United States Patent [19]

Baer

[11] **3,722,153**

[45] **Mar. 27, 1973**

[54] **STRUCTURAL SYSTEM**

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[21] Appl. No.: **34,163**

[52] **U.S. Cl.** **52/81,** 52/648, 52/DIG. 10

[51] **Int. Cl.** **E04b 1/32**

[58] **Field of Search** 52/81, 80, 79; 135/1

[56] **References Cited**

UNITED STATES PATENTS

3,114,176	12/1963	Miller	52/81
3,468,082	9/1969	Hadley	52/81
3,611,620	10/1971	Perry	52/81
2,987,318	6/1961	Hammer	52/81

FOREIGN PATENTS OR APPLICATIONS

1,116,877	1956	France	52/81
1,458,056	1966	France	52/DIG. 10
1,196,348	1965	Germany	52/81
1,484,634	6/1967	France	52/81

OTHER PUBLICATIONS

Mathematical Models by Cundy & Rollett, Oxford Press 1961. pages 76–83, 86–99, 120, 121 Table II, and 144–151

Space Arid Structures by John Borrego; MIT Press; 1968; pages 19, 88, 89 and 90.

Dome Book 2, pages 10–12, 1971 distributed by Random House

Useful Curves and Curved Surfaces, Architectural Record; April 1958; pages 245, 247, 249

Mathematical Models; 1962, Oxford University Press, page 111

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Assistant Examiner—Henry E. Raduazo
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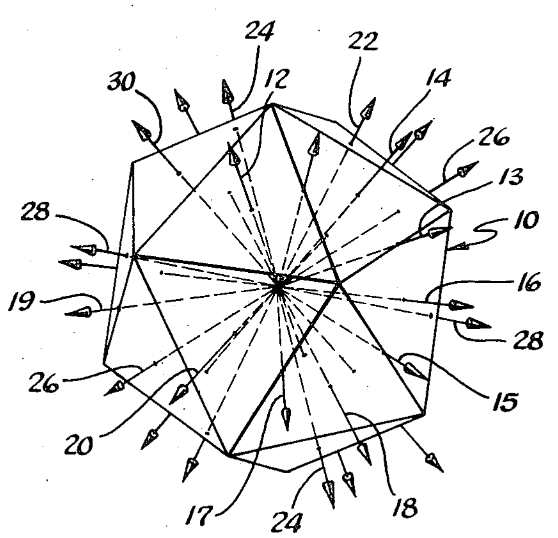
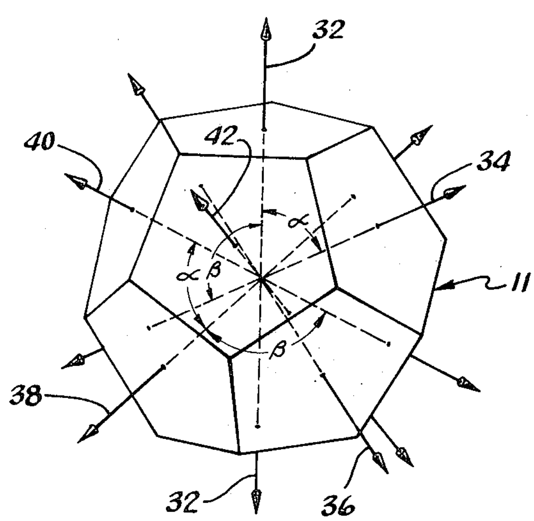
[57] **ABSTRACT**

A structural system in which the structural members are interconnected so that they are parallel to the lines of the star of the faces of an icosahedron. The system includes structural members parallel to the lines of the star of the vertices of an icosahedron and also includes structural members parallel to the lines of the star of the midpoints of the edges of an icosahedron.

10 Claims, 15 Drawing Figures





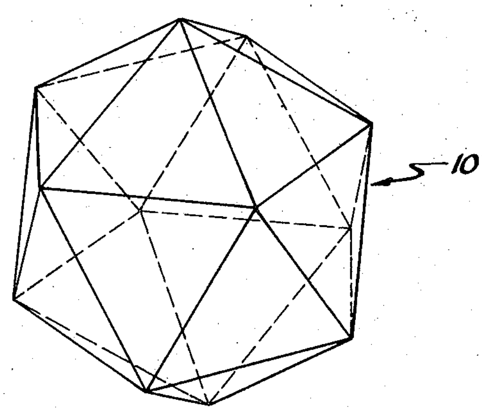


FIG_1

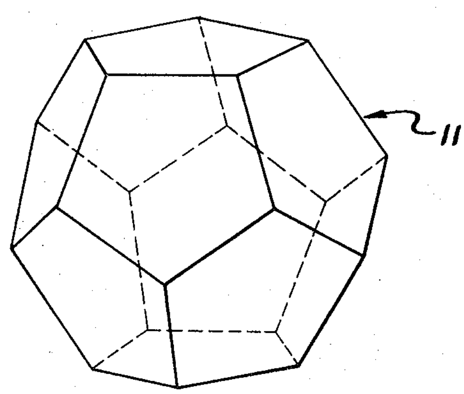


FIG_2

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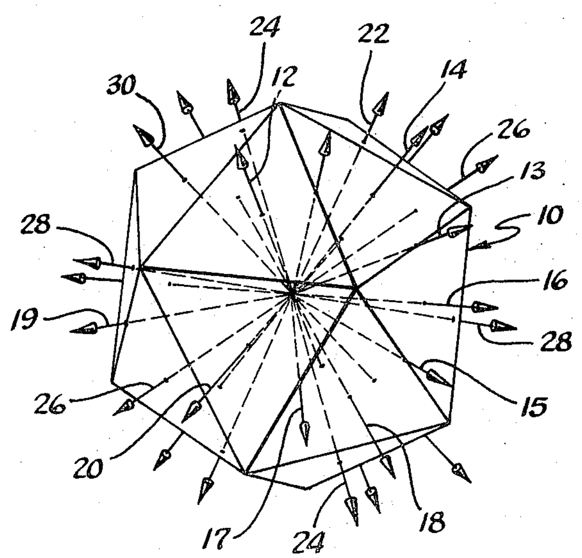

