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SINC/COSINC TRANSFORM—A CONTINUOUS FOURIER TRANSFORM

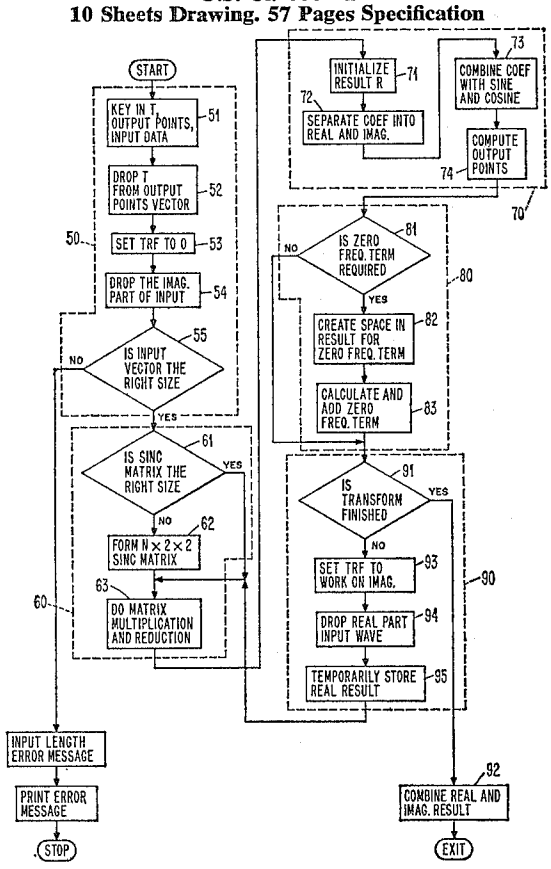
Robert F. Abramson, Arvada, Colo., assignor to International Business Machines Corporation, Armonk, N.Y.

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Int. Cl. G06f *7/38, 15/34*

U.S. Cl. 444—1

10 Sheets Drawing. 57 Pages Specification



The Fourier transform of a non-periodic wave is calculated by programmed computer using matrix multiplication. The non-periodic wave to be transformed is converted to a staircase waveform where the amplitude of each step in the staircase waveform is the average amplitude of the original wave. Walsh coefficients of the staircase waveform are then computed by matrix multiplication of the Walsh matrix times sample values for the wave where the sample values are the various amplitudes of the staircase wave during separate step or sample intervals. Thereafter, the continuous Fourier transform of a non-periodic wave is calculated by computing the Fourier transform of the Walsh functions. The latter computation is simplified by multiplying the Walsh coefficients by a sinc array and a cosinc (defined hereinafter) array. A further and ultimate simplification of the entire process is accomplished by multiplying the Walsh matrix by an intermingling of the sinc and cosinc arrays. The result is a matrix consisting mostly of zeroes and hereinafter defined as the sinc matrix. Multiplication of the input wave values by the sinc matrix will yield directly sine/cosine coefficients of the Fourier sine/cosine transforms for the staircase waveform that simulates the input wave. With the coefficients of the Fourier sine/cosine transforms defined, the Fourier transform at any frequency may be computed.

