March 15, 1932. W. R. BENNETT 1,849,656

TRANSMISSION NETWORK

Filed June 29, 1929

FIG. 1

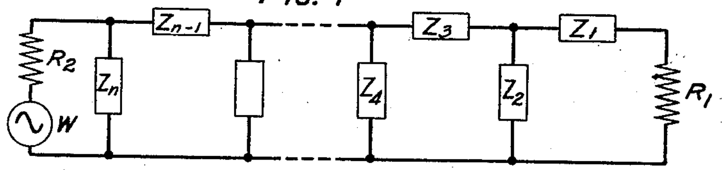


FIG. 2

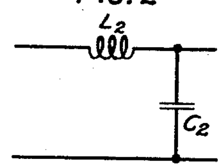


FIG. 3

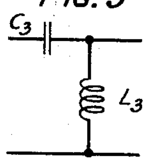


FIG. 4

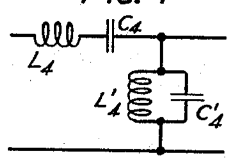
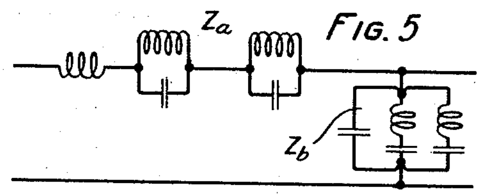
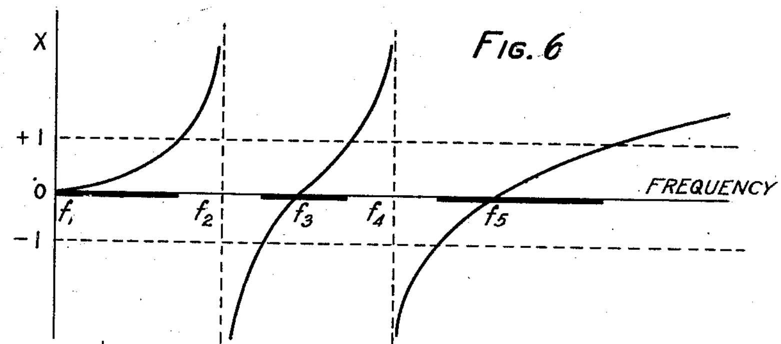
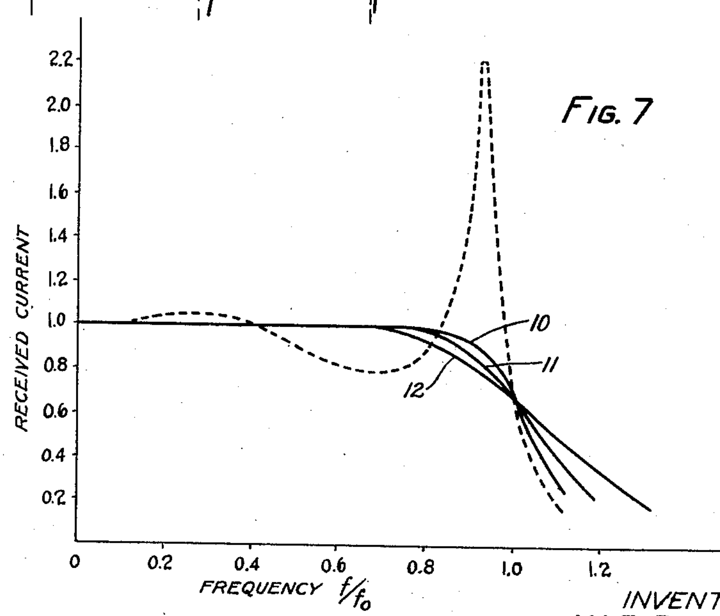


FIG. 5







INVENTOR
W. R. BENNETT

BY G. H. Stevenson

ATTORNEY

Patented Mar. 15, 1932 1,849,656

UNITED STATES PATENT OFFICE

WILLIAM R. BENNETT, OF JERSEY CITY, NEW JERSEY, ASSIGNOR TO BELL TELEPHONE LABORATORIES, INCORPORATED, OF NEW YORK, N. Y., A CORPORATION OF NEW YORK

TRANSMISSION NETWORK

Application filed June 29, 1929. Serial No. 374,669.

