

Feb. 7, 1950

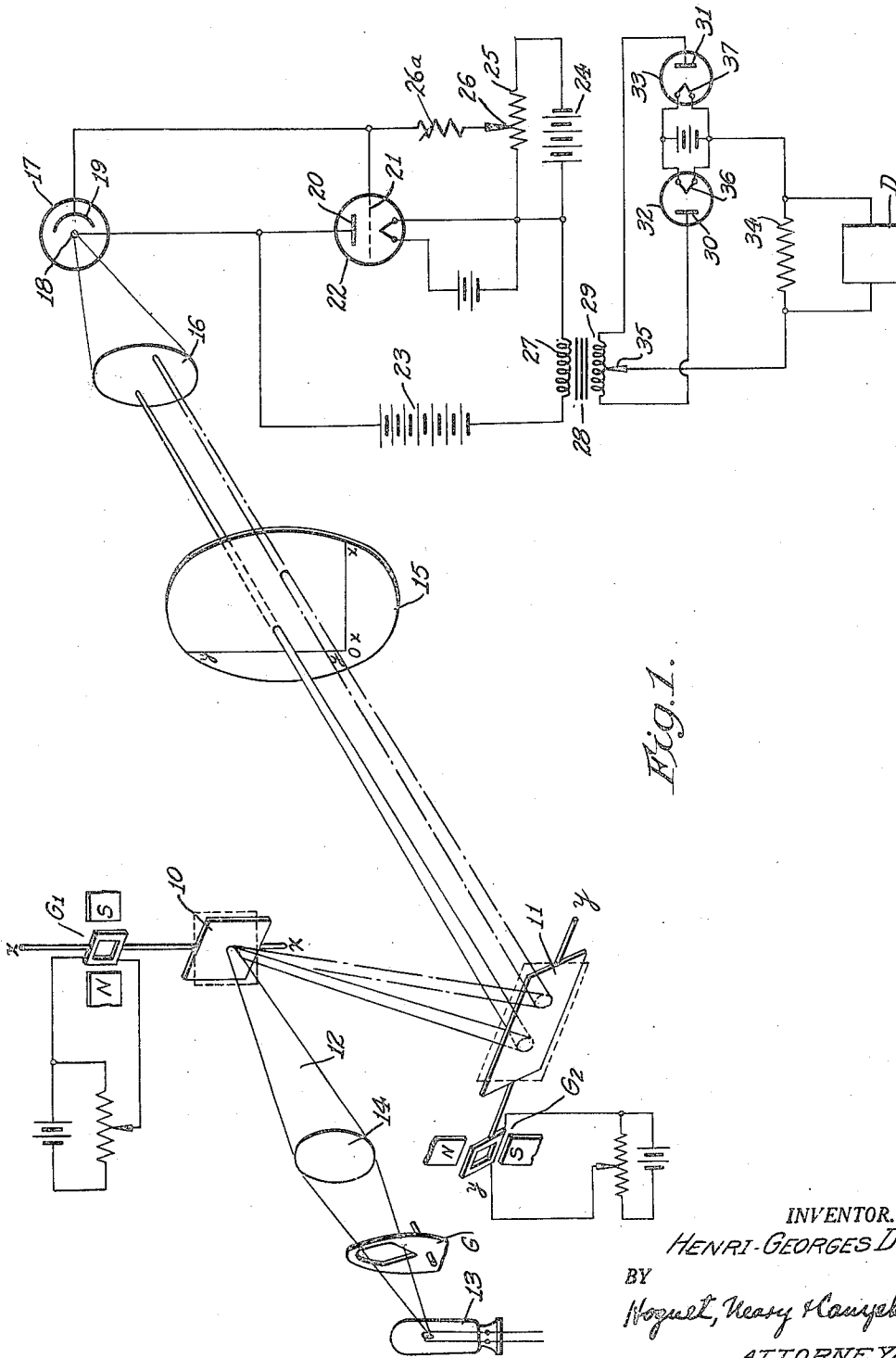
HENRI-GEORGES DOLL

2,497,042

ELECTROOPTICAL FUNCTION SYNTHESIZER

Filed Oct. 19, 1943

6 Sheets-Sheet 1



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Fig. 2.

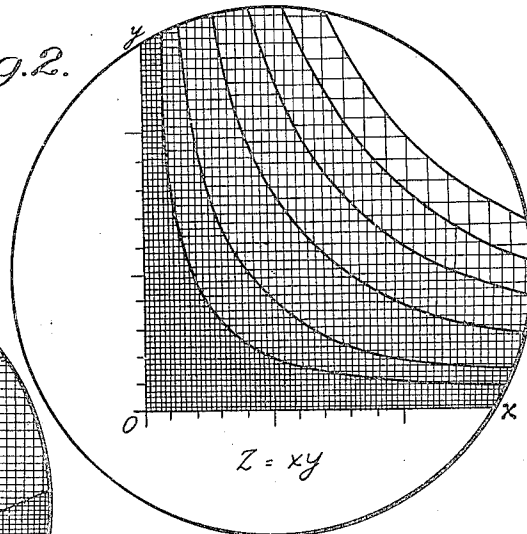


Fig. 3.

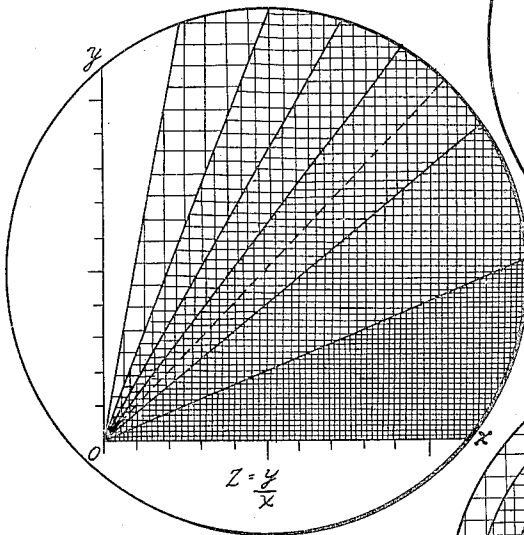


Fig. 4.