SINC/COSINC TRANSFORM — A CONTINUOUS FOURIER TRANSFORM

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FIG. 1A

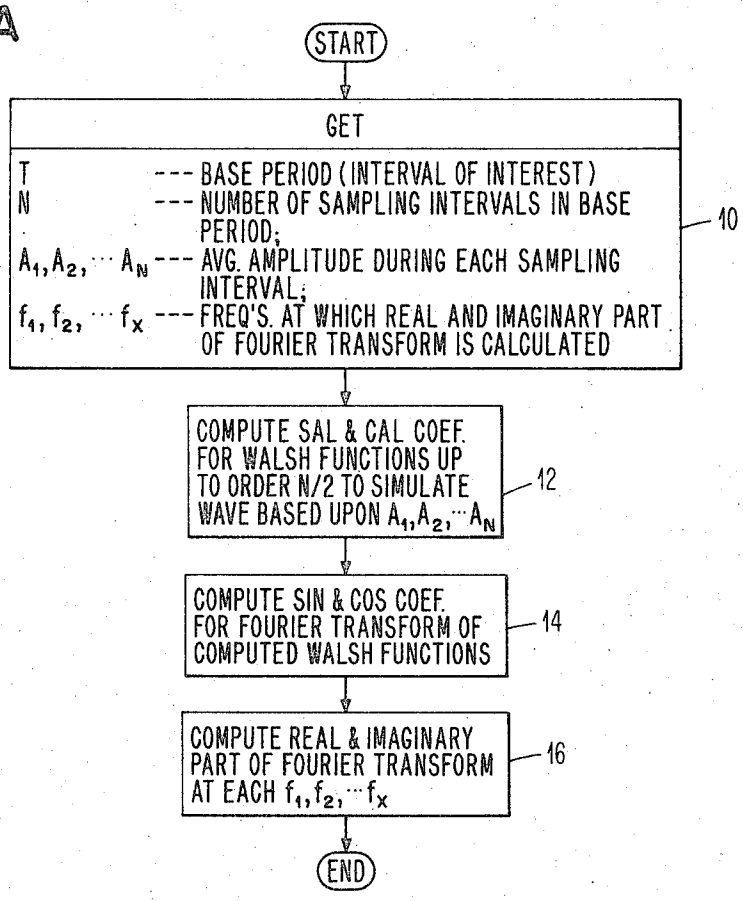


FIG. 1B

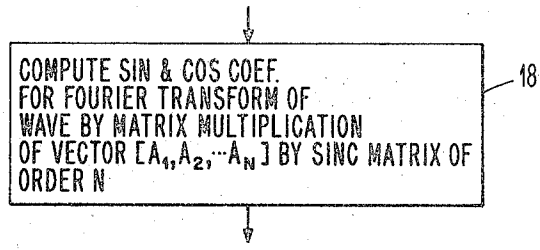
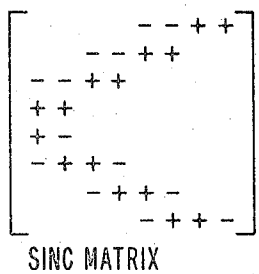


FIG. 2



SINC MATRIX

FIG. 3

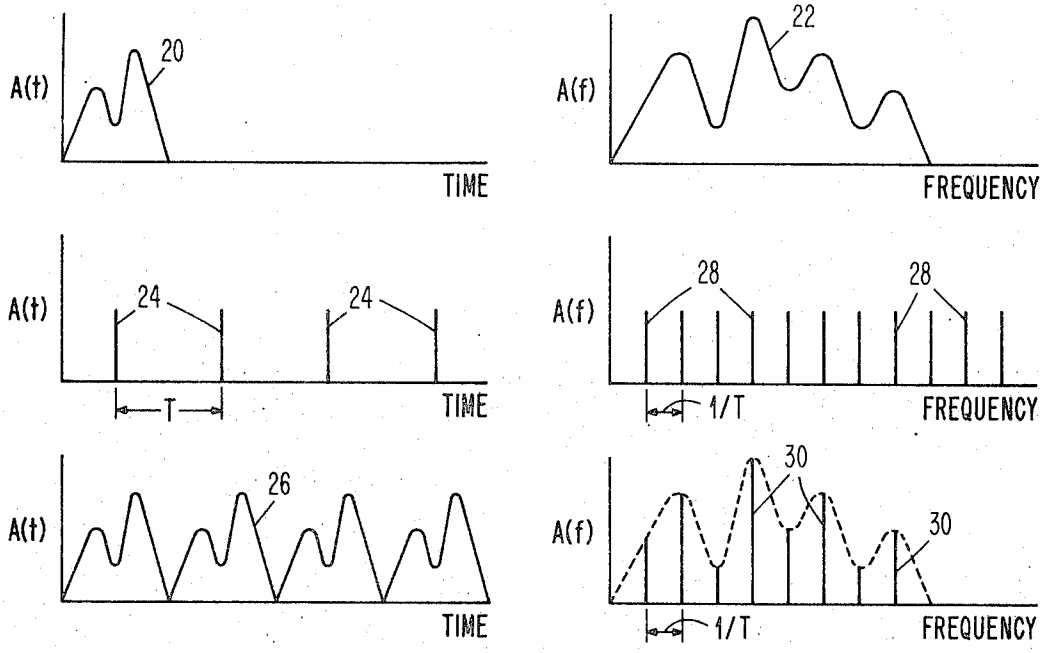


FIG. 12B

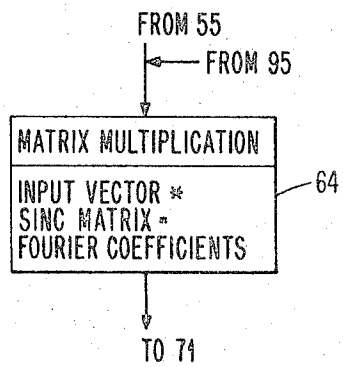
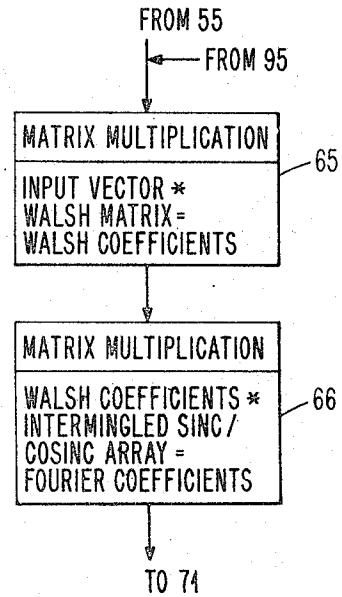


