${ }_{(19)}$ United States
(12)

Patent Application Publication Hoffiman et al.
(10) Pub. No.: US 2002/0094132 A1
(43) Pub. Date:

Jul. 18, 2002
(54) METHOD, APPARATUS AND COMPUTER PROGRAM PRODUCT FOR GENERATING PERSPECTIVE CORRECTED DATA FROM WARPED INFORMATION

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Appl. No.: $\quad 10 / 056,476$
(22)

Filed: Jan. 24, 2002

Related U.S. Application Data
(63) Continuation of application No. 09/200,172, filed on Nov. 25, 1998, now patented.

## Publication Classification

(51) Int. Cl. ${ }^{7}$ $\qquad$ G06K 9/36 382/276

## ABSTRACT

Apparatus, methods, and computer program products that provide fast and accurate means of mapping one data space into another by precisely mapping grid points between the data spaces and then by performing a bilateral-bilinear interpolation to map the points bounded by the precisely mapped grid points. The precisely mapped grid points define boundary lines that bound a data region in a source space. Each scan line mapped to the source space is dependent on the slopes of the bounding lines of the data region.


$100 \% \quad \begin{aligned} & \text { Fig. 1A } \\ & \\ & \text { (Prior Art) }\end{aligned}$


Fig. 1C
(Prior Art)


Fig. 2

Fig. 3A

|  | 301 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 309 |  |  |  |  |  |  |
| 305 |  |  |  |  |  |  |



Fig. 3B


Fig. 3C


Fig. 4A

Fig. 4B


$500 \uparrow$

Fig. 5

${ }_{600} \uparrow$

## Fig. 6

# METHOD, APPARATUS AND COMPUTER PROGRAM PRODUCT FOR GENERATING PERSPECTIVE CORRECTED DATA FROM WARPED INFORMATION 

Background of the Invention

[0001] 1. Field of the Invention
[0002] This invention relates to the field computer technology for mapping one data space to another data space.
[0003] 2. Background
[0004] Computers are often used to map data that exists in a source data space to a destination data space. This type of mapping is often used in "virtual reality" and "telepresence" applications. The data in the source data space can represent a warped image that is obtained by a distorting lens such as a fisheye lens or a catadioptric lens. The data in the destination data space can be presented by a presentation device such as a video screen, computer monitor or printer. The problem is how to rapidly generate the data for the destination data space from the source data space.
[0005] One approach is to backward map the coordinates of a point in the destination data space to coordinates in the source data space and to obtain the value for the point in the destination data space from the mapped point in the source data space. Precisely mapping each point (for example, by using floating point calculations to perform the mapping) is expensive in either memory or computation, or both.
[0006] Another approach is to precisely map a grid of points from the destination data space to the source data space. These grid points bound regions (patches) that contain pixels that have similar mapping as the grid points that bound the region. Thus, the precisely mapped grid points are used to determine coefficients for a mapping that can be applied to each point in the region. Each of the grid points in the destination data space has a corresponding grid point in the source data space. Thus, the destination grid point and the corresponding source grid point are referred to as a "point pair."
[0007] By using all four of the destination and source point pairs that bound a region, a perspective transformation can be computed and used to find the corresponding pixel in the source data space. Thus,

$$
\begin{aligned}
& x_{s}=\frac{a x_{d}+b y_{d}+t_{x}}{e x_{d}+f y_{d}+1} \\
& y_{s}=\frac{c x_{d}+d y_{d}+t_{y}}{e x_{d}+f y_{d}+1}
\end{aligned}
$$

[0008] Can be used to perform the mapping where ( $\mathrm{x}_{\mathrm{s}}, \mathrm{y}_{\mathrm{s}}$ ) is the is the resulting coordinate in the source data space, $\left(\mathrm{x}_{\mathrm{d}}\right.$, $\mathrm{y}_{\mathrm{d}}$ ) is the coordinates of the pixel in the destination data space, and $a, b, c, d, e, f, t_{x}$ and $t_{y}$ are the perspective transform coefficients. This calculation includes at least six multiply operations, two division operations, and six add operations. The multiply and division operations are computationally expensive. Although this equation does not generate mapping artifacts between the regions, the additional computational overhead is often prohibitive.
[0009] Another approach is to generate a look-up table that provides the $x_{s}$ and $y_{s}$ coordinates when given the $x_{d}$ and $y_{d}$ coordinates. With high resolution images and high-resolution presentation devices, these look-up tables become unwieldy even with modern computers.
[0010] Another approach is to approximately map the region using a less computationally expensive formula. For example, an affine transformation can be used to perform this mapping. In this circumstance, three of the four sets of destination and source point pairs are used to compute the affine coefficients for the transformation. Then coordinates that specify a pixel that in the destination data space can be used to find the corresponding pixel in the source data space. Thus,

$$
\begin{aligned}
& x_{\mathrm{s}}=a x_{\mathrm{d}}+b y_{\mathrm{d}}+c \\
& y_{\mathrm{s}}=d x_{\mathrm{d}}+e y_{\mathrm{d}}+f
\end{aligned}
$$