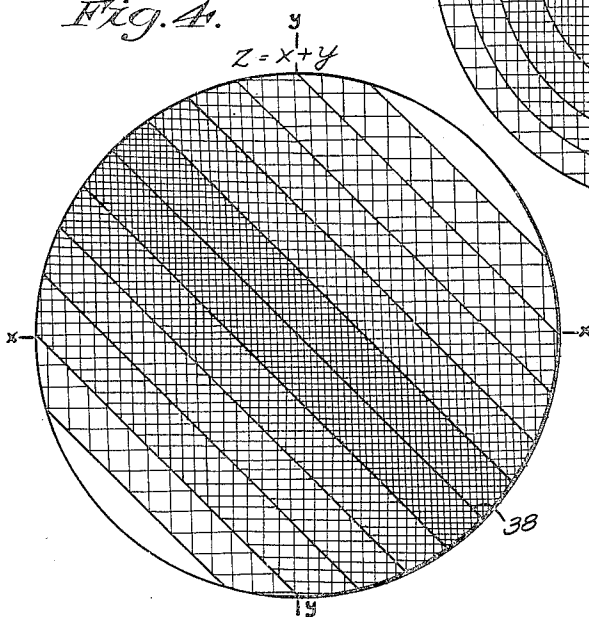
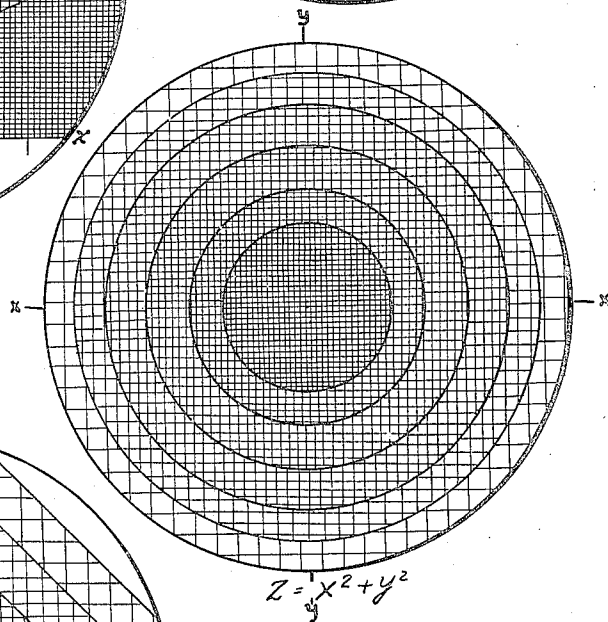


Fig. 5.

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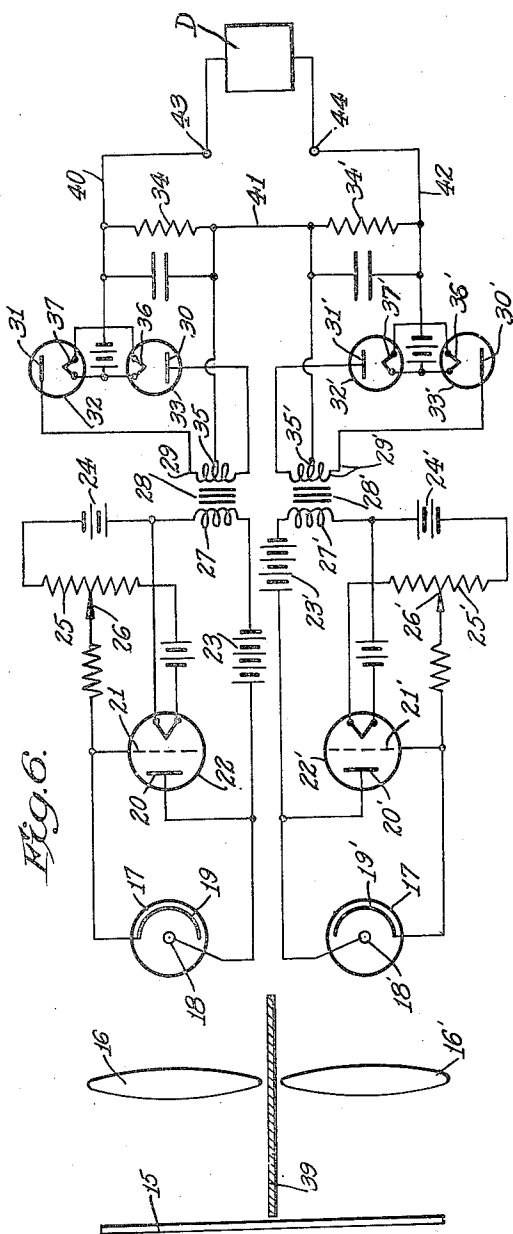
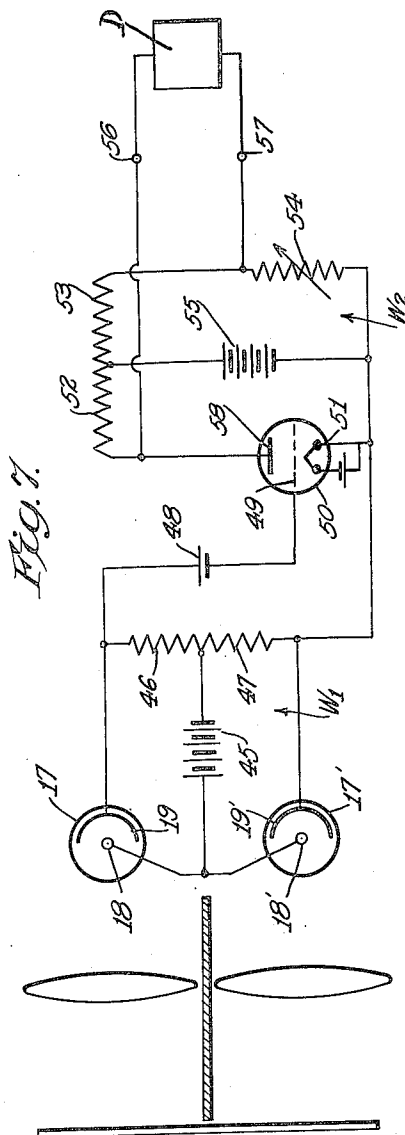
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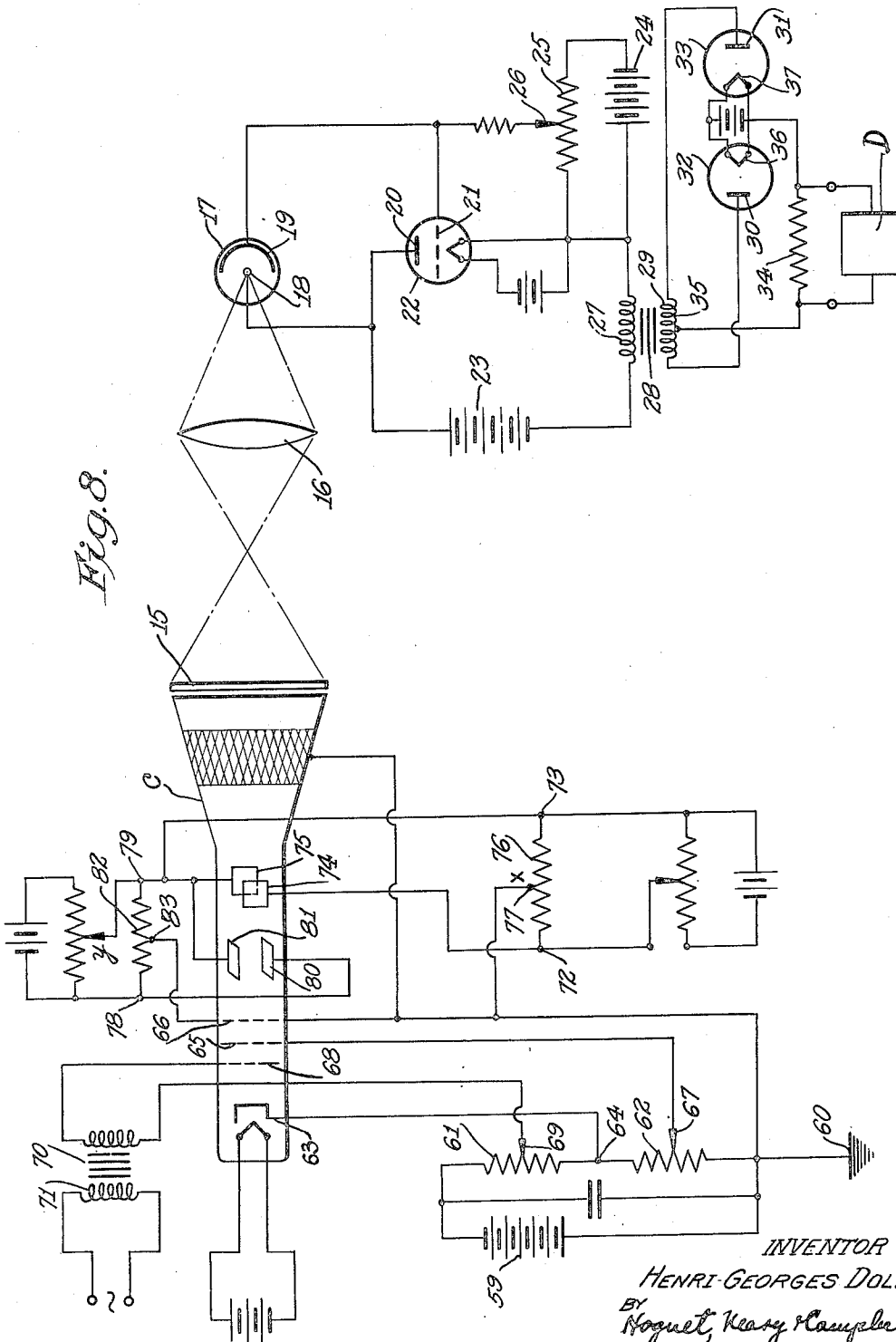
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ELECTROOPTICAL FUNCTION SYNTHESIZER

