

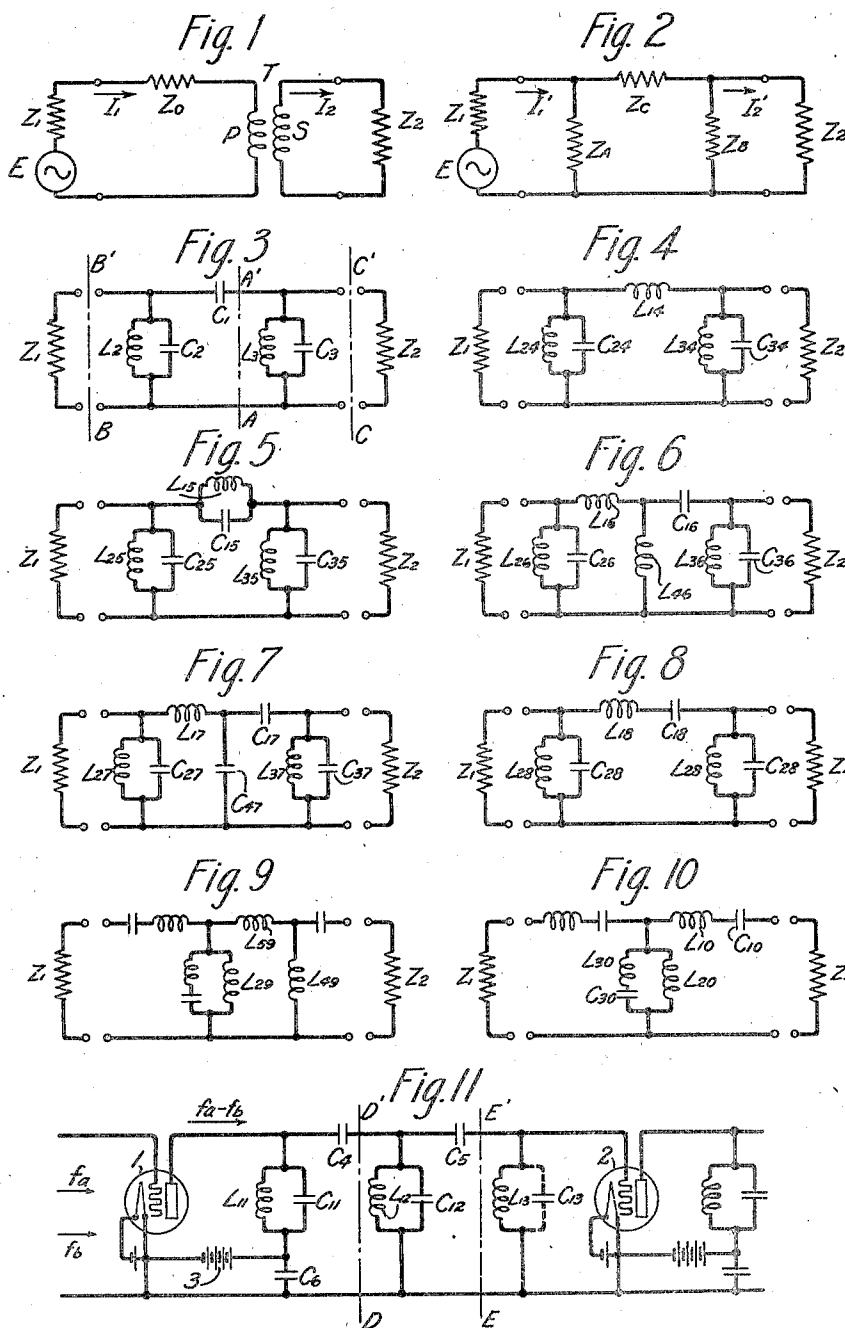
April 16, 1929.

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1,708,950

ELECTRIC WAVE FILTER

Original Filed May 12, 1925



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UNITED STATES PATENT OFFICE.

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ELECTRIC WAVE FILTER.

Application filed May 12, 1925, Serial No. 29,690. Renewed March 16, 1928.

This invention relates to wave filters and more particularly to filters having wave transforming properties.

The object of the invention is to permit highly efficient and selective transmission of a band of waves having definite limiting frequencies, between terminating impedances of different magnitudes.