This invention relates to wave transmission networks, and more particularly to artificial lines having broad band frequency selective properties.

In wave transmission systems involving a uniform line, or an artificial line of iterative character, a necessary condition for uniform efficiency of transmission is that the terminal apparatus with which the line is associated should have an impedance substantially equal to the characteristic impedance of the line at all frequencies in the range it is desired to transmit. In certain cases this can be done with great exactness, but where the line is an iterative structure of reactive impedance elements, as in case of a coil loaded line or of a broad band wave filter, the condition can be fulfilled only approximately. This is for the reason that the characteristic impedances of these structures are not constant with frequency, except at frequencies remote from the transmission band limits, and are strongly variable at frequencies close to the band limits. When such lines are used, as is ordinarily the case, with terminal apparatus of constant resistive impedance, wave reflections occur at the ends of the line and give rise to irregularities in the transmission characteristic. These irregularities are most noticeable at points corresponding to the natural frequencies of the system and may be regarded as due to insufficiency of the damping of the resonances by the resistive terminal impedance. For many purposes the resonance effects are of little importance since their magnitude is greatly diminished by the effects of dissipation in the branch impedances, but in other cases, where the internal dissipation is very small or where the operating conditions require a high degree of uniformity, the irregularities due to undamped resonance may be very objectionable.

By this invention broad band selective systems are provided in which the transmission is free from irregularities of the type described. This result is obtained by departing from the usual iterative type of structure and instead, arranging the selective circuit as a tapered line in which the impedance values of the elements vary from section to section in accordance with a prescribed law, as hereinafter described. The structures of the invention comprise artificial lines of the ladder, or series-shunt type which, in schematic form, have general resemblances to broad band wave filters of known type, but differ therefrom in certain important respects. One characteristic feature of the structures is that the series branch impedances increase in value progressively from both ends towards the center, while the shunt branch impedances vary in an inverse manner. Viewed in the light of the ordinary wave filter theory this method of tapering corresponds to varying both the characteristic impedance and the width of the transmission band from section to section along the line. The resulting characteristic may be regarded as being due to the internal reflections neutralizing those occurring at the ends of the line, however, the operation of the circuits of the invention cannot readily be analyzed in terms of the ordinary filter theory and the above statement of the principle of operation is therefore intended to be merely suggestive.

The nature of the invention and its mode of operation will be more fully understood from the following detailed description and by reference to the attached drawings, of which

Fig. 1 is a schematic diagram illustrating the general form of the transmission networks of the invention;

Figs. 2, 3, 4 and 5 show various specific forms of line sections applicable to Fig. 1;

Fig. 6 is a diagram illustrating certain points in the theory of the invention; and

Fig. 7 illustrates the type of transmission characteristic obtained by the invention.

The circuit illustrated in Fig. 1 comprises terminal resistances R_1 and R_2 of equal value which may be constituted by resistance elements or may represent the impedances of long transmission lines or of power consuming loads, a wave source W included in series with R_2, and an artificial line of the ladder type having n branch impedances Z_1, Z_2, Z_3, . . . Z_n, disposed alternately in series and in shunt between R_1 and R_2. The branch impedances are made up of substantially pure reactance elements, the series impedances being all of similar character and the shunt impedances being similar to each other, but having a frequency variation inverse to that of the series impedances. In accordance with the invention the series impedances Z_1, Z_3, etc., are given the values,

$$\begin{aligned} Z_1 &= a_1XR, \\ Z_3 &= a_3XR, \quad (1) \\ Z_{n-1} &= a_{n-1}XR, \end{aligned}$$