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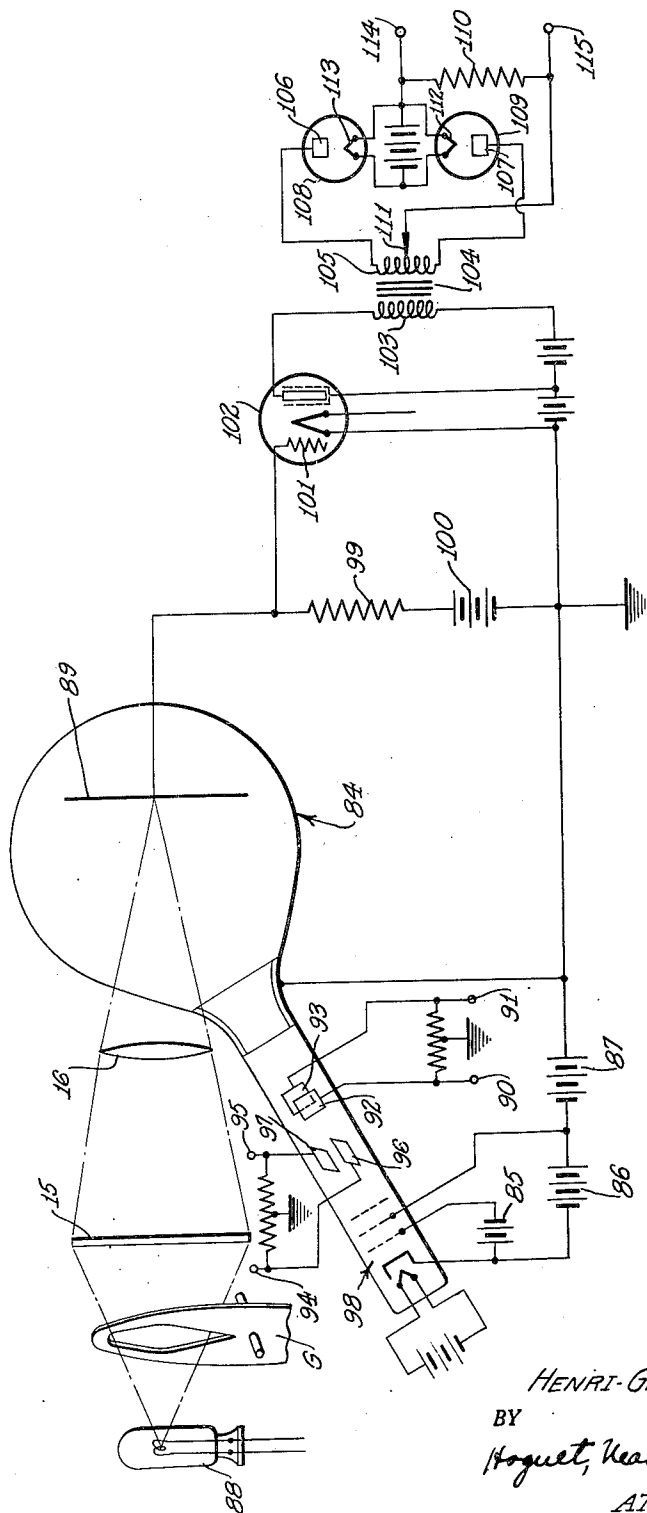
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Fig. 9.