[0011] Where ( $\mathrm{x}_{\mathrm{s}}, \mathrm{y}_{5}$ ) is the resulting coordinate in the source data space, $\left(\mathrm{x}_{\mathrm{d}}, \mathrm{y}_{\mathrm{d}}\right)$ is the coordinates of the pixel in the destination data space, and $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}$ and f are the affine coefficients for the grid region bounded by the precisely mapped grid points. This calculation includes four multiply operations and four add operations for each pixel in the patch and so is still computationally expensive. An additional problem with this approach is that an affine transformation often generates a very poor approximation to the perspective transformation. The affine transformation only uses three of the four point pairs that bound the region. Thus, the affine transformation can generate mapping artifacts (such as discontinuities) along edges of the quadralateral defining the region in the source space.
[0012] FIG. 1 A illustrates a quadralateral patch in source space, indicated by general reference character 100 , used to show mapping artifacts generated by the affine transformation. The quadralateral patch in source space $\mathbf{1 0 0}$ is bounded by grid points (such as a point A 101, a point B 103, a point C 105, and a point D 107). Applying an affine transformation to this region (using the points 101, 103, and 105 and the corresponding points in the destination space) would generate the patch bounded by the point A 101, the point B 103, the point C 105 and a point D'109 instead of approximating the original patch.
[0013] FIG. 1B illustrates a presentation of a correctly mapped image, indicated by general reference character 150, that represents a magnified image presented on a presentation device. This image does not have any mapping artifacts. It can be generated by precisely mapping each point in the destination data space to the source data space. It can also be generated by precisely mapping grid points, and using the perspective transformation previously discussed to map the points in each region defined by the grid. Compare FIG. 1B with FIG. 1C.
[0014] FIG. 1C illustrates a presentation of an incorrectly mapped image, indicated by general reference character 160, that shows mapping artifacts 161 that can result from the use of the affine transformation. As can be seen, these mapping artifacts include discontinuities in lines. Other mapping artifacts include (without limitation) texture discontinuities and color discontinuities.
[0015] It would be advantageous to use a fast mapping algorithm that also provides a good approximation for a precise perspective-correct transformation that maintains
continuity across patch boundaries without the computational or memory overheads associated with the prior art. Devices and computers that use these methods will operate more efficiently than those that use prior art methods will.

## SUMMARY OF THE INVENTION

[0016] The invention provides a fast and accurate means of mapping one data space into another by precisely mapping grid points between the data spaces and then by performing a bilateral-bilinear interpolation to map the points bounded by the precisely mapped grid points.
[0017] One aspect of the invention is a computer-controlled method that includes the step of determining a region in a destination data space. The region is bounded by a plurality of grid points. It defines a first plurality of data points in the destination data space. The method precisely maps the plurality of grid points in the destination data space to a plurality of mapped grid points in a source data space. The source data space contains, or is associated with, a second plurality of data points. The plurality of mapped grid points define a plurality of boundary lines that represent the boundary of the region as mapped into the source data space. The method also applies a bilateral-bilinear interpolation algorithm to approximately map the first plurality of data points to the second plurality of data points.
[0018] Another aspect of the invention is an apparatus that includes a central processing unit (CPU) and a memory coupled to the CPU. The apparatus also includes a region determination mechanism that is configured to determine a region in a destination data space. The region is bounded by a plurality of grid points. The region defines a first plurality of data points within the destination data space. The apparatus also includes a precise mapping mechanism that is configured to precisely map the plurality of grid points determined by the region determination mechanism to a plurality of mapped grid points in a source data space. The source data space contains (or associates) a second plurality of data points. The plurality of mapped grid points define a plurality of boundary lines that represent the boundary of the region as mapped into the source data space. The apparatus also includes a bilateral-bilinear interpolation mechanism that is configured to approximately map the first plurality of data points in the region to the second plurality of data points using the plurality of mapped grid points.
[0019] Yet another aspect of the invention, is a computer program product that includes a computer usable storage medium having computer readable code embodied therein for causing a computer to map a destination data space to a source data space. When executed on a computer, the computer readable code causes the computer to effect a precise mapping mechanism, a region determination mechanism, and a bilateral-bilinear interpolation mechanism. Each of these mechanisms having the same functions as the corresponding mechanisms for the previously described apparatus.
[0020] Still another aspect of the invention is a computer program product embodied in a carrier wave transmitting computer readable code therein for causing a computer to map a destination data space to a source data space. When executed on a computer, the computer readable code causes the computer to effect a precise mapping mechanism, a region determination mechanism, and a bilateral-bilinear
interpolation mechanism. Each of these mechanisms having the same functions as the corresponding mechanisms for the previously described apparatus.