and the shunt impedances Z_2, Z_4, etc. are given the values,

$$Z_2=\frac{R}{a_2X},$$

$$Z_4=\frac{R}{a_4X}, \quad (2)$$

$$\cdots\cdots$$

$$Z_n=\frac{R}{a_nX},$$

where R is the common value of resistances R_1 and R_2, X is a frequency factor expressing the variation of the impedances with frequency and the factors a_1, a_2, etc. are numerical coefficients having the values

$$a_1=2\sin\frac{\pi}{2n},$$

$$a_2=2\sin\frac{3\pi}{2n}, \quad (3)$$

$$a_3=2\sin\frac{5\pi}{2n}, \text{ etc.,}$$

or in general,

$$a_r=2\sin\frac{(2r-1)\pi}{2n}.$$

The circuit is shown terminating at the right in a series impedance Z_1 and at the left in a shunt impedance Z_n, n being an even number. It is not necessary, however, that this particular arrangement be followed; the invention is not restricted to an even number of branches and each end of the line may terminate in a series or in a shunt impedance regardless of the termination of the other end. The coefficients a_1, a_2, etc. as given by Equation 3 are applied to the branches in order, counting from one end of the line, but it is immaterial which end is used as a starting point since the coefficients are equal in pairs as follows:

$$\begin{aligned} a_1&=a_n, \\ a_2&=a_{n-1}, \\ a_3&=a_{n-2}, \text{ etc.} \end{aligned}$$

With these proportions the current in resistance R_2 due to an E. M. F. E_2 in series with R_1 is given by the equation

$$|I|=\frac{E}{2R\sqrt{1+|X|^{2n}}} \quad (4)$$

where $|I|$ denotes the absolute magnitude, or modulus value, of the current, and $|X|$ is the modulus value of the frequency factor X.

The factor X, which expresses the frequency variation of the branch reactances is an imaginary quantity expressible as an odd rational function of frequency, but its modulus value is a positive real quantity. Numerically X is equal to each of the ratios

$$\frac{Z_1}{a_1R},\ \frac{Z_3}{a_3R},\ \ldots\ \frac{R}{a_2Z_2},\ \frac{R}{a_4Z_4},\ \ldots$$

that is, it defines, except for the numerical coefficients a_1, a_2, etc. the ratio of the values of the series impedances to the terminal resistance, or the ratio of the terminal resistance to the shunt reactances. The general form of the impedance of any reactance structure, and hence the general form of the frequency factor X, is discussed by R. M. Foster, A reactance theorem, Bell System Technical Journal, Vol. III, No. 2, April 1924, page 259.

From Equation (4) it follows that, so long as $|X|$ is less than unity, the value of the current is closely equal to

$$\frac{E}{2R}$$

but when $|X|$ exceeds unity the current becomes very small, particularly when the number n of the impedance branches is large. The frequencies at which X passes through the value unity thus demark the transmission and the attenuation ranges of the circuit, the former being defined by numerical values of X between

$$+\sqrt{-1} \text{ and } -\sqrt{-1}.$$

From the known properties of reactances it follows that as $|X|$ increases from zero to unity, the quantity on the right hand side of Equation (4) continuously diminishes from the maximum value

$$\frac{E}{2R}$$

to the value

$$\frac{E}{2\sqrt{2}R},$$

the rate of diminution being very small at first and then increasing, but showing no intermediate maxima or minima. The frequencies for which X is zero correspond to the resonances of the series branches and to the anti-resonances of the shunt branches. The transmission bands are therefore centered about the resonance frequencies of the series branches, except when these occur at zero frequency and infinite frequency.

The determination of the numerical coefficients a_1, a_2, etc., which define the taper of the line, is a lengthy and involved mathematical procedure, only the major steps of which will be described here.

The ratio of the voltage E of the source W to the current I in the resistance R_1 may be expanded as a power series,

$$\frac{E}{I}=A_0X^n+A_1X^{n-1}+A_2X^{n-2}\cdots+A_{n-2}X^2+A_{n-1}X+1, \quad (5)$$

in which the coefficients A_0, A_1, etc. are functions of the coefficients a_1, a_2 etc. of the impedance branches. The simple character of the power series is due to the fact that the series impedances and the shunt admittances are all proportional to the same frequency function X, thus permitting the ratio to be expanded as a function of a single variable.