FIG. 12C



SINC/COSINC TRANSFORM — A CONTINUOUS FOURIER TRANSFORM

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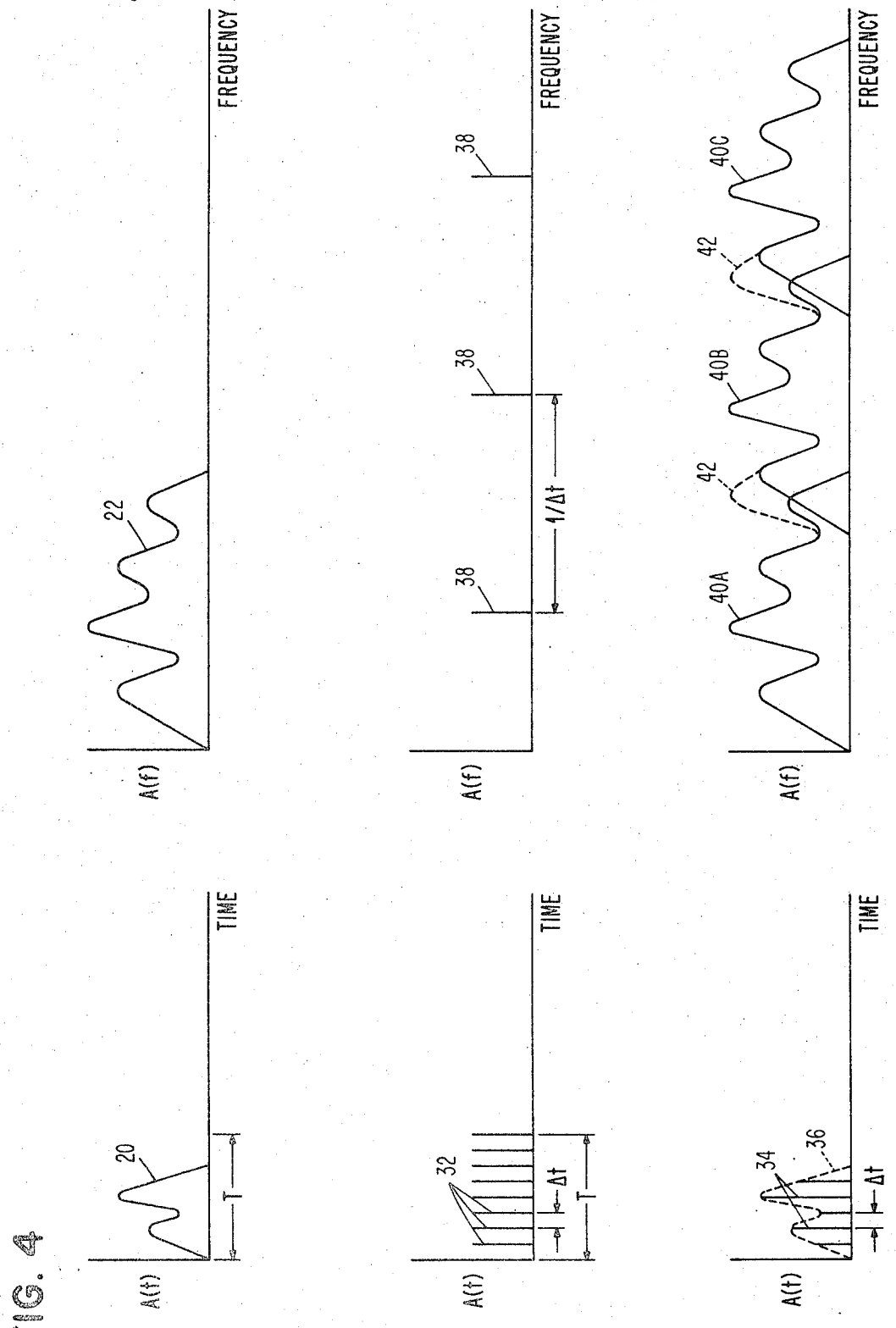


