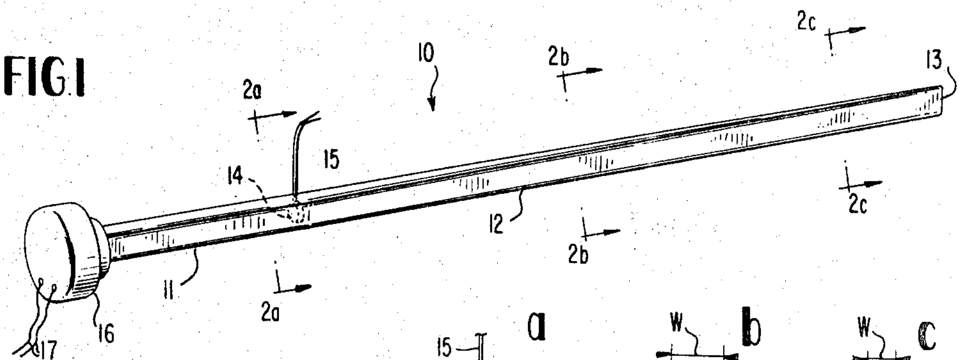
PHASE SHIFTERS

Original Filed June 9, 1964

FIG 1



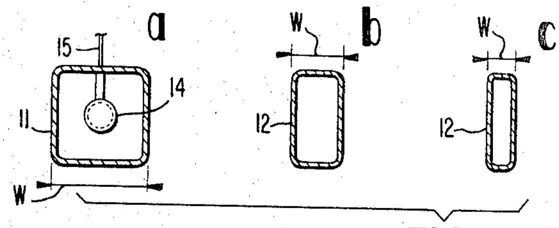


FIG 2

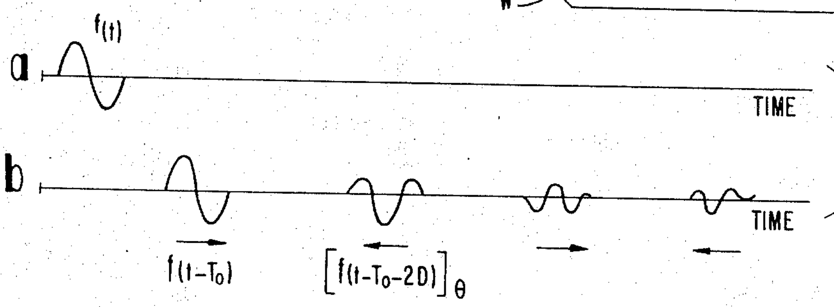
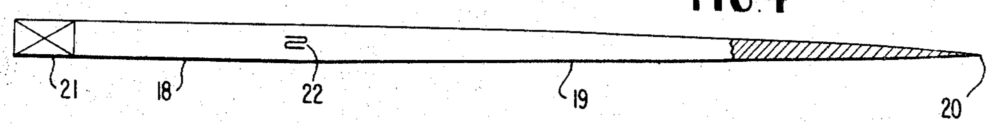