Equation (5) may be written in the form

$$\frac{E}{I}=A_0(X-X_1)(X-X_2)(X-X_3)\cdots(X-X_n), \quad (6)$$

where X_1, X_2, etc. are the n roots of the expression on the right of Equation (5) and the desired ratio, namely that of the absolute magnitudes of E and I, can be found by multiplying this expression by its conjugate. This gives

$$\left|\frac{E}{I}\right|^2=(-1)^nA_0^2(X^2-X_1^2)(X^2-X_2^2)\cdots(X^2-X_n^2). \quad (7)$$

If now, it is stipulated that the absolute magnitude of the voltage ratio shall have the value

$$\left|\frac{E}{I}\right|^2=|X|^{2n}+1, \quad (8)$$

the identity is established that

$$(-1)^nA_0^2(X^2-X_1^2)(X^2-X_2^2)\cdots(X^2-X_n^2)=|X|^{2n}+1, \quad (9)$$

which is satisfied for A_0 equal to unity and X_1^2, X_2^2 etc. respectively equal to the n, nth roots of ± 1, the + sign being taken if n is odd and the − sign if n is even. The values of the roots X_1, X^2, etc. may be written as follows:

$$X_r=\pm\left[\cos\frac{(2r-1)}{2n}\pi+j\sin\frac{(2r-1)}{2n}\pi\right](r=1\cdots n), \quad (10)$$

when n is even, and

$$X_r=\pm\left[\cos\frac{(2r-2)}{2n}\pi+j\sin\frac{(2r-2)}{2n}\pi\right],$$

when n is odd, the sign being chosen so that the real part corresponding to the damping constant, is negative.

These roots define the character of the transient, or free, oscillations of the circuit. The imaginary parts of the roots are proportional to the natural frequencies and the real parts to the corresponding damping constants. If plotted in the complex plane with the real parts as abscissæ and the imaginary parts as ordinates, it will be seen that the roots are represented by equal vectors spaced at equal angles, the angle between successive vectors being equal to

$$\frac{\pi}{n}.$$

The roots in their complex form are sometimes termed the complex damping constants of the circuit. A circuit for which the complex damping constants have the values indicated above possesses the unique transmission characteristics expressed by Equation 4. It is not free from natural or transient oscillations, but in its response to forced oscillations, resonance effects at the several natural periods are not in evidence. The response to forced oscillations resembles that of a singly resonant system but is much broader and flatter. In the more complex systems of the invention a plurality of transmission bands may exist, the response characteristics in each band being of the character of a single broad resonance, free from irregularities at the natural periods. The roots in such cases each define a plurality of natural periods, one for each transmission range.

The roots X_1, X_2, having been found, the coefficients A_1, A_2, etc. in Equation 5 can be determined by standard mathematical processes, these coefficients being related to the roots in the following manner;

$\frac{A_1}{A_0}$ is the sum of the roots;

$\frac{A_2}{A_0}$ is the sum of the products of the roots taken two at a time;

$\frac{A_3}{A_0}$ is the sum of the products taken three at a time, and so on.

Having determined the necessary values of the coefficients of Equation (5) to make it correspond to the stipulated form of the voltage ratio, the next step is to determine the values of the impedance coefficients a_1, a_2, etc. so that the physical network will have the prescribed characteristic. To do this Equation (5) may be rewritten with the coefficients A_1, A_2, etc. fully expanded, that is, expressed in terms of the impedance coefficients a_1, a_2, etc. of which they are functions. Two expressions are thus established for the ratio

$$\frac{E_1}{E_2},$$

the one involving the numerical values of the coefficients A_1, A_2, necessary to obtain the prescribed characteristic, and the other involving the coefficients expanded as functions of a_1, a_2, etc. Equating the coefficients of corresponding powers of X, a sufficient number of relationships is set up to enable the individual values of the impedance coefficients to be found.

