Oct. 16, 1962 L. G. WHITESELL 3,058,662

ELECTRIC ANALOG MULTIPLIER

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Fig. 2

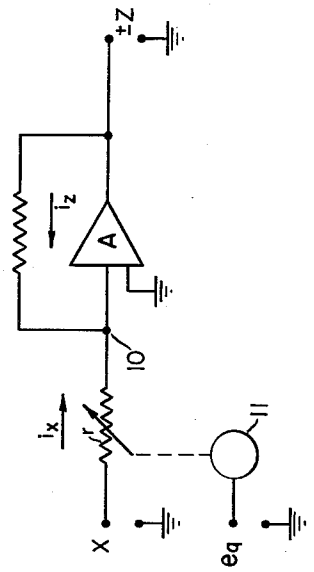


Fig. 3

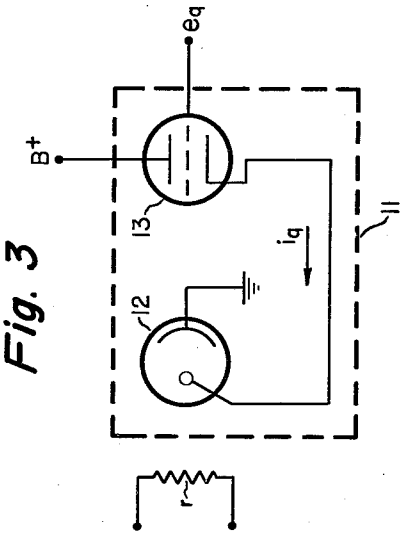


Fig. 1

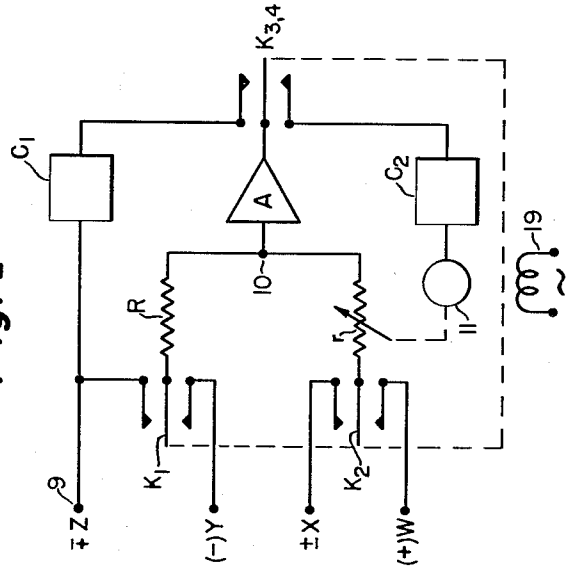
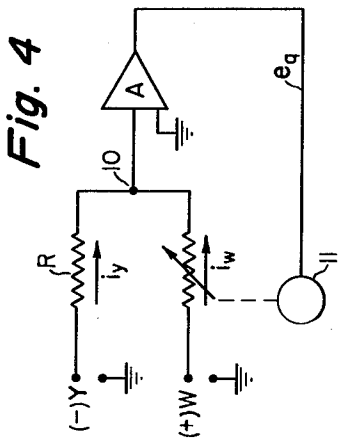