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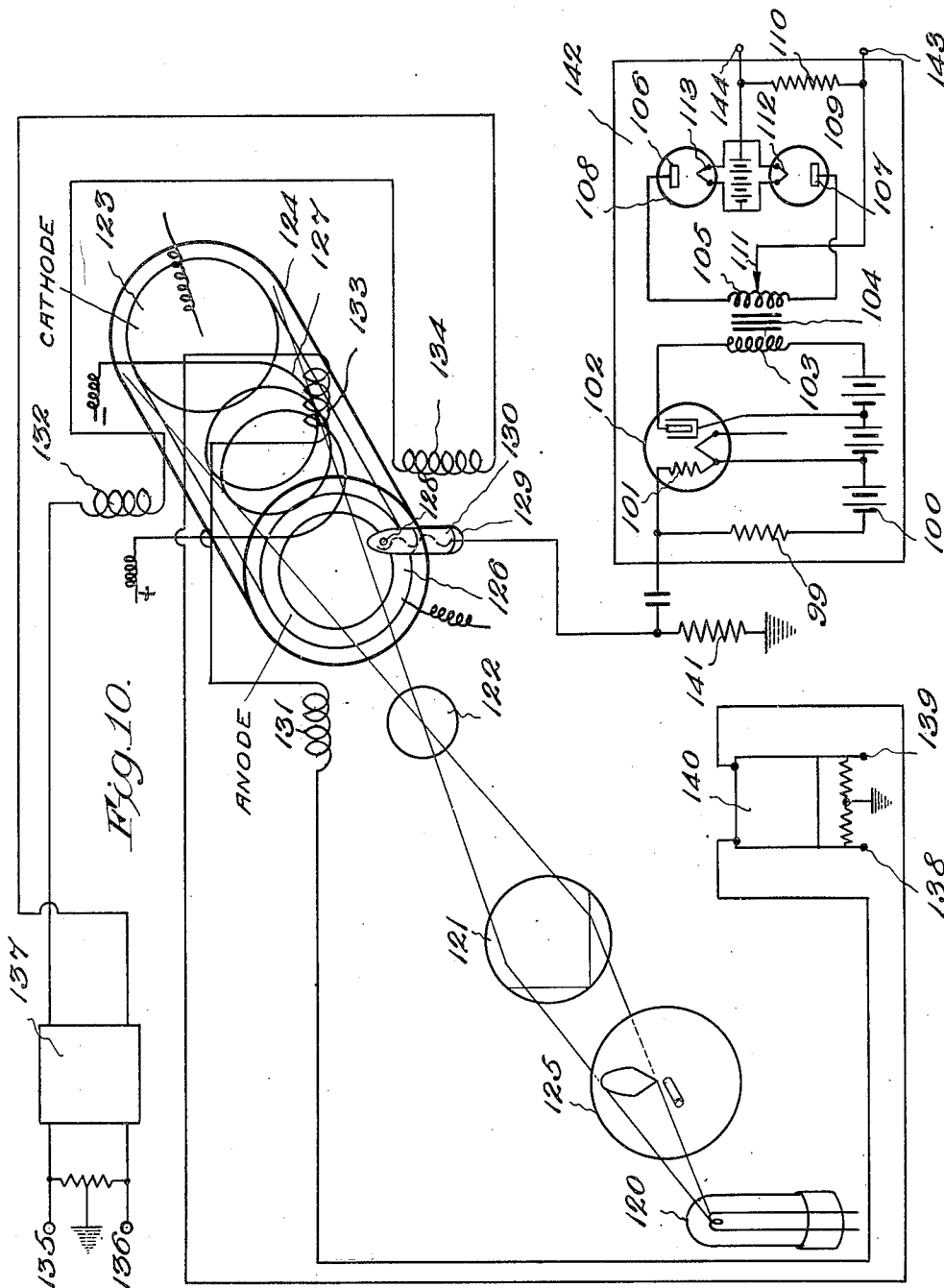
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ELECTROOPTICAL FUNCTION SYNTHESIZER

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6 Sheets-Sheet 6



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2,497,042

ELECTROOPTICAL FUNCTION SYNTHESIZER

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Application October 19, 1943, Serial No. 506,827

3 Claims. (Cl. 235—61)

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The present invention relates to mathematical solvers and more particularly to new and improved apparatus for providing instantaneous solutions for values of a function of a plurality of variables where the instantaneous values of the variables are known.

Heretofore mechanical solving apparatus has been employed for this purpose. In such apparatus, a large number of moving mechanical parts such as gears, levers and cams, for example, are usually employed for performing desired mathematical operations on the variables involved in the function whose value is to be obtained. Since mechanical elements of this sort have mass, they introduce inertia effects which produce an undesirable time lag between the introduction of the values of the variables into the solver and the reaching of a solution. Also, such elements must be made to a high degree of precision to avoid back-lash and other mechanical difficulties that might introduce errors into the solutions obtained.

It is an object of the present invention, accordingly, to provide a new and improved mathematical solver which is free from the above noted defects of the prior art.

Another object of the invention is to provide a new and improved solver in which few or no moving mechanical parts are employed for performing mathematical operations on the variables involved in the function whose value is to be obtained.

A further object of the invention is to provide a new and improved solver which receives electrical values proportional to the variables involved in the function whose value is to be determined, and provides instantaneous electrical values proportional to the values of said function.

A still further object of the invention is to provide a new and improved solver which is accurate, light in weight, and instantaneous in operation.

Still another object of the invention is to provide a new and improved solver of the above character which is capable of solving for negative and positive values of the function whose value is to be determined.

Broadly speaking, the objects of the invention are attained by modifying the intensity of a beam of radiant energy in accordance with instantaneous values of the variables on which the function depends, and directing the modified beam to a photosensitive device which provides an electrical output proportional to the instantaneous value of the function.

The modification of the intensity of the beam

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may be accomplished in a number of different ways. For example, the beam may be transmitted through a screen laid out as a two-variable coordinate system, the radiant energy transmission coefficient of which at each point is proportional to the value of the function corresponding to the values of the variables at that point. If the beam is displaced parallel to the screen axes, proportionally to the instantaneous values of the respective variables, it is clear that the intensity of the radiant energy impinging on the photosensitive device at any instant will be proportional to the value of the function corresponding to the instantaneous values of the variables at that instant.

A moving radiant energy beam of the desired character may be obtained according to the invention by directing radiant energy to a reflecting surface which is rotated about an axis in accordance with values of one of the variables, and directing the reflected light to a second reflecting surface which is rotated, in accordance with values of a second variable, about an axis at right angles to the axis of the first reflecting member.