By carrying out the steps indicated above for a number of cases, corresponding to progressively increasing values of n from 2 to 10, the general values of the coefficients a_1, a_2, etc. given in Equations 3 were arrived at. Further computations in which these values were applied to circuits having greater numbers of branches have shown that they apply for all values of n as large as may be desired.

The design of circuits to transmit oscillations between prescribed limits of frequency may be carried out in a direct manner by giving the series branches an appropriate structure and assigning such values to the elements that the reactances have the magnitudes $\pm aR$ at the specified limits, the coefficients a being those appropriate to the order of the branch. The design of the shunt branches follows from the reciprocal relationship involved. However the following procedure, which is based on the design of uniform wave filters is generally simpler.

A uniform line of recurrent structure, having series branches of impedance 2XR and shunt branches of impedance

$$\frac{R}{2X},$$

has its transmission band limits defined by the condition

$$X = \pm\sqrt{-1}$$

which is the same as the condition determining the band limits in the circuits of the invention. Such a line, like the circuits of the invention, is characterized by an inverse relationship between its series and its shunt impedances, the product of any pair of these impedances being constant and equal to R^2. The design of wave filters of this type, to which the name "constant K" has been given is described in U. S. Patent 1,227,113, issued May 22, 1917 to G. A. Campbell, and in U. S. Patent 1,509,184 issued September 23, 1924 to O. J. Zobel. The design formulæ of these patents may be used to compute the impedance elements of a typical section of the "constant K" line having the prescribed cut-off frequencies, and, from the impedances of this typical section, the design of the corresponding circuit of the invention may be computed by virtue of the relationship mentioned above. The impedances of the series branches are equal to half the series impedance of the related "constant K" section multiplied by the appropriate impedance coefficients of Equation 3, and the impedances of the shunt branches are found by taking twice the shunt impedance of the iterative structure and applying the appropriate coefficients.

The application of the foregoing principles will be illustrated in connection with specific circuits adapted to pass a single band of frequencies, the line structures being illustrated by Figs. 2, 3 and 4 respectively.

Fig. 2 represents a typical section of a low-pass structure, the series branches comprising simple inductances and the shunt branches simple capacities. Assuming the section illustrated to represent the r^{th} and the $(r-1)^{th}$ branches, the impedances have the values

$$j\omega L_2 = a_r XR$$

and

$$\frac{1}{j\omega C_2} = \frac{R}{a_{r-1}X}$$

respectively where ω denotes 2π times the frequency. If the cut-off frequency for which $|X|$ is unity, be denoted by f_0 it follows that

$$L_2 = \frac{a_r R}{2\pi f_0}, \quad (11)$$

$$C_2 = \frac{a_{r-1}}{2\pi f_0 R}, \quad (12)$$

and also that

$$X = j\frac{f}{f_0}. \quad (13)$$

The form of the transmission characteristic of the low-pass type of circuit is shown in Fig. 7 in which the magnitude of the received current is plotted against the frequency function f/f_0. The uniform curves 10, 11 and 12, representing the characteristics of circuits having 3, 5 and 7 branches respectively, illustrate the high degree of uniformity that exists throughout the transmission range. For comparison purposes dotted curve 15 shows the value of the voltage ratio in the prototype "constant K" wave filter having five branches. These curves are computed on the assumption that there is no energy dissipation in the energy branches, the effect of dissipation being to add to the curves for the circuits of the invention a gradual downward slope with frequency, and to reduce the high peak in curve 15.

Fig. 3 shows a typical section of a high-pass structure, the series impedance in this case being capacities and the shunt impedances inductances. Capacity C_3, representing in general the r^{th} branch impedance has the value

$$C_3 = \frac{1}{a_r 2\pi f_0 R} \quad (14)$$

and inductance L_3 of the $(r-1)^{th}$ branch has the value

$$L_3 = \frac{R}{a_{r-1}2\pi f_0}. \quad (15)$$

The frequency factor in this case has the value

$$X = j\frac{f_0}{f}. \quad (16)$$

The structure of Fig. 4 is of the band-pass type, the series branches consisting of simple resonant circuits, and the shunt branches consisting of simple anti-resonant circuits. The series branches are all resonant at the same frequency, since their impedance values contain the same frequency factor X and the shunt branches are likewise anti-resonant at the same frequency. If the two cut-off frequencies be denoted by f_1 and f_2 respectively the frequency factor has the value

