

lie on the same hyperbola S' whose equation is

$$\frac{x^2}{R^2}-\frac{y^2}{b^2}=1 \tag{8}$$

it can be shown that

$$D=R\left(\sqrt{1+\frac{K^2}{n^2}}-1\right) \tag{9}$$

where n represents the ratio of the distance C to half the width of the span, i.e.

$$n=\frac{2C}{W} \tag{10}$$

From Equation 9 it follows that

$$n=\frac{RK}{\sqrt{2DR+D^2}} \tag{11}$$

whence, for the selected relationship

$$D=\frac{W}{4}$$

$$n=\frac{K}{\sqrt{m+\frac{m^2}{4}}} \tag{12}$$

where

$$m=\frac{W}{2R} \tag{13}$$

with the numerical values specified in connection with FIGS. 1–4, $m=1/150$ so that $m\approx 12.2$ K. Substituting the minimum and maximum values of K computed above for $L(20)=10$ m and $L(20)=20$ m, we find that, in this embodiment, n ranges between approximately ⅓ and ⅔, i.e.

$$\frac{W}{6}\leq C\leq\frac{W}{3}$$

Thus, the intersection G between the principal generatrix **21'** or **22'** and either of the minor sides of the rectangular outline of the shell **20** will always remain substantially within the middle third of a respective half of such minor side. This fact, as will be readily apparent from FIG. 7, enables the convenient accommodation of two families of prestressing elements **21**, **22** between the longitudinal edges of the shell.

In the manufacture of our improved shell **20** it will simply be necessary to place the reinforcements **21–23** in a suitably shaped mold, with a proper pretensioning of the rods **21** and **22** against the mold, and to pour the shell around these reinforcements in a manner known per se. It may be mentioned that the thickness of the shell need not be uniform but may, if desired, be somewhat increased toward the longitudinal edges of the span, particularly in the case of long structures; this thickness may vary, for example, from 6.5 cm. at the center to about 10 cm. at the edges.

The shells so produced may be disposed side by side, in direct contact or with interposition of simple slabs, e.g. as disclosed in copending applications Ser. No. 846,611, filed October 15, 1959, and Ser. No. 81,210, filed December 12, 1960, by Wilhelm J. Silberkuhl and Ernst Haeussler.

We claim:

1. A concrete structure adapted to be used in roof construction and the like, comprising an approximately horizontal concrete shell of substantially constant thickness substantially in the shape of a section of a one-sheet hyperboloid of generally rectangular outline with a length ranging substantially between five and ten times its width, said shell being upwardly cambered in longitudinal direction of the rectangle with a radius of curvature so chosen that the elevation of the shell vertex above the lowest points of the shell at the minor sides of the rectangle ranges between substantially 1.5% and 0.5% of the length of said shell, two sets of tensioned elongated prestressing elements extending within said shell substantially along straight-line generatrices of said hyperboloid from respective halves of one minor side of the rectangle to opposite halves of the other minor side, each of said sets including a principal prestressing element passing substantially through the center of said shell and emerging substantially within the middle third of a respective half of each of said minor sides, and piers supporting said shell along said minor sides, the center of the underside of said shell being located substantially at a level which passes through the outermost cross-sections of said shell at said minor sides along the lower half of the altitude of said cross-sections, said altitude ranging between substantially one fifth and one third of the width of said minor sides.

2. A structure according to claim 1 wherein the length of said shell in said longitudinal direction ranges between substantially 10 and 25 m., the length of said minor sides ranging between substantially 3 and 4 m., said radius of curvature being of the order of 180 m., and the thickness of said shell being substantially 5 cm. at least near the longitudinal center line thereof.

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