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ANALOG COMPUTERS FOR FORMING THE INTEGRAL OF ONE VARIABLE
WITH RESPECT TO ANOTHER VARIABLE
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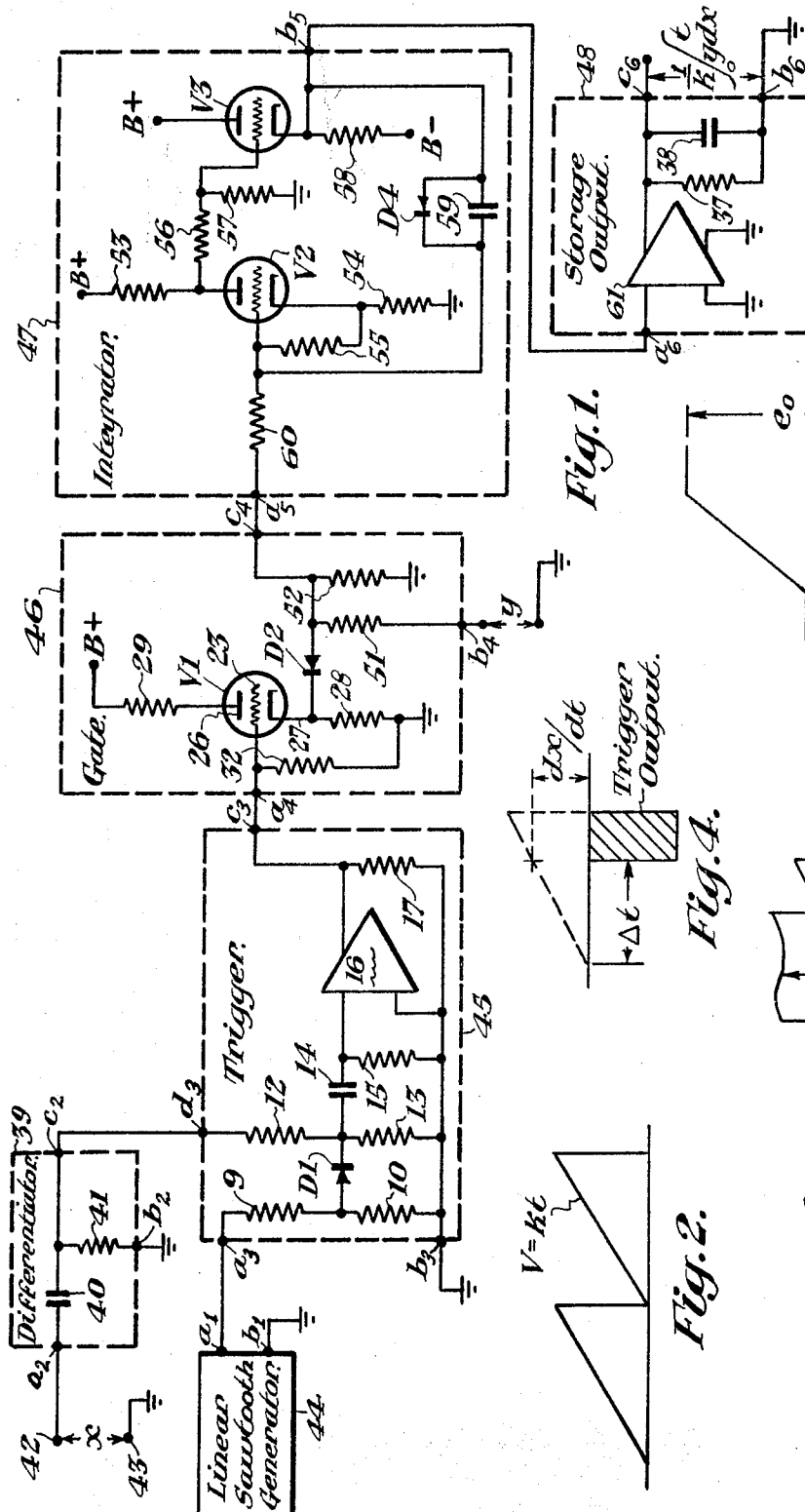


Fig. 1.

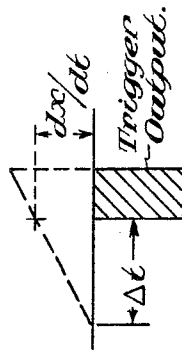


Fig. 4.