## DESCRIPTION OF THE DRAWINGS

[0021] FIG. 1A illustrates a mapping artifact resulting from an affine transformation;
[0022] FIG. 1B illustrates a presentation of an image without mapping artifacts;
[0023] FIG. 1C illustrates a presentation of an image with mapping artifacts;
[0024] FIG. 2 illustrates a computer system capable of using the invention in accordance with a preferred embodiment;
[0025] FIG. 3A illustrates a gridded destination data space in two-dimensions in accordance with a preferred embodiment;
[0026] FIG. 3B illustrates a gridded source data space with a mapped destination data space in two-dimensions in accordance with a preferred embodiment;
[0027] FIG. 3C illustrates a gridded destination data space with a mapped destination data space in three-dimensions in accordance with a preferred embodiment;
[0028] FIG. 3D illustrates a gridded source data space with a mapped destination data space in three-dimensions in accordance with a preferred embodiment;
[0029] FIG. 4A illustrates a gridded patch in two-dimensions in accordance with a preferred embodiment;
[0030] FIG. 4B illustrates the gridded patch of FIG. 4A as mapped into the source data space in accordance with a preferred embodiment;
[0031] FIG. 5 illustrates an overview of the process used to backward map pixels in a destination data space to a source data space in accordance with a preferred embodiment; and
[0032] FIG. 6 illustrates a bilateral-bilinear interpolation algorithm that backward maps pixels in a destination region to a source data space in accordance with a preferred embodiment.

## DESCRIPTION OF THE PREFERRED EMBODIMENTS

## [0033] Notations and Nomenclature

[0034] The following 'notations and nomenclature' are provided to assist in the understanding of the present invention and the preferred embodiments thereof.
[0035] Procedure-A procedure is a self-consistent sequence of computerized steps that lead to a desired result. These steps are defined by one or more computer instructions. These steps are performed by a computer executing the instructions that define the steps. Thus, the term "procedure" can refer to a sequence of instructions, a sequence of instructions organized within a programmed-procedure or programmed-function, or a sequence of instructions organized within programmed-processes executing in one or more computers.
[0036] Operating Environment
[0037] FIG. 2 illustrates a computer, indicated by general reference character 200, that incorporates the invention. The computer 200 includes a processor 201 having a central processor unit (CPU) 203, a memory section 205, and an input/output ( $\mathrm{I} / \mathrm{O}$ ) section 207. The I/O section 207 is connected to a presentation device 211, a disk storage unit 213 and a CD-ROM drive unit 215. The CD-ROM drive unit 215 can read a CD-ROM medium 217 that typically contains a program and data 219. The CD-ROM drive unit 215 (along with the CD-ROM medium 217 ) and the disk storage unit 213 comprise a filestorage mechanism (a filesystem). Some embodiments of the invention include a network interface 221 that connects the computer 200 to a network 223. An application program 225 executes from the memory section 205. The application program 225 can be loaded into the memory section 205 over the network 223 or from the filesystem. In one embodiment of the invention, the application program 225 includes computer code that causes the computer to perform the inventive steps. The CD-ROM drive unit 215 (along with the CD-ROM medium 217 ) are illustrative of mechanisms that can be used to read computer code from a removable media. One skilled in the art will understand that not all of the displayed features of the computer $\mathbf{2 0 0}$ need to be present for the invention.

