April 8, 1958 T. G. MIHRAN 2,830,223

SCALLOPED BEAM AMPLIFICATION

Filed April 22, 1954 3 Sheets-Sheet 1

Fig. 1

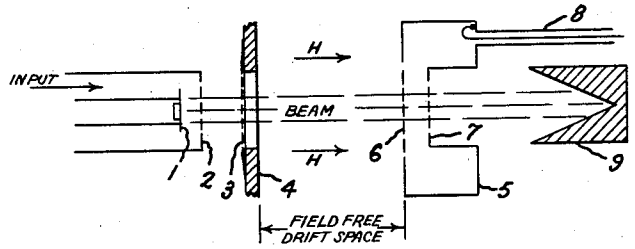


Fig. 2a. Fig. 2b. Fig. 2c. Fig. 2d. Fig. 2e.

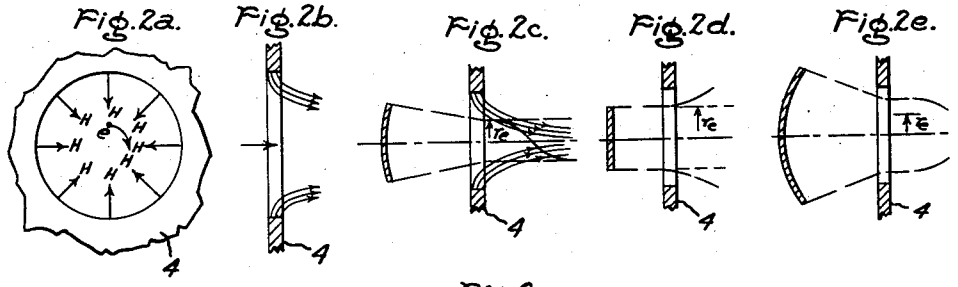


Fig. 6.

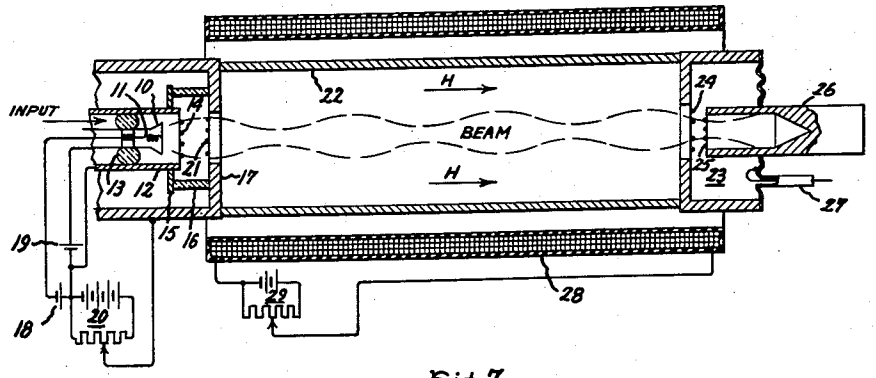
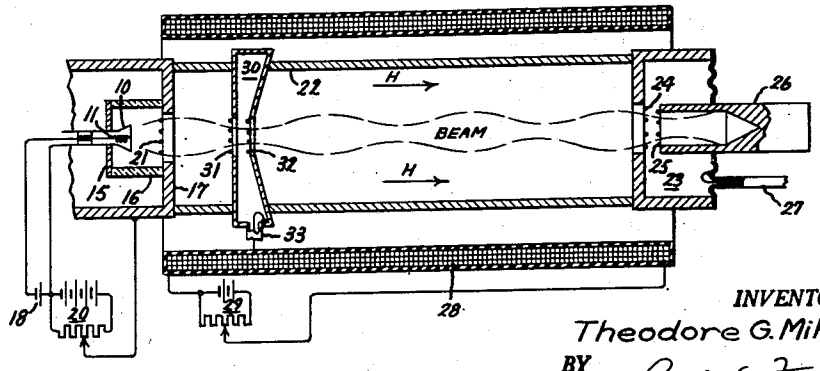


Fig. 7.



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SCALLOPED BEAM AMPLIFICATION

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Fig. 3.

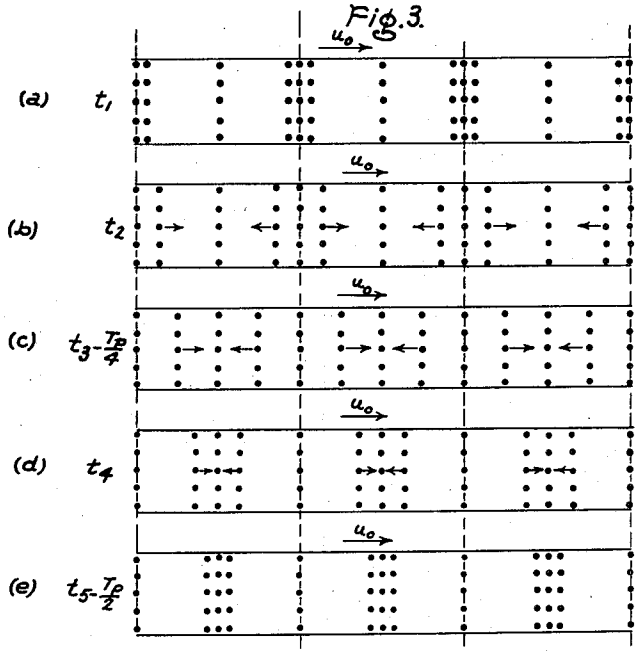
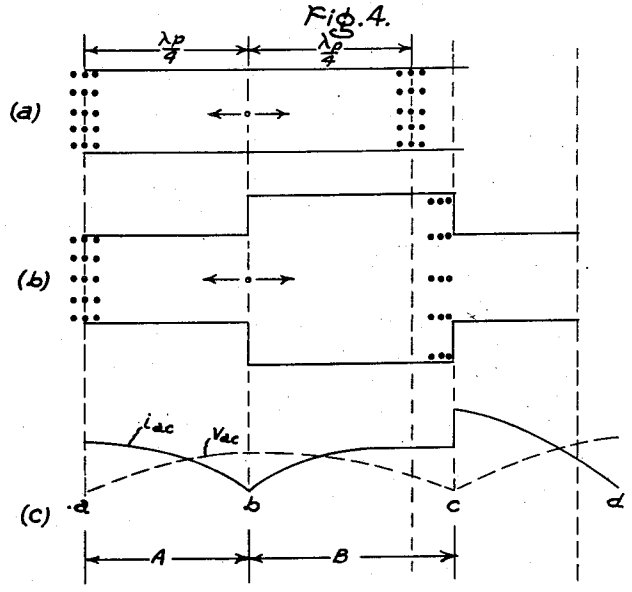