The principles involved in the invention are broadly disclosed in copending application 751,748 filed Nov. 24, 1924 (which became Patent 1,681,554, Aug. 21, 1928) of which the present application is a continuation in part, with particular reference to types of filters embodying one species of the invention. In the present specification a number of examples will be illustrated and described of filters embodying a different specific form of the broad invention, the scope of the present invention being pointed out in the claims.

The type of wave filter with respect to which invention is disclosed is that invented by G. A. Campbell and described in his U. S. Patent No. 1,227,113 issued May 22, 1917,—the unique feature of which is that it permits the transmission through a system of a band of waves between definite limits of frequency and discriminates sharply against waves having frequencies outside the band. For further information regarding the theory of this filter and the design of many specific forms reference is made to the following published articles; Physical theory of the electric wave filter—G. A. Campbell, Bell System Technical Journal, Vol. I, No. 2, November 1922; The theory and design of uniform and composite wave filters,—O. J. Zobel, Bell System Technical Journal, Vol. II No. 1, January 1923; and, Transmission characteristics of electric wave filters—O. J. Zobel, Bell System Technical Journal, Vol. III, No. 4, October 1924.

In these references the principle is developed that the properties of an included portion of an infinite transmission line, consisting of a sequential arrangement of homologous four-terminal networks, may be identical with the self properties of the individual networks. It is also shown, particularly in

the reference last mentioned, that the transmission properties of a system comprising a finite number of similar networks terminated in any arbitrary impedances may be readily computed in terms of certain simple over-all properties of the networks, whereby the laborious process is avoided of obtaining a solution in terms of the parameters of the elementary components of the networks.

By this method of attack the rules for proportioning networks to secure various types of broad band selectivity have been discovered and set forth in simple form, and networks of many types have been devised having band filter properties of various kinds.

Although the filter characteristics of the individual networks, or sections, are exactly reproduced only in the infinitely extended line, it has been found that they are reproduced with great accuracy in a finite system comprising a limited number of sections connected between resistive terminating impedances, and are present to a controlling degree even when only one section is included between the terminating resistances. The case of resistive termination is one of great practical interest and importance since wave generators and receivers must necessarily be substantially free from reactance to achieve high efficiency in performing their functions.

Hitherto filter systems have been designed and constructed to operate efficiently between equal terminating impedances and in the event that the terminating impedances were necessarily unequal one or more transformers have been used to transform the impedances to equal values suitable for terminating the filter.

In accordance with the present invention the transformation is accomplished by the filter itself and the need for additional transforming devices is eliminated.

The invention will be fully understood from the following detailed description taken in connection with the accompanying drawings of which

Figures 1 and 2 illustrate a theorem upon which the invention is based,

Figures 3, 4 and 5 illustrate specific forms of wave transforming filter sections having

the same configurations as their non-transforming prototypes,

Figures 6 and 7 illustrate other specific forms of wave transforming filters of which the non-transforming prototype is shown in Figure 8,

Figures 9 and 10 show respectively an additional form of transforming filter and the non-transforming prototype;

Figure 11 illustrates in schematic form an application of the invention to the coupling of amplifiers in tandem.

In the network shown in Fig. 1 there is included between two terminating impedances Z_1 and Z_2 a series resistance Z_0 and a transformer T having certain ideal properties. Broadly these properties are that the transformer neither stores nor dissipates wave energy in its windings and transforms the wave intensity uniformly at all frequencies. Specifically the windings should be of infinite inductance and of zero resistance and should be perfectly coupled together. The inductances, although each is infinitely great, should have a finite ratio depending on the transformation ratio desired.

The network of Fig. 2, in which the combination of Z_0 and the ideal transformer is replaced by a Π network of impedances Z_a , Z_b and Z_c , is equivalent to that of Fig. 1 with respect to the transmission of waves between the terminating impedances Z_1 and Z_2 provided that the following relationships

exist between the impedances of the two systems;

$$\begin{aligned} Z_c &= \phi Z_0 \\ Z_a &= \frac{\phi}{\phi-1} Z_0 \quad (1) \\ Z_b &= -\frac{\phi^2}{\phi-1} Z_0 \end{aligned}$$

in which ϕ is the step-up voltage transformation ratio of the ideal transformer, or ϕ^2 is the ratio of the inductance of the secondary winding S to that of the primary winding P .