## [0038] Data Space

[0039] One aspect of the invention maps points between two data spaces. FIG. 3A through FIG. 3D illustrate some of the possible data spaces that can be mapped by this aspect of the invention.
[0040] FIG. 3A illustrates a gridded destination data space 300 showing a first grid point 301, a second grid point 303, a third grid point $\mathbf{3 0 5}$, and a fourth grid point 307. These points are bounding points for the destination data space. Each of the intersections in the destination data space (for example a fifth grid point $\mathbf{3 0 9}$ ) is precisely mapped to a source data space. The grid points bound regions that contain data points that will be approximately mapped to the source data space. For example, the third grid point 305 and the fifth grid point 309 are two of the four grid points that bound a region 311 that contains points having mappings that will be interpolated. A bilateral-bilinear interpolation algorithm performs this approximate mapping. The bilat-eral-bilinear interpolation algorithm is subsequently described with respect to FIG. 5 and FIG. 6 as applied to patches in a two-dimensional data space.
[0041] FIG. 3B illustrates a gridded source data space 350 indicating how the destination data space is mapped to the source data space. The resolution of the gridded destination data space $\mathbf{3 0 0}$ and the gridded source data space $\mathbf{3 5 0}$ need not be the same. The gridded source data space $\mathbf{3 5 0}$ can contain (or reference) warped image data that represents a true image that has been warped by a lens. One skilled in the art will understand that a physical lens need not be used to generate the warped image as ray-tracing techniques through a virtual lens can also be used to generate the warped image. A virtual lens can be used to generate images in a virtualspace. Once the image is generated, the invention can be used to present the image.
[0042] One aspect of the invention backward maps the destination data space to the source data space using a
mapping that generates a perspective corrected image in the destination data space. One step in this mapping process precisely maps the first grid point $\mathbf{3 0 1}$, the second grid point 303 , the third grid point 305 , the fourth grid point 307 , the fifth grid point $\mathbf{3 0 9}$, and other grid points to the gridded source data space 350. These grid points map to a mapped first grid point 301', a mapped second grid point 303', a mapped third grid point 305 ', a mapped fourth grid point $307^{\prime}$, a mapped fifth grid point $309^{\circ}$ and other grid points respectively in the source data space. Thus, the region 311 is mapped to a mapped region 311'.
[0043] Notice that the gridded destination data space 300 when mapped into the gridded source data space $\mathbf{3 5 0}$ need not result in a parallelogram-the slopes of each of the lines defined by the mapped grid points can be different.
[0044] FIG. 3C illustrates a 3-D gridded destination data space, indicated by general reference character $\mathbf{3 6 0}$, that has a first plane 361 (bounded by the first grid point 301, the second grid point $\mathbf{3 0 3}$, the third grid point $\mathbf{3 0 5}$ and the fourth grid point $\mathbf{3 0 7}$ ) and a second plane 363 (sp) (bounded by a sp-first grid point $\mathbf{3 6 5}$, a sp-second grid point 367 , a sp-third grid point $\mathbf{3 6 9}$ and another point that cannot be seen in FIG. 3C).
[0045] FIG. 3D illustrates a 3-D gridded source data space, indicated by general reference character 370, that indicates how the 3-D gridded destination data space $\mathbf{3 6 0}$ is mapped to the 3-D gridded source data space 370. A mapped first plane $\mathbf{3 6 1}$ is bounded by the mapped first grid point 301', the mapped second grid point 303 ', the mapped third grid point $305^{\prime}$, and the mapped fourth grid point $307^{\prime}$. A mapped second plane $363^{\prime}$ (msp) is bounded by a mspsecond grid point $367^{\prime}$, and a msp-third grid point $369^{\prime}$, and two other points that cannot be seen in FIG. 3D
[0046] FIG. 3C and FIG. 3D show how grid points can be imposed on three-dimensional spaces. Once the grid points are precisely mapped, the points contained in the region (the volume) between and including the first plane $\mathbf{3 6 1}$ and the second plane 363 can be interpolated by extending the subsequently described techniques. Similar techniques can be applied to n -dimensional spaces.
[0047] Although the bilateral-bilinear interpolation algorithm is applicable to $n$-dimensional spaces, subsequent discussion of the algorithm is directed to two-dimensional spaces containing image data. Each region is a two-dimensional patch containing points that represent pixels. One skilled in the art will understand how to modify the described algorithm to be applicable to higher dimensional spaces, for non-image data, and to a source data space that references the data. Such a one will also understand that the invention can be used (without limitation) to map a viewport onto spherical, cylindrical, and panoramic spaces.
[0048] FIG. 4A illustrates a patch in destination data space, indicated by general reference character 400, bounded by a first grid point 401, a second grid point 403, a third grid point 405 and a fourth grid point 407. The destination patch $\mathbf{4 0 0}$ contains a number of pixels (in the illustration, 36 pixels) of which a pixel 409 is but one. The bilateral-bilinear interpolation algorithm efficiently generates data values for the pixels contained in the destination patch 400. In this particular illustration, the 36 pixels are arranged in six scan lines. Each scan line is six pixels long.