FIG. 4

SINC/COSINC TRANSFORM — A CONTINUOUS FOURIER TRANSFORM

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FIG. 5A

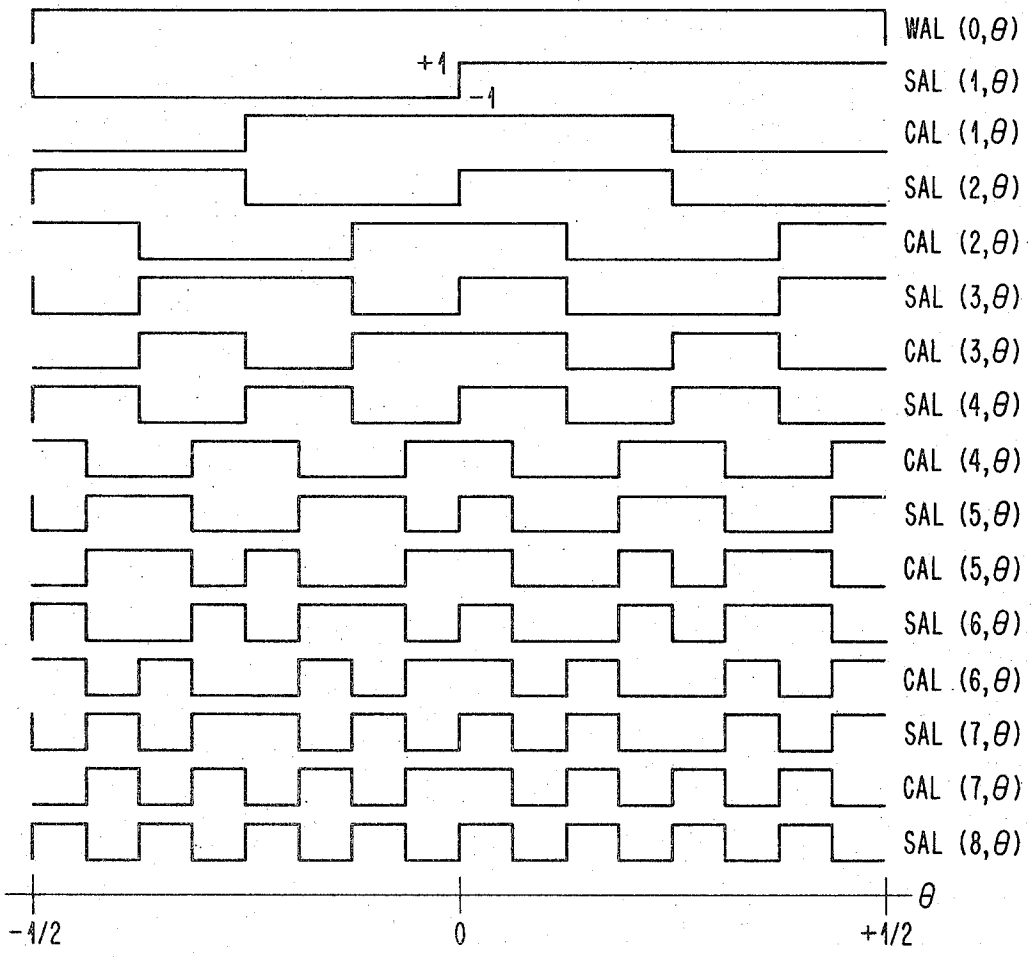


FIG. 5B
WALSH MATRIX

+	−	−	+	+	−	−	+	+	−	−	+	+	−	− +
+	−	−	+	+	−	−	+	−	+	+	−	−	+	+ −
+	−	−	+	−	+	+	−	−	+	+	−	+	−	− +
+	−	−	+	−	+	+	−	+	−	−	+	−	+	+ −
+	−	+	−	−	+	−	+	+	−	+	−	−	+	− +
+	−	+	−	−	+	−	+	−	+	−	+	+	−	+ −
+	−	+	−	+	−	−	+	−	+	−	+	−	+	− +
+	−	+	−	+	−	+	−	+	−	+	−	+	−	+ −
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+ +
+	+	+	+	+	+	+	+	−	−	−	−	−	−	− −
+	+	+	+	−	−	−	−	−	−	−	+	+	+	+ +
+	+	+	+	−	−	−	+	+	+	+	−	−	−	− −
+	+	−	−	−	−	+	+	+	+	−	−	−	+	+ +
+	+	−	−	−	−	+	+	−	−	+	+	+	−	− −
+	+	−	−	+	+	−	−	−	−	+	+	−	−	+ +
+	+	−	−	+	+	−	−	+	+	−	−	+	+	− −

FIG. 6

COSINE TRANSFORM

$$a(c0)=\frac{1}{\pi f}\left[\text{SIN}\left(\pi f\frac{8T}{8}\right)\right]$$

$$a(c1)=\frac{1}{\pi f}\left[2\text{SIN}\left(\pi f\frac{4T}{8}\right)-\text{SIN}\left(\pi f\frac{8T}{8}\right)\right]$$

$$a(c2)=\frac{1}{\pi f}\left[2\text{SIN}\left(\pi f\frac{2T}{8}\right)-2\text{SIN}\left(\pi f\frac{6T}{8}\right)+\text{SIN}\left(\pi f\frac{8T}{8}\right)\right]$$