In another embodiment of the invention, electrical values proportional to the instantaneous values of two variables may be applied to the horizontal and vertical deflecting plates, respectively, of a cathode ray oscilloscope. The trace on the oscilloscope screen provides a beam of radiant energy of desired characteristics. Other devices, such as a Farnsworth dissector tube or a television tube of the type known as an iconoscope, may also be used for this purpose, as described in greater detail hereinafter.

A function of three variables may be solved by further modifying the radiant energy beam in accordance with values of the third variable. Functions of four or more variables may also be solved according to the invention by employing a plurality of solvers and supplying the output of one solver as one of the variable inputs to a second solver.

The invention may be better understood from the following detailed description of several representative embodiments, taken in conjunction with the accompanying drawings, in which:

Figure 1 illustrates schematically solving apparatus constructed according to the invention;

Figures 2, 3, 4 and 5 illustrate different types of screens corresponding to different types of functions that may be solved with the apparatus shown in Figure 1;

Figures 6 and 7 are schematic diagrams of cir-

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culits enabling solutions of proper sign to be obtained for both positive and negative functions;

Figure 8 illustrates schematically another embodiment of the invention which utilizes a cathode ray oscilloscope;

Figure 9 illustrates schematically a further embodiment employing an iconoscope; and

Figure 10 illustrates schematically a still further embodiment utilizing a Farnsworth dissector tube.

In the embodiment illustrated in Figure 1 of the drawings, a beam of radiant energy is displaced in two mutually perpendicular directions by a combination of reflecting surfaces having mutually perpendicular axes of rotation.

Referring to Figure 1, the solving apparatus comprises a pair of reflecting surfaces 10 and 11 such as small mirrors, for example, which are mounted for rotation about the mutually perpendicular axes $x-x$ and $y-y$, respectively, in accordance with magnitudes of the variables x and y . To this end, the mirrors 10 and 11 may be directly connected to mechanical elements whose displacements represent magnitudes of the variables x and y , or they may be attached to the moving coils of two galvanometers G_1 and G_2 which are energized by electrical currents proportional to the magnitudes of the variables x and y , respectively, as shown. A light beam 12 is provided by a suitable light source 13 and is directed by means of a conventional type lens 14 upon the mirror 10 from which it is reflected to the mirror 11 and reflected from the latter to a screen 15, upon which the light beam is focused.

Disposed in the path of the beam reflected from the mirror 11 is a screen 15, to be described hereinafter, which modifies the intensity of the light in accordance with the magnitudes of the variables x and y . The modified light beam is focused by means of a conventional type lens 16 upon a conventional type photosensitive element 17 which may be a photoelectric cell, for example. If a light beam of constant intensity is employed, the photoelectric cell 17 should preferably be shielded to prevent stray light from reaching it, and a conventional type direct current amplifier should be provided for amplifying its output. It is more convenient, however, to modulate the light beam by applying alternating current to the source 13, or by placing an interrupter G in the path of the beam 12, for example. Where this is done, a conventional alternating current amplifier may be provided for the photoelectric cell 17, as described below.

The electrodes 18 and 19 of the photoelectric cell 17 are connected to the plate electrode 20 and the control grid electrode 21, respectively, of a conventional type electronic amplifying tube 22. The plate electrode 20 of the tube 22 and the anode 18 of the photoelectric cell 17 are both supplied with electrical energy from a suitable source 23. Grid bias for the electronic tube 22 is provided by a potentiometer comprising a source of voltage 24 and a resistor 25, the variable contact 26 of which is connected to the control grid 21 of the tube 22 through a resistance 26a.

In the plate circuit of the tube 22 is connected the primary winding 27 of a transformer 28, the secondary winding 29 of which is connected to the anodes 30 and 31, respectively, of the rectifying tubes 32 and 33, respectively. The output of the rectifiers 32 and 33 is impressed across a load resistance 34, which is connected at one end to the mid-tap 35 of the transformer secondary winding 29 and at its other end to the filaments 36 and 37 of the rectifiers 32 and 33,

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respectively. The resistance 34 can be shunted advantageously by means of a condenser to reduce the A. C. component (principally second harmonic) of the rectified current.

The screen 15 is provided with a horizontal axis $x-x$ and a vertical axis $y-y$, as shown, and its transparency at each point of the coordinate system is such that the amount of light transmitted at that particular point is directly proportional to the function $z=f(x, y)$ for which solutions are to be made.

In operation, the mirrors 10 and 11 are adjusted so that when the magnitudes of the variables x and y are both zero, the light beam 12 passes through the origin O of the $x-x, y-y$ coordinate system. If the mirrors 10 and 11 are then rotated about their respective axes in accordance with the magnitudes of the variables x and y , respectively, for any given values x_0 and y_0 of the variables the light beam 12 will impinge upon the screen 15 at the corresponding point (x_0, y_0) , and the amount of light reaching the photoelectric cell 17 will be directly proportional to the value of the function z corresponding to those particular values of the variables x and y . The electrical output of the solver is received by a device D which may be a suitably calibrated indicator or a device to be actuated in accordance with values of the function solved for.

When a modulated light beam is used in order to permit the use of alternating current amplifiers, the output gives the average instantaneous value of the function z .

Representative screens corresponding to a number of simple well known functions of two variables are shown in Figures 2-5, inclusive. Figure 2 illustrates a screen representing the function $z=xy$. The screens in Figures 3, 4 and 5 correspond to the functions

$$z = \frac{y}{x}$$

$$z = x + y$$

and

$$z = x^2 + y^2$$

respectively. Similar screens may be constructed for any function $z=f(x, y)$ provided that values of the function can be calculated for the different values of the variables x and y so that the transparency which the screen must have at each point thereon can be determined.

Transparent screens of the type shown in Figures 2-5, inclusive, may be constructed in any desired fashion. In the ideal case, the transparency should vary continuously from point to point. However, screens of this type are sometimes difficult to build and expensive. Nevertheless, they must be so constructed where extreme accuracy is desired.

From many applications, however, it is sufficient to determine the instantaneous value of the function $z=f(x, y)$ only approximately. In some instances, an accuracy of plus or minus 5 or 10% may be entirely adequate. In such cases, the transparency of the screen need not be determined to any greater degree of accuracy, and a satisfactory screen may be constructed by dividing its surface into different zones varying progressively in transparency by 5 or 10%, for example. For screens of this type it is necessary only to calculate the lines of equal transparency which constitute the boundaries between two successive zones as shown in Figures 2-5, inclusive, and to give the successive zones the desired degrees of transparency. It will be apparent that

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the higher the degree of accuracy required, the smaller the step from one zone to another must be, and the greater the number of zones required for a given screen surface.