FIG. 3A

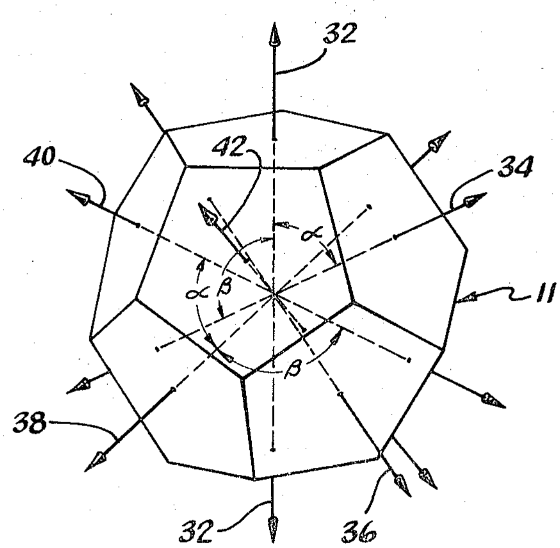

FIG. 3B

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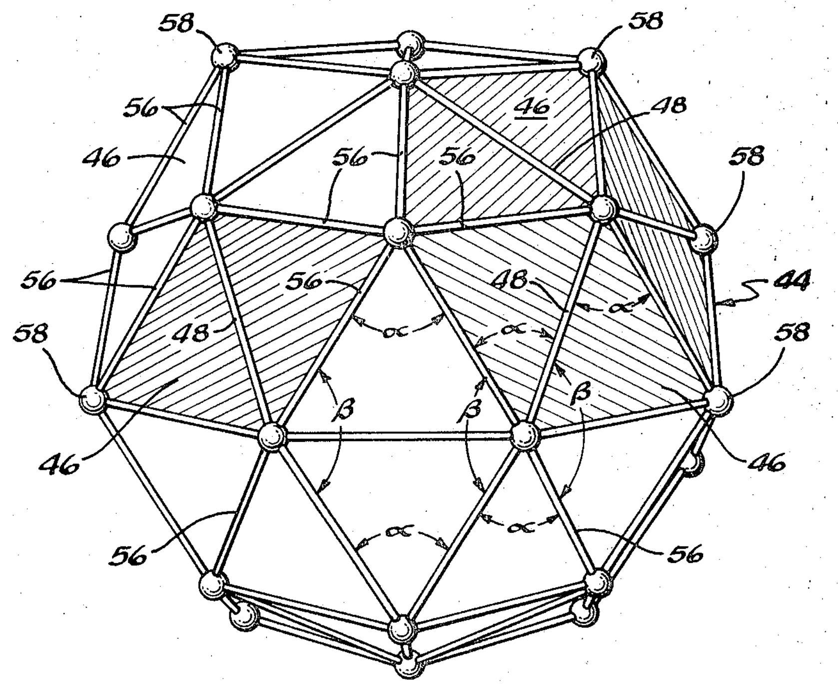

FIG_4

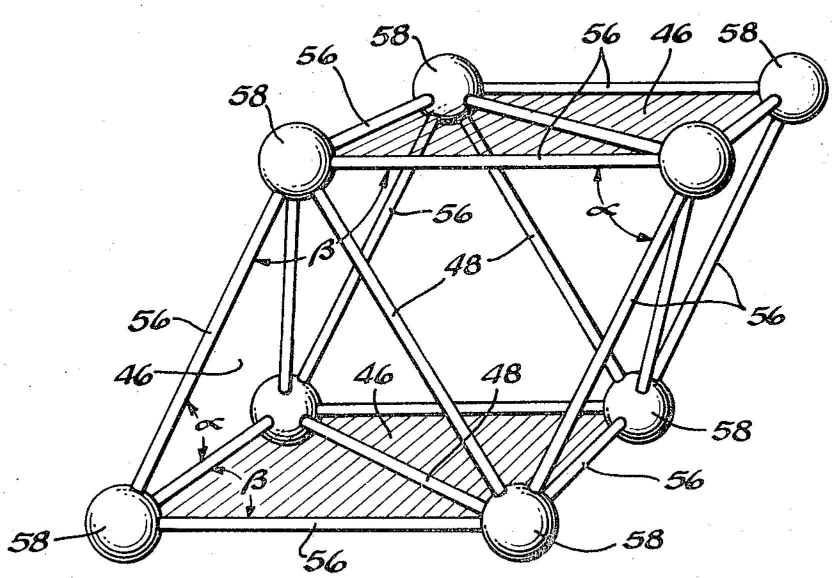

FIG_6A

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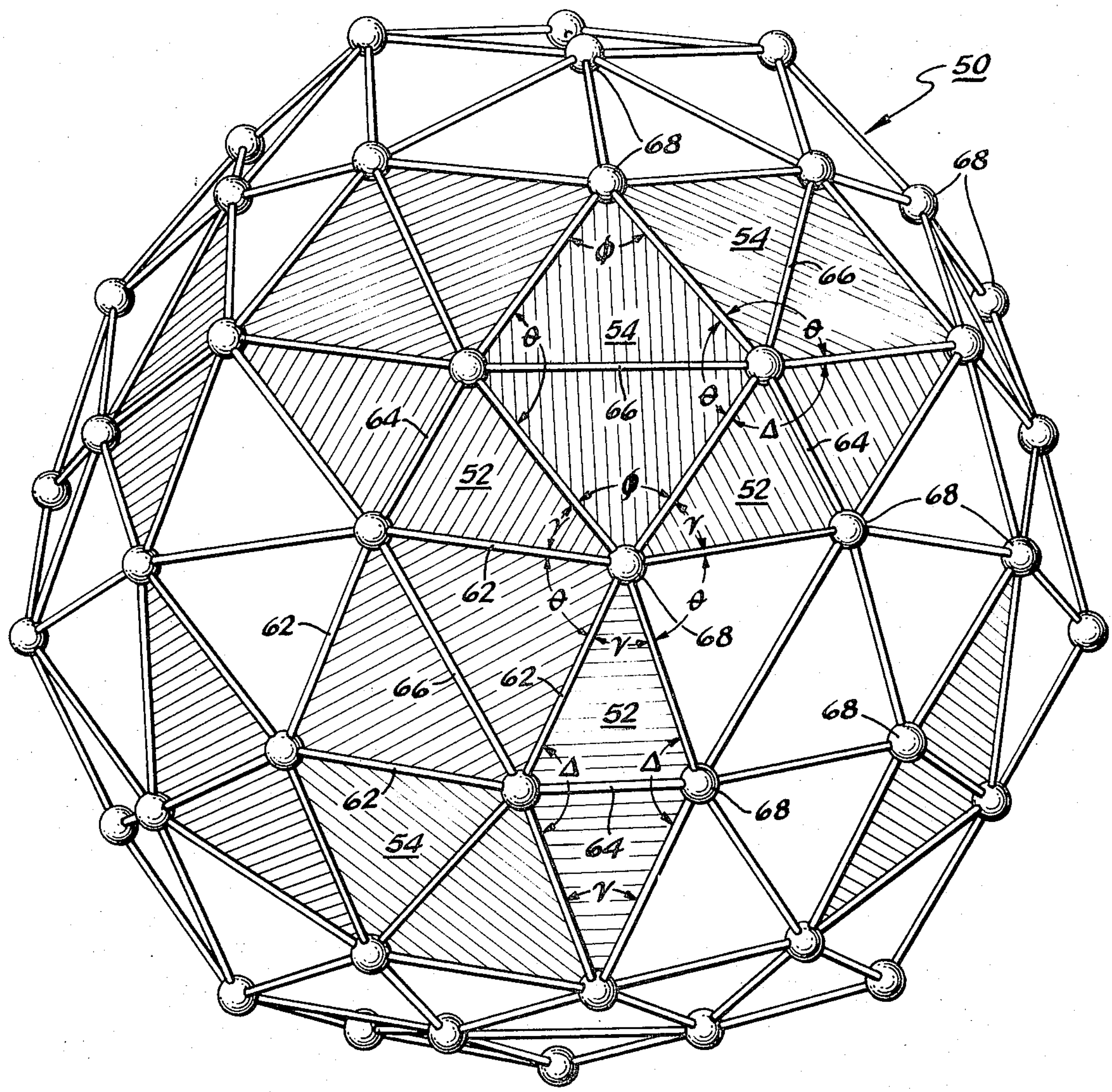