$$a(c3)=\frac{1}{\pi f}\left[2\text{SIN}\left(\pi f\frac{2T}{8}\right)-2\text{SIN}\left(\pi f\frac{4T}{8}\right)+2\text{SIN}\left(\pi f\frac{6T}{8}\right)-\text{SIN}\left(\pi f\frac{8T}{8}\right)\right]$$

$$a(c4)=\frac{1}{\pi f}\left[2\text{SIN}\left(\pi f\frac{T}{8}\right)-2\text{SIN}\left(\pi f\frac{3T}{8}\right)+2\text{SIN}\left(\pi f\frac{5T}{8}\right)-2\text{SIN}\left(\pi f\frac{7T}{8}\right)+\text{SIN}\left(\pi f\frac{8T}{8}\right)\right]$$

$$a(c5)=\frac{1}{\pi f}\left[2\text{SIN}\left(\pi f\frac{T}{8}\right)-2\text{SIN}\left(\pi f\frac{3T}{8}\right)+2\text{SIN}\left(\pi f\frac{4T}{8}\right)-2\text{SIN}\left(\pi f\frac{5T}{8}\right)+2\text{SIN}\left(\pi f\frac{7T}{8}\right)-\text{SIN}\left(\pi f\frac{8T}{8}\right)\right]$$

$$a(c6)=\frac{1}{\pi f}\left[2\text{SIN}\left(\pi f\frac{T}{8}\right)-2\text{SIN}\left(\pi f\frac{2T}{8}\right)+2\text{SIN}\left(\pi f\frac{3T}{8}\right)-2\text{SIN}\left(\pi f\frac{5T}{8}\right)+2\text{SIN}\left(\pi f\frac{6T}{8}\right)-2\text{SIN}\left(\pi f\frac{7T}{8}\right)+\text{SIN}\left(\pi f\frac{8T}{8}\right)\right]$$

$$a(c7)=\frac{1}{\pi f}\left[2\text{SIN}\left(\pi f\frac{T}{8}\right)-2\text{SIN}\left(\pi f\frac{2T}{8}\right)+2\text{SIN}\left(\pi f\frac{3T}{8}\right)-2\text{SIN}\left(\pi f\frac{4T}{8}\right)+2\text{SIN}\left(\pi f\frac{5T}{8}\right)\right.$$
$$\left.-2\text{SIN}\left(\pi f\frac{6T}{8}\right)+2\text{SIN}\left(\pi f\frac{7T}{8}\right)-\text{SIN}\left(\pi f\frac{8T}{8}\right)\right]$$

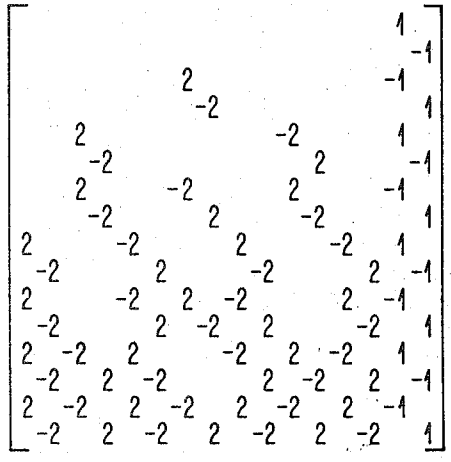
FIG. 8

SINC ARRAY

							1
			2				-1
	2				-2		1
	2		-2		2		-1
2		-2		2		-2	1
2		-2	2	-2		2	-1
2	-2	2		-2	2	-2	1
2	-2	2	-2	2	-2	2	-1

FIG. 10

INTERMINGLED SINC AND COSINC



SINC/COSINC TRANSFORM — A CONTINUOUS FOURIER TRANSFORM

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FIG. 7
SINE TRANSFORM

$$b(s1)=\frac{1}{\pi f}\left[1-\cos\left(\pi f\frac{8T}{8}\right)\right]$$

$$b(s2)=\frac{1}{\pi f}\left[1-2\cos\left(\pi f\frac{4T}{8}\right)+\cos\left(\pi f\frac{8T}{8}\right)\right]$$

$$b(s3)=\frac{1}{\pi f}\left[1-2\cos\left(\pi f\frac{2T}{8}\right)+2\cos\left(\pi f\frac{6T}{8}\right)-\cos\left(\pi f\frac{8T}{8}\right)\right]$$

$$b(s4)=\frac{1}{\pi f}\left[1-2\cos\left(\pi f\frac{2T}{8}\right)+2\cos\left(\pi f\frac{4T}{8}\right)-2\cos\left(\pi f\frac{6T}{8}\right)+\cos\left(\pi f\frac{8T}{8}\right)\right]$$

$$b(s5)=\frac{1}{\pi f}\left[1-2\cos\left(\pi f\frac{T}{8}\right)+2\cos\left(\pi f\frac{3T}{8}\right)-2\cos\left(\pi f\frac{5T}{8}\right)+2\cos\left(\pi f\frac{7T}{8}\right)-\cos\left(\pi f\frac{8T}{8}\right)\right]$$

$$b(s6)=\frac{1}{\pi f}\left[1-2\cos\left(\pi f\frac{T}{8}\right)+2\cos\left(\pi f\frac{3T}{8}\right)-2\cos\left(\pi f\frac{4T}{8}\right)+2\cos\left(\pi f\frac{5T}{8}\right)-2\cos\left(\pi f\frac{7T}{8}\right)+\cos\left(\pi f\frac{8T}{8}\right)\right]$$