One skilled in the art will understand that the destination patch $\mathbf{4 0 0}$ need not be square and may include more or fewer pixels than the $\mathbf{3 6}$ used in the illustration. The grid points are mapped to the source data space as is shown with respect to FIG. 4B.
[0049] FIG. 4B illustrates a mapped patch in source data space, indicated by general reference character 420, indicating some of the parameters used by the bilateral-bilinear interpolation algorithm. The mapped patch $\mathbf{4 2 0}$ is bounded by the mapped first grid point 401', the mapped second grid point 403 ', the mapped third grid point $405^{\prime}$, and the mapped fourth grid point 407 each of which have been precisely mapped to the source data space from the corresponding points in the destination data space. The data that is used to generate the value for the pixel 409 in the destination data space is located at a mapped pixel contribution area 409 . The mapped pixel contribution area $409^{\prime}$ contains pixels of a warped image at a resolution possibly different from the pixel resolution in the destination data space. Techniques known in the art are used to determine the value of the pixel 409 based on the information within the mapped pixel contribution area $409^{\prime}$.
[0050] The mapped grid points define lines that bound the mapped patch 420. Thus, the mapped first grid point 401 and the mapped third grid point 405 define a second boundary line 421; the mapped second grid point 403' and the mapped fourth grid point $407^{\prime}$ define a third boundary line 423; the mapped first grid point $401^{\prime}$ and the mapped second grid point 403 ' define a first boundary line $\mathbf{4 2 5}$ and a mapped third grid point 405 ' and a mapped fourth grid point $\mathbf{4 0 7}$ ' define a final boundary line $\mathbf{4 2 7}$. One skilled in the art will understand that a different geometry can be used other than the one described.
[0051] A first slope $\mathbf{4 2 8}$ represents the slope of the first boundary line 425 . A second slope 429 represents the slope of the second boundary line 421 and in the two-dimensional case includes delta-x and delta-y components. A third slope 431 represents the slope of the third boundary line 423. A final slope $\mathbf{4 3 5}$ represents the slope of the final boundary line 427.
[0052] The bilateral-bilinear interpolation algorithm operates by determining the second slope $\mathbf{4 2 9}$ and the third slope 431 for the boundary lines. The second slope 429 and the third slope $\mathbf{4 3 1}$ need not be the same. Each of these slopes is used to determine a respective delta-x and delta-y value that is dependent on the number of scan lines in the destination patch $\mathbf{4 0 0}\left(\mathrm{N}_{\mathrm{yd}}\right)$. Next, each pixel in the first scan line in the destination patch $\mathbf{4 0 0}$ is iterated. To do this, a delta-x and delta-y representing the first slope $\mathbf{4 2 8}$ is determined responsive to the number of pixels in the scan line contained by the destination patch $400\left(\mathrm{~N}_{\mathrm{xd}}\right)$, and the coordinates of the starting pixel and the ending pixel. As each destination pixel is iterated the mapped pixel contribution area $409^{\prime}$ in the source data space is evaluated to determine a value for the destination pixel. As the destination pixel is advanced, the corresponding position in the source data space is advanced by the delta-x and delta-y corresponding to the first slope 428. Once the first scan line has been processed subsequent scan lines in the destination patch 400 are processed. The starting coordinates for a subsequent scan line in the mapped patch $\mathbf{4 2 0}$ is advanced by the delta-x and delta-y corresponding to the second slope

429 and the ending position of the subsequent scan line in the mapped patch $\mathbf{4 2 0}$ is advanced by the delta-x and delta-y corresponding to the third slope 431. Thus, the subsequent slope for each mapped scan line changes dependent on the slope of the second boundary line 421 and the third boundary line 423. Thus for example, a subsequent slope 437 can be (and usually is) different from the first slope 428, the final slope 435 and any other subsequent slope.
[0053] Each subsequent scan line in the destination patch 400 is iterated (such that the last subsequent scan line is the final boundary line 427 ).
[0054] One skilled in the art will understand that the bilateral-bilinear interpolation algorithm, previously summarized and subsequently described in detail, assures that adjacent patches correctly align. Thus, the bilateral-bilinear interpolation algorithm does not generate mapping artifacts as shown in FIG. 1A and FIG. 1C.