FIG_5

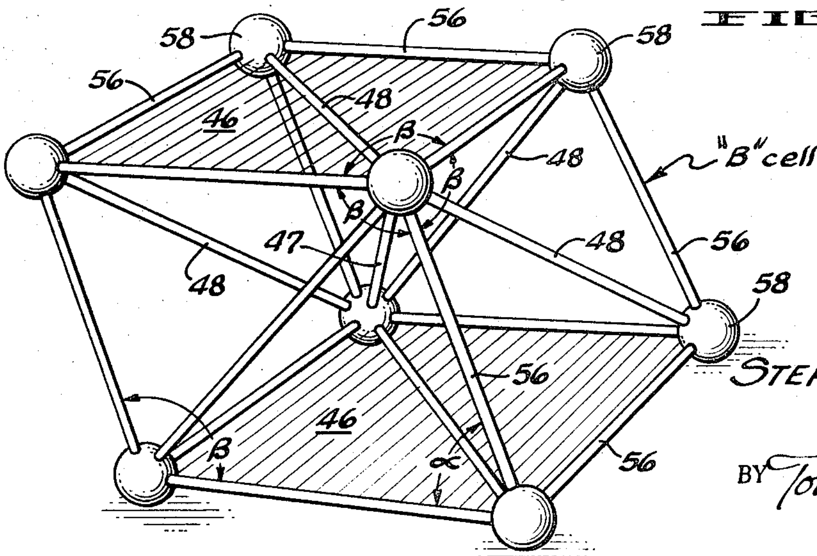

FIG_6B

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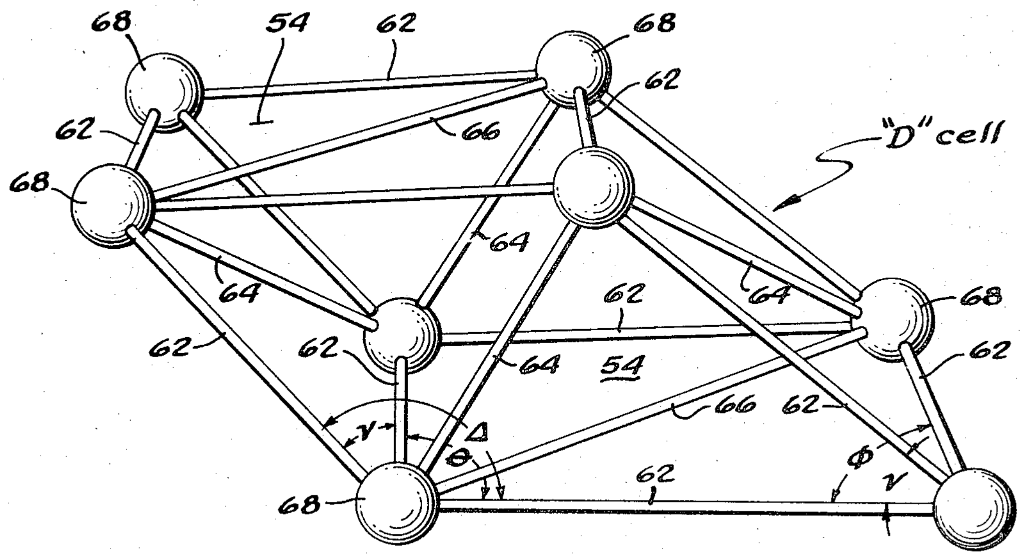

FIG_7D

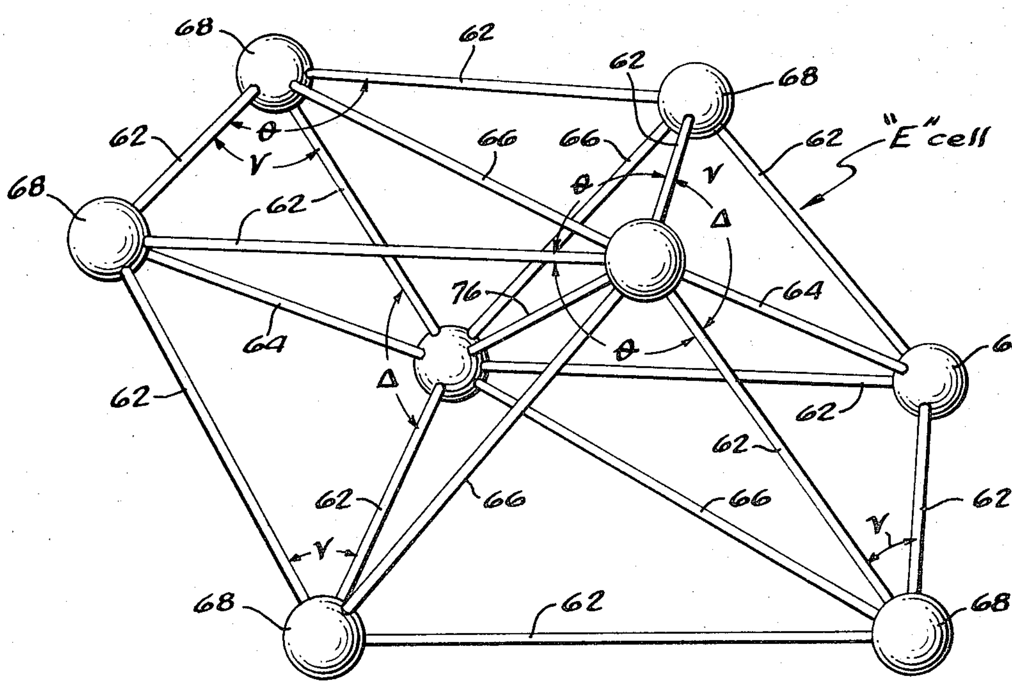

FIG_7E

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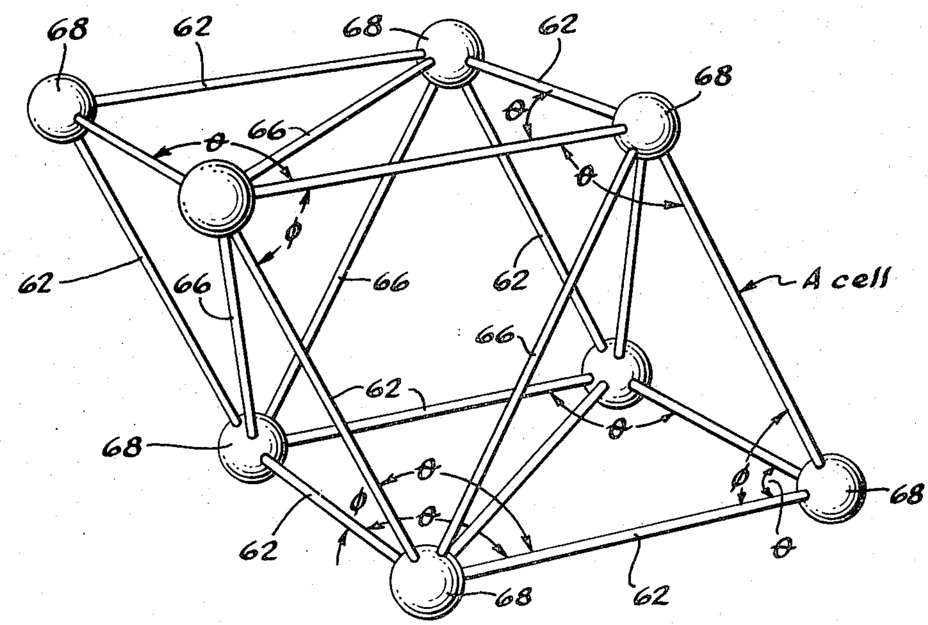