FIG 3

FIG 4



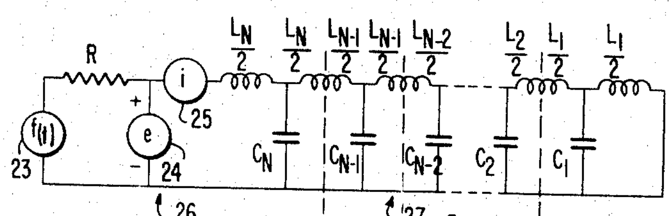


FIG 5

FIG 7

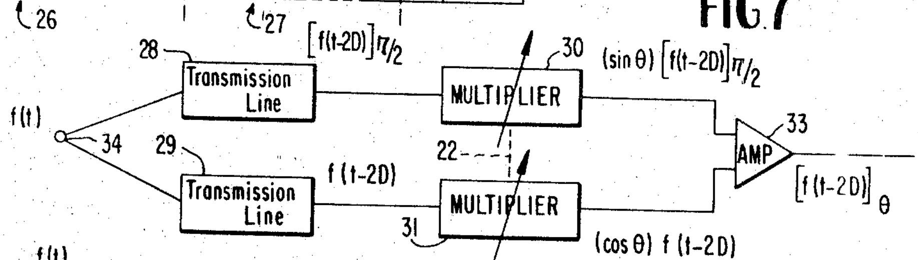
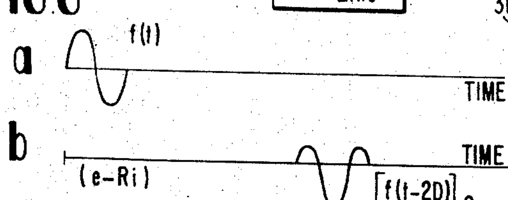


FIG 6



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United States Patent Office

3,546,604
Patented Dec. 8, 1970

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3,546,604

PHASE SHIFTERS

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Original application June 9, 1964, Ser. No. 373,684. Divided and this application Feb. 19, 1969, Ser. No. 813,366

Int. Cl. H03k *5/18*

U.S. Cl. 328—155 8 Claims

ABSTRACT OF THE DISCLOSURE

A phase shifter using two reflecting transmission lines. The lines may be mechanical, acoustical or lumped electrical elements generally tapered physically or electrically. An input signal is applied in common to both lines which may include sine and cosine multipliers with the outputs being summed.

This application is a division of application Ser. No. 373,684 entitled Phase Shifter and filed June 9, 1964.

The invention relates generally to phase shifters and more specifically, to methods and apparatus for modifying a given waveform by shifting each frequency component thereof through the same phase angle.

Heretofore, phase shifters have suffered from the inability to shift a wide band of frequencies without distorting the relative amplitudes of the different frequencies. The present invention overcomes this problem by providing a wave transmission line which is tapered in accordance with a certain formula to be explained below. Composite signals, which are signals comprising a number of frequencies, are applied to the transmission lines of my invention, travel the length of the line and are reflected at the reflecting end. Upon reflection, the signal travels back to the input end. The signal which has traveled the length of the transmission line twice is detected. The detected signal is thus delayed with respect to the input signal by an amount equal to twice the length of the transmission line, and contains each frequency of the input signal phase shifted by some predetermined amount. The relative amplitudes of the detected frequencies are the same as the relative amplitudes of the frequencies in the input signal.

A composite phase shifted signal is produced by transmitting a composite input signal to a detector via a reflecting transmission line which is tapered. My invention has applicability to tapered acoustical transmission lines, tapered mechanical transmission lines, tapered electrical transmission lines, and tapered electromagnetic transmission lines.

The transmission lines of my invention are tapered between the input end and the reflecting end by varying a selected parameter of the transmission line. That is, the selected parameter k, which may be either width, cross sectional area, or electrical inductance and capacitance is varied along the length of the transmission line in accordance with the formula $k_x = K(\sin A_x)^p$, where k_x is the value of the selected parameter at any point x distance from the reflecting end, K is the value of the parameter at the input end of the transmission line, A_x is a value which is proportional to the distance x, and p is any desired number. The resulting phase shift imparted to a signal by the transmission line is

$$\theta = p\frac{\pi}{2}$$

The parameter which is selected to be varied in accordance with the above formula depends on the type of transmission line used, e.g., for the acoustical transmission line, width or height is the selected parameter; for the mechanical transmission line, cross sectional area is the selected parameter; for the electrical transmission line, the inductances and capacitances are the selected parameters.

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My invention further encompasses the use of two transmission lines for achieving a variable phase shifter.

It is therefore a primary object of my invention to provide a phase shifter for shifting each frequency component of a given waveform the same phase angle without changing the relative amplitude of the frequencies.

A further object of my invention is to provide an acoustical transmission line for phase shifting a given waveform, the transmission line varying in width along its length.

Another object of my invention is to provide a mechanical phase shifter transmission line which varies in cross sectional area along its length and serves to phase shift a given waveform through a specified phase angle.

Another object of my invention is to provide an electrical transmission line which modifies a given waveform by shifting each frequency component by the same phase angle.