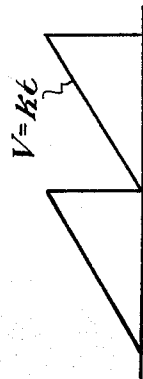


Fig. 2.

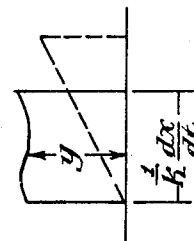


Fig. 5.

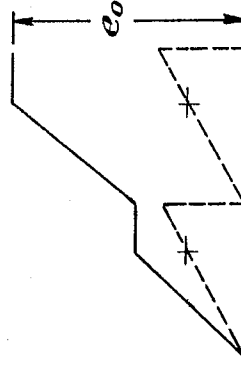


Fig. 6.

Fig. 3.

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ANALOG COMPUTERS FOR FORMING THE INTEGRAL OF ONE VARIABLE WITH RESPECT TO ANOTHER VARIABLE

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Original application May 15, 1959, Ser. No. 813,498, now Patent No. 3,104,319, dated Sept. 17, 1963. Divided and this application May 2, 1963, Ser. No. 277,581 4 Claims. (Cl. 235-183)

My invention relates to analog computers, and in particular to an improved analog computer for continuously generating a desired function of two variables.

This application is a division of my copending patent application for Letters Patent of the United States, Ser. No. 813,498, filed May 15, 1959, now Patent No. 3,104,319, for Analog Computers, and assigned to the assignee of the present application.

Known analog computers have in general been based either on moving parts, such as potentiometers driven by servomechanisms, or on circuits incorporating nonlinear elements in which the current or voltage characteristics of the elements approximate desired transformation functions. Computers of the latter type have in general either been limited in range or exceedingly complex. Computers of the former type are inherently relatively slow in operation; in addition, they require more or less frequent attention because of the wear inherent in moving parts.

It is an object of my present invention to provide an analog computer which has no moving parts and in which linearity is achieved without the use of elaborate circuitry because the circuits employed inherently operate in accordance with the functions to be generated.

As is well known in the art, it is possible to generate common functions of two variables, such as products and quotients, if an increment of time can be selected in accordance with the value of one of the variables so that the other variable may be integrated over this time increment. It is one object of my invention to provide a simple and reliable means for rapidly and continuously generating such time increments incorporating a novel combination of stable and reliable circuits.

It is another and more particular object of my invention to provide an analog multiplier for continuously developing the product of two variables with high linearity over any selected range of the variables.

Many devices are known in the art for continuously generating the integral of one variable with respect to time. However, the generation of the integral of one variable in respect to a second variable which is not time is a more difficult problem, which, so far as I am aware, has not been successfully solved. Accordingly, it is a third and more particular object of my invention to provide a simple device for rapidly and continuously forming the integral of one variable with respect to another.

Other objects and further advantages of my invention will become apparent to those skilled in the art as the description proceeds.

In carrying out my invention, I provide a conventional linear saw-tooth generator for repeatedly generating a triangular wave form at a frequency which is relatively high with respect to the expected rate of variation of the variables to be transformed. The output of this generator is compared with the time rate of change of one of the variable inputs, which is assumed to be provided in the form of a variable direct voltage, in a trigger circuit. The trigger circuit is so arranged that a large output pulse is provided when and only when the magnitude of the saw-tooth voltage exceeds the magnitude of the input. In this manner, the time of one cycle of the

saw-tooth generator is divided into a first interval which is proportional to the magnitude of the time rate of change of one of the input variables and a second interval which constitutes the remainder of the period of the saw-tooth. Thus, the initial object of my invention, to provide a time increment proportional to the magnitude of the variable, is achieved.

In carrying out multiplication, in accordance with the embodiment of my invention, I apply the pulse derived as described above to control a gate to which the second input variable is applied, so that the second variable is passed through the gate during the time in which the pulse is absent, which time, as described above, is proportional to the first interval of the one variable. The portion of the second variable that is passed through the gate is integrated, and the value of the integral stored. As will later appear, the output is directly proportional to the integral of the second variable with respect to the first variable.

The above and other aspects of my invention will be best understood from the following description when read in connection with the accompanying drawings. I shall first describe one embodiment of my invention in detail, and shall then point out the novel features thereof in claims.