$$X=j\frac{\frac{f}{f_m}-\frac{f_m}{f}}{\frac{f_2}{f_m}-\frac{f_m}{f_2}} \quad (17)$$

where

$$f_m=\sqrt{f_1 f_2}.$$

The design in this case is most readily developed by camparison with the prototype "constant K" wave filter, the values of the impedance elements of which are given by

$$L_{40}=\frac{R}{\pi(f_2-f_1)}$$

$$C_{40}=\frac{f_2-f_1}{4\pi f_1 f_2 R}$$

for the series branches, and

$$L'_{40}=C_{40}R^2,$$

$$C'_{40}=\frac{L_{40}}{R^2},$$

for the shunt branches. These values lead at once to the impedance values for the circuit of the invention

$$L_4=\frac{a_r R}{2\pi(f_2-f_1)}=\frac{a_r L_{40}}{2},$$

$$C_4=\frac{f_2-f_1}{A_r 2\pi f_1 f_2 R}=\frac{2C_{40}}{a_r}, \quad (18)$$

$$L'_4=\frac{R(f_2-f_1)}{a_{r-1}2\pi f_1 f_2},$$

and

$$C'_4=\frac{R}{a_{r-1}2\pi(f_2-f_1)},$$

the series elements L_4, C_4, as before, being assumed to represent the r^{th} branch and the shunt elements L'_4 C'_4 constituting the $(r-1)^{th}$ branch of the artificial line.

The circuits of the invention may also be adapted for multi-band transmission, an example of a structure of this type being shown in Fig. 5. Only a single section is illustrated, but this suffices to show the character of the branch impedances. The particular structure shown has three pass bands, one of which starts at zero frequency. The series impedance Z_a comprises an inductance and two anti-resonant circuits all in series, This combination is resonant at three frequencies, including zero, and is anti-resonant at three frequencies, including infinity. The shunt impedance Z_b is inversely related to Z_a, being anti-resonant at the resonance frequencies of Z_a and vice versa. It comprises a condenser and two series resonant circuits all connected in parallel. The three transmission bands are located about the three resonance frequencies of the series branch impedance, and may be placed roughly in their desired positions in the frequency scale by properly placing the resonance frequencies.

$$L'_{40}=C_{40}R^2,$$

Additional bands may be obtained by adding anti-resonant circuits to the series branch and corresponding resonant circuits to the shunt branch. The formation of the pass bands is illustrated by Fig. 6, which shows the form of the frequency factor X for the circuit of Fig. 5. The value of X increases from zero at the first resonant frequency, $f_1=0$, to infinity at the first anti-resonant frequency f_2 and passes through zero and infinity at the successive frequencies f_3, f_4, f_5 and $f_6=\infty$. The transmission ranges are defined by values of X between -1 and $+1$ and are indicated in the figure by the thickened portions of the horizontal axis.

Explicit design formulæ for multi-band circuits become extremely complicated and of little use for practical purposes. It is generally simpler to proceed towards the final design by a trial method involving successive approximations, the first step of the process being the determination of the "constant K" prototype, as in the example discussed above. The form of the branch impedances having been determined for the prescribed number of bands, a trial design of the series impedance may be made, with the resonant frequencies placed according to the desired band locations. A plot of the impedance will indicate the positions of the band limits and, if these are incorrect, the constants may be varied in successive trials until a good approximation to the desired positions is obtained. The design of the shunt impedance follows from the inverse relationship, and the final design is obtained by applying the impedance coefficients a_1, a_2 as already indicated.

What is claimed is:

1. A frequency selective transmission line comprising a plurality of reactive impedances disposed alternately in series and in shunt relation, said series impedances being of similar type whereby their reactances are constantly related at all frequencies, said shunt impedances being of inverse type to said series impedances, and the reactances of said series impedances and the susceptances of said shunt impedances being proportional to the quantity

$$\sin\frac{2r-1}{2n}\pi$$

where r denotes the order of the impedance counting from one end of the line and n is the total number of impedances.