A further object of my invention is to provide a variable phase shifter for altering a given waveform by shifting each frequency component through a phase angle and providing a control for selecting the desired phase angle.

Other objects of my invention will be pointed out in the following detailed description and claims and illustrated in the drawing, in which:

FIG. 1 shows an acoustical transmission line in accordance with my invention;

FIGS. 2*a*, 2*b*, and 2*c* illustrate cross sectional views of the acoustical transmission line of FIG. 1;

FIGS. 3*a* and 3*b* are time graphs, helpful to illustrate the operation of my invention;

FIG. 4 shows a mechanical transmission line embodying my invention;

FIG. 5 shows an electrical transmission line embodying my invention;

FIGS. 6*a* and 6*b* are time graphs helpful to explain the operation of FIG. 5; and

FIG. 7 is a modification of my invention applicable for use with the transmission lines of FIGS. 1, 4, and 5, to provide variable phase shift.

In FIG. 1 there is shown an acoustical transmission line **10** which is hollow and may be air-filled. It has an input end **11**, a reflecting end **13**, and a tapered portion **12** therebetween. A loud-speaker **16** is connected to apply the input waveform to the transmission line, and a detector **14**, for example a microphone, is positioned at the junction of the input portions **11** and the tapered portion **12** to receive the phase shifted signal after reflection. Output leads **15** are connected to detector **14**, and input leads **17** are connected to speaker **16**.

I have found that by constructing the line **10** so that the width w of the tapered portion **12** varies in accordance with

$$\sin\left(\frac{\pi}{2}\frac{x}{L}\right)^p$$

a phase shift which is equal to

$$\left(p\frac{\pi}{2}\right)$$

will be imparted to all frequencies of a signal waveform.

In the formula for the taper of the transmission line, x is the distance at any point from the reflecting end **13**; L is the total length of the tapered portion **12**, and p is a number selected in accordance with the desired shift.

FIGS. 2*a* through 2*c* are cross sectional views of transmission line **10** of FIG. 1 taken respectively along lines 2*a*, 2*b*, and 2*c* shown in FIG. 1. In FIG. 2*a* the width w of the transmission line is at a maximum. The width tapers toward zero as shown by FIGS. 2*b* and and 2*c*.

The acoustical transmission line is an effective phase shifter for a wide band of frequencies in the acoustic range; however, that band of frequencies is not without some limits. The upper limit on frequency is set by the requirement that the shortest wave length be large compared to the maximum width of the transmission line. With a 5:1 ratio as an arbitrary definition of "large," a one-half inch width allows two and one-half inches for the shortest usable wave length, or a frequency of about 5000 cycles per second. (The speed of sound in air is about 1100 feet per second.) The lower limit on frequency is set by the requirement that the rate of taper be not too abrupt. This can be stated quantitatively as follows. Looking at FIG. 1, it may be seen that the tapered portion **12** tapers from $(\sin 0)^p$, where x equals 0, to

$$\left(\sin\frac{\pi}{2}\right)^p$$

where x equals L. (x equals 0 at reflecting end **13**, and x equals L at the junction of portions **11** and **12**.) Thus, the wave guide tapers through a quarter of the sine wave, or 90°, for the entire length L, and the term 4L may be considered as the "taper wave length." The lower limit requirement is met if the wave length of the lowest frequency, λ_1, is short compared with the "taper wave length." With 8:1 is an adequate ratio, and a value of fifty inches for L, λ_1 equals twenty-five inches and the lower frequency equals approximately 500 cycles per second. It is to be understood that these upper and lower frequencies are specified only by way of example.

Referring to the time graphs in FIG. 3, the operation of the acoustical transmission line in FIG. 1 will be explained. It should be stated at the outset that the acoustical transmission line is suitable for phase shifting only non-continuous signals, because for a continuous wave the portion traveling toward the reflecting end would interfere with the portion traveling from the reflecting end, and the interference would be detected at microphone **14**. FIG. 3*a* illustrates the signal introduced into input end **11** by loud-speaker **16**. FIG. 3*b* illustrates the signal which pass microphone **14**. The arrows beneath the signals in FIG. 3*b* indicate the direction of travel of the particular signal.