Fig. 4.



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SCALLOPED BEAM AMPLIFICATION

Filed April 22, 1954 3 Sheets-Sheet 3

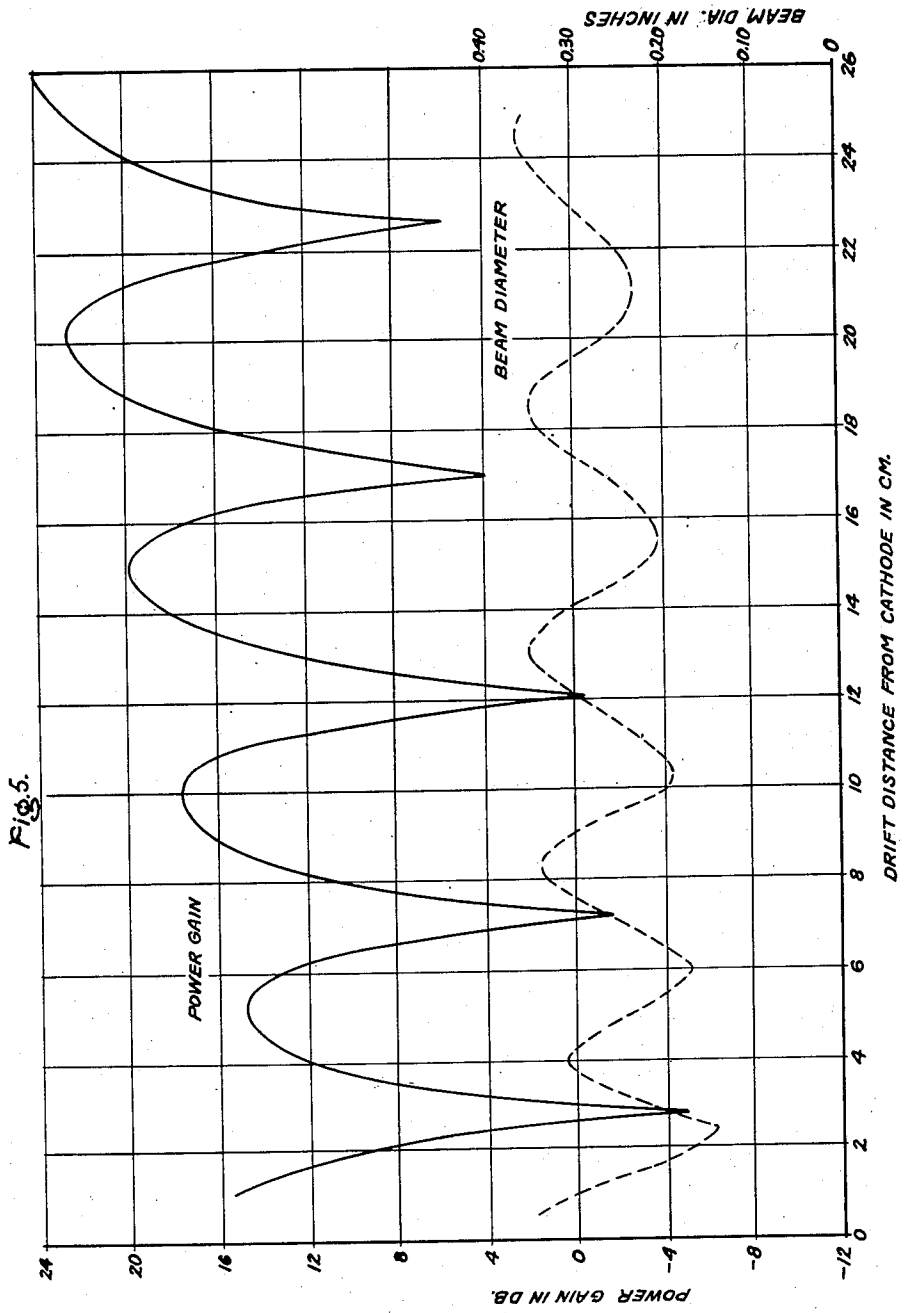


Fig. 5.

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2,830,223

SCALLOPED BEAM AMPLIFICATION

Theodore G. Mihran, Schenectady, N. Y., assignor to General Electric Company, a corporation of New York

Application April 22, 1954, Serial No. 424,918

1 Claim. (Cl. 315—5.35)

This invention relates to an apparatus for amplifying high frequency electromagnetic signals.