After the boundaries between adjacent zones have been calculated for any given function, the zones between them may be made of different transparency in any desired manner. For example, a suitable screen may be made from a photographic plate on which zones of different transparency are made by varying the time of exposure for the different zones. However, it is simpler and generally adequate to use a system of cross-etching, the density of which varies from one zone to another as shown in Figures 2-5, for example. In such case the light beam 12 in the apparatus shown in Figure 1 should not be too sharply focused, and it should cover sufficient surface so that the amount of light transmitted through the screen 15 will truly represent the average transparency at that particular location.

For the functions $z=xy$ (Figure 2) and

$$z = \frac{y}{x}$$

(Figure 3) it has been assumed that the variables x and y have only positive values, and that the function has no negative values. For this reason, the origin of the x - y coordinate system has been displaced away from the center of the screen so as to obtain a more efficient use of its surface. Where this is done, the mirrors 10 and 11 should be adjusted so that the beam 12 in its initial position will lie on the origin of the screen 15.

It will be noted that in the case of the function $z=x+y$ (Figure 4), both x and y can be either positive or negative so that z can also be positive or negative. Thus, on the screen shown in Figure 4, the entire area above the diagonal line 38 represents positive values of z , whereas the entire area below that line represents negative values at z . For a function of this type, both positive and negative values may be obtained by employing two independently operating circuits like that of Figure 1 and by dividing the optical field into positive and negative sections. Such a solution is described in detail in connection with Figures 6 and 7.

The embodiment shown in Figure 6 is intended to be employed where a modulated light beam is used for the purpose of obtaining an alternating current output from a photoelectric cell. In this embodiment, the upper positive area of the screen 15, which may be of the type shown in Figure 4, is isolated from the lower negative area thereof by means of an opaque screen 39, which is disposed perpendicularly to the screen 15 along the line 38.

Located on one side of the opaque screen 39 is the lens 16 and photoelectric cell 17 with its associated circuit. On the other side of the opaque screen 39 is disposed a second lens 16' and photoelectric cell 17' and its associated circuit. Since the circuit for the photoelectric cell 17' is identical with that for the photoelectric cell 17, corresponding parts in the former have been designated by corresponding primed reference characters. The output resistors 34 and 34' are connected in series by means of conductors 40, 41 and 42, such that the output at the terminals 43 and 44 is the difference between the voltages appearing across the resistors 34 and 34'.

If the value of the function z is positive so that the beam impinges upon the upper positive portion of the screen 15, only the photoelectric

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cell 17 will be operative so that the voltage appearing at the terminals 43 and 44 will be equal to the voltage developed across the resistor 34. If, on the other hand, the value of the function z is negative so that the beam impinges upon the lower negative portion of the screen 15, only the photoelectric cell 17' will be energized, and the voltage at the terminals 43 and 44 will be substantially equal to the voltage developed across the resistor 34'. This voltage will be equal in magnitude but opposite in polarity to the voltage developed across the resistor 34 when the value of the function z is positive and has the same absolute value.

It will be apparent, therefore, that the circuit shown in Figure 6 provides outputs at the terminals 43 and 44 which have either positive or negative polarities depending upon whether the instantaneous values of the function z are positive or negative.

If an unmodulated light beam is employed, then the circuit shown in Figure 7 may be used. In this modification, the anodes 18 and 18' of the photoelectric cells 17 and 17' should be connected together and to the positive terminal of a source of voltage 45, the negative terminal of which is connected to the common point between two resistors 46 and 47. The other ends of the resistors 46 and 47 are connected to the cathodes 19 and 19' of the photoelectric cells 17 and 17', thereby forming a Wheatstone bridge W_1 . The cathode 19 of the photoelectric cell 17 is also connected in series with a source of biasing voltage 48 to the control grid 49 of a conventional type electronic amplifying tube 50, the cathode 51 of which is connected to the cathode 19' of the photoelectric cell 17'.

The plate circuit of the electronic tube 50 constitutes one of the arms of a second Wheatstone bridge W_2 , the other arms of which comprise the resistances 52, 53 and 54. A source of voltage 55 is connected to the cathode 51 of the tube 50 and to the mid-point between the resistors 52 and 53, and the output of the bridge W_2 is provided at the terminals 56 and 57 which are connected to the plate electrode 58 of the tube 50 and the common point between the resistors 53 and 54, respectively.

In operation, both of the bridges are adjusted to be in balance when no light is falling upon the photoelectric cells 17 and 17'. The first bridge is balanced by changing the value of one or both of the resistances 46 and 47; the second bridge is balanced by changing the value of resistance 54. If, however, light falls on either the photoelectric cell 17 or 17', the second bridge W_2 , including the tube 50, is unbalanced providing an output at the terminals 56 and 57 which may be either positive or negative depending upon whether the light beam impinges upon the upper positive half of the screen 15 or the lower negative half thereof. In this embodiment, it is desirable to shield the optical system so as to prevent stray light from influencing the results obtained.

Similarly, the density screen can be arranged to give an output equal to the function z plus a , in which a is a positive constant sufficient to make $z+a$ always positive. The output of the system would then give a potential difference corresponding to the $z+a$ function and the quantity a can then be subtracted by means of a direct current potentiometer placed in series with the output to produce either a positive or negative function.

This system of handling the case where the function z can be either positive or negative can be used not only in the apparatuses utilizing two rotating mirrors, but also with other apparatus hereinafter described.

Instead of using mirrors having mutually perpendicular axes of rotation, as shown in Figure 1, a cathode ray oscilloscope may be employed, as shown in Figure 8, for providing a light beam that can be displaced in mutually perpendicular directions in accordance with the values of the variables x and y .

Referring to Figure 8, a conventional type cathode ray oscilloscope is shown at C with its screen disposed adjacent the screen 15. The oscilloscope C is energized in the conventional manner from a source of electrical energy 59, the positive terminal of which is grounded at 60. The cathode 63 of the oscilloscope C is connected to the common point 64 between two resistors 61 and 62 connected across the voltage source 59, and the accelerating electrodes 65 and 66 are connected to the points 67 and 60, as shown in the figure. Negative bias for the control grid 68 is provided by connecting it to the variable contact 69 on the resistor 61.

