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GAUSSIAN WAVEFORM GENERATOR

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Fig. 2

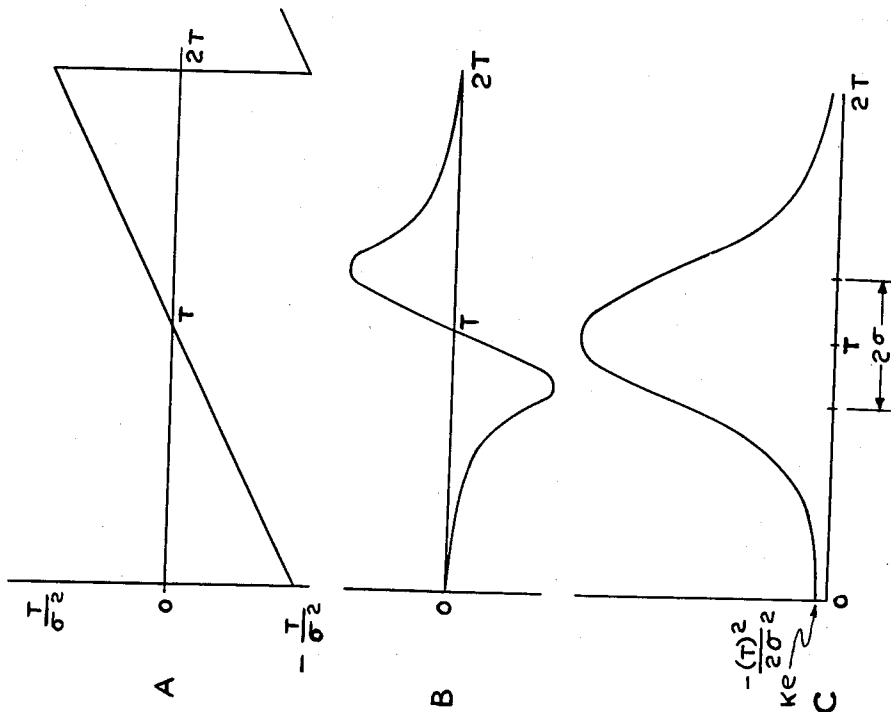
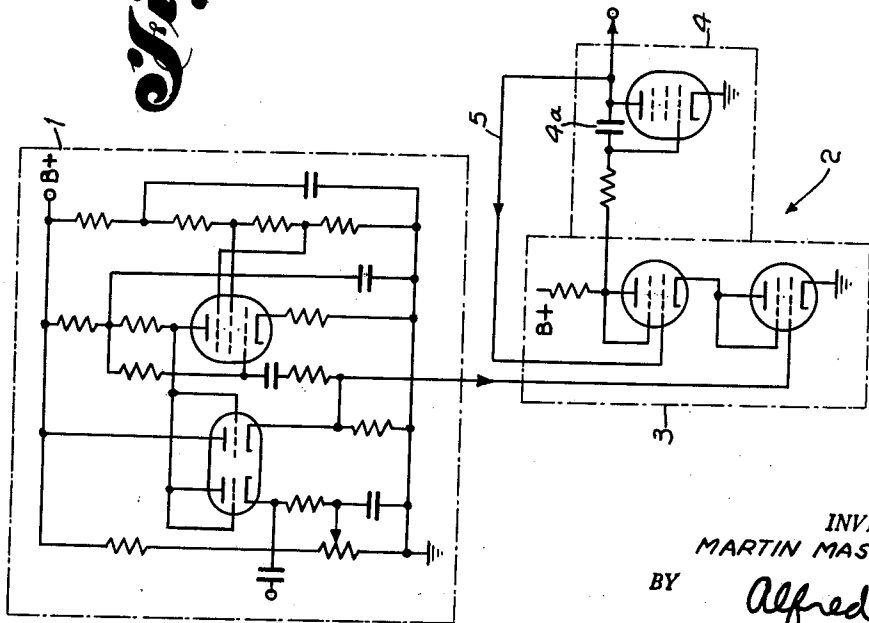


Fig. 1



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GAUSSIAN WAVEFORM GENERATOR

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2 Claims. (Cl. 235-197)

This invention relates to waveform generators and more particularly to a waveform generator for producing Gaussian-shaped waveshape.