FIG_7A

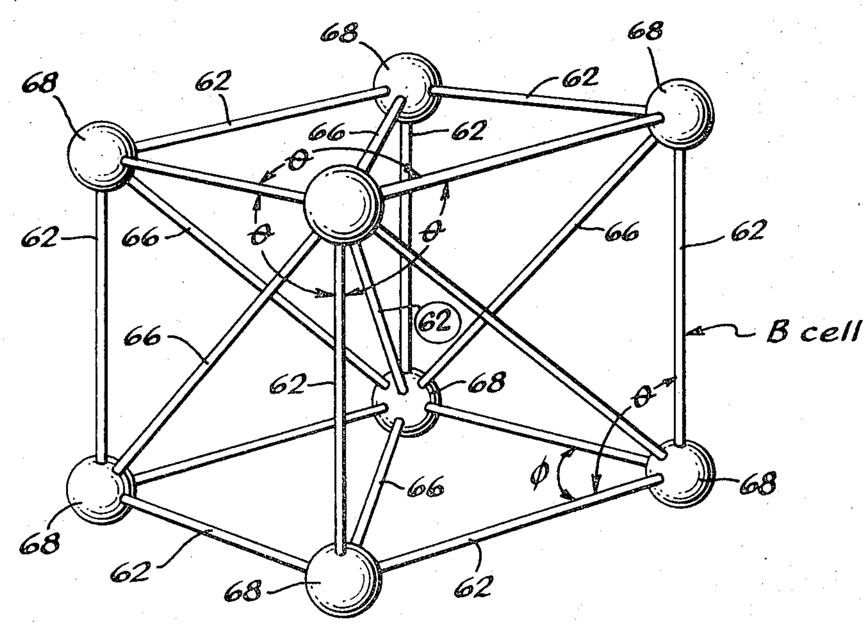

FIG_7B

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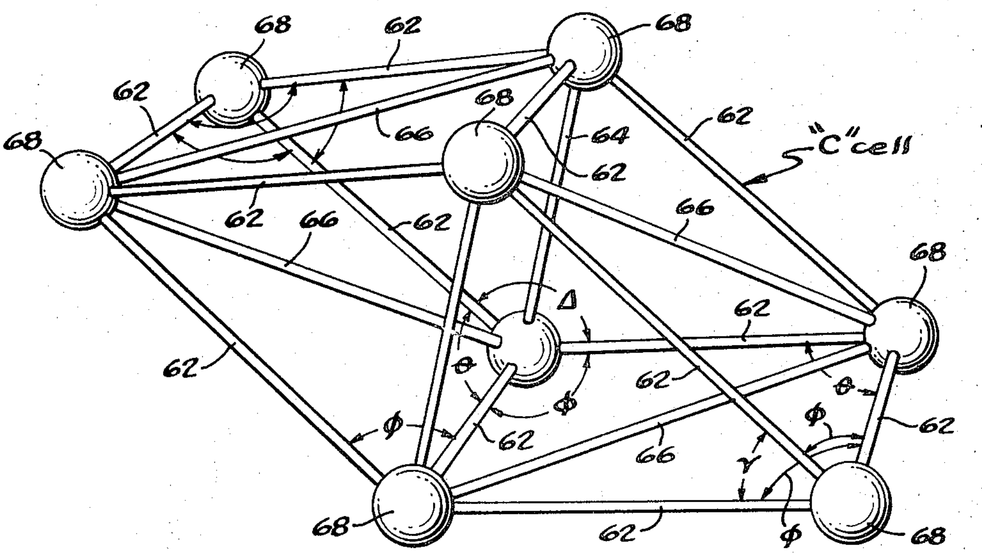