In the drawings,

FIG. 1 is a schematic wiring diagram of an embodiment of my invention;

FIG. 2 shows a typical saw-tooth wave form;

FIG. 3 illustrates typical wave forms of a pair of input variables;

FIG. 4 is a composite wave form diagram illustrating the manner in which a trigger pulse is formed in the apparatus of my invention;

FIG. 5 is a typical wave form of a gate pulse in the device shown in FIG. 1; and

FIG. 6 is a typical wave form diagram illustrating the output of the integrator in the device of FIG. 1.

Referring now to FIG. 1, I have shown an embodiment of my invention which is adapted to provide the integral of one variable with respect to another. As shown, the apparatus essentially comprises a linear saw-tooth generator 44, a differentiator 39, a trigger 45, a gate 46, an integrator 47, and a storage output device 48. Saw-tooth generator 44 may be any one of a number of conventional and well-known circuits, such as are employed, for example, in oscilloscope circuits. Such circuits are so well known in the art as not to require further detailed description. For example, pages 260, 261, 262, and 264 of Radar Electronic Fundamentals, published in 1944 by the Bureau of Ships, United States Navy (Navships 900,016), show typical circuits which could be employed in my invention if so desired. The only essential requirement for generator 44 is that its output be substantially linear and of constant slope and that the duration of the rising portion of the voltage be moderately constant as illustrated in FIG. 2. As will appear, the apparatus does not depend for operation on the overall duration of cycles of the saw-tooth output, and some variation in the frequency of the generator could, therefore, be tolerated. The maximum amplitude of the saw-tooth voltage should exceed the maximum amplitude of the input voltage, and may be any suitable value substantially higher than this maximum amplitude.

I provide means 39 for differentiating the variable x before it is applied to the trigger circuit. The differentiator itself is shown as a simple RC combination comprising a capacitor 40 and a resistor 41, which operates in a manner well known in the art to produce a voltage across resistor 41 proportional to the time rate of change of the applied signal voltage x .

Trigger 45 may be any conventional high gain com-

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parator circuit which produces a relatively large square pulse when one of two applied inputs exceeds the other. As here shown, this circuit includes a comparison network comprising resistors 9, 10, 12 and 13 and diode D1, an isolating condenser 14 and a high gain amplifier 16 having input and output resistors 15 and 17, respectively. Amplifier 16 may be of any conventional construction, but preferably is provided with an odd number of stages having a relatively high overall gain so that a very small applied input voltage will produce a large, square, negative pulse at the output. In operation, a first voltage is applied across resistors 9 and 10 of the comparator and a second voltage is applied across resistors 12 and 13. Condenser 14 and resistor 15 are so selected that relatively low frequency changes in the voltage applied across resistors 12 and 13 will not cause any appreciable input to amplifier 16. Preferably, resistors 10 and 13 are chosen to be larger than resistors 9 and 12. For example, resistors 9 and 12 might be 1 megohm and resistors 10 and 13 might be 10 megohms. When the voltage applied across resistors 9 and 10 is less than the voltage applied across resistors 12 and 13, the presence of diode D1 blocks any conduction through the path including resistor 9 and resistor 13 in series. However, as soon as the voltage applied across resistors 9 and 10 exceeds the voltage applied across resistors 12 and 13, the voltage across resistor 10 will exceed the voltage across resistor 13 and the current between resistor 10 and the elements connected to the cathode of diode D1 causing the voltage drop to appear across resistor 13. The voltage across resistor 13 will thus increase and a signal will be applied to amplifier 16 which will continue to be applied as long as the voltage across resistors 9 and 10 continues to increase. During this interval of conduction, a negative output pulse will appear at terminal c_3 of trigger 45.

Gate 46 may be any conventional gate circuit of the type through which a first voltage is linearly transmitted until a second voltage is applied which cuts off the gate, as by effectively shunting the first voltage. However, in the preferred form of my invention this unit also functions to provide a reset pulse, and is thus designed to have a constant negative output voltage when not passing a positive signal. For example, as shown, a conventional vacuum tube V1 may be employed for this purpose. As shown, tube V1 has a cathode 27, a grid 23 and a plate 26. Operating voltage for the tube is supplied by any suitable means, as indicated by the B+ symbol representing a source having its positive terminal connected to plate 26 through resistor 29 and its negative terminal connected to ground. For reasons to appear, the cathode is returned to ground through a cathode resistor 28. Grid 23 is returned to ground through resistor 32.

As shown, the variable input signal voltage y is applied between input terminal b_4 and ground. Input terminal b_4 is also connected to ground through resistors 51 and 52 in series. The junction of resistors 51 and 52 is directly connected to output terminal c_4 , and is connected to cathode 27 through a diode D2.