Loud-speaker **16** introduces a composite waveform signal $f(t)$ into the input portion **11** of acoustical transmission line **10**. The signal $f(t)$ travels toward the reflecting end **13** of the transmission line and arrives at the junction of portions **11** and **12** at time T_o. This waveform is described by the formula $f(t-T_o)$ and is shown in FIG. 3*b*.

The composite signal travels the length of tapered portion **12**, is reflected at reflecting end **13**, and travels back towards microphone **14**. The signal arriving at microphone **14** has been further delayed by an amount equal to 2D, where D is the one-way delay time due to portion **12** of transmission line **10**. This signal also has been phase shifted by an angle θ equal to

$$p\frac{\pi}{2}$$

The composite wave is defined by the formula

$$[f(t-T_o-2D)]_\theta$$

detected by microphone **14**. The two remaining waveforms in FIG. 3*b* indicate that the composite waveform continues to be reflected until it is either attenuated or absorbed completely.

In order to prevent the composite signal from being re-reflected at the input end by the loud-speaker **16** after the desired signal is detected by microphone **14**, an absorber (not shown) may be placed in the input portion **11** and positioned to absorb the signal after it travels back towards loud-speaker **16**.

FIG. 4 shows a slender rod transmission line for delaying and phase shifting a composite mechanical wave sig-

nal. Numeral **21** designates a crystal transducer which transmits longitudinal waves down the length of the transmission line in the same manner as loud-speaker **16** transmits sound waves down acoustical transmission line **10**. The transmission line consists of an input end **18**, a reflecting end **20**, and a tapered portion **19** therebetween. A detector **22** is positioned at the junction of input end **18** and portion **19** and may be any type of detector which is capable of detecting reflected longitudinal waves, e.g., a strain-gage detector. The mechanical wave guide is similar in construction and operation to the acoustical wave guide shown in FIG. 1, except that the mechanical wave guide is solid and the cross sectional area throughout portion **19** is tapered in accordance with the formula:

$$a_x = A\left[\sin\left(\frac{\pi}{2}\frac{x}{L}\right)\right]^q$$

a_x is the cross sectional area of tapered portion **19** at any point x distance from reflecting end **20**, A is the cross sectional area at the junction of portion **18** and **19**, x is the distance from the reflecting end **20**, and L is the length of tapered portion **19**.

The time graphs in FIGS. 3, which show the composite waveform at different times during the phase shifting operation, apply equally as well to the merchanical phase shifter shown in FIG. 4.

As an example, if it is assumed that the rod is made of aluminum for which the speed of longitudinal waves is about 18,000 feet per second, then the mechanical phase shifter shown in FIG. 4 would operate effectively for frequencies between 8,000 and 80,000 cycles per second to shift these frequencies by an angle

$$\theta = p\frac{\pi}{2}$$

FIG. 5 shows a further embodiment of my invention as applied to electrical transmission lines. For electrical transmission lines, the tapering is accomplished by varying electrical parameters or components rather than by varying dimensional components. The delay line comprises an input portion **26** and a tapered portion **27**. The electrically tapered portion **27** is a lumped constant delay line made up of N inductance-capacitance sections. As shown in FIG. 5, the first section ($n=1$) comprises an inductance L_1 and a capacitance C_1; the second section ($n=2$) comprises an inductance L_2 and a capacitance C_2; the n^{th} section comprises an inductance L_n and a capacitance C_n. In each section, the inductance L_n is separated by connection to capacitor C_n into two equal parts, $L_n/2$.