Fig. 4



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Fig. 5

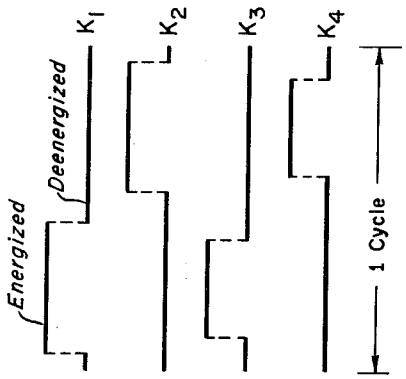


Fig. 6

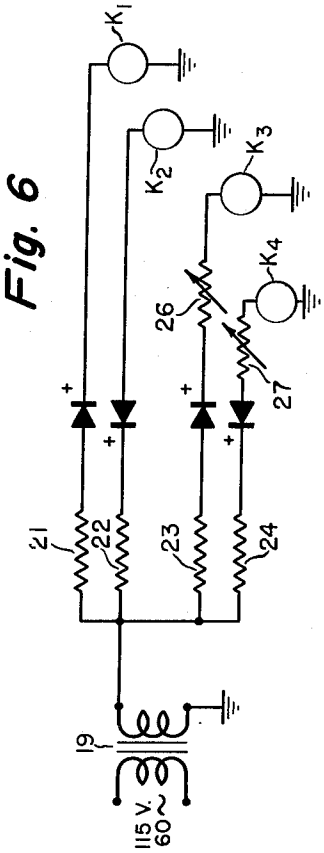
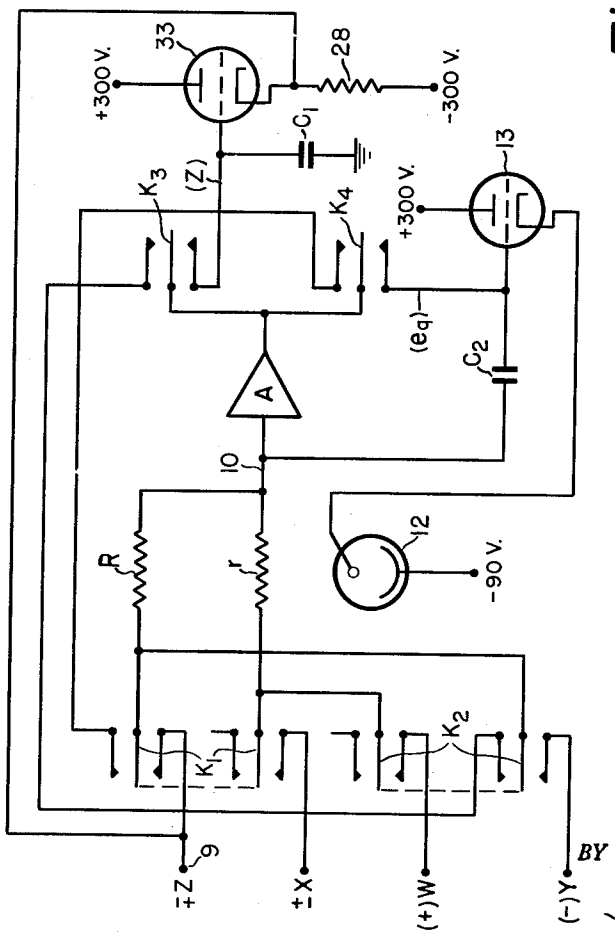


Fig. 7



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United States Patent Office 3,058,662 Patented Oct. 16, 1962

3,058,662

ELECTRIC ANALOG MULTIPLIER

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Filed Feb. 29, 1960, Ser. No. 11,804
9 Claims. (Cl. 235—194)

This invention relates to electronic multipliers, and more particularly to analog multipliers capable of delivering output signals which are the products or quotients of the amplitudes of input signals.

A primary object of the invention is to provide an electronic analog multiplier which produces an output signal which is the quotient of the product of the amplitudes of two input signals by the amplitude of a third input, whether the several inputs be of variable or fixed amplitude. Another object is to provide an electronic analog multiplier which compares with servomultipliers in simplicity, stability, and accuracy, while at the same time avoiding the servomultiplier's limitation of speed and mechanical wear in the potentiometers. A further object is to provide an electronic analog multiplier which presents no adjustment or maintenance problems in contrast with most conventional multipliers of this type. Still another object is to provide novel components and combinations of such components suitable for use with electronic analog multipliers which afford superior accuracy and simplicity, characteristics which they carry forth in other applications. An additional object is to provide simplified and rugged circuitry for these components and combinations of components. Other and more particular objects will be apparent as the description proceeds.

In accordance with the invention, an electronic analog multiplier is provided which is capable of delivering to an output terminal an output voltage signal which is the quotient of the product of the amplitudes of first and second input signals by the amplitude of a third input signal. The inventive multiplier essentially comprises an operational amplifier having a fixed resistance and a variable resistance connected into its summing junction. The variable resistance is made responsive to a control signal voltage, which in accordance with the preferred form of the invention operates to vary the intensity of a light source which emits light onto the variable resistance, the variable resistance having a resistance value which varies in response to the intensity of light which it receives.