The components are so selected that with amplifier 16 in trigger 45 producing no output, conduction in tube V1 causes a voltage to appear at cathode 27 which is greater than the largest value of the variable voltage y . Thus, in this condition diode D2 is blocked, and a voltage proportional to y appears at output terminal c_4 . When trigger 45 produces the large negative output pulse previously described, tube V1 is driven abruptly to cutoff and cathode 27 falls to substantially ground potential. Diode D2 now conducts, shunting the signal y through the relatively small cathode resistance 28, and thus terminal c_4 of gate 46 is referenced to ground.

Integrator 47 may be any conventional integrator, and here being shown as a two-stage amplifier comprising triodes V2 and V3, the first stage V2 acting as a plate

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follower and the second stage V3 acting as a cathode follower.

Triode V2 has its plate connected to the positive terminal of a suitable source of potential indicated by B+, the negative terminal of which may be assumed to be grounded, through a suitable plate resistor 53. The cathode of tube V2 is grounded, as shown, through a suitable cathode resistor 54, and the grid is returned to the cathode through a suitable grid resistor 55.

The output of tube V2 is resistance coupled to the grid of tube V3 through a potential divider comprising resistors 56 and 57 in series, as shown. Tube V3 acts as a conventional cathode follower, having its plate directly connected to B+ and its cathode connected to a suitable negative source B- through cathode resistor 58. The constants are selected so that with no input to the grid of tube V3, output terminal b_5 is at ground potential. Integration of positive input signals is accomplished in a conventional manner, by means of a capacitor 59 connected between the cathode of tube V3 and the grid of tube V2. The input signal is applied through resistor 60 to the grid of tube V2. Capacitor 59 gradually charges as the voltage builds up at the cathode of tube V3, but due to the degenerative feedback supplied to the input through capacitor 59, the time constant of the integrator is greatly increased. To prevent capacitor 59 from discharging below ground potential, a diode D4 is connected across it as shown.

The integrating action just described is conventional.

Storage output device 48 may be any one of a number of suitable conventional devices, but is here shown simply as an RC circuit for producing an output voltage proportional to the peak value of the pulses from integrator 4. Since it may be desirable to provide a positive output signal, a stage 61, which may be a conventional phase inverter, is shown in output device 48. The function of this unit is simply to invert the phase of pulses from integrator 47, and it may be entirely conventional in structure.

The output of amplifier 61 is connected to ground through a simple averaging circuit 37 and capacitor 38 in parallel. The values of R and C are selected such that, after several cycles of the pulses from integrator 47, the potential at output terminal c_6 is proportional to the peak value of the pulses from integrator 47. Should this peak value change, the output voltage will accordingly follow the change with a lag of a few cycles. Normally, the basic frequency of the pulses will be set by saw-tooth generator 44, the frequency of which is chosen to be greatly in excess of the frequency at which the variables will be expected to change. Normally, for the usual physical variables, if the frequency of the saw-tooth generator is in the order of kilocycles or tens of kilocycles, this condition will easily be satisfied.

In considering the operation of the embodiment of FIG. 1, reference will be made to FIGS. 2 through 6, which show typical wave forms at various stages in the equipment.

The output of saw-tooth generator 44 is generally of the form shown in FIG. 2. As indicated, the rise portion of this wave form may be represented by the linear relationship $v=kt$, where v is the voltage, t is the time, and k is a proportionality constant dependent on the slope. As shown in FIG. 3, the variable voltages x and y are direct voltages which are assumed to vary very slowly with respect to the frequency of saw-tooth generator 44, as illustrated by a comparison of FIGS. 2 and 3. (The relative variations have been considerably exaggerated in the drawings for purposes of illustration.)

As will appear from the above description, the voltage x is applied to the terminals 42 and 43, and a differentiated output is taken from terminal c_2 of differentiator 39. When the differentiated output of differentiator 39 is applied to terminal d_3 of trigger 45 and the output of saw-tooth generator 44 is applied to input terminal a_3 of

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trigger 45 no output will appear during the time Δt shown in FIG. 4, in which the saw-tooth output is less than the voltage dx/dt . When the saw-tooth exceeds dx/dt , and for the remainder of the rise time of the saw-tooth, the trigger will produce a negative output pulse as shown in FIG. 4. In other words, the voltage applied to terminal d_3 of trigger 45 will be proportional to dx/dt , and the time during which the trigger output is not present, Δt will thus be proportional to $(1/k)dx/dt$. This output pulse will be applied to gate 46 together with the variable voltage y . Thus, as shown in FIG. 5, the output of the gate will be positive pulses having a height y and a width $(1/k)dx/dt$.