My invention is directed to a continuous interaction device which amplifies a radio frequency signal applied to an electron beam, the diameter of which varies periodically to define successive regions of varying direct current density. Amplification is obtained by interaction of the modulation produced by the radio frequency signal voltage and the beam of varying diameter. This mechanism of amplification is relatively insensitive to wide swings in operating frequency. My invention is essentially independent of boundary conditions; that is, a signal can be applied to a control grid, a velocity modulation cavity or a helix in order to modulate and vary the axial radio frequency electron density in an electron beam. Such an electron beam can be caused to traverse a field free drift space or region and be collected by a conventional energy extracting apparatus. By properly selecting the strength of the magnetic field in the field free drift space and the position of the electron source to properly shape the beam, amplification in the field free drift space may be realized. Therefore, amplification is accomplished without the necessity of an accompanying physical circuit which renders conventional electron beam amplifying apparatus mechanically more complex and less efficient. It is, therefore, an object of my invention to provide an apparatus for amplifying electromagnetic energy which is independent of any interaction of the electron beam with an external structure.

It is a further object of my invention to provide an apparatus for amplifying electromagnetic energy which is relatively insensitive to variations in the frequency of the electromagnetic energy to be amplified.

Another object of my invention is to provide an apparatus for amplifying electromagnetic energy whereby the amplification takes place in a field free drift region.

It is an object of my invention to provide an apparatus which is adapted to amplify electromagnetic energy at short wavelengths.

It is also an object of my invention to provide an apparatus to amplify an applied signal by interaction of the signal with an electron beam of varying diameter.

An amplifier in accordance with my invention may consist of an electron tube structure comprising an electron gun for producing a stream of electrons. The electron stream is first accelerated and then caused to pass through an equipotential region or field free drift space to a collector electrode. A magnetic beam shaping field is provided to form the stream of electrons in the field free drift space into successive regions of varying direct current electron density. The electron stream is modulated by a radio frequency signal to form electron groups or bunches in the electron stream. The bunches are then properly oriented with respect to the regions of varying

direct current electron density to obtain amplification by interaction between the electron stream and the electron bunches.

The invention itself may best be understood by reference to the following description taken in connection with the drawings in which Figures 1, 2*a*–2*e*, inclusive, 3 and 4 are diagrammatic illustrations used to explain the operation of my invention; Figure 5 is a graph of power gain and beam diameter as a function of drift distance from the cathode in an actual apparatus incorporating my invention; Figures 6 and 7 illustrate structure incorporating my invention.

In order to describe my invention, it will be necessary from time to time to refer to terms and phenomena which will be described in the immediate succeeding paragraphs in the form of definitions.

Referring now to Figure 1 of the drawing there is illustrated a thermionic cathode **1**, which emits electrons, a control electrode **2**, an accelerating electrode **3**, a magnetic shield **4**, a movable output cavity **5** with electron permeable members **6** and **7** and output coupling means **8**. Suitable voltages are applied to the accelerating electrode so that electrons emitted by the cathode travel along the lines indicated by the word "beam" and are collected by collector **9**. Output cavity **5** can be adjusted axially along the electron beam. The magnetic shield **4**, accelerating electrode **3**, movable output cavity **5** and collector **9** are maintained at the same direct current potential. Therefore, the region from accelerating electrode **3** to collector **9** may be considered a field free drift space, i. e. no direct current accelerating potential is applied to an electron in the beam once it leaves accelerating electrode **3** and travels to collector **9**. A magnetic field is provided having flux flowing in the direction indicated by the arrows H. Magnetic shield **4** is maintained at the same direct current potential as electrode **3** and also prevents any magnetic flux from affecting the cathode and control grid. If a radio frequency signal is applied between cathode **1** and grid **2**, a density modulation of the electrons in an axial direction results.

With the above brief description of Figure 1 the term plasma wavelength may be defined. The application of a radio frequency signal between electrode **2** and cathode **1** results in a variation in the electron density in an axial direction along the beam. It should be appreciated at this point that all electrons leaving cathode **1** and accelerated by accelerating electrode **3** travel at an average velocity determined by the electric field gradient between cathode **1** and accelerating electrode **3** and that the variation in electron density as a result of the applied radio frequency input is an instantaneous variation superimposed upon the beam of electrons traveling from cathode **1** to collector **9**.

These variations in the axial electron concentration may be considered as groups or bunches which are separated by a distance which is defined as an electron wavelength, i. e. if no degrouping or debunching forces are present due to the natural repulsive effects of particles with like charge, a series of bunches travels down the beam between accelerating electrode **3** and collector **9**. The distance between these bunches is determined by the frequency of the applied modulating signal voltage and the direct current beam velocity. Actually the bunches tend to disperse due to natural repulsion forces so that at some point along the beam the electrons in any given bunch leaving the accelerating electrode **3** are completely debunched and have a maximum axial velocity. This distance between a group of electrons which are completely bunched and the same group when completely debunched is defined as one-quarter plasma wavelength. This plasma wavelength is effectively independent of the modulating frequency

and, in the one dimensional case to be described below, is completely independent of modulating frequency. Plasma wavelength may also be defined in terms of a mathematical expression, as will be given below, and also in terms of physical phenomena as twice the distance between two regions of maximum or minimum electron concentration.

The plasma frequency must first be derived for the condition wherein the natural tendency of electrons to repel each other due to like electrical charge and to reform in new groups or bunches occurs in one dimension only. That is, the beam of Figure 1 may be considered to have infinite dimensions in a plane perpendicular to a line normal to the face of cathode **1** running from the cathode **1** to collector **9** and perpendicular to the surface of the drawing and that all action of electrons in a bunch or concentrated region occurs along an axis parallel to a line drawn from cathode **1** to collector **9**.