FIG. 7C

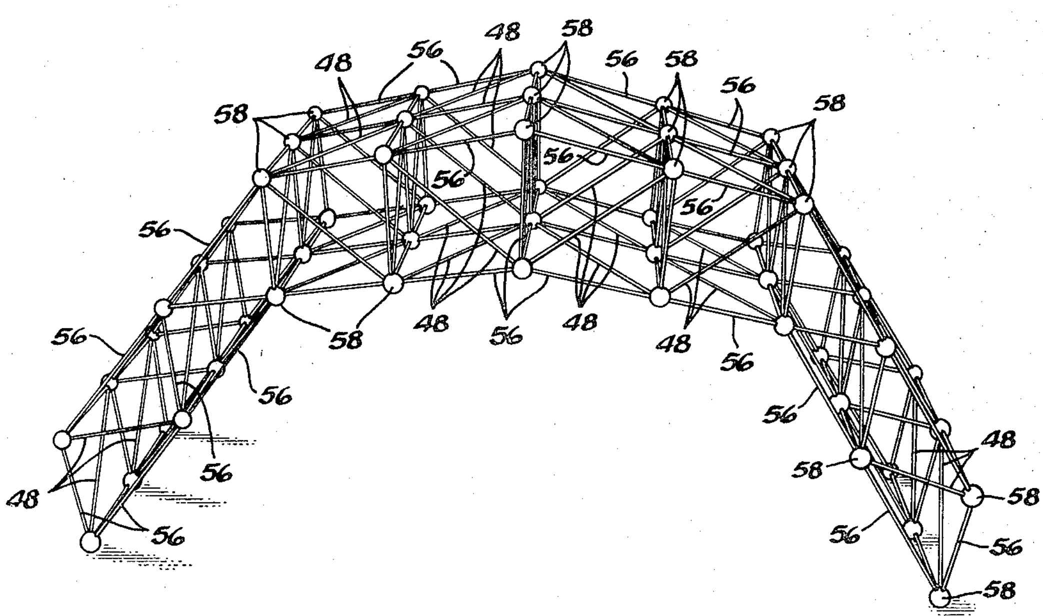

FIG. 8

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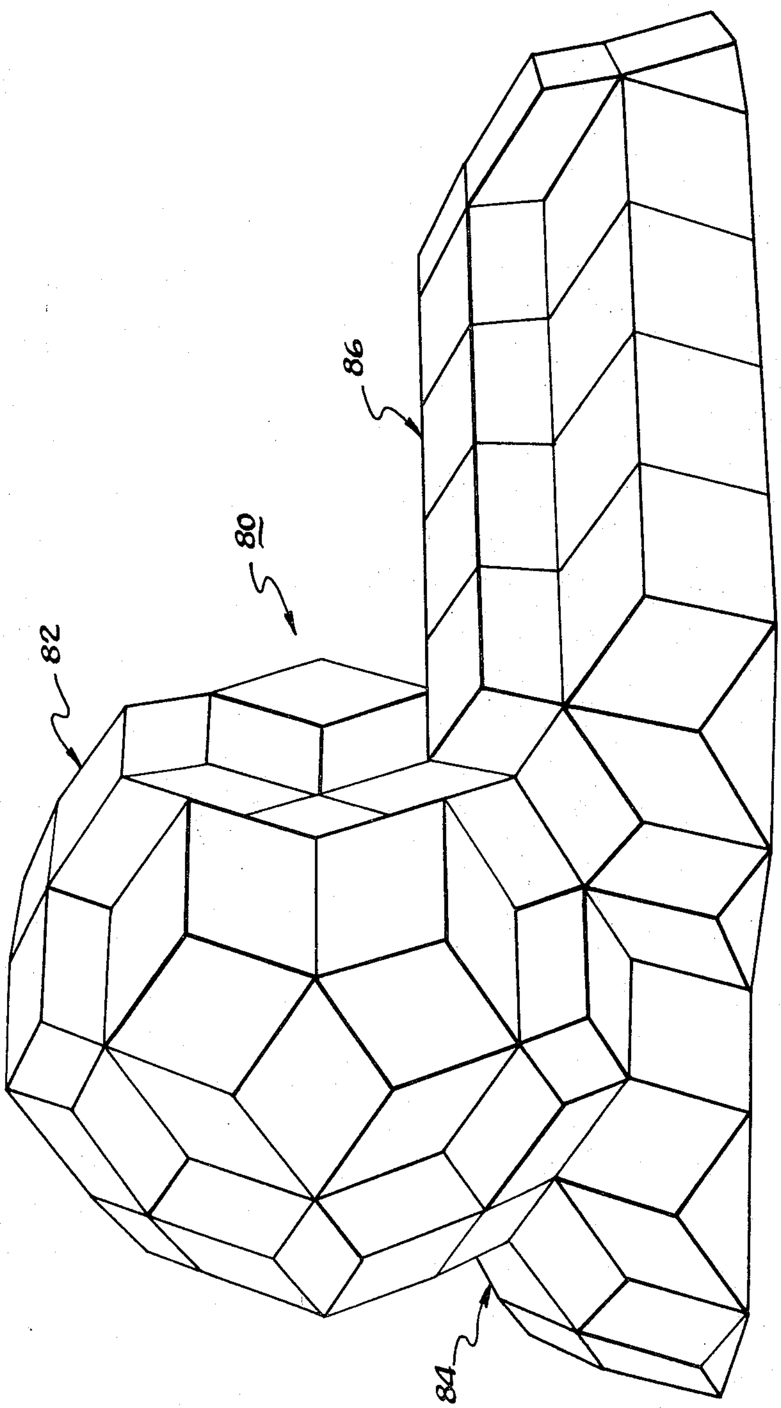

FIG_9

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STRUCTURAL SYSTEM

This invention relates to building structures and, more particularly, to a structural system utilizing the five-fold symmetries of the icosahedron and its dual, the dodecahedron.

The icosahedron and the dodecahedron are the two most complex of the five regular Platonic polyhedra, the others being the tetrahedron, the cube, and the octahedron. The regular Platonic polyhedra are the only ones having faces which are equal regular polygons and having equal vertices or corners. These polyhedra have fascinated man since before the time of the ancient Greeks.

Plato believed that all objects are composed of four elements: earth, air, fire, and water. The fundamental particles of fire were supposed to have had the shape of the tetrahedron, those of air the octahedron, those of earth the cube, and those of water the icosahedron. The fifth shape, the dodecahedron, was reserved by God for the shape of the Universe itself. These shapes are important and fascinating because they are the distilled patterns of our space; they are conjunctions of regularity and complexity. Our world is filled with many variations and combinations of a relatively few different patterns. There are only three dimensions and five regular polyhedra. The icosahedron with 20 sides and the dodecahedron with 12 sides have more corners and more faces than the other three. They are the only regular polyhedra in which the number "five" occurs: there are five triangles which meet to form each of the vertices of the icosahedron and there are five sides of each face in the dodecahedron.

The tetrahedron, cube, and octahedron and their symmetries often appear in nature and in man's work. Crystals of sodium bromate are in the form of tetrahedra; sodium chloride, common salt, has cubic crystals; and alum crystals are in the form of octahedra. These three simple shapes have been used extensively by man. The cube of course is used as a container because of its convenient geometric properties, that is, cubic containers will pack together and the edges of a cube may be extended to form different shapes having the same symmetries. The tetrahedron and the octahedron have been used extensively in constructing space frames and rigid structures.

The dodecahedron and the icosahedron have not been so utilized. Because each of them has five-fold symmetry, they cannot occur as crystals. Crystals are built up of molecules which are located in systems of regular points and it is impossible for such a system to have five-fold symmetry. This may be demonstrated if one attempts to cover a plane surface with regular pentagonal tiles. There will always be gaps between some of the tiles in such a case. Other regular tiles having three, four or six sides (triangular, square, and hexagonal) will completely cover a plane surface. There are crystals, for instance, $MoAl_{12}$ which contain icosahedral elements within their component structure but their basic symmetries are not icosahedral. In these cases the icosahedral elements merely "go along for the ride."

The icosahedron and the dodecahedron also appear in the structure of the skeletons of certain tiny sea animals called radiolarians, for example, Circogonia icosahedra and Circorrhegma dodecahedra. Among man-made objects the icosahedron and dodecahedron are most frequently used as decorations or ornaments. Christmas decorations are sometimes made in the shape of stellated dodecahedra. Some desk calendars are made in the shape of a dodecahedron with each month occupying one of the 12 faces. During the last 15 years the icosahedron and dodecahedron have appeared as surface patterns in the arrangements of members of the now famous Geodesic Domes of R. Buckminster Fuller. Reference is made, for example, to the illustrations in Fuller's U.S. Pat. No. 2,682,235 of June 29, 1954.

The present invention is radically different from the process one would use in making icosahedral ornaments or Goedesic domes. The present invention involves utilizing the five-fold symmetry of the dodecahedron and icosahedron for a novel crystal-like structural system. It has already been stated that five-fold symmetry is impossible in natural crystal growth because there is no intelligence to guide the growth of the structure. My invention is a system and structure which allows one to exploit the simplicity and elegance of the natural crystal in order to construct structures far more complex and beautiful than possible with inorganic crystalline shapes.

Definition of Terms

In order to more readily comprehend the explanation which follows, certain of the terms which may be unfamiliar are defined below.

Zonohedron — a polyhedron with every face a polygon composed of pairs of equal parallel opposite sides.

Zone — a set of edges in a zonohedron that are all parallel to one another.

Zome — a man made structure derived from zonohedra, where the zones of the zonohedron may be stretched or shrunk or removed to produce, if desired, an asymmetric dome shaped structure.

Stretching a Zone — any zonohedron or zone may have its shape altered without altering angles by stretching the set of lines and, thus, a band of faces girdling the figure. The stretching may be accomplished by lengthening the individual members or by adding more components.

Six Zone Truss — a truss system whose principal members are substantially equal in length and parallel to the lines of the star formed by the diameters through the vertices of an icosahedron (or through the midpoints of the pentagonal faces of a dodecahedron).

Ten Zone Truss — a truss system whose principal members are substantially equal in length and parallel with the lines in a star created by the diameters through the midpoints of the triangular faces of an icosahedron.

Thirty One Zone System — a structural system utilizing members which are parallel to the lines in the three stars of the vertices, face midpoints, and edge midpoints of the icosahedron or its dual the dodecahedron.