It should be noticed that the sum of the impedances Z_a , Z_b and Z_c is zero and that Z_b and Z_a are in the ratio $-\phi$.

The proof of the equivalence of the two circuits may be established by the application of Kirchhoff's laws to determine in each case the current in the impedance Z_2 corresponding to an impressed electromagnetic force E in series with Z_1 .

In the network of Fig. 1 the assumption of ideal transformer properties gives the solution

$$I_2 = \frac{E}{\phi(Z_1 + Z_0) + \frac{Z_2}{\phi}} \quad (2)$$

I_2 denoting the current in the impedance Z_2 . The direct solution of the network of Fig. 2 gives for the received current, denoted by I'_2 , the value

$$I'_2 = \frac{EZ_a Z_b}{(Z_1 + Z_a)\{(Z_a + Z_b + Z_c)(Z_2 + Z_b) - Z_a^2\} - Z_a^2(Z_2 + Z_b)} \quad (3)$$

which, when simplified by substituting the values given by Equations 1 for Z_a , Z_b and Z_c and by noting that the sum of these impedances is zero, becomes identical with the value given by Equation 2.

By a similar procedure it may also be shown that the two input currents I_1 and I'_1 are equal, thereby completely establishing the equivalence of the two networks with respect to wave transmission from Z_1 to Z_2 .

The equivalence of the two for wave transmission in the opposite direction follows directly from the principle of reciprocity, or it may be separately established by the methods already described.

The impedances Z_a , Z_b and Z_c are all of like type and of the same type as Z_0 ; for example, if Z_0 consists of an inductance Z_a , Z_b and Z_c would also be of the nature of inductances. But, one of the inductances, either Z_a or Z_b depending upon whether the ratio ϕ is less or greater than unity, must be negative, and consequently to construct a physical circuit exactly in accordance with Fig. 2 it would be necessary to have a new kind of circuit element, namely a negative inductance.

The construction of networks in accordance with Fig. 2 in its most general sense would require not only negative inductances but also negative capacities and resistances. It has been customary to speak of a capacitive reactance as negative and of an inductive reactance as positive, and out of this there has grown a practice of regarding an inductance as a negative capacity and vice versa. A capacity, however, can correspond to a negative inductance at one wave frequency only and therefore cannot be used as a negative inductance element in the transforming network. The negative elements required to make the network of Fig. 2 universally equivalent to that of Fig. 1 must in each case correspond to constant positive elements, and as such elements are not at present available the construction of the transforming network as a separate physical entity is practically impossible.

If, however, the network is included in an extended system it may be so construed that the negative elements can be absorbed in or overbalanced by positive elements of the proper type which form essential parts of the system. Under these circumstances the

desired transforming effect may be fully realized.

Wave filters of the types described in the references mentioned in an earlier part of this specification are especially adapted to modification by means of the transforming network the result of the modification being the production of new types of filter in which the waves are transformed in intensity while the selective characteristics remain unaltered.

Examples of wave transforming filters are shown in Figs. 3 to 7 inclusive and Fig. 9. In each example a single section is shown included between unequal terminating impedances Z_1 and Z_2 . The meaning of the term section as applied to recurrent wave filters is defined in the aforementioned references in which the various types of section with respect to the mode of termination are also described to filters of the so-called ladder type which comprise a tandem arrangement of similar series impedances and similar shunt impedances. In a ladder type structure of the repeated, or uniform, type the most important sections are the mid-series and the mid-shunt sections. The mid-series section in its generalized form is a T-network comprising two equal series impedances, each being equal to one half of the impedance of a complete series branch of the repeated structure, and a shunt impedance equal to the impedance of a complete shunt branch of the repeated structure. The mid-shunt section is obtained, as it were, by cleaving the repeated structure through the middle of two successive shunt impedances; it therefore consists of a Π network having two shunt impedances each equal to twice the impedance of a complete shunt branch and a full series branch impedance connected therebetween.

Important characteristics of the uniform filter sections are the iterative impedance and the propagation constant. These are most conveniently defined with reference to an infinitely long uniform filter for the reason that the properties of the infinite filter are most simply related to the self properties of the individual section.