This assembly of operational amplifier, fixed resistance, variable resistance, and means for varying the variable resistance, is arranged in two different circuit configurations, with means being provided for cyclically effecting alternation between the two configurations. In the first configuration, the fixed resistance is connected in feedback from the output of the operational amplifier into its summing junction. The variable resistance receives the first input signal and transmits it via this resistance to the summing junction. At this time, control of the variable resistance is effected by means of a control signal, originating from the second circuit configuration. In this first configuration, the output signal of the operational amplifier is taken as the output of the electronic analog multiplier.

In the second circuit configuration, the fixed and variable resistance are again connected into the summing junction of the operational amplifier, but the fixed resistance receives the second input signal (which is to be multiplied by the first input signal) while the variable resistance receives the third input signal (which is to be divided into the first and second inputs). The control signal, which governs the resistance of the variable resistance, is at this time connected to receive the output signal of the operational amplifier.

Thus, signals derived in the second configuration are employed to produce the output signal during the first configuration.

Further details and advantages of the invention will be apparent from the ensuing description when read in conjunction with the attached drawings wherein:

FIGURE 1 is a schematic circuit diagram showing one embodiment of the inventive electronic multiplier with appropriate switch means for effecting alternation of the circuit components between the first and second figurations, as described above;

FIGURE 2 shows in schematic form the first circuit configuration;

FIGURE 3 is a circuit suitable for changing a variable resistance in response to a control signal, as may be used in the analog multiplier;

FIGURE 4 schematically shows the second multiplier configuration;

FIGURE 5 shows the sequence of switching operations as may be employed in cyclically effecting alternation of the circuit components of the first configuration (FIGURE 2) and the second configuration (FIGURE 4);

FIGURE 6 is a schematic circuit of a system for effecting this cyclic alternation; and

FIGURE 7 is the preferred form of the inventive electronic analog multiplier.

For the purpose of facilitating an understanding of the operation of the inventive system, attention is first invited to FIGURE 2. Operational amplifier A is associated with a fixed resistance R connected from the output of amplifier A to the summing junction **10** of the amplifier. An input resistance, in this particular case variable resistance *r*, is likewise connected into summing junction **10** and receives a first input voltage.

Operational amplifier A is the basic element of an electronic analog computer, and amplifies an input D.C. voltage received at its summing junction **10** by a large factor, ranging from several thousand to several million. The input connection to amplifier A is usually made directly to the grid of a vacuum tube so that essentially no current can flow into or out from this grid.

In the circuit shown in FIGURE 2, if amplifier A has a high gain, junction **10** will be very nearly at ground potential if the output Z at output terminal **9** is at a voltage within the operating limits of amplifier A. Thus, essentially all of the input voltage X is impressed across resistance *r*. This produces a current i_x. Similarly, voltage Z is impressed across resistance R, thereby producing i_z. Since no current can enter or leave the amplifier input grid, i_x must equal i_z. This implies the relation

$$Z=R/rX$$

If resistance *r* is made to have a resistance value which is a function of a control voltage e_g, then, from the above formula, the following relationship exists:

$$Z=\frac{Re_gX}{K}$$

In this way, an operational amplifier is made to perform as a multiplier.

The preferred method of making resistance *r* a function of control D.C. voltage, or a second input voltage, is to employ a resistance which varies in response to the intensity of light which it receives. Then, if the control voltage is employed to operate a light source which emits light onto the variable resistance, the resistance of the latter will be responsive to the control signal. Many light-variable resistances are well known and are usually referred to as "photoresistive cells." Typical photoresistive cells are made of semiconductor materials such as

the elements selenium, silicon and germanium; thallous sulfide; cadmium or lead sulfide, selenide or telluride. A suitable light source is a glow modulator tube of the type used in facsimile equipment.

A suitable circuit for a photoresistive cell and a glow modulator tube is shown in FIGURE 3. Glow modulator tube **12** radiates light onto cadmium sulfide resistance r. Tube **12** receives a current signal, e_g, from cathode follower **13**, which converts the input voltage signal into a proportional current signal i_g. Since the resistance of photoresistive cell r varies with the amount of light received, since the amount of light varies with the control current signal i_g, and since the control signal i_g varies with the control voltage signal e_g, the value of resistance r is thus responsive to the control signal e_g.