IN THE DRAWINGS

FIG. 1 is a perspective view of an icosahedron;

FIG. 2 is a perspective view of a dodecahedron;

FIG. 3A is a perspective view of an icosahedron showing the 10 diameters through the midpoints of the faces comprising the 10 zone star of the icosahedron and some of the 15 diameters bisecting the edges thereof;

FIG. **3B** is a perspective of a dodecahedron showing the six diameters through the midpoints of the faces comprising the six zone star of the dodecahedron;

FIG. **4** is a perspective view of a triacontahedron, a simple 30 sided structure utilizing the principles of the present invention;

FIG. **5** is a perspective view of an enneacontahedron, a 90 sided structure showing an application of the 10 zone system of the present invention;

FIGS. **6**A and B show two basic cells of the six zone truss systems;

FIGS. 7A – E show five basic cells of the 10 zone truss systems;

FIG. **8** is a perspective view of an extended truss forming an arch utilizing the basic cells of the six zone system; and

FIG. **9** is a perspective view of a structure illustrating the application of panels of the 10 zone structural system to a domed structure.

Description of the Preferred Embodiment

Referring now more particularly to the drawings in which the same reference numerals refer to identical parts in each of the several views, the present invention utilizes the relationships and angles of the 20 faces, 30 edges, and 12 vertices of the icosahedron shown in FIG. **1** and of the dual of the icosahedron, the dodecahedron, illustrated in FIG. **2**. The dodecahedron has 12 faces, 20 vertices, and 30 edges. These two polyhedra are duals or reciprocals of each other. Simply stated, the principle of duality is that each of the vertices of a regular polyhedron may be replaced by a polar plane. Thus, the polyhedra illustrated have many of the same symmetries. Much of the following description will be equally applicable to both the icosahedron and the dodecahedron because of their relatiOnship as duals.

An icosahedron is illustrated generally at **10** in FIGS. **1** and **3**A. The 10 diameters which pass through opposite faces of the icosahedron (and through the opposite vertices of its dual, the dodecahedron) are numbered **12, 14, 16, 18, 20, 22, 24, 26, 28,** and **30** and are shown in FIG. 3A. It can be demonstrated that these lines which shall be here called the star of the faces of the icosahedron or 10 zone star have a complex angular relationship to one another. The lines from two sets of angles with each other, the angles being: arc cos ⅓ (70° 31′ 44″) and its supplement 109° 28′ 16″ and arc sin ⅔ (41° 48′ 38″) and its supplement 138° 11′ 22″.

Also illustrated in FIG. **3**A are some of the diametral lines which bisect the edges of the icosahedron. In order to preserve clarity only five of these lines are shown, **11, 13, 15, 17,** and **19**. Since there are 30 edges in the icosahedron there are a total of 15 such diametral lines. Because the faces of the icosahedron are all regular, each of the 15 diametral lines through the midpoints of the edges of the icosahedron will bisect four of the angles formed by the lines of the 10 zone star. In the description which follows, the 15 diametral lines will be referred to as the "edge star".

In FIG. **3**B the six zone star is shown. The six zone star, or the star of the vertices of the icosahedron (or of the faces of the dodecahedron **11**) as illustrated is made up of the six diameters through the vertices of an icosahedron, or the six diameters through the midpoints of the faces of a dodecahedron. These are the lines numbered **32, 34, 36, 38, 40,** and **42**. These lines all make the same angle with each other: arc tan 2 (63° 26′ 06″) and its supplement 116° 33′ 54″. In the present invention the structural members used to carry out the principles of the invention are parallel to at least one of the lines of these stars. It is an integral of the invention that structural members forming a plane may be replaced by structural planar faces. Such structural faces may be stressed skins, prestressed concrete slabs, rigid plastic sheets, honeycomb sandwich panels, or any of the myriad building material usable for making such sheets. In the use of sheets for practicing the invention, the edges of the sheets are parallel to the lines of the stars and are rigidly interconnected to form a continuous skin for a structure.

The previously described edge star, **15** lines intersecting the edges of the icosahedron, is also related to the lines of the six zone star. It has already been noted that the six zone star and the 10 zone star are interrelated because of the duality of the icosahedron and dodecahedron. Thus, the 15 diametral lines through the midpoints of the edges of the icosahedron are identical to the 15 lines through the midpoints of the edges of the dodecahedron although not separately illustrated as such. Each of the 15 edge star lines will also bisect two of the angles formed by the lines of the six zone star. As will be shown in the description below, the lines of the 15 edge bisectors may be utilized to strengthen structural units conforming to the six zone and 10 zone alignments. In addition, because of the interrelationship between the 15 edge bisectors and both the six zone and 10 zone stars the 15 edge bisector lines form a link between 10 zone and six zone structures and permit utilization of combinations of the two systems for even more intricate and beautiful structural designs.

A triacontahedron, a 30 sides zonohedron associated with the star of the faces of the dodecahedron is shown in FIG. **4**. This figure may be used to illustrate the beginnings of the structural system of the present invention. The triacontahedron shown generally at **44** in FIG. **4** is made up of a plurality of diamond shaped faces **46**, the edges of which are aligned with the star lines of the dodecahedron so that the acute angles of this figure are 63° 26′ 06″ and the obtuse angles are 116° 33′ 54″. For purposes of clarity in the illustrations the angle 63° 26′ 06″ will hereinafter be referred to as angle α and 116° 33′ 54″ as angle β. In FIG. **4** the diamond shaped faces are bisected by a diagonal **48** which in the case of an open framework such as is illustrated would considerably stiffen the structure. It is contemplated as an integral part of the present invention that each of the faces **46** of a structure such as is illustrated in FIG. **4** could be made of an integral planar sheet as previously described, each of the edges of which is parallel to at least one of the lines of the six zone star, 10 zone star, or of the 15 lines of the edge star.

Each of the diagonals **48** which, as has been noted, serve to form a series of triangles to stiffen the structure already illustrated, is parallel to one of the 15 lines of the previously described edge star. The diagonals **48** triangulate (divide into triangles) the diamond shaped faces of the triacontahedron **44** and as will be shown are parallel to diagonals also of the enneacontahedron illustrated in FIG. 5. The enneacontahedron is a 90 sided polyhedron having a plurality of diamond shaped faces of two kinds: a slender diamond **52** and a larger diamond shape **54**. Each of the edges of the diamond shaped faces comprising the enneacontahedron shown in FIG. 5 is parallel to one of the 10 lines making up the 10 zone star of the faces of the icosahedron (and the vertices of the dodecahedron).

Each of the diamond shaped faces **46** of the triacontahedron **44** comprises four edges **56** of equal length. If the length is expressed as L, the length of the diagonal **48** is approximately 1,0514L. The polyhedron structure **44** may be made up of a plurality of interconnected structural elements **56** and **48** to result in an open, airy form. The structural elements may be pipes, rods, beams, or any other elongate form. In FIG. 4, the elements are shown interconnected by means of ball-shaped connectors **58**. The connectors may be any device which permits the elements to be joined together in the manner and at the angles illustrated. The connectors in this application may be in the shape of dodecahedra with the structural members welded or otherwise attached to the faces thereof.

The lines of the edge star are arranged in such a way that each bisects one 41° 48′ 38″, one 138° 11′ 22″, and one 109° 28′ 16″ angle formed by the lines of the 10 zone star, and each is perpendicular to two of the lines of the 10 zone star. Also, each lines of the edge star bisects one 63° 26′ 06″ angle and one 116° 33′ 54″ angles of the six zone star and each is perpendicular to two lines of the six zone star. The lines of the edge star are related to each other is such a fashion that all angles between lines of the edge star are multiples of 36°, 60°, or 90°.