The cathode ray beam should preferably be modulated so as to enable an alternating current amplifier to be used with the photoelectric cell 17. To this end, alternating current is impressed upon the control grid 68 from a transformer 70, the primary winding 71 of which is connected in the circuit of the control grid 68.

An electrical value proportional to the variable x is impressed upon the terminals 72 and 73 which are connected to the vertical deflecting plates 74 and 75, respectively, of the oscilloscope C. Connected across the terminals 72 and 73 is a resistance 76, the mid-tap 77 of which is connected to ground. In similar fashion, a voltage proportional to the variable y is impressed upon the terminals 78 and 79, which are connected to the horizontal deflecting plates 80 and 81, respectively, of the oscilloscope C. A resistor 82 is connected across the terminals 78 and 79, the mid-tap 83 of which is connected to ground.

The fluorescent spot produced on the screen of the cathode ray oscilloscope C by the cathode ray beam impinging thereon, provides a beam of radiant energy which is transmitted through the screen 15 and is focused by the lens 16 upon the photoelectric cell 17. Since the latter and its electrical circuit are identical with the corresponding electrical equipment in Figure 1, corresponding reference characters have been used to designate corresponding parts throughout.

Initially, the cathode ray oscilloscope C is adjusted in the conventional manner so that when the variables x and y are zero in magnitude, the cathode ray spot will lie at the origin of the coordinate system on the screen 15. Electrical values directly proportional to instantaneous values of the variables x and y are then impressed upon the vertical and horizontal deflecting plates, respectively, of the oscilloscope C, thereby causing the fluorescent spot to move to corresponding positions on the screen 15. The light transmitted through the screen 15 at any given point corresponding to particular values of the variables x and y is directly proportional to the value of the function for those particular values of the variables. Accordingly, the voltage impressed across the resistor 34 is directly proportional to the instantaneous values of the func-

tion z corresponding to instantaneous values of the variables x and y .

If solutions are to be made of functions having both positive and negative values, either of the circuits shown in Figures 6 and 7 may be substituted for the photoelectric cell circuit shown in Figure 8, as described above.

In the embodiment shown in Figure 9, a television tube of the type known as an iconoscope is employed for the purpose of solving for values of a function of two variables. In this figure, the iconoscope is designated by the reference character 84, and it is energized by suitable sources of electrical energy 85, 86 and 87 in the conventional manner. Light from a suitable source 88 is directed through a screen 15 of the type shown in Figures 2-5, inclusive, and is focused by means of a lens 16 upon the photosensitive screen 89 of the iconoscope 84 so that the light pattern on the photosensitive screen 89 corresponds to the transparency of the screen 15. The light from the source 88 should be modulated, either by supplying alternating current to the source 88 or by interposing an interrupter G in the path of the beam, as shown in the figure.

Electrical values proportional to the magnitudes of the variable x are impressed upon the terminals 90 and 91 which are connected to the vertical deflecting plates 92 and 93, respectively, of the iconoscope 84. In similar fashion, electrical values proportional to the magnitudes of the variable y are impressed upon the terminals 94 and 95 which are connected to the horizontal deflecting plates 96 and 97, respectively. It will thus be seen that the electron beam provided by the electron gun 98 will be displaced vertically in accordance with instantaneous values of y and horizontally in accordance with instantaneous values of x .

Initially, the position of the electron beam is adjusted so that when the variables x and y are both zero, the beam impinges on the origin of the x - x , y - y coordinate system of the image of the screen 15 formed on the photosensitive screen 89.

As is known in the art, the intensity of the current flowing in the circuit of the photo-sensitive screen 89 varies with the intensity of the modulated light falling upon the screen at the point where the electron beam impinges thereon. Accordingly, if instantaneous values of x and y are applied to the vertical deflecting plates 92 and 93 and the horizontal plates 96 and 97, respectively, of the iconoscope 84, the current flowing in the circuit of the photosensitive element 89 will vary instantaneously in accordance with the value of the function z , as determined by the characteristics of the screen 15. This current is passed through a resistance 99 which is connected in series with a battery 100.

The voltage developed across the resistance 99 is impressed upon the control grid electrode 101 of a conventional type electronic amplifying tube 102; in the plate circuit of which is connected the primary winding 103 of a transformer 104. The secondary winding 105 of the transformer 104 is connected to the anodes 106 and 107 of conventional type rectifying tubes 108 and 109, respectively. The output of the rectifiers 108 and 109 is impressed upon a load resistance 110, one end of which is connected to the mid-tap 111 of the transformer 104, and the other end of which is connected to the filaments 112 and 113 of the rectifying tubes 109 and 108, respectively. The voltage appearing at the terminals 114 and 115,

respectively, will be directly proportional to instantaneous values of z which is the desired function of x and y .

In all of the embodiments described above, it has been assumed that the value solved for was a function of only two variables x and y . However, it is a relatively simple matter, in accordance with the invention, to solve a function of three variables. For example, this may be accomplished in the embodiment shown in Figure 8 by modulating the voltage applied to the control grid 68 in accordance with the third variable. If the potential applied to the control grid 68 at each instant is such that the intensity of the electron beam is proportional to the function $g(u)$ of the variable u and if, simultaneously, the electron beam is given a horizontal deflection proportional to the variable x and a vertical deflection proportional to the variable y , the amount of light transmitted through the screen 15 at each instant will be proportional to the function $z = g(u) \cdot f(x, y)$.

Solutions can also be obtained for more than three variables by using a plurality of solvers and utilizing the output of one to control the radiant energy beam of the second. For example, if a voltage proportional to the variable u is applied to the vertical deflection plates 74 and 75 of the cathode ray tube C in Figure 8, and a voltage proportional to the variable v is applied to the horizontal deflecting plates 80 and 81, the output of the photoelectric cell 17 will be proportional to the function $g(u, v)$. Similarly, if electrical values proportional to the variables y and x are applied to the horizontal and vertical deflecting plates of a second oscilloscope solver of the same type, the output of the photoelectric cell thereof will be proportional to the function $f(x, y)$, assuming the intensity of the electron beam to be constant.

By applying the voltage which appears across the resistor 34, which voltage is proportional to the function $g(u, v)$, to the control grid of the second oscilloscope, the intensity of the electron beam thereof will be proportional to the function $g(u, v)$, so that the output of the second solver will be proportional to the function $z = g(u, v) \cdot f(x, y)$.

The embodiment of the invention shown schematically in Figure 10 utilizes a Farnsworth image dissector tube. In this form of the invention, light from a source 120 passes through a screen 121 of the type shown in Figure 2 of the drawing, for example, and is focused by means of a lens 122 upon a photosensitive cathode 123 of the image dissector tube 124. The light intensity at any point on the cathode 123, therefore, corresponds to the transparency of the corresponding point on the screen 121.