The plasma angular velocity may be defined by the equation

$$\omega_p=\sqrt{\frac{e}{m}\frac{1}{\epsilon_0}\frac{i_0}{u_0}} \tag{1}$$

where e and m are the charge and mass respectively of an electron, ϵ_0 is the dielectric constant of free space, i_0 is current density in amperes per square meter and u_0 is the direct current velocity of the charges. The direct current velocity is that velocity given to an electron by the electric field between the cathode **1** and the accelerating grid **3**. Equation 1 may be rewritten in terms of the plasma frequency so as to read

$$f_p=\frac{1}{2\pi}\sqrt{\frac{e}{m}\frac{1}{\epsilon_0}\frac{i_0}{u_0}} \tag{2}$$

The period of the plasma oscillation is equal to the reciprocal of the frequency, therefore

$$T_p=2\pi\sqrt{\frac{m}{e}\frac{\epsilon_0}{}\frac{u_0}{i_0}} \tag{3}$$

The wavelength is equal to the velocity times the period; therefore $\lambda p=u_0T_p$ or substituting from Equation 3

$$\lambda_p=2\pi u_0\sqrt{\frac{m}{e}\frac{\epsilon_0}{}\frac{u_0}{i_0}} \tag{4}$$

When the beam is cut down to a finite size, i. e. the cross-section thereof is of finite size, the plasma frequency is lowered due to a relative weakening of the axial electron forces which is caused by the diversion of a portion of the repulsion forces in directions other than along the beam axis. The effective plasma frequency can therefore be defined as f_q where

$$f_q=[F]f_p \text{ where } F<1 \tag{5}$$

and

$$\lambda_q=\frac{1}{[F]}\lambda_p \tag{6}$$

F is a function of

$$\frac{d\ (\text{beam diameter})}{i\ (\text{tube diameter})} \text{ and } \gamma_d$$

where

$$\gamma_d=\frac{2\pi f}{\mu_0}\cdot d \tag{7}$$

and where f is equal to the modulating frequency or frequency of the signal to be amplified. It is noted that the function F is affected by variations in the modulating frequency; however, the effect is relatively small, especially if the beam diameter or the modulating frequency is large.

Before discussing some of the methods of obtaining a beam of cyclically varying diameter defining successive regions of high and low direct current electron density which is referred to in this specification as a scalloped beam, it will be necessary to define the action of an electron in a magnetic field. This definition will initially define electron motion without space charge effects. When an electron is projected into a magnetic field in a direction transverse to the lines of flux it will tend to spiral and will travel through 360° of such spiral at a frequency which is called the cyclotron frequency. The cyclotron frequency may be defined as

$$\omega_c=\frac{e}{m}B \tag{8}$$

where e and m are the charge and mass respectively of an electron and B is the flux per unit area in the drift space. Expressed in terms of the cyclotron frequency, the above equation may be written as

$$f_c=\frac{1}{2\pi}\frac{e}{m}B \tag{9}$$

and the period may be expressed as

$$T_c=\frac{2\pi m}{eB} \tag{10}$$

If the electron is projected into the field with a direct current velocity u_0, the electron travels in a helix of constant diameter and the axial distance between like portions of the helical path may be defined as the cyclotron wavelength and may be written in the form of an equation as

$$\lambda_c=u_0T_c \text{ or } \lambda_c=u_0\frac{2\pi m}{eB} \tag{11}$$

If space charge effects are taken into consideration, a portion of the magnetic field is utilized in overcoming the natural repulsion effects of the electrons in the beam and the portion of the magnetic field available to cause the electron to spiral is reduced by a factor K. Since the conditions necessary for a scalloped beam include an appreciable space charge effect the scalloped beam wavelength may be defined as

$$\lambda_s=\lambda_c\frac{1}{K} \text{ or } \lambda_s=u_0\frac{2\pi m}{eBK} \tag{12}$$

where the initial velocity u_0 may be expressed by the equation

$$u_0=\frac{2e}{m}V_0 \tag{13}$$

and where V_0 is the direct current potential which accelerates the electrons to a velocity u_0. In a preferred form of the apparatus of my invention, K has a value which lies in the range from 1 to 0.707. It should be noted that there must be a force or velocity component of the electron which is transverse to the electric field in order to realize spiraling of the electrons at the scalloped beam frequency. This force or velocity component results from the natural repulsion effects between adjacent electrons.

An example of several methods for obtaining a scalloped beam may be described by reference to Figures 2*a* to 2*e* in which the structure of 2*a* may be considered the defocusing aperture in magnetic shield **4** through which the electron beam passes. The direction of the magnetic field is illustrated by arrows H. At the entrance of aperture **4** there is a component of the magnetic field which is perpendicular to the direction of propagation of any one electron *e* emitted by cathode **1**. Such an electron is indicated by the dot labeled *e* and the arrow indicates the direction which *e* will tend to take if it is projected into the above defined magnetic field in a direction into and perpendicular to the surface of the drawing. The electron will acquire an angular velocity which may be defined as $\omega_s=\omega_cK$ were ω_s is the scalloped beam angular velocity.

In order to produce a scalloped beam certain critical starting conditions are necessary. It may now be assumed that electrons leave a cathode that is magnetically shielded and are suddenly introduced into a magnetic field by means of a defocusing aperture in a magnetic

shield and accelerating anode as illustrated in Figure 2c. In going through the fringing portion of the magnetic field, the electrons acquire an angular velocity as has been previously described, which interacts with the uniform portion of the magnetic field present in the drift space and produces an inward force on the electrons. With a proper choice of magnetic field strength B, this force can be made to cancel exactly the outward space-charge and centrifugal forces of the beam. Under these conditions all electrons travel down the beam in helices about the axis of symmetry and the outline of the beam is smooth as shown in Figure 2c. It is noted that the defocusing action of the aperture must be taken into account in order to present a parallel electron beam to the magnetic field.