With the structure of the triacontahedron **44** now noted, the growth of the system from such a starting point, as with a crystalline array may be described. In FIG. 6A, one of the basic cells of the six zone truss is shown, the other basic cell of the six zone system being illustrated in FIG. 6B. The six zone A cell comprises a plurality of structural elements **56** of the same length L as the elements of the triacontahedron **44** of FIG. 4. The diagonal members **48** used previously are used in the A cell for strengthening the structural unit. The connectors **58** are again identical to those utilized for the triacontahedron. As might be expected from the use of the identical elements, there are six faces in the A cell, each being the same as face **46** of FIG. 4. The angles in the faces are the angles of the six zone star the angles α and β.

The six zone B cell in FIG. 6B is in the obtuse configuration, that is, the obtuse angles β of three faces meet at two diagonally opposite vertices. As a result, these vertices tend to be closer together than in the more conventional A cell. The short diagonal **47** between the obtuse vertices is approximately .562 times the length of one of the edge members **56**. The short diagonal member **47** while strengthening a six zone cell is actually a member of the 10 zone system, being parallel to one of the lines of the 10 zone truss system.

Since the faces of the A and B cells of the six zone system are congruent to the faces of the triacontahedron, a series of suitably arranged A and B cells may be superimposed on the faces of that polyhedron to cause the structure to grow in any of the face directions.

As previously noted there are two sets of faces making up the surface of the enneacontahedron, the slender diamond **52** and a larger diamond shape **54** which is also known as a Maraldi diamond. The relationship among the diamond faces is more clearly seen in FIG. 5. The angles of the two diamonds **52** and **54** are as follows:

In the slender diamond **52**, the acute angle is 41° 48′ 38″ which will be referred to as γ and its supplement is 138° 11′22″ which will be referred to as Δ and the angles of the Maraldi diamond **54** are 70° 31′ 44″ which will be referred to as angle ϕ and its supplement is 109° 28′ 16″ which will be referred to as θ. Each of the diamond faces **52** and **54** of the enneacontahedron **50** comprises four edges **62** of equal length. If the length is expressed as L the length of the short diagonal **64** is approximately 0.714L and the length of the long diagonal **66** of the Maraldi diamond **54** is 1.155L. The polyhedral structure **50** may be made up of a plurality of interconnected structural elements **62** with additional reinforcing members **64** and **66**. As with the polyhedral structure **44**, the structural elements of the enneacontahedron **50** may be pipes, rods, beams or any other elongate forms. They may be interconnected by means of ball-shaped connectors **68** or any other device which permits the elements to be joined together at the specific angles of the 10 zone system.

The five basic cells of the 10 zone structural system of the present invention are illustrated in FIGS. 7A through 7E. These cells are the basic building blocks of the 10 zone system and illustrate the variety of angular relationships which the structural elements of the 10 zone system may have when interconnected. It is possible to join one cell to another at one of the edges thereof in order to expand the structural units in any direction. Thus, in the construction of a truss utilizing the basic cells of the present invention one need merely add additional rows of similar truss cells end to end to gain length, side by side to gain lateral strength, and several rows deep for greater stiffness.

Each of the cells of the 10 zone system illustrated in FIGS. 7A through 7E comprises a plurality of primary structural elements **62**, the primary elements being joined together in the form of four sided rhomboidal planar figures in the shape of the slender diamonds **52** and the Maraldi diamonds **54** making up the faces of the enneacontahedron and each of which utilize the basic angles of the 10 zone system. Included within the cell members illustrated are additional structural elements **64** and **66** for strengthening the structural units.

The rhomboidal faces of the five cells of the 10 zone system are made up of combinations of the slender and Maraldi diamonds **52** and **54** previously described as making up the face of the enneacontahedron **50**. In FIG. 7A the A cell of the 10 zone system is illustrated, each of the faces of the A cell is a Maraldi diamond **54**. In the case of the A cell the faces are joined together in the acute configuration. For convenience of reference, because of the perspective of the illustration the angles of the diamond faces are shown. Thus, in the A cell FIG. 7A the acute configuration means that in two corners thereof the three acute angles ϕ meet together as illustrated. In the B cell illustrated in FIG. 7B, each of the faces of the cell are Maraldi diamonds **54** in the obtuse configuration. This means that at two of the corners of the A cell the three obtuse angles θ meet to yield the configuration of the B cell. One of the interesting features of the configuration of the B cell is that the central diagonal (62) reinforcing the cell between the two corners having the three obtuse angles meeting is of the same length as all of the edge members. Not only is the central diagonal (62) of the B cell of the same length as each of the edge members **62** of the B cell, but it is parallel to one of the lines of the 10 zone star. Thus, it is a standard member of the 10 zone system.

The C cell illustrated in FIG. 7C comprises two parallel slender diamonds **52** and four Maraldi diamonds **54** arranged into parallel sets. The arrangement of these rhomboidal faces is in the acute configuration, that is, in which two of the corners have the acute angles of the diamond faces meeting each other. Thus, in FIG. 7C the rear left corner of the C cell has two vertices with ϕ angles and one with the γ angle meeting, and this combination of angles is repeated in the front right hand corner of the cell illustrated.

In the 10 zone D cell illustrated in FIG. 7D the cell comprises a combination of four of the slender diamonds **52** and two of the Maraldi diamonds **54**. The Maraldi diamonds are the top and the bottom faces of the cell as illustrated and all the other faces are the slender diamonds having the γ and Δ angles. As has already been noted, each of the edge members of the D cell, in the same manner as the A, B, and C cells, comprises the primary structural element **62** and in addition utilizes the additional structural elements **64** and **66** as diagonals for reinforcement purposes.

In the ten zone E cell illustrated in FIG. 7E the cell comprises a combination of four Maraldi diamonds **54** and two of the slender diamonds **52** in the obtuse configuration. This configuration is similar in many respects except for the angles to the obtuse configuration of the six zone B cell illustrated in FIG. 6B. In the 10 zone E cell the slender diamonds are at the extreme left and right of the cell. The obtuse angles at which the structural elements making up the cell meet in two of the corners are illustrated by the angles θ and Δ. Because of the obtuse configuration of the E cell the structure is not as rigid as would be desired. Therefore, an extra structural element **66** may be connected between the connection points of the obtuse angles. This extra structural member **76** is approximately 0.419 of the length of the primary structural elements **62** and is disposed at an angle which makes it part of the six zone system. Thus, it may be seen in this particular structural unit of the present system that the six zone system and the 10 zone system are complementary and, in certain cases structural elements parallel to the six zone start may be used in structures whose primary elements are parallel to the 10 zone star.

It should be borne in mind in viewing the cells of the six and 10 zone systems that a diagonal bisector through the obtuse angles of the cells will subdivide the cell into a non-regular octahedron and two non-regular tetrahedra. Each of the acute parallelepiped cell structures forms a strong truss cell when thus subdivided. The cells in the obtuse configuration are suitable as shells only. They do not subdivide in the same manner as the acute cells and thus require the "special" diagonals which have previously been described. The effect of the "special" diagonals is to form six non-regular tetrahedra having a common edge which is the "special" diagonal.