The values of the inductances and capacitances are chosen in accordance with the following formulae:

$$L_n = L_N\left(\sin\frac{\pi}{2}\frac{n}{N}\right)^p$$

$$\frac{1}{C_n}\frac{1}{C_N}\left(\sin\frac{\pi}{2}\frac{n}{N}\right)^p$$

where

L_N=the value of the inductance of the N^{th} section
L_n=the value of the inductance of the n^{th} section
C_N=the value of the capacitance of the N^{th} section
C_n=the value of the capacitance of the nth section
N=the length, in sections, of the transmission line
n=the distance, in sections, from the reflecting end of the transmission line=**1**, **2**, . . . N

By varying the circuit parameters in this manner, a composite signal $f(t)$ applied to the input of the electrical transmission line will be delayed, reflected, and phase shifted by an angle equal to

$$p\frac{\pi}{2}$$

The electrical transmission line, unlike the acoustical and mechanical transmission lines, is operable on continuous waves as well as non-continuous waves, because re-reflections of the input signal at the input end may be prevented by placing a resistance R equal to the characteristic impedance of the lumped constant transmission line in the input portion **26**.

The operation of the electrical transmission line of FIG. 5 may be explained with reference to FIG. 5 and the time graphs of FIG. 6. A composite signal $f(t)$ is applied at the input **23**. This signal is shown in FIG. 6*a*. The signal travels the length of the transmission line toward the reflecting end, is reflected, and travels the length of transmission line towards absorbing resistor R. The reflected signal $[f(t-2D)]\theta$, where D equals the one-way delay time of portion **27**, is detected after having been phase shifted an amount θ equal to

$$p\frac{\pi}{2}$$

The detector consists of a voltage pick-off **24** and a current pick-off **25**. The detected current i is converted to a voltage iR by any convenient means, such as a transformer with a proper resistive value in the secondary. The detected voltage e is subtracted from the voltage iR in a different amplifier (not shown). This type of detector arrangement allows only the reflected signal to be detected as the input signal will subtract to zero. The input signal $f(t)=e+iR$ subtracts to zero because R equals the characteristic impedance of transmission line and therefore $e=-iR$.

The electrical phase shifter of FIG. 5 is not completely without limits in bandwidth for which it is operable. The period of the highest frequency signal must be considerably longer than the delay time per section of the transmission line. The period of the lowest frequency signal must be considerably shorter than four times the total delay in the transmission line. An example of the upper and lower frequency limits for which the delay line will give acceptably constant phase shifts are 300 to 5000 cycles per second.

The above-described acoustical, mechanical, and electrical phase shifters are satisfactory for phase shifting the frequencies of a composite wave equal amounts. However, for each desirable phase shift a new transmission line must be constructed.

A composite wave signal $f(t)$ may be variably phase shifted any amount θ by the following steps:

(1) Delay the signal $f(t)$ an amount equal to 2D to produce a signal $f(t-2D)$.

(2) Simultaneously with Step 1, delay the signal an amount equal to 2D and phase shift the resulting signal by 90° to produce a signal

$$[f(t-2D)]\frac{\pi}{2}$$

(3) Multiply the signal derived from Step 1 by cosine θ and the signal derived from Step 2 by sine θ and add the resulting signals.

$$\cos\theta f(t-2D)+\sin\theta[f(t-2D)]\frac{\pi}{2}=[f(t-2D)]_\theta$$

Thus, it can be seen that the result is the original signal delayed and phase shifted by any angle θ. By merely varying the multipliers $\cos\theta$ and $\sin\theta$, the phase shift angle θ may be varied. This result can be achieved by the use of two transmission lines of the acoustical, mechanical, or electrical type, and the addition circuitry shown in FIG. 7.

If it is assumed that acoustical phase shifters are used, the two acoustical phase shifters are each of exactly the same length. The length provides a two-way transmission time of 2D.

The first transmission line is not tapered at all and the amount of phase shift imparted to a signal $f(t)$ would be zero. The detected signal from this phase shifter is $f(t-2D)$.

The second phase shifter has its width tapered in accordance with the formula

$$\left(\sin\frac{\pi}{2}\frac{x}{L}\right)^2$$

The detected signal from this phase shifter is

$$[f(t-2D)]\frac{\pi}{2}$$

The detected signals from the first and second phase shifters are multiplied respectively by the cosine and sine of θ. Variable potentiometers, which are calibrated in terms of sine θ and cosine θ, are suitable as multipliers. By summing the outputs of the multiplier, the desired signal $[f(t-2D)]_\theta$ is achieved. To change the phase shift, an operator need only vary the potentiometers.