The iterative impedance is the impedance looking into the infinitely extended filter. If the filter be extended infinitely in both directions it may be conceived that a cleavage is made at some arbitrary point and the impedance of one or both portions is measured at the cleavage. There is an indefinite number of iterative impedances which depend upon the particular point at which the cleavage is made. If it is made at mid-shunt the impedances of both portions of the doubly extended line will be equal, and so also if the cleavage is made at mid-series. This symmetry is uniquely characteristic of the two points of section and in consequence the mid-

series and mid-shunt iterative impedances acquire special importance.

Another distinctive characteristic of the mid-section terminations is that in selective circuits, such as wave filters, in which the elements are purely reactive the character of the iterative impedance is closely related to the selectivity characteristic of the circuit. Within the limits of the pass-bands the iterative impedances are purely resistive, and in the frequency ranges in which waves are attenuated the iterative impedances are purely reactive.

In addition to this it should be noted that the mid-shunt and mid-series sections of a uniform filter are symmetrical with respect to the input and the output terminals.

The wave transforming filter sections of the present invention, on account of their transforming properties, are unsymmetrical structures with respect to their input and output terminals. Their mode of operation and the method of determining the design is, however, most readily understood by considering how they are related to symmetrical sections of uniform filters.

Referring to Fig. 3, the network included between the dotted lines B, B' and C, C' may be considered as a connecting link between an extended low impedance filter, represented by the generalized impedance Z_1 , and an extended high impedance filter represented by Z_2 . These two filters may be assumed to comprise uniform sections of the same general form as the connecting circuit.

The connecting circuit could be constructed in two parts, one part being designated simply as an extension of the low impedance filter and the other to match the high impedance filter. The two parts could then be coupled together, for example at the section AA', through a transformer having an appropriate transformation ratio and possessing as fully as possible the properties of an ideal transformer.

If this is done the connecting circuit will include a combination of the type illustrated in Fig. 1, namely a series impedance constituted by the series condenser and an ideal transformer. It will cause no change in the characteristics of the system, therefore, if the equivalent network of three capacities in accordance with Fig. 2 is substituted for the combination. The negative capacity of the transforming network may, in this case, be combined with the positive capacity of one of the shunt branches to produce a positive resultant.

To elucidate further the procedure outlined in the foregoing paragraphs, design formulae for the filter section of Fig. 3 will be developed in detail.

Since the procedure is not affected by the particular nature of the terminating imped-

ances Z_1 and Z_2 it may be assumed that they represent transmission lines having resistive impedances R_1 and R_2 respectively. The filter section will first be computed as a symmetrical mid-shunt section suitable for connecting two equal resistances of the value R_1 after which the modification required to give the proper wave transformation will be determined.

The designations C_1, L_2, C_2, L_3 and C_3 , of the circuit elements refer to the final structure and the symbols denote also the respective capacity and inductance values. The same symbols accompanied by a prime mark will be used to denote the coefficients of the initial structure.

The coefficients of the symmetrical section designed to the impedance R_1 are given by the following formulæ in which f_1 and f_2 denote respectively the lower and the upper limiting frequencies of the transmission band.

$$\left. \begin{aligned} C'_1 &= \frac{f_1 + f_2}{4\pi f_1 f_2 R_1} \\ C'_2 = C'_3 &= \frac{f_1}{2\pi f_2 (f_2 - f_1) R_1} \\ L'_2 = L'_3 &= \frac{(f_2 - f_1) R_1}{2\pi f_1 f_2} \end{aligned} \right\} \quad (4)$$

The derivation of these formulæ is explained by O. J. Zobel, Theory and design of uniform and composite electric wave filters, Bell System Technical Journal Volume II No. 1, January 1923, the formulæ relating to Fig. 3 being shown in Appendix II Section V.

If, now, it is assumed that the wave transforming property is to be secured by inserting an ideal transformer at the section AA', the best value of the transformation ratio ϕ is that corresponding to the maximum flow of energy through the system, and is defined by the equation

$$\phi = \sqrt{\frac{R_2}{R_1}}$$

The insertion of the transformer requires that the part of the filter section to the right of AA' be increased in impedance in the ratio $1:\phi^2$. This requires that the value of L'_3 be multiplied by ϕ^2 and the value of C'_3 be divided by ϕ^2 .