There is a definite relation between the beam current I_0, the beam voltage V_0, the magnetic field strength B, and the beam radius r_e which must be satisfield in order to obtain a beam having this smooth outline. This relationship may be developed in accordance with equations and phenomena well known in the art to render an expression in terms of meter-kilogram-second units known as the conditions necessary for what is customarily termed Brillouin flow. This relationship may be expressed in the form of Equation 14.

$$I_0=1.45\times10^6B^2V_0^{1/2}r_e^2 \qquad (14)$$

Not only must this relationship hold, but the electrons must enter this region in essentially parallel flow, i. e. traveling in a direction parallel to the beam axis and at a radius r_e. If these conditions are not fulfilled the electrons tend to deviate from the equilibrium radius.

One method of obtaining a scalloped beam is by introducing the electrons in a non-parallel electron beam such as illustrated in Figure 2d. Figure 2d shows an electron gun with a given angle of electron beam convergence which is placed behind an aperture with a different defocusing angle so that the electrons in the beam tend to diverge when introduced into the magnetic field.

It is well known in the art that when an electron beam is projected down a tube having finite dimensions that the force on an individual electron tending to deviate from a parallel path along the beam is at a minimum at the equilibrium radius of the beam. This equilibrium radius has been defined as r_e. This force, which tends to return the electron to a position at the equilibrium radius, increases as an individual electron deviates from this equilibrium radius. Therefore, an electron or group of electrons projected through an aperture such as the aperture illustated in Figure 2d will be in a potential field that limits the radial travel of the electron and tends to return the electron toward the equilibrium radius. Upon reaching the equilibrium radius, however, the electron has acquired an inward radial velocity. This causes the electron to overshoot the equilibrium radius and again the potential field brings it to rest, this time at a radius somewhat less than r_e. The electron is again returned toward the equilibrium radius and reaches the equilibrium radius with the same velocity it had at the beginning of the cycle. The cycle repeats as the electron travels along the beam toward the collector. In other words, the electrons oscillate about the equilibrium radius. The wavelength of this oscillation has been previously developed and is defined by Equation 12.

Another method of obtaining a scalloped beam is illustrated in Figure 2e. In this instance a magnetic field other than that specified in Equation 14 is utilized so that the electron beam has a radius, in this case greater than the equilibrium radius r_e which is defined by Equation 14. This means that parallel flow electrons are being injected into a magnetic field at a radius other than the equilibrium radius. The electrons are in a potential field and are acted upon to return to the equilibrium radius. Upon reaching it, they have acquired kinetic energy which carries them through it to a smaller radius, and again oscillations take place about the equilibrium radius and the result is a scalloped beam the wavelength of which is defined by Equation 12. Thus a scalloped beam can be obtained with an initial parallel flow beam merely by the use of a magnetic field strength other than that given by Equation 14.

The potential field which has been mentioned in the above paragraphs which tends to return electrons on either side of the equilibrium radius to the equilibrium radius may be better understood by applying the analogy of a simple pendulum. At the upper end of the pendulum travel the energy in the pendulum is all in the form of potential energy. At the bottom end of the pendulum's travel the pendulum has maximum velocity and zero potential energy. At some intermediate point in the travel of the pendulum on either side of the bottom dead center position the pendulum will possess both kinetic and potential energy. If we now substitute an electron for the pendulum and assume the bottom dead center position as the condition at the equilibrium radius it may be seen that for the above-described intermediate positions of the pendulum or electron there will be a force which is composed of both potential and kinetic energy which will cause the electron to swing from a region outside to a region inside the equilibrium radius.

It is noted that these are merely a few examples of manners in which a scalloped beam may be obtained and that any method of obtaining a scalloped beam may be used in the practice of my invention. For example, when the cathode is not shielded, i. e. when the magnetic field threads through the cathode, beam scalloping will always occur. There is no critical magnetic field to prevent radial motion of electrons in the beam.

Figures 3 and 4 illustrate the mechanism of my invention. Figures 3a to 3e illustrate the action of electrons throughout one-half plasma wavelength in a beam which has a constant direct current density. For the purpose of this discussion the constant forward velocity μ_0 is neglected since the effect of this velocity merely shifts the illustrated electron distribution to the right hand side of the drawing at a constant rate. It must be understood that the electrons illustrated by the dots in Figures 3 and 4 are those electrons which are in a maximum bunched condition and which are formed by an applied modulating signal voltage. These bunches define regions of high radio frequency current density. The velocity of an electron relative to the direct current electron velocity is referred to as radio frequency velocity. It is noted that Figures 3 and 4 do not show those bunches which may be present when the electron wavelength is less than the plasma wavelength and that a complete illustration would also show other bunches in various stages of debunching or rebunching between the illustrated bunches. In the absence of such an applied modulating signal there would be a uniform distribution of electrons present in the drift space which would be moving from the left hand side of Figure 3 to the right hand side with an average velocity u_0 as indicated by the arrow and which are referred to as direct current. The horizontal boundary lines may be assumed to define the outer surface or envelope of a beam of uniform direct current density. The dots illustrate bunches of electrons which are separated by one-half plasma wavelength and which may be formed by any conventional density modulating or velocity modulating apparatus. There is no direct current potential difference between the ends of the illustrated electron beams. The phenomena about to be described occurs in a field free drift space. The electrons in these bunches have a natural tendency to separate due to the natural repulsion effects of particles with like charge. Therefore, in Figure 3b at some time t_2 the electrons in each of the bunches have started to drift apart giving up their high potential energy which they had when in a bunch and converting this potential energy into kinetic energy. At a time t_3, as illustrated

in Figure 3c, approximately one-quarter plasma wavelength later, the electrons have given up all their potential energy and are essentially uniformly distributed along the tube or electron beam. In this condition it may be assumed that the electrons have a maximum velocity and that the instantaneous alternating current is zero. At a time t_4 as illustrated in Figure 3d the electrons have started to regroup or bunch at a region between the original bunches. The electrons are then giving up their kinetic energy due to the natural repulsion forces between the electrons and their velocity is decreasing while the instantaneous radio frequency current density is increasing. At a time t_5 which is equal to one-half plasma period after the initial time t_1 the electrons have completely rebunched, have effectively zero velocity and the instantaneous radio frequency current density is at a maximum.