In FIG. 7, a signal $f(t)$ is applied at input **34** to transmission lines **28** and **29**. Transmission lines **28** and **29** may be of the electrical, acoustical, electromagnetic, or mechanical type. Transmission line **28** is designed, in accordance with my invention, to delay the signal by an amount 2D and impart a $\pi/2$ phase shift to the signal. Transmission line **29** is designed, in accordance with my invention, to delay the signal by an amount 2D and impart a 0° phase shift to the signal, i.e., no phase shift. The output $[f(t-2D)]_{\pi/2}$ from transmission line **28** is multiplied by $\sin\theta$ in variable multiplier **30**, and the output $f(t-2D)$ from transmission line **29** is multiplied by $\cos\theta$ in variable multiplier **31**. Dotted line **22** indicates that multipliers **30** and **31** may be ganged together for purposes of having one control vary the angle θ. The multipliers may be potentiometers. The outputs from multipliers **30** and **31** are summed in amplifier **33** which provides the desired output signal $[f(t-2D)]_\theta$.

Thus it can be seen that, by the modification shown in FIG. 7, an operator need only turn a dial, or adjust the potentiometers, to achieve a desired phase shift. The above explanation of a variable phase shifter applies equally as well to electrical, electromagnetic, or mechanical transmission lines.

It will be obvious to those skilled in the art that the above-described embodiments are meant to be merely exemplary and that they are susceptible of modification and variation within the spirit and scope of my invention. Therefore, the invention is not to be limited except as defined by the following claims.

I claim:

1. A variable phase shifter for phase shifting a signal $f(t)$ any desired angle θ comprising:

(a) first reflecting transmission line means having an input and an output for delaying said signal,

(b) second reflecting transmission line means having an input and an output for delaying and imparting a 90° phase shift to said signal,

(c) third means for simultaneously applying said signal to the inputs of said first and second means, and

(d) fourth means connected to the outputs of said first and second means for combining the signals appearing at the outputs of said first and second means.

2. The system as claimed in claim **1**, wherein said fourth means comprises:

(a) first multiplier connected to the output of said first means for multiplying said delayed and phase shifted signal by cosine θ,

(b) second multiplier connected to the output of said second means for multiplying said delayed and phase shifted signal by sine θ, and

(c) a summing amplifier connected to the outputs of said first and second multipliers.

3. The system as claimed in claim **1** wherein said second reflecting transmission line means comprises a reflecting transmission line having a parameter which varies in accordance with sine A where A is proportional to the distance from the reflecting end of said transmission line.

4. The system as claimed in claim 3, wherein said fourth means comprises:

(a) first and second multipliers connected respectively to the outputs of said first and second means, and

(b) a summing amplifier connected to the outputs of said first and second multipliers.

5. A system for variably phase shifting a signal $f(t)$ any desired angle θ comprising:

(a) a first acoustical transmission line having an input end, a closed reflecting end, and a microphone located near said input end, said transmission line having constant width and height along its length,

(b) a second acoustical transmission line having an input end, a closed reflecting end, and a microphone located near said input end, said transmission line having a width which varies between said reflecting end and said microphone in accordance with the formula

$$w_x = W \sin\frac{\pi}{2}\frac{x}{L}$$

where w_x is the width of said transmission line at a point x distance from the reflecting end, W is the width of said transmission line between said input end and said microphone, L is the distance between said reflecting end and said microphone, and x is the distance of any point from said reflecting end,

(c) means for simultaneously applying said signal to the input ends of said first and second acoustical transmission lines,

(d) first variable potentiometer connected to the microphone of said first transmission line for multiplying the signal picked up by said microphone by cosine θ, and second variable potentiometer connected to the microphone of said second transmission line for multiplying the signal picked up by said microphone by sine θ, and

(e) a summing amplifier connected to the outputs of said first and second potentiometers.