The integrator operating on the pulses from gate 46 will have an output proportional to the integral of y over a time proportional to the rate of change of x . Mathematically, this relationship may be stated as follows:

$$\Delta t = \left(\frac{1}{k}\right) \frac{dx}{dt}$$

$$e_0 = \int_0^t \left(\frac{1}{k}\right) \frac{dx}{dt} y dt$$

This equation represents the output of the integrator for one cycle. Since the integrator is not reset by the process, the effect is to integrate these integrations over time. Thus, the output of the integrator is essentially a double integral equal to

$$e_0 = \int_0^t \int_0^t \left(\frac{1}{k}\right) \frac{dx}{dt} y dt dt$$

By known methods, since y is essentially constant over each of the one-cycle integrations, this expression can be reduced to

$$e_0 = \int_0^t y \int_0^t \left(\frac{1}{k}\right) \frac{dx}{dt} dt dt$$

$$e_0 = \int_0^t y \left(\frac{1}{k}\right) \frac{dx}{dt} dt$$

$$e_0 = \left(\frac{1}{k}\right) \int_0^t y dx$$

Accordingly, it is clear that this embodiment of my invention will function to produce an output voltage of the type shown in FIG. 6 which follows the desired integral of y with respect to x . As noted above, the output of the saw-tooth generator will be chosen to be much higher in frequency than the possible variations of the input variables, so that the discontinuities shown in FIG. 6 for illustration purposes will be inconsequential in practice.

While I have shown only one embodiment of my invention in detail, it will be apparent to those skilled in the art that many modifications and changes can be made without departing from the scope of my invention. Accordingly, I do not wish to be limited to the details shown, but only by the scope of the following claims.

Having thus described my invention, what I claim is:

1. In an integrator, in combination, differentiating means controlled by a first signal to produce a second signal in accordance with the time rate of change of said first signal, means for generating a linear triangular signal, triggering means controlled by said second signal and said linear triangular signal for producing a trigger pulse during the time said linear triangular pulse exceeds said second signal, gate means controlled by said trigger

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pulse to permit the passage of a third signal during the absence of said trigger pulse, and integrating means connected to said gate means to integrate the passed portion of said third signal.

2. An integrator, comprising, in combination, differentiating means controlled by a first signal to produce a second signal in accordance with the time rate of change of said first signal, generator means for producing a series of saw-tooth pulses, triggering means controlled by said second signal and said series of saw-tooth pulses for repeatedly producing trigger pulses during the time when the amplitude of said series of saw-tooth pulses exceeds the amplitude of said second signal, gate means controlled by said trigger pulses for repeatedly blocking a third signal during the presence of said series of triggering pulses and for permitting the passage of said third signal during the absence of said triggering pulses in order to produce a series of gated pulses having amplitudes proportional to the amplitude of said third signal and durations proportional to the time rate of change of said first signal, and integrating means controlled by said gated pulses for producing an output signal in accordance with the integral of said third signal with respect to said first signal.

3. Integrating means, comprising, in combination, differentiating means, means for applying a first variable signal to said differentiating means to produce a rate signal, an oscillator for producing a linear saw-tooth signal, comparator means controlled by said saw-tooth signal and said rate signal for producing a pulse when said saw-tooth signal exceeds said rate signal, gate means having signal transmitting and signal blocking conditions, means for applying a second variable signal to said gate means to produce an output in accordance with said second variable signal in the signal transmitting condition of said gate means, means for applying said pulse to said gate means to actuate it to its signal blocking condition, and an integrator connected to said gate means to integrate the transmitted portions of said second variable signal.

4. An integrator, comprising, in combination, first and second sources of voltage varying in accordance with a first and a second variable, respectively, differentiating means controlled by said first source for producing a first signal in accordance with the time rate of change of said first voltage, generating means for producing a saw-tooth signal, triggering means controlled by said first signal and said saw-tooth signal for producing a first output during a first portion of said saw-tooth signal and a second output during a second portion of said saw-tooth signal, gating means controlled by said triggering means for permitting the passage of said second voltage during the presence of said first output and for blocking the passage of said second voltage during the presence of said second output and integrating means for integrating the passed portion of said second voltage to produce a second signal in accordance with the integral of said second variable with respect to said first variable.

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