The next step of substituting, for the combination of series capacity C'_1 and the ideal transformer the equivalent transforming network, leads directly to the final design.

The series capacity C_1 is equal to the capacity C'_1 first computed decreased in the ratio $1:\phi$. Since the transforming network comprises capacity elements the impedances are increased inversely as the capacities are reduced, hence the multiplying factor involving the ratio ϕ in Equation 1 must be inverted to give the capacity values in terms of C'_1 .

The left hand shunt capacity C_2 is equal

to the resultant of capacity C'_2 in parallel with a positive capacity having the value $C'_1(\phi - 1) \div \phi$, and the right hand shunt capacity C_3 is the resultant of the parallel combination of the capacity $C'_3 \div \phi^2$ and a negative capacity $C'_1(1 - \phi) \div \phi^2$.

The values may now be written down as follows:

$$\left. \begin{aligned} C_1 &= C'_1 \div \phi, \\ C_2 &= C'_1 + \frac{\phi - 1}{\phi} C'_1, \\ C_3 &= \frac{C'_2}{\phi^2} + \frac{(1 - \phi) C'_1}{\phi^2} \end{aligned} \right\} \quad (6)$$

By means of Equation (4) these values may be expressed in terms of the line resistance and the cutoff frequencies, for example:

$$C_3 = \frac{1}{4\pi f_2 R_1 \phi^2} \times \frac{2f_1^2 + (1 - \phi)(f_2^2 - f_1^2)}{f_1(f_2 - f_1)} \quad (7)$$

An inspection of Equation (7) shows that for large values of ϕ_0 the resultant capacity may become negative and that there is consequently a limiting value of the transformation ratio which cannot in practice be exceeded. The limiting value, denoted by ϕ_0 , is given by the equation

$$\phi_0 = \frac{f_2^2 + f_1^2}{f_2^2 - f_1^2} \quad (8)$$

This value of the limiting ratio is not general but refers only to the network of Figure 3 and to certain closely related networks. The value in any other specific case may be computed quite readily by equating the magnitude of the negative element in the transforming network to the magnitude of the positive element with which it is to be combined.

The reasoning process described above in connection with the design of the section renders it self-evident that the wave selection properties of the filter are maintained. Certain impedance properties of the section will now be considered.

It may be assumed that the impedance Z_1 comprises an infinite sequence of symmetrical mid-shunt sections of the same schematic type as the coupling section and designed in accordance with Equation (4) for an impedance R_1 . The impedance Z_2 may likewise be assumed to comprise a similar filter designed for an impedance R_2 .

Under such circumstances the impedances measured on both sides of the section BB' will be equal and so also will the impedances on both sides of the section CC'. At the latter section however the impedances will be ϕ^2 times greater than at the former section, that is the impedances at both sections will have the same kind of characteristic with respect to frequency but for any specified frequency they will be in the ratio $1:\phi^2$.

This is readily seen since the transforming section is in effect equivalent to a uniform section of impedance K_1 , for example, followed by an ideal transformer or voltage ratio ϕ .

The section therefore operates as if it had different iterative impedances at its opposite terminals, the impedances being in the ratio ϕ^2 . By definition, however, the iterative impedance of a network is the impedance it exhibits when terminated at its remote terminals by an infinite line of repeated sections, or, in other words, when terminated by its own iterative impedance. But the transforming section is not intended to operate under this condition, consequently its actual iterative impedances have not great significance.

Under the ideal operating conditions described above the transforming section couples together two infinite filters without there resulting any reflection losses at the junctions.

There are therefore two impedances which may be used to terminate the unsymmetrical section, one at the one end and one at the other end, so as to produce a system free from reflection loss. These impedances have been termed the "image impedances" of the section.

It has been shown in one of the references already mentioned, Transmission characteristics of wave filters—O. J. Zobel, Bell System Technical Journal, Volume III, No. 4, October, 1924, that the image impedance for one end of a four terminal network is the geometric mean of the open circuit and short circuit impedances measured at that end. The image impedances are these characteristic properties of the network itself without reference to any system in which it may be connected. In the special case of a symmetrical network the image and the iterative impedances are identical.