2. A broad band frequency selective system comprising an artificial line of ladder structure, each series branch of said line including an inductance and each shunt branch including a capacity, the values of said inductances and said capacities being proportional to the quantity

$$\sin \frac{2r-1}{2n}\pi$$

where r denotes the order of a branch counting from one end of the line, and n is the total number of branches.

3. A broad band frequency selective system comprising an artificial line of ladder structure, each series branch of said line including a capacity, and each shunt branch including an inductance, the values of said capacities and said inductances being inversely proportional to the quantity

$$\sin \frac{2r-1}{2n}\pi$$

where r denotes the order of a branch counting from one end of the line, and n is the total number of branches.

4. A broad band frequency selective system comprising a ladder type artificial line each series branch of which comprises an inductance and a capacity in series and each shunt branch of which comprises an inductance and a capacity in parallel, said series branches being all resonant at a given frequency and said shunt branches being anti-resonant at the same frequency, and the inductances of said series branches and the capacities of said series branches being proportional to the quantity

$$\sin \frac{2r-1}{2n}\pi$$

where r denotes the order of a branch counting from one end of the line, and n is the total number of branches.

5. In combination with a wave source and a terminal load having equal resistive impedances, a network in accordance with claim 1 the branch impedances of which are proportioned with respect to the resistance of the terminal impedances and to the limiting frequencies of a preassigned range whereby transmission through the network in the preassigned frequency range is substantially free from irregularities due to wave reflection at the terminals.

6. A wave transmission network comprising a plurality of reactive impedances disposed alternately in series and in shunt to constitute a multi-section ladder type line, said series impedances being of similar type whereby their reactances are constantly related at all frequencies, said shunt impedances being of inverse type to said series impedances, and the reactances of said series impedances and the susceptances of said shunt impedances increasing symmetrically from both ends of the line towards the center.

7. A wave transmission network comprising a plurality of reactive impedances disposed alternately in series and in shunt to constitute a multi-section ladder type line, said series impedances being of similar type, said shunt impedances being of inverse type to said series impedances, and the reactances of said series impedances and the susceptances of said shunt impedances increasing from both ends of the line towards the center, the rate of increase diminishing towards the center.

8. A broad-band wave filter comprising a plurality of sections, the transmission bands of said sections having a frequency range in common and said sections being so proportioned that their respective band widths vary progressively from both ends towards the center of the network.

9. A broad-band wave filter comprising a plurality of sections, the transmission bands of said sections having a frequency range in common and said sections being so proportioned that their respective band widths diminish progressively from both ends towards the center of the network.

10. A transmission network comprising a plurality of sections, each of said sections being adapted to transmit selectively oscillations in a broad frequency range and said sections being so proportioned with respect to each other that the transmission bands of adjacent sections have a frequency range in common and vary progressively in width from both ends towards the center of the network.

11. The method of eliminating transmission irregularities in a broad-band multi-section wave filter which comprises progressively increasing the transmission band widths of adjacent sections from the center of the filter towards both ends.

12. A low pass wave filter comprising a plurality of sections, each section including series inductance and shunt capacity, the inductances and capacities of said sections being so proportioned that the cut-off frequencies of the respective sections increase progressively from the center of the filter towards both ends.

13. The method of eliminating transmission irregularities in a broad-band multi-section wave filter which comprises progressively increasing the transmission band widths of adjacent sections from the center of the filter towards both ends and varying the values of the nominal characteristic impedances from one section to another.

14. A broad-band wave filter comprising a plurality of sections, the transmission bands

of said sections having a frequency range in common, and said sections being so proportioned that their band widths vary progressively from both ends toward the center of the network and their nominal characteristic impedances change in value from section to section.

In witness whereof, I hereunto subscribe my name this 28th day of June, 1929.

WILLIAM R. BENNETT.