## [0055] Data Space Mapping

[0056] FIG. 5 illustrates a mapping process, indicated by general reference character $\mathbf{5 0 0}$ used to backward map data points in a destination data space to data points in a source data space. The mapping process $\mathbf{5 0 0}$ initiates at a 'start' terminal 501 and continues to an 'initialization' procedure 503. The 'initialization' procedure 503 performs initialization steps for the mapping process $\mathbf{5 0 0}$. These steps can include steps for allocating memory for the source data space, allocating memory for the destination data space, determining the resolution of the presentation device (if any) used to present the destination data, and the spacing of grid points in the destination data space. Next, the mapping process $\mathbf{5 0 0}$ continues to a 'load source data' procedure $\mathbf{5 0 5}$ that inputs the source data into the source data space. The source data can be read (without limitation) from a file, a scanner device, a video device, from a network, a medical diagnostic tool or other similar device. In addition, the source data can represent a portion of a video data stream (the video data stream can be compressed; in addition the video stream can be live video, stored video or computer generated video). The 'load source data' procedure $\mathbf{5 0 5}$ need not complete before the mapping process $\mathbf{5 0 0}$ continues to a 'determine grid points' procedure 507. The 'determine grid points' procedure $\mathbf{5 0 7}$ uses the resolution and the size of the destination data space and possibly other parameters to determine the size of the region. Depending on the configuration of the source data space and the destination data space, the region can be n-dimensional. The region defines the data points that will be interpolated instead of being precisely mapped. The bilateral-bilinear interpolation algorithm can be applied to n-dimensional spaces. When the region is two-dimensional, the region is referred to as a patch.
[0057] A 'precisely map grid points' procedure $\mathbf{5 0 8}$ precisely maps the grid points that bound the selected region in the destination data space to points in the source data space. The 'precisely map grid points' procedure $\mathbf{5 0 8}$ uses well known transformations that can include floating point multiplication and division operations to precisely locate points in the source data space that correspond to the grid points in the destination data space.
[0058] Once the grid points that bound the region have been precisely mapped, the mapping process $\mathbf{5 0 0}$ continues
to an 'iterate region' procedure $\mathbf{5 0 9}$ that iterates each region in the destination data space that is to be interpolated. A 'get grid point coordinates in source data space' procedure $\mathbf{5 1 1}$ obtains the grid points that bound the iterated region. Then a 'map points in region' procedure $\mathbf{5 1 3}$ applies a bilateralbilinear interpolation algorithm to approximately map the points in the region to a portion of the data in the source data space. The bilateral-bilinear interpolation algorithm is subsequently described with respect to FIG. 6.
[0059] The mapping process $\mathbf{5 0 0}$ repeats to the 'iterate region' procedure 509 until all the regions in the destination data space are iterated. The resulting data in the destination data space can then be presented by a 'present destination space data' procedure 514. This presentation can be accomplished (without limitation) by visually presenting the information by using a presentation device such as a printer or monitor, by providing a printout of the data, or by subsequent processing of the data using other mechanisms. The mapping process $\mathbf{5 0 0}$ completes through an 'end' terminal 515.
[0060] One skilled in the art will understand that the precisely mapped grid points define lines in the source data space that can serve as boundary lines for the mapped region in the source data space.
[0061] FIG. 6 illustrates a bilateral-bilinear interpolation algorithm process, indicated by general reference character $\mathbf{6 0 0}$ that is invoked from the 'map points in region' procedure $\mathbf{5 1 3}$ of FIG. 5. A preferred embodiment is directed towards mapping data between two two-dimensional data spaces. This embodiment can be used to generate a perspective corrected image from a warped image that was generated from a true image projected through a lens (such as a fisheye lens or a catadioptric lens). One skilled in the art will understand that a physical lens need not be used to generate the warped image as ray-tracing techniques through a virtual lens can also be used to generate the warped image.
[0062] The bilateral-bilinear interpolation algorithm process 600 initiates at a 'start' terminal 601 and continues to an 'initialize' procedure 603. The 'initialize' procedure $\mathbf{6 0 3}$ determines the slopes for the boundary lines in the source data space that define the limits of the scan lines in the patch. The slope is defined by a delta-x and delta-y that depend on the number of scan lines in the patch. The 'initialize' procedure $\mathbf{6 0 3}$ also defines the starting and ending coordinates in the source data space for the first scan line that is to be interpolated. For a patch bounded by points $\mathrm{P}_{0}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)$, $\mathrm{P}_{1}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right), \mathrm{P}_{2}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$, and $\mathrm{P}_{3}\left(\mathrm{x}_{3}, \mathrm{y}_{3}\right)$ (these points corresponding to the mapped first grid point 401', the mapped second grid point 403 , the mapped third grid point $\mathbf{4 0 5}^{\prime}$, and the mapped fourth grid point 407' of FIG. 4B) in the source data space and where the patch in the destination data space includes $\mathrm{N}_{\mathrm{yd}}$ scan lines, each scan line containing $\mathrm{N}_{\mathrm{xd}}$ pixels, the 'initialize' procedure 603 can include steps similar to:
[0063] $\mathrm{dx} 1=(\mathrm{x} 2-\mathrm{x} 0) / \mathrm{nyd}$; // determine slope of left line
[0064] dyl=(y2-y0)/nyd;
[0065] dxr=(x3-x1)/nyd; // determine slope of right line
[0066] dyr=(y3-y1)/nyd;
[0067] startx=x0; // set starting coordinates

```
[0068] starty=y0;
[0069] endx=x1; // set ending coordinates
[0070] endy=y1;
```