The light from the source can be modulated either by supplying alternating current to the source 120 or by interposing an interruptor 125 between the light source 120 and the screen 121.

In the Farnsworth tube, electrons are emitted from each point on the cathode 123 in proportion to the intensity of the light falling upon that point. Inasmuch as the light intensity at any point corresponds to the transparency of the screen 121, the electron intensity at any point corresponds to the function $f(x, y)$.

The tube 124 includes an anode 126 to which a potential may be applied to attract the electrons emitted from the cathode 123. A focusing coil 127 within the tube produces a magnetic field coaxial with the tube. When the strength of the

magnetic field is correctly adjusted, an electron image of the cathode 123 is produced at a plane parallel to the cathode 123 and in the plane of the opening 128 in the tube member 129 which projects into the tube 124. The electrons passing through the aperture 128 enter a series of electron multipliers 130 within the member 129 which increase the sensitivity of the tube.

The circuit connections to the anode, focusing coil and electron multiplier are normal and follow the manufacturers' recommendations.

Outside the tube 124 are placed four coils 131, 132, 133 and 134. The coils 132 and 134 are connected in series in such a way that current passing through them produces a uniform magnetic field in a vertical direction. The coils 131 and 133 likewise are connected in series to produce a uniform horizontal field. The effect of these magnetic fields is to change the direction of the electrons moving from the cathode 123 toward the anode 126 to the right or left and up and down. In this way, any part of the electron image can be made to appear at the aperture 128.

Electrical values proportional to the magnitude of the variable x are impressed upon the terminals 135 and 136 of the amplifier and current control circuit 137 which is connected to the coils 132 and 134. The impressed electrical values will produce a current in the coils 132 and 134 causing a vertical magnetic field of the correct direction and strength to displace the electron image horizontally by an amount corresponding to the x coordinate.

Similarly, electrical values proportional to the magnitude of the variable y are impressed upon the terminals 138 and 139 of the amplifier and current control circuit 140 which is connected in series to the coils 131 and 133. The horizontal field produced will displace the electron image vertically by an amount corresponding to the coordinate y .

Initially the circuits 137 and 140 are adjusted so that the part of the electron image corresponding to the origin of the x - y coordinate system of the image of the screen 121 formed on the cathode 123 arrives at the aperture when both the variables x and y are zero.

Inasmuch as the density of the electron image is a function $f(x, y)$, the average number of electrons entering the aperture 128 for any instantaneous quantities impressed upon the terminals 135, 136, and 138 and 139 will vary instantaneously in accordance with the value of the function z .

The output of the electron multipliers 130, which also varies with the value of the function z , develops a voltage across the resistance 141. This voltage is impressed upon an amplifier and rectifier 142 like that described in connection with the iconoscope and disclosed in Figure 9. The voltage appearing at the output terminals 143 and 144 will be directly proportional to the average instantaneous value of z .

While the use of the image dissector and the iconoscope have been described in connection with a screen of a type which gives only positive functions, it is possible to utilize these devices to deal with functions which can be either positive or negative. Thus, by illuminating a part of a screen with a light having one frequency and the other part of the screen with a light modulated to a different frequency and providing a filter before the output, the two frequencies can be separated to give two outputs which can be added

algebraically, to produce an output which is either positive or negative.

From the foregoing, it will be apparent that solvers of the type described above may be employed for solving all types of functions of two variables and many types of functions of more than two variables. It will further be apparent that solvers constructed according to the invention include a minimum number of moving parts so that errors arising from inertia effects and other mechanical difficulties are reduced to a minimum. In those embodiments including a cathode ray oscilloscope or an iconoscope or an image dissector which utilizes an electron beam, inertia effects are completely eliminated, enabling solutions of desired functions to be obtained instantaneously.

Solvers constructed according to the invention may be adapted for a wide variety of uses. For example, they may be employed in the solution of problems incident to gun fire control as well as in aircraft control systems of the type disclosed in my copending application for "Automatic control system for vehicles," filed July 17, 1943, Serial No. 495,093 now forfeited. Other uses will occur to those skilled in the art.

While several specific embodiments have been described above, the invention is not intended to be limited thereto. Those embodiments are susceptible to numerous changes in form and detail within the scope of the appended claims.

I claim:

1. An electrical solver for solving for values of a function of at least two variables comprising, means providing a beam of radiant energy, means for displacing said beam in different directions in proportion to the magnitudes of said respective variables, an optical screen in the path of said beam for modifying the intensity of said beam in accordance with the instantaneous value of the function corresponding to the instantaneous values of the variables, said screen having one portion representing positive values of said function and another portion representing negative values of said function, photosensitive means disposed to receive said modified beam, and electrical means connected to said photosensitive means for providing in a circuit an electrical value of one polarity proportional to positive values of said function, and an electrical value of different polarity proportional to negative values of said function.

2. An electrical solver for solving for values of a function of at least two variables comprising, means providing a beam of radiant energy, means for displacing said beam in different directions in proportion to the magnitudes of said respective variables, an optical screen in the path of said beam for modifying the intensity of said beam in accordance with the instantaneous value of the function corresponding to the instantaneous values of the variables, said screen having one portion representing positive values of said function and another portion representing nega-

tive values of said function, a plurality of photosensitive means disposed to receive radiant energy from said portions of the optical screen, respectively, means for periodically varying the intensity of said beam, amplifying means for said respective photosensitive means, rectifying means connected to said amplifying means, means connecting the outputs of said rectifying means, in series opposition to electrically responsive means.

3. An electrical solver for solving for values of a function of at least two variables comprising, means providing a beam of radiant energy, means for displacing said beam in different directions in proportion to the magnitudes of said respective variables, an optical screen in the path of said beam for modifying the intensity of said beam in accordance with the instantaneous value of the function corresponding to the instantaneous values of the variables, said screen having one portion representing positive values of said function and another portion representing negative values of said function, a plurality of photosensitive means disposed to receive radiant energy from said portions of the optical screen, respectively, an electrical network connected to said photosensitive means so as to constitute a Wheatstone bridge of which said photosensitive means constitute two adjacent arms, an electronic amplifying tube having control grid and cathode electrodes electrically connected to the output terminals of said Wheatstone bridge, a second electrical network connected in the plate-cathode circuit of said tube so as to constitute a second Wheatstone bridge of which said tube plate-cathode circuit comprises one of the arms, and means receiving the output of said second Wheatstone bridge.

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