The system of FIGURE 3 when combined as control means **11** in FIGURE 2, affords an electronic analog multiplier capable of multiplying input signal X by input e_g. However, for purposes requiring the highest degree of accuracy and stability, as well as for applications wherein the quotient of a third signal is required, a more elaborate circuit is necessary.

The circuit shown in FIGURE 4, when combined with the circuit of FIGURE 2, affords the additional accuracy, stability and versatility.

Referring to FIGURE 4, the operational amplifier A is shown in a configuration where fixed resistance R receives a second input signal Y and is connected into summing junction **10** of operational amplifier A. The third input signal W, which is to be divided into the product of X and Y, is transmitted to variable resistance r which likewise is connected into summing junction **10**. The magnitude of resistance r is controlled by controller **11** which receives the control signal e_g as the output of operational amplifier A.

Since by the nature of the operational amplifier, i_w must equal $-i_y$, the output e_g of amplifier A will cause variable resistance r to adjust in magnitude until the sum of the currents at summing junction **10** is zero. This will result in a ratio of the resistances being equal to the ratio of the two voltages

$$\frac{R}{r}=\frac{-y}{w}$$

Notice that the components used in FIGURE 2 and FIGURE 4 are the same. Only the arrangement of connections and voltages is different. If a means is provided for switching the connections so that circuits of FIGURES 2 and 4 are alternately produced, and especially if a memory for the values of e_g and Z from one switching cycle to the next is also included, the resistance ratio, $R/r=y/w$ produced by FIGURE 4 can be used to provide the relation between X and Z in FIGURE 2. The value of Z obtained by such an arrangement is:

$$Z=\frac{-R}{r}\cdot X \qquad \text{(FIGURE 2)}$$

$$\frac{R}{r}=\frac{-y}{w} \qquad \text{(FIGURE 4)}$$

so

$$Z=+\frac{X\cdot y}{w}$$

Thus the multiplier will not only provide a product, but it can provide a quotient or a combined product and quotient.

Referring to FIGURE 1, a switching arrangement which is capable of affording the foregoing characteristics is shown. With the relay contacts K_1, K_2, and K_3, K_4 closed downward, the circuit is in the second configuration (FIGURE 4), and resistance r is adjusted until the ratio of fixed resistance R to variable resistance r is equal to the negative ratio of input signal y to input signal w. With the contacts closed upward, the circuit is in the first configuration (FIGURE 2) and the ratio of resistance R to r acts to produce an output voltage Z which is the product of the amplitudes of input voltage signals X and y divided by the amplitude of input signal w.

In order to store output signal Z and control signal e_g during switching of the relays, memory units C_1 and C_2, respectively, are provided.

Turning now to FIGURE 7, a schematic diagram is provided of the circuit of FIGURE 1, embodying as control element the circuit of FIGURE 3 and employing as memory units C_1 and C_2 the capacitors C_1 and C_2. These capacitors provide the memory action, storing voltages X and e_g between switching cycles. By being returned to the summing junction **10** of operational amplifier A, C_2 serves the additional function of providing a phase lead connection to compensate for the time lag due to the finite mobility of carriers in the cadmium sulfide variable resistance r. This provides excellent stability for the circuit.

Relays K_1 and K_3 operate to provide the circuit configurations of FIGURE 2. Relays K_2 and K_4 give the connections for the circuit of FIGURE 4. During the transition interval when neither pair of relays is energized, provision is made through normally closed contacts to connect the output and input of amplifier A through resistance R. This avoids permitting amplifier A to drift into saturation.

Turning now to FIGURE 5, the preferred sequence of energizing and deenergizing the several relays is shown. This sequence minimizes switching noise in the output voltage Z.

Control over the relay switching sequence can be established by means of the circuit shown in FIGURE 6. Transformer **19** is connected into a 60-cycle A.C. line, and the output is taken through four parallel connections to feed each of the four relay coils K_1, K_2, K_3, and K_4. K_1 receives power through fixed resistance **21** and a selenium rectifier, while relay coil K_2 receives power through a similar fixed resistance **22** and another selenium rectifier connected in the opposite polarity. Relay coils K_3 and K_4 are connected in similar manner to relays K_1 and K_2, with the addition of variable resistances, **26** and **27**, respectively, which are used in regulating the portion of each cycle during which relays K_3 and K_4 are energized. Alternatively, a single, 3-pole, double throw, break-before-make type of relay may be employed.