6. A system for variable phase shifting a signal $f(t)$ any desired angle θ comprising:

(a) a first mechanical transmission line having an input end, a closed reflecting end, and a detector located near said input end, said transmission line having constant cross sectional area along its length,

(b) a second mechanical transmission line having an input end, a closed reflecting end, and a detector located near said input end, said transmission line having a cross sectional area which varies between said reflecting end and said detector in accordance with the formula

$$a_x = A \sin\frac{\pi}{2}\frac{x}{L}$$

where a_x is the cross sectional area of said transmission line at a point x distance from the reflecting end, A is the cross sectional area of said transmission line between said input end and said detector, L is the distance between said reflecting end and said detector, and x is the distance of any point from said reflecting end,

(c) means for simultaneously applying said signal to the inputs of said first and second mechanical transmission lines,

(d) first potentiometer connected to the detector of said first transmission line for multiplying the signal detected by said detector by cosine θ and second potentiometer connected to the detector of said second transmission line for multiplying the signal detected by said detector by sine θ, and

(e) a summing amplifier connected to the outputs of said first and second potentiometers.

7. A system for variably phase shifting a signal $f(t)$ any desired angle θ comprising:

(a) a first lumped constant transmission line having an input end, a reflecting end and detecting means located at said input and to detect reflected signals, said transmission line comprising inductance-capacitance sections of equal values,

(b) a second lumped constant transmission line having an input end, a reflecting end and detecting means located at said input end to detect reflected signals, said transmission line comprising N inductance-capacitance sections having inductances and capacitances whose values vary in accordance with the formulae

$$L_n = L_N \sin\frac{\pi}{2}\frac{n}{N}$$

$$\frac{1}{C_n}\frac{1}{C_N}\sin\frac{\pi}{2}\frac{n}{N}$$

where L_n and C_n are respectively the values of the inductance and capacitance of the n^{th} section, L_N and C_N are respectively the values of the inductance and capacitance in the section nearest the input end of said transmission line, and n equals 0, 1, 2, 3, . . . ,

(c) means for simultaneously applying said signal to the inputs of said first and second transmission lines,

(d) a first variable potentiometer connected to the detecting means of said first transmission line for multiplying the signal detected by said detecting means by cosine θ, and a second variable potentiometer connected to the detecting means of said second transmission line for multiplying the signal detected by said detecting means by θ, and

(e) a summing amplifier connected to said first and second potentiometers.

8. A method for phase shifting a signal $f(t)$ any desired angle θ, comprising the steps:

(a) delaying said signal to form a delayed signal,

(b) delaying and phase shifting by 90° said signal to form a delayed phase shifted signal,

(c) multiplying said delayed signal by cosine θ to obtain a first multiplied signal,

(d) multiplying said delayed phase shifted signal by sine θ to obtain a second multiplied signal, and

(e) adding said first and second multiplied signals.

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H. A. DIXON, Assistant Examiner

U.S. Cl. X.R.

307—295; 328—143; 333—34

PO-1050
(5/69)

UNITED STATES PATENT OFFICE
CERTIFICATE OF CORRECTION

Patent No. 3,546,604 Dated Dec. 8, 1970

Inventor(s) James E. White

It is certified that error appears in the above-identified patent and that said Letters Patent are hereby corrected as shown below:

Col. 2, line 56: $\sin\left(\frac{\pi}{2}\frac{x}{L}\right)^p$ should read -- $\left(\sin\frac{\pi}{2}\frac{x}{L}\right)^p$ --

Col. 2, line 9: "amplitude" should read --amplitudes--

Col. 2, line 67: "phase" omitted should read --phase shift.--

Col. 4, line 15: supra "q" should read --p--

Col. 4, line 56: "C_nC_N" should read --$C_n=C_N$--

Col. 6, line 3: supra number "2" should read --)^1--

Col. 8, line 8: "N" omitted should read --comprising N indu tance--

Col. 8, line 28: "N" omitted should read --3,...N,--

Signed and sealed this 14th day of September 1971.

(SEAL)
Attest:

EDWARD M.FLETCHER,JR.
Attesting Officer

ROBERT GOTTSCHALK
Acting Commissioner of Patent