In certain types of pulse transmitters it is desirable to utilize a minimum spectrum of minimum rise time for the radiated signal consistent with the transmission of the desired information. Examples of systems utilizing minimum spectrum are pulse code modulation systems, radar and loran transmitters, certain types of aerial navigation systems, certain types of frequency shift keying systems, and certain types of facsimile links. It is desirable in most of these systems to have a minimum pulse rise time with minimum output signal bandwidth. The pulse shape having minimum bandwidth for the minimum pulse rise time is that shape defined by the Gaussian distribution. Heretofore the prior art schemes for producing Gaussian waveforms have generally related to the use of filters to act on square wave or similar type pulse to produce an output therefrom having a shape defined substantially by the Gaussian distribution. These prior art schemes have certain disadvantages. One disadvantage is that to obtain a reasonable approximation of the Gaussian error curve many filter sections are required, and the resulting pulse is only approximately of Gaussian shape. Also, the use of filters cause a time delay.

One object of the present invention is to provide an improved waveform generator to produce a waveform more substantially of Gaussian shape than heretofore obtainable.

Another object of the present invention is to provide a waveform generator employing simple circuitry for producing a substantially Gaussian shaped waveform.

Still another object of the present invention is to provide a waveform generator for producing Gaussian pulses from a source of sawtooth voltage.

A feature of the present invention is a Gaussian waveform generator comprising means to produce a sawtooth waveform signal, and means coupled to the waveform signal producing means to modify the sawtooth waveform signal to produce a Gaussian waveform output signal.

Another feature of the present invention is a Gaussian waveform generator of the type described where the modifying means includes at least a multiplier and an integrator.

Still another feature of the present invention is a Gaussian waveform generator of the type described including a feedback path coupled between the output of the integrator and the multiplier.

The above mentioned and other features and objects of this invention will become more apparent by reference to the following description taken in conjunction with the accompanying drawings, in which:

FIG. 1 is a schematic diagram of an embodiment for producing a Gaussian shaped waveform in accordance with the principles of this invention;

FIG. 2 is a series of waveforms useful in explaining the operation of the embodiment of FIG. 1.

It is known that a Gaussian pulse may be represented by the expression

$$g(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-T)^2}{2\sigma^2}} \quad (1)$$

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where σ is defined as the standard deviation, t is defined as time, e is the base of the natural logarithm, T is the time at which it is desired to peak the Gaussian pulse, and $g(t)$ is the general symbol for a quantity as a function of time. A Gaussian distribution of current (i) may then be expressed as

$$i(t) = Ke^{-\frac{(t-T)^2}{2\sigma^2}} \quad (2)$$

where K is a constant equal to

$$\frac{1}{\sqrt{2\pi}\sigma}$$

The differential of $i(t)$ of Equation 2 would appear as

$$\frac{di}{dt} = \frac{-i(t) \cdot (t-T)}{\sigma^2} \quad (3)$$

By integrating Equation 3 as it appears, the expression for $i(t)$ becomes

$$i(t) = - \int_0^t \frac{(t-T)}{\sigma^2} \cdot i(t) dt \quad (4)$$

It is to be noted that $i(t)$ still represents the expression for a Gaussian current pulse and

$$\frac{(t-T)}{\sigma^2}$$

represents a linear variation of current with time. Therefore, it can be said from an examination of Equation 4 that a Gaussian pulse may be produced by multiplying a linear variation of current with time, such as a sawtooth signal, with a Gaussian variation of current with time and integrating the product of the two currents.

Referring to FIG. 1, a Gaussian waveform generator is shown comprising means 1 to produce a sawtooth waveform signal, and means 2 coupled to signal producing means 1 to modify the sawtooth waveform signal therefrom to produce a Gaussian waveform output signal.