[0071] Next, the bilateral-bilinear interpolation algorithm, process $\mathbf{6 0 0}$ continues to an 'iterate scan line in patch' procedure 605 that iterates each scan line in the patch in the destination data space. When all the scan lines in the patch have been iterated, the bilateral-bilinear interpolation algorithm process 600 completes through an 'end' terminal 607. The number of iterations to iterate each scan line in the patch is the value of nyd.
[0072] An 'initialize working variables' procedure 609 initializes the variables used for the iteration of the pixels in the iterated scan line. These initializations include determining the slope for the iterated scan line based on the coordinates of the start point and the end point of the scan line in the source data space. The start point of the scan line substantially lies on the boundary line defined by $\mathrm{P}_{0}$ and $\mathrm{P}_{2}$. The end point of the scan line substantially lies on the line defined by $P_{1}$ and $P_{3}$. Thus, these lines bound each scan line. The slope of the scan line is determined using the start point, the end point, and the number of pixels in the scan line in the patch. The 'initialize working variables' procedure 609 can include steps similar to:

```
[0073] dx=(endx-startx)/nxd; // determine scan line
    slope
[0074] dy=(endy-starty)/nxd;
[0075] sx=startx;
[0076] sy=starty;
```

[0077] An 'iterate each pixel in scan line' procedure 611 iterates each pixel in the destination scan line. To iterate each pixel in the scan line requires $\mathrm{N}_{\mathrm{xd}}$ iterations. When all the pixels in the line have been iterated, the bilateral-bilinear interpolation algorithm process 600 continues to an 'advance to next scan line in patch' procedure 613. The 'advance to next scan line in patch' procedure $\mathbf{6 1 3}$ advances the startx, starty, endx and endy values by dxl, dyl, dxr, and dyr respectively. Thus, the 'iterate each pixel in scan line' procedure $\mathbf{6 1 1}$ determines a subsequent starting point and a subsequent ending point that bound a subsequent line that has a subsequent line slope. Thus, the subsequent line depends on the slope of the boundary lines. The 'advance to next scan line in patch' procedure 613 can include steps similar to:
[0078] startx $+=$ dxl; // determine new scan line start
[0079] starty+=dyl; // coordinates
[0080] endx $+=d x r$; // determine new scan line end
[0081] endy+=dyr; // coordinates
[0082] One skilled in the art will understand that the interpolation of each scan line thus depends on the slope of the patch's bounding lines. Thus, the end position of each scan line in one region substantially matches the start position of each scan line for an adjacent patch. The result is that there are no interpolation artifacts between adjacent patches (or regions).
[0083] Once the 'advance to next scan line in patch' procedure $\mathbf{6 1 3}$ completes, the bilateral-bilinear interpolation
algorithm process 600 continues to the 'iterate scan line in patch' procedure 605 to iterate the next scan line in the patch or to exit if all scan lines have been iterated.
[0084] A 'set pixel' procedure 615 obtains the data value for the pixel in the destination data space from the data area specified by sx and sy in the source data space. One skilled in the art will understand how to obtain the value of a destination pixel from the source data space given coordinates in the source data space. Such a one will also understand how to combine data values in the source data space to generate the data value in the destination data space.
[0085] Once the 'set pixel' procedure 615 completes, the bilateral-bilinear interpolation algorithm process 600 continues to an 'advance to next pixel in scan line' procedure 617. The 'advance to next pixel in scan line' procedure 617 advances $s x$ and sy by dx and dy respectively. Next, the bilateral-bilinear interpolation algorithm process 600 continues to the 'iterate each pixel in scan line' procedure 611 until each pixel in the scan line has been iterated.
[0086] One skilled in the art will understand that the invention improves the mapping between two data spaces while still maintaining high performance.
[0087] From the foregoing, it will be appreciated that the invention has (without limitation) the following advantages:
[0088] 1) The invention dramatically reduces mapping artifacts when mapping from one data space to another.
[0089] 2) The invention provides a high performance mapping capability between data spaces.
[0090] 3) The invention provides a superior real-time presentation of a visual image when the source data space contains warped image data.
[0091] Although the present invention has been described in terms of the presently preferred embodiments, one skilled in the art will understand that various modifications and alterations may be made without departing from the scope of the invention. Accordingly, the scope of the invention is not to be limited to the particular invention embodiments discussed herein.