In a conventional drift tube amplifier a standing wave of radio frequency current will be established, very much in the fashion described in the analysis of Figure 3 of the drawing. This standing wave will take the form as illustrated in Figure 4a in which there will be regions of maximum axial electron concentration separated by one-half plasma wavelength and with an intermediate region therebetween at which the electron concentration is at a minimum and the electron velocity at a maximum. Figure 4b illustrates a beam of electrons the direct current density of which varies rapidly at certain points along the field free drift space due to a change in beam diameter. Figure 4c illustrates the current and electron velocity characteristics of electrons in a beam the direct current density of which varies inversely in accordance with the diameter of the envelope of Figure 4b.

It may be seen that the maximum electron density is the same in each of the regions illustrated in Figure 4a. This is because the potential energy given up when the electrons separate and reach maximum velocity is completely converted into kinetic energy and the kinetic energy is then completely reconverted into potential energy in forming the next bunch. This analysis is sound if it is assumed that no other forces are acting on the electron other than those along a line parallel with the beam axis.

An understanding of the term electric restoring force may be had by assuming a region with a given average number of electrons per unit volume. If a series of bunches or groups of electrons are suddenly introduced into this region the electrons in each bunch will immediately start to separate or debunch due to the natural repulsion effects of particles with like charge. The electrons in the bunches will tend to form new bunches as has been described in regard to the illustrations of of Figs. 3a to 3e. The force tending to debunch the electrons is a function of the average electron concentration in the region as well as the radio frequency electron concentration. If we now assume a fixed initial radio frequency electron density in the bunches, the electric restoring force will be proportional to the direct current electron density only. If the given average number of electrons per unit volume is increased, the tendency of the electrons to debunch is increased and the tendency to bunch is decreased. If the direct current electron density is decrease the electrons have less tendency to debunch and conversely electrons traveling toward each other with a given velocity will tend to form a tighter bunch. For purposes of this specification, a region of low direct current electron density is termed a region of low electric restoring field, i. e. the tendency of the electric field to slow down electrons having a given velocity is low so that highly concentrated bunches of electrons are formed. Conversely a region of high direct current electron density is said to have a high electric restoring field.

Referring now to Figures 4b and 4c it will be assumed that a current maximum occurs in region A. It should be noted that the currents and velocities herein mentioned are instantaneous or alternating currents as distinct from the direct current component of the beam. As a radio frequency signal bunch in region A progresses toward region B, debunching forces will gradually convert the potential energy of the bunch into kinetic energy. One-quarter of a plasma wavelength later at point b there is a current null and a velocity maximum. If the direct current density is reduced suddenly at this point as is illustrated in Figure 4b by the expanded envelope of the direct current beam, the radio frequency or alternating current velocity of the electrons remains unchanged and as further drifting occurs the kinetic energy of the electrons is gradually reconverted into potential energy. However, because of the reduced direct current density in region B and resulting reduced electrical restoring force when the electrons reach a current maximum at point c they are closer together axially than they were at point a. It should be noted that with the reduced direct current density and reduced electric restoring or debunching force the electrons drifted further before giving up all of the kinetic energy and reaching a point of maximum concentration. The electrons at point c are more tightly compressed than at point a, and constitute an increased total radio frequency current in the beam. At point c the direct current density can be returned to its initial value without altering the total radio frequency current in the beam. No energy is expended in reconcentrating the beam, since, for the purpose of this analysis, only the repulsion effects along an axis parallel with the beam axis are being considered. It is further noted that where the electron wavelength is less than the beam radius the repulsion or fringe effects perpendicular to the axis of the beam are relatively low and, therefore, this theoretical assumption that no energy is required to increase the direct current density of the beam is substantially accurate.

However, in an apparatus having finite dimensions, energy is extracted from the electron beam in reconcentrating the beam. This energy is transferred to the radio frequency field and contributes to the observed gain.

It may therefore be seen that the radio frequency current in an electron beam can be enhanced by arranging to have high direct current density in the region of the beam where debunching occurs and low direct current density where rebunching occurs. This process may be repeated for as many cycles as desired along the drift space and will result in an exponential growth of the current standing wave maxima.

In actual practice it is not possible to achieve a perfectly stepped beam of varying diameter defining regions of varying direct current density in accordance with the steps shown in Fig. 4. However, it has been found that very similar results may be obtained by using a scalloped beam which is produced in the manner previously described. For optimum gain, the scallop wavelength should equal one-half the plasma wavelength. This is in agreement with the description of Figure 4, since the condition illustrated in Fig. 4 would occur when the scalloped beam approximates most nearly the ideal stepped beam. Furthermore, the small diameter section of the scalloped beam should coincide with the debunching portion and the large diameter section with the rebunching portion of the standing wave of radio frequency current. That is, the radio frequency current null should occur when the beam diameter is near its equilibrium value and increasing. This phase relationship is exhibited clearly in Figure 4c.