A wide range of cyclical frequencies can be employed for effecting circuit alteration between the two configurations, but it is usually convenient to employ 60-cycle line frequency. Experimental results from an embodiment constructed according to FIGURE 7 indicate exceptional high accuracy and stability. The noise level was no greater than about 50 millivolts, and the various input signals can have a frequency of up to about 10 cycles per second or even higher. The output drift rate is determined by the quality of operational amplifier A and a good chopper stabilized amplifier greatly aids in this function.

The experimental model employed a fixed resistance R of 1.0 megohm, resistance **28** of 25,000 ohms, C_1 of 0.005 UF, C_2 of 0.01 UF, tubes **13** and **33** of type 12AT7 or 6201 twin triodes with both sections in parallel. Amplifier A was a Philbrick model K2–X, the glow tube was a Sylvania type IB59/R1130B, and the cadmium sulfide cell was a Clairex type CL–2. Relays K_1 and K_2 were Stevens-Arnold millisec type B–12, and relays K_3 and K_4 were type B–11.

From the foregoing description it is evident that there has been provided an electronic analog multiplier which fully satisfies the objects of the invention. While the invention has been described in connection with one specific embodiment thereof, it is clear that many alternatives, modifications, and variations will be apparent to those skilled in the art in light of the foregoing description. Accordingly, it is intended to embrace all such alternatives, modifications and variations as fall within the spirit and broad scope of the invention.

I claim:

1. An electronic analog multipler apparatus capable of delivering to an output terminal an output signal which is the quotient of the product of the amplitudes of first and second input signals by the amplitude of a third input signal, comprising in combination: an operational amplifier having a summing junction and delivering the output signal; a fixed resistance and a variable resistance each connected into said summing junction, said variable resistance being responsive to a control signal; means for alternatively connecting (*a*) the first input signal into said variable resistance and the output signal of said operational amplifier into both said fixed resistance and said output terminal, and connecting (*b*) the second input signal into said fixed resistance, the third input signal into said variable resistance, and the output signal from said operational amplifier as a control signal for said variable resistance; and means for cyclically effecting said alternation.

2. Apparatus of claim 1 including means for storing the output signal and the control signal during said alternation.

3. Apparatus of claim 2 wherein said means for storing the signals are capacitors, the capacitor for storing the control signal being connected to the summing junction of the operational amplifier.

4. An electronic analog multipler apparatus capable of delivering to an output terminal an output signal which is the quotient of the product of the amplitudes of first and second input signals by the amplitude of a third input signal, comprising in combination: an operational amplifier having a summing junction and delivering the output signal; a fixed resistance and a variable resistance each connected into said summing junction, said variable resistance being responsive to the intensity of light received thereby; a light source emitting light to said variable resistance in response to a control signal thereto; means for alternatively connecting (*a*) the first input signal into said variable resistance and the output signal of said operational amplifier into said both fixed resistance and said output terminal, and connecting (*b*) the second input signal into said fixed resistance, the third input signal into said variable resistance, and the output signal from said operational amplifier as a control signal to said light source; and means for cyclically effecting said alternation.

5. Apparatus of claim 4 including means for storing the output signal and the control signal during alternation.

6. Apparatus of claim 5 wherein said means for storing the signals comprise capacitors, the capacitor for storing the control signal being connected to the operational amplifier summing junction.

7. Apparatus of claim 4 including a cathode follower connected to said light source for converting a control voltage signal to a control current signal for said light source.

8. An electronic analog multiplier apparatus capable of delivering to an output terminal an output signal which is the product of the amplitudes of two input signals, comprising in combination: an operational amplifier having a summing junction and delivering the output signal; a fixed resistance connected from the output of said operational amplifier into the summing junction thereof; a variable resistance receiving one of said input signals and connected into said summing junction, said variable resistance being responsive to the intensity of light received thereby; and a light source receiving the second of said input signals and emitting light to said variable resistance of an intensity proportional to said second input signal, whereby the output signal at the output terminal of said operational amplifier is the product of the amplitudes of said first and said second input signals.

9. Apparatus of claim 8 including a cathode follower connected to said light source for converting a control voltage signal to a control current signal for said light source.

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