What is claimed is:

1. A computer controlled method including steps of:
determining a region in a destination data space, said region bounded by a plurality of grid points and said region defining a first plurality of data points in said destination data space;
precisely mapping said plurality of grid points to a plurality of mapped grid points in a source data space associating a second plurality of data points, wherein said plurality of mapped grid points define a plurality of boundary lines that represent the boundary of said region as mapped into said source data space; and
applying a bilateral-bilinear interpolation algorithm to map said first plurality of data points to said second plurality of data points.
2. The computer controlled method of claim 1 further including presenting said first plurality of data points using a presentation device.
3. The computer controlled method of claim 1 wherein said second plurality of data points represents a warped image and said first plurality of data points represents a perspective corrected image.
4. The computer controlled method of claim 3 wherein said warped image represents a true image warped by a lens, said perspective corrected image substantially representing said true image.
5. The computer controlled method of claim 4 wherein said lens is a catadioptric lens.
6. An apparatus having a central processing unit (CPU) and a memory coupled to said CPU, said apparatus including:
a region determination mechanism configured to determine a region in a destination data space, said region bounded by a plurality of grid points and said region defining a first plurality of data points in said destination data space;
a precise mapping mechanism configured to precisely map said plurality of grid points determined by the region determination mechanism to a plurality of mapped grid points in a source data space associating a second plurality of data points, wherein said plurality of mapped grid points define a plurality of boundary lines that represent the boundary of said region as mapped into said source data space; and
a bilateral-bilinear interpolation mechanism configured to map said first plurality of data points in said region to said second plurality of data points using said plurality of mapped grid points.
7. The apparatus of claim 6 further including a presentation device configured to present said first plurality of data points.
8. The apparatus of claim 6 wherein said second plurality of data points represents a warped image and said first plurality of data points represents a perspective corrected image.
9. The apparatus of claim 8 wherein said warped image represents a true image warped by a lens, said perspective corrected image substantially representing said true image.
10. The apparatus of claim 9 wherein said lens is a catadioptric lens.
11. A computer program product including:
a computer usable storage medium having computer readable code embodied therein for causing a computer to map a destination data space to a source data space, said computer readable code including:
computer readable program code configured to cause said computer to effect a region determination mechanism configured to determine a region in said destination data space, said region bounded by a plurality of grid points and said region defining a first plurality of data points in said destination data space;
computer readable program code configured to cause said computer to effect a precise mapping mechanism configured to precisely map said plurality of grid points determined by the region determination mechanism to a plurality of mapped grid points in said source data space associating a second plurality of data points, wherein said plurality of mapped grid points define a
plurality of boundary lines that represent the boundary of said region as mapped into said source data space; and
computer readable program code configured to cause said computer to effect a bilateral-bilinear interpolation mechanism configured to map said first plurality of data points in said region to said second plurality of data points using said plurality of mapped grid points.
12. The computer program product of claim 11 further including computer readable program code configured to cause said computer to drive a presentation device configured to present said first plurality of data points.
13. The computer program product of claim 11 wherein said second plurality of data points represents a warped image and said first plurality of data points represents a perspective corrected image.
14. The computer program product of claim 13 wherein said warped image represents a true image warped by a lens, said perspective corrected image substantially representing said true image.
15. The computer program product of claim 14 wherein said lens is a catadioptric lens.
16. A computer program product including:
a computer data signal embodied in a carrier wave having computer readable code embodied therein for causing a
computer to map a destination data space to a source data space, said computer readable code including:
computer readable program code configured to cause said computer to effect a region determination mechanism configured to determine a region in said destination data space, said region bounded by a plurality of grid points and said region defining a first plurality of data points in said destination data space;
computer readable program code configured to cause said computer to effect a precise mapping mechanism configured to precisely map said plurality of grid points determined by the region determination mechanism to a plurality of mapped grid points in said source data space associating a second plurality of data points, wherein said plurality of mapped grid points define a plurality of boundary lines that represent the boundary of said region as mapped into said source data space; and
computer readable program code configured to cause said computer to effect a bilateral-bilinear interpolation mechanism configured to map said first plurality of data points in said region to said second plurality of data points using said plurality of mapped grid points.

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