A typical curve is illustrated in Figure 5 of the drawing. The solid curve shows a standing wave of power output, the maximum amplitude of which increases exponentially with distance. This curve was taken with a sliding cavity apparatus similar to that illustrated by Figure 1 of the drawing. The current wave was initiated by a space charge control section operating at 1000 mega-

cycles. A radio frequency signal of 1000 megacycles was applied between the control electrode **2** and cathode **1** to density modulate the electron beam flowing from cathode **1** through the field free drift space to collector **9**. The output power was measured with a movable cavity such as cavity **5** of Figure 1. The distance of the cavity from the cathode could be varied from 1 centimeter to 25.8 centimeters. The direct current beam voltage was set at 500 volts and the cathode current at 50 milliamperes. Under these conditions a gain of 13.4 decibels was observed with the cavity snug against the accelerating anode. As the cathode to output cavity distance was increased a standing wave of output power was obtained with each successive maximum several decibels above the maximum of the preceding wave. With the cavity placed 25.8 centimeters from the cathode, an overall gain of 24.0 decibels was measured, of which 10.6 decibels was due to the scalloped beam amplification phenomena. During these measurements, the magnetic field was held constant at 92 gauss. The dotted curve of Figure 5 shows how the diameter of the beam varied along the drift space under these conditions. It should be noted that corresponding to the five half plasma wavelengths of radio frequency power, there are five scallops in the beam. A half plasma wavelength is the distance from one radio frequency current minimum to another radio frequency current minimum. It should also be noted that the phasing between the plasma wave and the direct current scallops is very close to the optimum phasing developed in the discussion of Figure 4. The scallops illustrated by the dotted curve result from the shaping effect of the magnetic field on the electron beam and should not be confused with the transverse debunching scallops sometimes observed in klystrons operating at relatively high radio frequency current levels.

The mathematical relationship between the effective plasma wavelength and the scalloped wavelength may be developed in the following fashion. The basic relationship for optimum interaction may be defined by the equation

$$\gamma_q = 2\gamma_s \tag{15}$$

Substituting from Equations 4 and 6 the expression for B_{SBA}, the flux necessary to obtain optimum scalloped beam amplification may be written as

$$B_{SBA} = \frac{2[F]}{K}\sqrt{\frac{m}{e}\frac{i_0}{u_0}\frac{1}{\epsilon_0}} \tag{16}$$

An idea of the gain that can be expected from this phenomena may be obtained by a mathematical analysis of Figure 4*c* in which the amplitudes of the velocity and current density waves which exist in the beam may be defined by equations

$$|v_A| = \left|v_b \cos\frac{\omega_{qA}}{u_0}Z\right| \tag{17}$$

$$|i_A| = \left|jv_b\frac{\omega}{\omega_{qA}}\frac{I_{dA}}{\mu_0}\sin\frac{\omega_{qA}}{u_0}Z\right| \tag{18}$$

In these equations v_A is the instantaneous radio frequency or alternating current velocity of an electron at any point in region A; v_b is the radio frequency or alternating current velocity of an electron at the beginning of region B and is also the maximum electron velocity; u_0 is equal to the direct current or average electron velocity; i_A is the instantaneous radio frequency or alternating current density in region A; ω_{qA} is the effective plasma radio frequency in region A, I_{dA} is the direct current density in the same region and Z is the distance along the axis of the beam. Boundary conditions may be established by assuming that at point *a*

$$\omega_{qA}\frac{Z}{u_0} = \pi/2$$

$$v_a = 0 \tag{19}$$

$$|i_a| = \left|v_b\frac{\omega}{\omega_{qA}}\frac{I_{dA}}{u_0}\right| \tag{20}$$

It should be noted at this point, that the values given for v_a and i_a are absolute values and that in order to carry out this analysis, a boundary condition must be selected for

$$\omega_{qA}\frac{Z}{u_0}$$

which will render the velocity zero at point *a*. It is further noted that this analysis is based on the one dimensional action of the electrons along the Z axis or beam axis only and that finite beam size is taken into account by the use of the effective plasma frequency rather than the one dimensional plasma frequency. It may be assumed that the signal bunch will drift until the current density modulation is completely converted into velocity modulation at point *b*. At this point the magnitude of the radio frequency velocity is simply v_b and the radio frequency current density is zero since

$$\omega_{qA}\frac{Z}{u_0} = \pi$$

By expanding the beam the current density is reduced to I_{dB} which is the direct current density in the B region of Figure 4*c*. This does not change the radio frequency velocity at this point but it does change the effective plasma frequency in region B. Therefore, the current density wave in this region is given by the equation—

$$|i_b| = \left|jv_b\frac{\omega}{\omega_{qB}}\frac{I_{dB}}{u_0}\sin\frac{\omega_{qB}}{u_0}Z\right| \tag{21}$$

If this current density wave is allowed to drift a quarter-plasma-wavelength to point *c* it will reach a maximum value of

$$|i_c| = v_b\frac{\omega}{\omega_{qB}}\frac{I_{dB}}{u_0} \tag{22}$$

It is convenient to return the direct current beam density to its original value at this point since this may be done without altering the total radio frequency current in the beam.