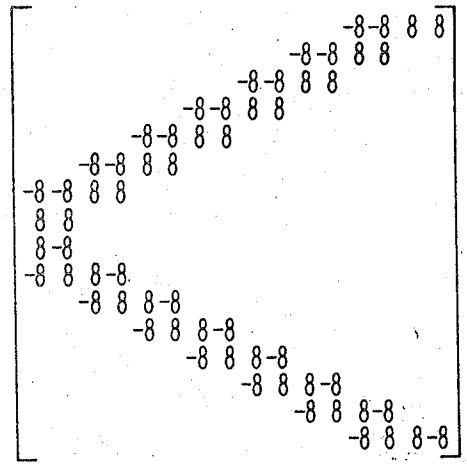
$$b(s7)=\frac{1}{\pi f}\left[1-2\cos\left(\pi f\frac{T}{8}\right)+2\cos\left(\pi f\frac{2T}{8}\right)-2\cos\left(\pi f\frac{3T}{8}\right)+2\cos\left(\pi f\frac{5T}{8}\right)-2\cos\left(\pi f\frac{6T}{8}\right)+2\cos\left(\pi f\frac{7T}{8}\right)-\cos\left(\pi f\frac{8T}{8}\right)\right]$$

$$b(s8)=\frac{1}{\pi f}\left[1-2\cos\left(\pi f\frac{T}{8}\right)+2\cos\left(\pi f\frac{2T}{8}\right)-2\cos\left(\pi f\frac{3T}{8}\right)+2\cos\left(\pi f\frac{4T}{8}\right)-2\cos\left(\pi f\frac{5T}{8}\right)+2\cos\left(\pi f\frac{6T}{8}\right)-2\cos\left(\pi f\frac{7T}{8}\right)+\cos\left(\pi f\frac{8T}{8}\right)\right]$$

FIG. 9
COSINC ARRAY

							-1
			-2				1
	-2				2		-1
	-2		2		-2		1
-2		2		-2		2	-1
-2		2	-2	2		-2	1
-2	2	-2		2	-2	2	-1
-2	2	-2	2	-2	2	-2	1

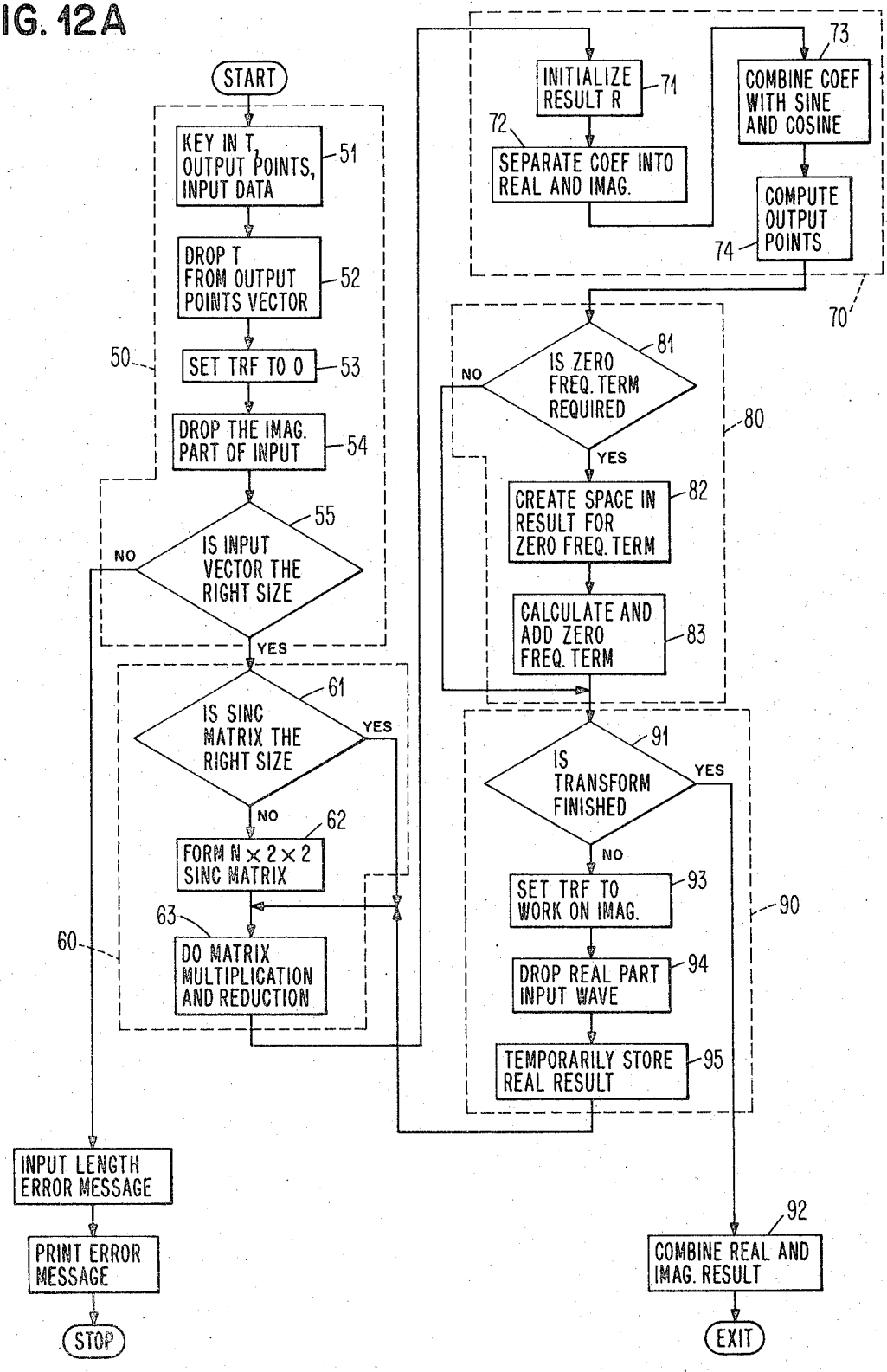
FIG. 11
SINC MATRIX
(UNFACTORED)