In FIG. **8** a truss utilizing the angles of the six zone system is illustrated. Each of the primary structural members of the truss is equivalent to the primary structural elements **56** of the six zone cells shown in FIG. **6**A and **6**B. The ball connectors **58** are used in the same manner to connect the primary structural elements **56** and, in addition the reinforcing structural members, the diagonals **48** are used to give rigidity to the overall structure. It should be noted that because of the arrangement of the primary structural members in one of the six angular directions of the six zone system it is possible for the structural elements to join together in cellular arrays with smooth transitions from one direction to another. The same, of course, applies to the use of structural elements of the 10 zone system, or of a combination of elements of the six and 10 zone systems with elements of the edge star relationship in what I have called the 31 zone system. Cellular arrays of incredible complexity and beauty are available by simply adding units one to another in the directions dictated by the angular relationships of the zonal systems of the present invention.

An even more striking and beautiful structure utilizing the ten zone system is shown in FIG. **9**. FIG. **9** is a building utilizing as its basic components a plurality of diamond shaped structural panels the edges of which conform to the angles of the 10 zone system. Thus, each of the faces of the building structure **80** is either a slender diamond **52** or a Maraldi diamond **54** as found in the enneacontahedron previously described. By joining the structural panels along the lines of the 10 zone system an extraordinary structure results. It should be noted that the structure **80** comprises a generally spherical dome **82** resting on a partial spherical dome **84** and fused with an elongate partially cylindrical structure **86**.

An additional feature of the structural system of the present invention resides in the ability to form a plurality of blocks or hollow, solid forms in the shape of the previously described cells of the six, 10, or 31 zone systems. It is thus possible to utilize such blocks as, for example, a child's game in which the child is able to combine the blocks into myriad possible shapes by utilizing the 15 planes found in the six zone system or the 45 planes of the 10 zone system. In addition to use as toys, the blocks of the present structural system could be made in extremely large sizes to permit their use as building blocks for the construction of buildings of normal size. The possible applications of the resent invention are limited only by the ingenuity of the mind which sets about its utilization.

The concept of using blocks in combinations related to the six and 10 zone solid-cell blocks was mentioned, but not pursued, in a book published in Germany in 1938: Gerhard Kowalewski, *Der Keplerische Koerper und Andere Bauspiele*. Kowalewski was concerned with two of the regular polyhedra unknown to the ancient world and discovered by Kepler: the two stellated dodecahedra and particularly with coloring of the various facets of the constructs. As far as I have been able to determine no one previously has discovered the interrelationship of the six and 10 zone cells heretofore described and their utilization for both practical and esthetic applications.

Because of the previously described interrelationship of the six, 10, and 15 zone system—either separately or when combined into the 31 zone system which utilizes all of the interrelated stars of the dodecahedron—building blocks having equal length edges, and in many cases, congruent faces, permit combining the building units into regular shapes, e.g., dodecahedra or icosahedra or into structures as complex as the domed structure in FIG. 9.

As is well known, the golden section is one of the regularly occurring relationships in polyhedra having five-fold symmetry. The golden section which is sometimes also called the divine proportion is a relationship between the portions of a line in which the ratio of the whole to the larger part is equal to the ratio of the larger to the smaller part. This is expressed mathematically as $\tau = (\sqrt{5+1})/(2) = 1.6180$. I have noted that there is a striking relationship among the rhomboidal faces of the cells previously described in which the golden section figures prominently.

Thus, if the equal edges of the planar faces **46** of a typical six zone cell are of a length a, and the short diagonal **48** of the face is length c, the long diagonal of the face is $c\tau$, where τ is the golden section. If the sides of the slender diamond **52** and of the Maraldi diamond **54** of the 10 zone system are of a length b, the short diagonal **66** of the Maraldi diamond is equal to length c, the long diagonal of the slender diamond **52** is $c\tau$ and its short diagonal **64** is c/τ. The specific relationship of the lengths a, b, and c is $a = c \cos 18°$ or $0.9510565c$, and $b = c \cos 30°$ or $0.8660254c$.

Other typical relationships among the edge members and the zonal cells are, e.g., the long diagonal from acute vertex to acute vertex in the six zone A cell is $b\tau^2$ where the edges **56** are of length a. The short diagonal **47** in the six zone B cell is b/τ where the edges **56** are of length a. The short diagonal **76** of the ten zone E cell is a/τ^2 where the edge members **62** are of length b.

In the previous description, each of the cells, and the rhomboidal faces thereof have been regular, that is, all the edges have been of the same length. It is an integral part of the present invention to utilize the angular relationships of the previously described rhomboidal faces but altering the shape thereof by elongating sets of edges. I have called this process "stretching a zone." Thus, a zonohedron may be elongated by elongating parallel edge members thus stretching a plurality of faces making up a band along or around the structure. The angular relationships among the structural members or the edges of the planar faces are identical to those in which equal length members or edges are utilized.

I call combinations of stretched (or shrunk) zones (rhomboidal faces) zomes. These are the resultant structures, non-regular polyhedral structures in which planar faces and other structural members are combined utilizing the angular relationships heretofore outlined. The zomes may be formed into combinations of dome shaped clusters in which symmetries of the system permit unlimited expansion and soaring, fantastic constructs that are both beautiful and livable at the same time.

While certain embodiments of the invention has been shown and described, it will be obvious that other adaptations and modifications can be made without departing from the true spirit and scope of the invention.

I claim:

1. A structural system comprising a plurality of interconnected elongate structural members, said members joined to each other to form a plurality of vertices, the angular relationship of the joined members at the vertices being such that at least one member is parallel to one of the lines of the star formed by diametral lines through the midpoints of opposite faces of an icosahedron; at least one member is parallel to one of the lines of the star formed by diametral lines through each of the opposite vertices of an icosahedron; and including additional elongate structural elements each of which is parallel to at least one of the lines of the star formed by diametral lines through the midpoints of opposite edges of an icosahedron.

2. The structural system of claim 1 in which the primary structural members comprise elongate structural members joined to each other so that each is parallel to at least one of the lines of the star formed by diametral lines through the midpoints of opposite faces of an icosahedron.

3. The structural system of claim 1 in which the primary structural members comprise elongate structural members joined to each other so that each is parallel to at least one of the lines of the star formed by diametral lines through each of the opposite vertices of an icosahedron.

4. The structural system of claim 1 in which the primary structural members comprise elongate structural members joined to each other so that each is parallel to at least one of the lines of the star formed by diametral lines through the midpoints of opposite edges of an icosahedron.

5. The structural system of claim 1 in which the structural element members comprise the edges of solid polyhedra.

6. The structural system of claim 2 in which the structural element members comprise the edges of solid polyhedra.

7. The structural system of claim 3 in which the structural element members comprise the edges of solid polyhedra.

8. The structural system of claim 4 in which the structural element members comprise the edges of solid polyhedra.

9. The structural system of claim 1 and including a plurality of connectors, the connectors adapted for joining the elongate structural members along the star angles formed by the diametral lines through the edge midpoints of an icosahedron, the diametral lines through the face midpoints of an icosahedron, and the diametral lines through the vertices of an icosahedron.

10. A structural system comprising a plurality of planar structural members interconnected at the edges thereof to form a structural surface, the edges of each of the joined planar members being such that at least one edge of a first member is parallel to one of the lines of the star formed by diametral lines through the midpoints of opposite faces of an icosahedron; at least one

edge of a second member is parallel to one of the lines of the star formed by diametral lines through each of the opposite vertices of an icosahedron; and including additional planar structural members each of which has one edge parallel to at least one of the lines of the star formed by diametral lines through the midpoints of opposite edges of an icosahedron.

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