The total radio frequency current is the product of the radio frequency current density and the beam area. If the total radio frequency current at point *c* is compared to that at point *a* it will be found that the ratio of the currents may be expressed by

$$\frac{I_c}{I_a} = \frac{\omega_{qA}}{\omega_{qB}}\frac{I_{dB}}{I_{dA}}\frac{d_B{}^2}{d_A{}^2} \tag{23}$$

where d_A and d_B are the beam diameters in regions A and B respectively. The effective plasma frequency ω_q is given by the equation

$$\omega_q = F(d,t,w,u_0)\left[\frac{eI_d}{m\epsilon_0 u_0}\right]^{1/2} \tag{24}$$

where *d* and *t* are the beam and drift tube diameters respectively. Substituting Equation 24 into Equation 23 an expression for current gain is obtained which may be expressed by the equation—

$$\frac{I_c}{I_a} = \text{current gain} = \frac{F_A}{F_B}\left(\frac{I_{dB}}{I_{dA}}\right)^{1/2}\left(\frac{d_B}{d_A}\right)^2 \tag{25}$$

If total current is kept constant, current density I_d is inversely proportional to the square of the beam diameter; therefore current gain may be expressed by the equation

$$\text{Current gain} = \frac{F_A}{F_B}\frac{d_B}{d_A} \tag{26}$$

Power gain is proportional to the square of the current gain and may be written in equation form as

$$\text{Power gain} = 20\log_{10}\frac{F_A}{F_B}\frac{d_B}{d_A}\text{ db} \tag{27}$$

Although Equation 27 gives the power gain for the stepped beam of Fig. 4*b*, it may readily be seen that it

also gives an upper limit for gain when a scalloped beam is used.

Figures 6 and 7 illustrate two types of apparatus incorporating my invention. Figure 6 illustrates an amplifier utilizing density modulation and constructed in accordance with my invention and Figure 7 illustrates an amplifier utilizing velocity modulation and constructed to permit practice of my invention. Like components in each of these two figures are designated by the same reference numerals.

Figure 6 illustrates a density modulated drift tube structure with a cathode **10** including a heater element **11** supported in tube **12** by glass beads **13**. Tube **12** has a fine wire grid **14** across the end thereof and is constructed so that a high frequency signal may be applied between the cathode structure **10** and the tube **12**. Tube **12** is supported by disk **15** and annular insulator **16**. The complete assembly is mounted on magnetic shield **17**. Power supply **18** provides current for heater **11**, source **19** provides grid bias for grid **14** and power supply **20** provides a direct current potential between cathode **10** and accelerating grid **21**. A tube **22** of stainless steel is mechanically and conductively coupled to the magnetic shield **17**. Tube **22** is provided at the end thereof with a tunable output cavity **23** which may be constructed of sheet copper. Cavity **23** is provided with grid structures **24** and **25**, a copper collector **26**, and an output line **27**. Tube **22** defines the field free drift space or region and is maintained, as is the rest of the structure from magnetic shield **17** to output cavity **23**, at the same direct current potential. Therefore, there is no direct current potential difference between the accelerating grid **21**, and collector **26**. Solenoid **28** surrounds the drift tube **22** and when energized by power supply **29**, provides a unidirectional magnetic field within the drift tube in the direction illustrated by the arrows H.

Figure 7 illustrates a structure which is similar to that illustrated in Figure 6 except that velocity modulation is utilized to produce bunches of electrons in the electron beam. Tunable cavity **30** consists of a metal chamber with grids **31** and **32** and input lead **33**. It is noted that there is no control grid as in the structure of Figure 6.

The apparatus illustrated in Figure 6 is operated by connecting the power supplies **18**, **19** and **20** to the tube structure so as to produce a beam between cathode **1** and the collector **26**. The power supply for solenoid **28** is then adjusted to produce a scalloped beam. A scalloped beam being considered to be one the diameter of the outer envelope of which varies periodically between the accelerating grid **21** and the collector **26**. The magnetic field is adjusted so that accelerating grid **21** and the entrance into the output cavity **23** will be located where the beam has an equilibrium diameter and where the beam is contracting. This is an initial adjustment of the scallops on the beam. A high frequency signal is then applied to the input of the tube to produce a varying field between cathode **10** and control grid **14**. This will produce density modulation in the electron beam. The magnetic flux is then varied by adjusting solenoid power supply **29** to produce scallops in proper phase relationship to the density modulation. The apparatus of Figure 7 is operated in a similar fashion. In the case of the apparatus of Figure 7 the modulator cavity **30** is placed at a point on the scalloped beam where the beam is at the equilibrium diameter and increasing in size, i. e. at a point where the direct current density is at an average value and the density is decreasing.

It will be readily appreciated that the illustrated apparatus of Figures 6 and 7 is given merely by way of example and that any method of producing a density or velocity modulated wave on an electron stream may be used in conjunction with the practice of my invention. The amplification is obtained by the interaction of the direct current beam and the modulated signal thereon and is not dependent upon interaction of a physical structure with the beam. In the practice of my invention it has been found that the phenomena accompanying it is relatively insensitive to frequency variations. An amplifier constructed in accordance with my invention will operate satisfactorily with frequency variations of as much as two to one. This wide band operation is accomplished by proper adjustment of the magnetic field and the provision of wide band input and output coupling circuits.

It may be readily appreciated that the method and practice of my invention may be applied to innumerable structures and therefore I aim in the appended claim to cover all variations in structure and application which fall within the true spirit and scope of the foregoing disclosure.

What I claim as new and desire to secure by Letters Patent of the United States is:

A high frequency electron beam amplification system comprising a source of electrons, means for accelerating electrons emitted by said source to provide an electron beam and means spaced from said source for collecting the electrons and establishing the path of said beam along a drift space substantially free from electric field, means establishing a magnetic field coupled with the beam path and having a direction substantially parallel to the beam path and an intensity correlated with the entrance conditions of the beam to the drift space, the average accelerating voltage and the average current density of the beam to provide a scalloped beam having a scallop wave length substantially equal to one-half the plasma wave length at the frequency of the signal to be amplified, modulating means coupled to the beam near the end thereof adjacent the source of electrons and positioned to produce a plasma wave having a minimum value located at a point of average and increasing radius of said scalloped beam, and output means coupled to said beam at substantially the maximum amplitude of the plasma wave.

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