For efficient operation the transforming section should be terminated in its own image impedances or in physical impedance structures that furnish a satisfactory approximation thereto. Successive transforming sections may be connected in tandem and each may have the same transformation ratio or different ratios may be employed but each succeeding section must start off from the impedance reached by the preceding one in order that reflection losses may be avoided.

In general sections having equal image impedances may be joined together to form any desired sequence, just as in the case of symmetrical filters sections having equal iterative impedances may be joined together.

In considering the flow of energy through these wave transforming filters it should be observed that the electromagnetic force and the current of transmitted waves will be transformed in inverse ratios. When the electromagnetic force is increased by the ratio being

greater than unity the current will be proportionately diminished and vice versa. The resultant electromagnetic force and current at the output end of such a filter may be found by first of all determining, by well known methods, the values that would result if the whole system were designed uniformly for the image impedance at the input end and then applying a single transformation ratio equal to the product of all the transformation ratios used in the system to the values so computed.

From the preceding explanation the transforming properties and the computation of the additional sections will be readily understood and the description of these will be limited to their most important characteristic features.

Formulae relating to the design of the symmetrical, or non-transforming, filters from which the individual transforming sections may be derived are given by O. J. Zobel, Theory and design of uniform and composite electric wave filters, Bell System Technical Journal, Vol. II, No. 1, January, 1923, reference being made particularly to Appendix II, Sections IV to XI inclusive.

In Figures 4 and 5 as in Fig. 3 the symbols designate the coefficients of the unsymmetrical transforming network. The coefficients of the symmetrical structures corresponding to these will be denoted by the same symbols accompanied by a prime mark. The symmetrical prototypes of Figs. 4 and 5 have the same structural arrangements of elements as the transforming sections. In each case it will be assumed that the symmetrical prototype is designed to match the lower impedance Z_1 .

In the network of Fig. 4 the series inductance is the basis of the transformation, and the shunt inductances of the Π type transformer equivalent are combined with the natural shunt inductances of the section to give the final inductances L_{24} and L_{34} .

Following the method outlined in connection with Fig. 3 the coefficients of Fig. 4 may be written down in terms of the coefficients of the symmetrical prototype. The values follow:

$$\left. \begin{aligned} L_{14} &= \phi L'_{14} \\ L_{24} &= \phi \frac{L'_{24} L'_{14}}{(\phi - 1) L'_{24} + \phi L'_{14}} \\ L_{34} &= \phi^2 \frac{L'_{34} L'_{14}}{(1 - \phi) L'_{34} + L'_{14}} \\ C_{24} &= C'_{24} \\ C_{34} &= C'_{34} \div \phi^2 \end{aligned} \right\} \quad (9)$$

The value of L_{34} becomes infinite for the special value of ϕ ($=\phi_0$) expressed by the equation

$$\phi_0 = 1 + \frac{L'_{14}}{L'_{24}} \quad (10)$$

for which condition the inductance may be dropped from the circuit.

In the section shown in Fig. 5 the series branch upon which the transformation is based consists of an inductance and a capacity connected in parallel. The shunt branches have the same configuration and the negative elements of the transformer equivalent may be absorbed therein.

The values of the coefficients in terms of those of the prototype structure are expressed by the following equations.

$$\left. \begin{aligned} L_{15} &= \phi L'_{15} \\ C_{15} &= C'_{15} \div \phi \\ L_{25} &= \frac{\phi L'_{15} L'_{25}}{(\phi - 1)L'_{25} + \phi L'_{15}} \\ C_{25} &= C'_{25} + \frac{\phi - 1}{\phi} C'_{15} \\ L_{35} &= \phi^2 \frac{L'_{15} L'_{35}}{(1 - \phi)L'_{35} L'_{15}} \\ C_{35} &= \frac{1}{\phi^2} \left(C'_{35} + (1 - \phi) C'_{15} \right) \end{aligned} \right\} \quad (11)$$

For one value of the transformation ratio the inductance L_{35} becomes infinite, and becomes negative when this value is exceeded. The capacity C_{35} becomes zero for another value of the ratio and negative for greater values. The limiting value of the transformation ratio is the lower of the two initial values.

The sections shown in Figs. 6 and 7 are based on the symmetrical prototype shown in Fig. 8. That of Figure 6 is derived from the prototype by making the series inductance L_{18} the basis of the transformation.