SINC/COSINC TRANSFORM — A CONTINUOUS FOURIER TRANSFORM

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FIG. 12A



SINC/COSINC TRANSFORM — A CONTINUOUS FOURIER TRANSFORM

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FIG. 15B

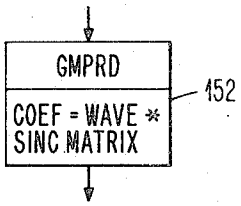


FIG. 15C

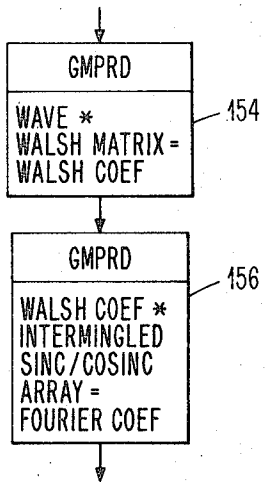


FIG. 15D

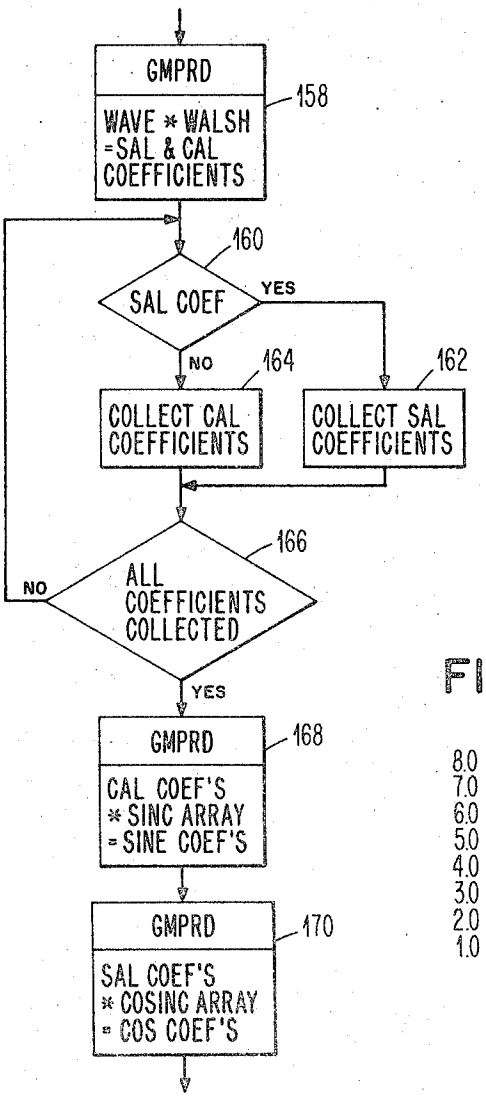
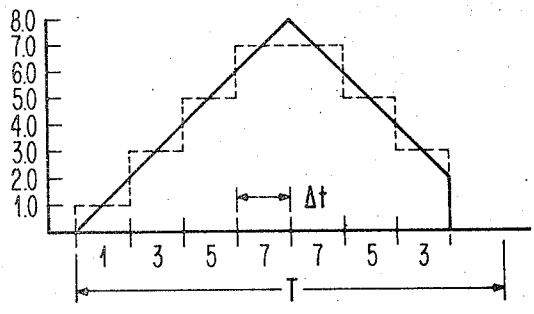