A sawtooth waveform generator 1, which for purposes of illustration is shown as a phantastron circuit, produces a sawtooth output signal as illustrated by curve A, FIG. 2. The sawtooth signal has a period of $2T$ and a slope of

$$\frac{1}{\sigma^2}$$

so that the expression for the linear relationship is the desired term

$$\frac{t-T}{\sigma^2} \quad (5)$$

Briefly, the operation of the Gaussian waveform generator is as follows. The output signal from sawtooth waveform generator 1 is coupled to multiplier 3. The output of multiplier 3 is coupled to integrator 4 to provide the desired Gaussian shaped pulse. The output of integrator 4 is fed back to multiplier 3 along feedback means 5 for multiplication with the sawtooth waveform to sustain the Gaussian output of integrator 4.

A more detailed explanation can be obtained by considering the initial cycle of the sawtooth signal. At time zero, the expression for a Gaussian pulse defined by Equation 2 becomes

$$i(t) = Ke^{-\frac{(T)^2}{2\sigma^2}} \quad (6)$$

This value of current is extremely small in comparison with the peak value of the Gaussian pulse, and may be initially produced by means of a small charge on capacitor 4a. This initial current which is present at time zero is applied to multiplier 3 along feedback means 5 and is multiplied by the initial value of the sawtooth wave from generator 1. The product of this multiplica-

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tion of initial sawtooth signal with initial Gaussian signal is then integrated by integrator 4 which results in an output signal which is the integral of the product of a linear signal with a Gaussian signal, and therefore Gaussian in form. The Gaussian output signal from integrator 4 is also fed back along feedback path 5 to be multiplied with the increasing value of sawtooth signal as time (t) increases.

As the sawtooth signal from generator 1 increases it is continually being multiplied with the Gaussian signal from feedback path 5 to produce a signal from multiplier 3 which when integrated, becomes a Gaussian shaped pulse. When time equals T , the sawtooth signal illustrated by curve A, FIG. 2 passes through a zero value and the Gaussian output pulse from integrator 4 reaches a peak value. As time continues to increase the sawtooth signal continues to rise in value and the Gaussian output pulse decreases in value until time equals $2T$, at which time the sawtooth signal will recycle and the Gaussian output pulse has decreased to the initial value expressed by Equation 6. The operation of the circuit of FIG. 1 for continued cycles of sawtooth signal is the same as has been described hereinabove. Relating the embodiment of FIG. 1 with the desired Gaussian Equation 4, it is seen that feedback means 5 is a source of Gaussian signal $i(t)$ and sawtooth generator 1 is a source of linear signal

$$\frac{t-T}{\sigma^2}$$

Multiplier 3 produces the product signal

$$\frac{(t-T)}{\sigma^2} \cdot i(t)$$

and integrator 4 performs the necessary integration of the product signal to produce an output signal substantially equal to

$$\int \frac{(t-T)}{\sigma^2} \cdot i(t) dt$$

which satisfies the condition for a Gaussian waveform signal.

The output signal from multiplier 3 is essentially the derivative of a Gaussian pulse and is illustrated by curve B, FIG. 2 and the Gaussian waveform produced is illustrated by curve C, FIG. 2. It is noted that control of the

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pulse width or deviation σ is determined by the slope of the sawtooth waveform from source 1 and the peak of the pulse occurs at the zero crossing of the sawtooth signal. If it is desired to peak the Gaussian wave at T , the sawtooth generator 1 should recycle in accordance with a time $2T$.

While I have described above the principles of my invention in connection with specific apparatus, it is to be clearly understood that this description is made only by way of example and not as a limitation to the scope of my invention as set forth in the objects thereof and in the accompanying claims.

I claim:

1. A Gaussian waveform generator comprising:
 - a sawtooth generator;
 - a multiplier having one of its inputs coupled to the output of said generator;
 - a single integrator coupled to the output of said multiplier; and
 - substantially linear means coupling the output of said integrator to the input of said multiplier.
2. A Gaussian waveform generator comprising:
 - means to produce a sawtooth waveform signal;
 - means, including a multiplier to one input of which said sawtooth signal is applied, responsive to said sawtooth signal and to a Gaussian pulse for producing a waveform that is the derivative of a Gaussian pulse;
 - a single integrator responsive to said derivative to produce a Gaussian output pulse; and
 - a substantially linear feed-back path coupled between the output of said single integrator and the input of said multiplier means.

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