It follows from Equation (1) that there can only be a step down transformation in the direction from Z_1 to Z_2 , otherwise the inductance L_{46} , which corresponds to Z_B of Fig. 2, would be negative and the network would be impracticable. The limiting value of the transformation ratio is that which makes the inductance L_{26} infinite, this being the inductance in which the negative element of the transformer equivalent is included.

The coefficients of the network of Fig. 6 are given in terms of the coefficients of the prototype network of Fig. 8 and the transformation ratio, by the following equations:

$$\left. \begin{aligned} L_{16} &= \phi L_{18} \\ L_{26} &= \phi \frac{L_{18} L_{28}}{(\phi - 1)L_{28} + \phi L_{18}} \\ L_{46} &= \frac{\phi^2 L_{18}}{(1 - \phi)} \\ C_{26} &= C_{28} \end{aligned} \right\} \quad (12)$$

The remaining coefficients C_{16} , L_{36} and C_{36} differ from the corresponding prototype coefficients in the ratio ϕ^2 , the capacities and the inductance being changed in inverse ratios.

In Fig. 7 is shown a second transforming network based on the prototype network of Fig. 8 but in this instance the series capacity C_{18} is used to give the transformer equivalent. This type of network must have a voltage step-up between Z_1 and Z_0 in order that the capacity C_{47} may be positive. The limiting value of the ratio is that which reduces to zero the value of capacity C_{37} , which analogously to L_{26} of Fig. 6, is the element that includes the negative element of the transformer equivalent. The relationships between the elements of the transforming network and its symmetrical prototype may be derived by the methods already described.

The wave filter sections of Figs. 3 to 8 inclusive are all of the mid-shunt type but the invention is not confined to this type alone as is illustrated by the example of a mid series transforming section of Fig. 9 and its symmetrical prototype shown in Fig. 10.

The shunt branch impedance of Figs. 9 and 10 differs from those of the other sections in that it includes one parallel branch consisting of a series resonant circuit. It might appear that since the series branches of Fig. 10 also consist of series resonant circuits the transformer equivalent might be based upon one complete series branch rather than on a single element. But while it is true that a pair of simple anti-resonant circuits connected in parallel may be replaced by a single anti-resonant circuit, and that two series resonant circuits connected in series are equivalent to a single resonant combination, it is not possible to replace a combination of two series resonant circuits in parallel by a single resonant circuit.

In Fig. 9 the transformation is based on the series inductance L_{10} of the prototype network, the transformer equivalent comprising L_{59} , L_{49} , and a negative component combined in L_{29} .

The foregoing description of the various examples of wave transforming filter sections discloses a general method of effecting wave transformation which is capable of application in many types of circuits besides those illustrated. The design procedure disclosed calls for the design in the first place of a non-transforming system having the desired energy propagation characteristics; it would be possible to derive formulae in each case for the direct computation but such a procedure would be much more complicated. Further, since the design of uniform, non-transforming systems is already a well-known art, any direct method of design would be less easily understood and applied than one based on a simple addition to the known principles.

As has already been pointed out the transformer equivalent Π network does not have a separate embodiment but it nevertheless

may be regarded as having a physical embodiment in combination with other elements.

An example of a practical application of the wave transforming section of Fig. 3 is shown in Fig. 11. The circuit illustrated represents a portion of a radio receiving system designed to operate in accordance with the so-called super-heterodyne principle. Space discharge device 1 is a demodulator, or detector upon which are simultaneously impressed a modulated wave of carrier frequency f_a representing the received radio wave, and an auxiliary wave of frequency f_b derived from a local source. The derived demodulation products are the carrier wave of intermediate frequency $f_a - f_b$ and the accompanying side frequencies corresponding to the modulations of the received radio wave. The wave filter included between the output terminals of 1 and the input terminals of a second space discharge device 2 is designed to select the desired waves and at the same time to step them up in voltage. By virtue of its band-pass properties the filter transmits all components of the intermediate frequency waves with equal efficiency and suppresses all other waves.

Space discharge tube 2 is an amplifier for the intermediate frequency waves and may be followed by additional stages coupled by similar selective circuits before the final detection of the signals is effected.