FIG. 13



SINC/COSINC TRANSFORM — A CONTINUOUS FOURIER TRANSFORM

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FIG. 14A

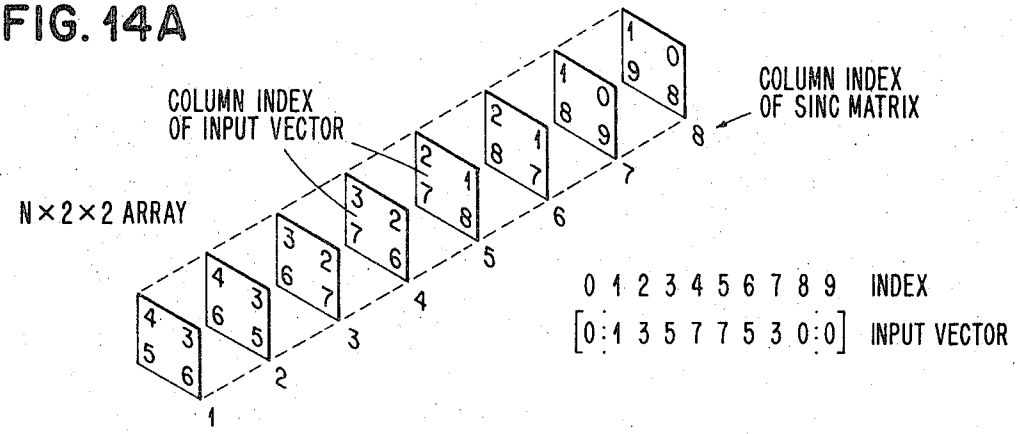


FIG. 14B

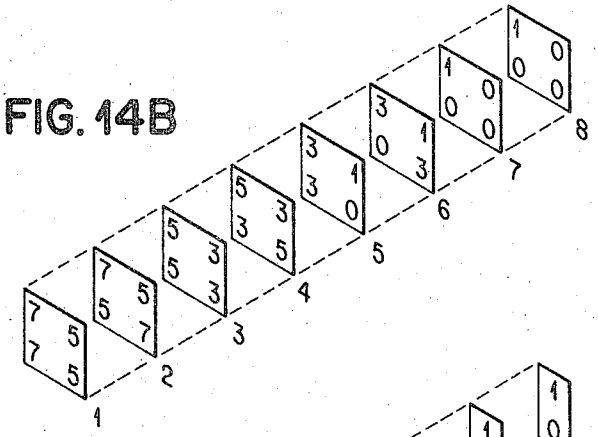


FIG. 14C

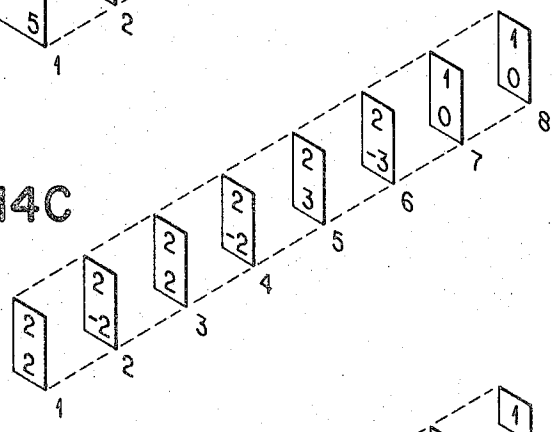
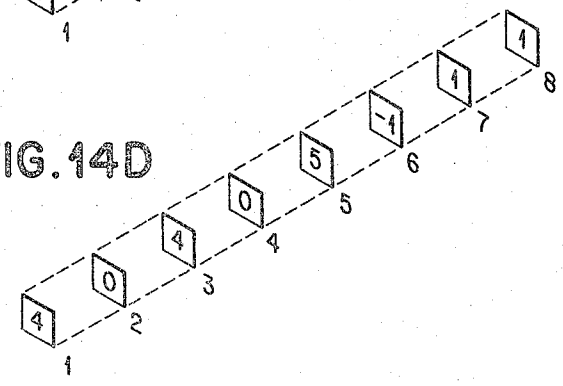


FIG. 14D



SINC/COSINC TRANSFORM — A CONTINUOUS FOURIER TRANSFORM

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FIG. 15A