The filter is designed to transmit waves between the limiting frequencies 45000 c. p. s. and 55000 c. p. s. and to be terminated at its input end in a resistance of 20,000 ohms, the space path resistance of demodulator 1. The requirements as to selectivity were such as to make it necessary to use two complete filter sections.

In order that the final filter elements may have such values as would render their manufacture simple and economical both sections have been designed as wave transforming sections, the result being equivalent to the insertion at the position DD' of an ideal transformer of ratio ϕ_1 and at the position EE' of a second transformer of ratio ϕ_2 .

A further design condition was set in that the input capacity of amplifier 2 was required to function as the final capacity C_{13} , a fixed value of 20 micro-microfarad being found to be the proper value. Satisfactory values of the transformation ratios were found by successive approximations, the values finally chosen being

$$\phi_1 = .45$$

$$\phi_2 = 5.0$$

The first transformation actually steps down the voltage, the over-all step-up transformation ratio being equal to 2.25.

Final computations based on Equations

(4) and (6) gave the following values for the coefficient.

$$L_{11} = 12.9 \text{ millihenrys}$$

$$L_{12} = 1.3 \text{ millihenrys}$$

$$L_3 = 64.5 \text{ millihenrys}$$

$$C_{11} = 453 \text{ micro-microfarads}$$

$$C_{12} = 1100 \text{ micro-microfarads}$$

$$C_{13} = 20 \text{ micro-microfarads}$$

$$C_4 = 359 \text{ micro-microfarads}$$

$$C_5 = 160 \text{ micro-microfarads}$$

Capacity C_6 functions verily as a low impedance shunt to the space current battery, and it should have a large capacity, for example 2 microfarads, so that its impedance is negligibly small at the frequencies of the waves transmitted through the filter.

What is claimed is:

1. An impedance transforming wave filter having a broad band pass characteristic, characterized in this that the filter includes as an impedance transforming means of constant transformation ratio a π network of impedances, at least one of the shunt impedances of the π network being combined with an adjacent shunt branch of the filter.

2. An impedance transforming wave filter having a broad band pass characteristic, and including as an impedance transforming means of constant transformation ratio a π network whose branches comprise similar impedances the coefficients of which are related by functions of the impedance transformation ratio, at least one of the shunt impedances of the π network being combined with an adjacent shunt branch of the filter.

3. An impedance transforming wave filter in accordance with claim 2 characterized in that the portions of the wave filter on opposite sides of the transforming means are proportioned to unequal characteristic impedances whose values are in the same ratio as the impedance transformation ratio of the transforming means.

4. An impedance transforming wave filter having a broad band pass characteristic, and including as an impedance transforming means a π network comprising shunt impedances Z_a and Z_b and a series impedance Z_c , the values of which impedances are related by the equations

$$Z_a = \frac{Z_c}{\Phi - 1}, \text{ and } Z_b = -\frac{\Phi Z_c}{\Phi - 1},$$

in which Φ is the square root of the impedance transformation ratio, and at least one of said shunt impedances being combined with an adjacent shunt impedance of the filter.

5. An impedance transforming wave filter in accordance with claim 4 characterized in this that the series impedance of the transforming network comprises a single reactive

element, the shunt impedances comprising similar elements, one of which has a negative coefficient, said negative element being combined with a similar positive element in an adjacent shunt impedance of the filter.

6. An impedance transforming wave filter in accordance with claim 4, characterized in this that the series impedance of the transforming network is a capacity, the shunt impedances being respectively positive and negative capacities and the negative capacity being combined with a positive capacity in an adjacent shunt branch of the filter.

7. An impedance transforming wave filter in accordance with claim 4, characterized in this that the portions of the wave filter on opposite sides of the transforming network are proportioned to characteristic impedances

which are unequal and in the constant ratio ϕ^2 .

8. An unsymmetrical wave filter section comprising two unequal shunt impedances proportionally related at all frequencies in a constant ratio ϕ^2 which is different from unity, in combination with a Π type coupling network comprising series and shunt impedances of similar type and of such values that their algebraic sum is zero, the shunt impedances of said coupling network being of opposite sign, and their magnitudes being related in the ratio ϕ , whereby waves may be transmitted without loss due to reflection between terminal impedances whose values are in the constant ratio ϕ^2 .

In witness whereof, I hereunto subscribe my name this 11 day of May, A. D., 1925.

EDWARD L. NORTON.