

July 28, 1953

O. G. BOWEN
METHOD AND APPARATUS FOR ANALYZING SHAPE AND
BENDING STRESSES IN STRUCTURAL ELEMENTS
Filed Feb. 13, 1950

2,646,716

Fig. 1.

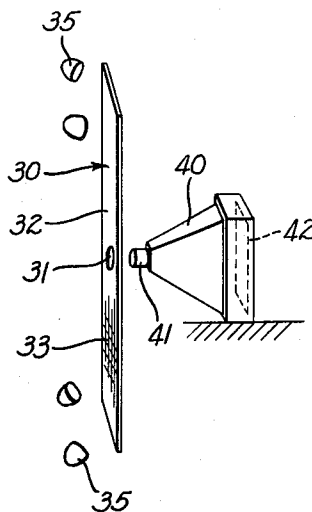
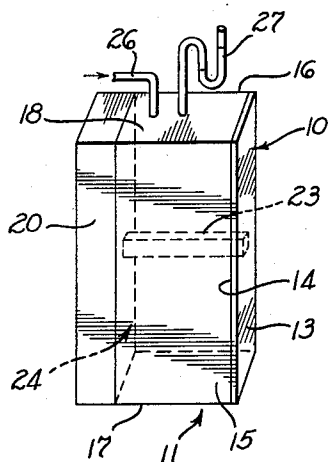


Fig. 2.

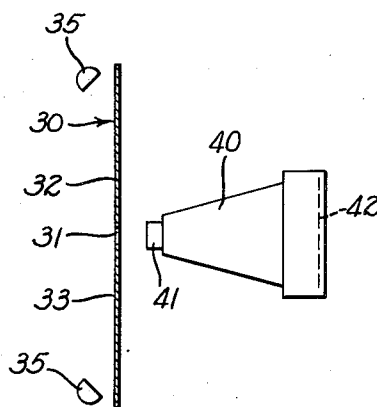
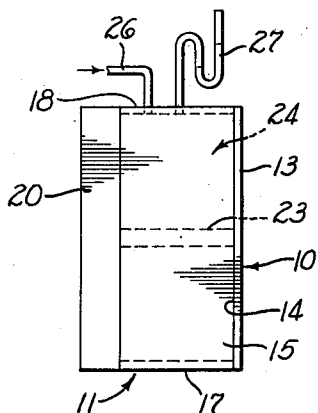


Fig. 3.

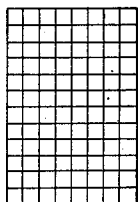
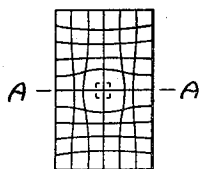


Fig. 4.



INVENTOR.
OLIVER GERALD BOWEN
BY HIS ATTORNEYS.
HARRIS, KIECH, FOSTER & HARRIS
BY *Lawrence F. Kiech*

UNITED STATES PATENT OFFICE

2,646,716

METHOD AND APPARATUS FOR ANALYZING
SHAPE AND BENDING STRESSES IN
STRUCTURAL ELEMENTS

Oliver Gerald Bowen, Los Angeles, Calif., assignor
to Presan Corporation, Los Angeles, Calif., a
corporation of California

Application February 13, 1950, Serial No. 144,014

11 Claims. (Cl. 88—14)

1

My invention relates to a method and apparatus for analyzing or determining the shape of a loaded element and more particularly for analyzing the bending stresses in an element when loaded.

It is an important object of the present invention to provide a novel method and apparatus employing optical principles and which can be used in the determination or analysis of shape or bending stresses in an element of a structure, typically a flat element such as a floor slab subject to bending due to the application of transverse loading.

The invention will be herein described with reference to the solution of a relatively simple problem, namely, the analysis of bending stresses in a reinforced-concrete floor slab of a rectangular building, the slab being supported by the four side walls and by a central column. This example has been chosen merely because of simplicity in explaining the principles of the invention. While the design of such a floor slab is relatively simple when employing known engineering formulas, the determination of the actual bending moments in a more complex building structure becomes extremely tedious and difficult. In certain irregularly-shaped or irregularly-supported floor slabs, use of conventional formulas and methods becomes practically impossible except to give rough approximations as concerns the placement and size of reinforcing rods. For example, the determination of the actual bending moments in a monolithic floor system which spans in more than one direction, represents an extremely difficult problem, particularly if the spacing of the supports or columns is irregular or non-uniform.

It is an object of the invention to provide a novel method and apparatus well suited to the solution of such a problem, as well as to the solution of problems involving stresses in other structural elements having flat surfaces and which may be irregularly supported to establish irregular stress patterns upon the application of transverse or other loads whether of a uniform or concentrated character.

It is another object of the present invention to provide a simplified method and apparatus for analyzing stresses in an irregularly supported element of a structure, involving the use of a model of such structure. The model should be of the same relative stiffness as the structure and should provide an element corresponding to the element of the structure to be analyzed or designed. The stress analysis can be made on the basis of the flexure of such an element of the

2

model or on the basis of flexure of the corresponding element of a completed or partially completed structure. Hereinafter, when speaking of such an element, it should be understood that the element may be a part of such a model or a part of such a structure.

It is an object of the present invention to analyze the shape or bending stresses in an element by making reflective or by mirrorizing (the latter term being used in a broad sense) the entire area of the element or some relatively large portion of the entire area which is to be critically examined. Onto this reflective surface is transmitted an optical pattern of known design to obtain a reflected optical pattern. When the element is loaded, the reflected optical pattern will differ from the transmitted optical pattern as a function of the bending of the element under load. If the element has a plane reflective surface, comparison of the transmitted and reflected optical patterns in corresponding zones will give a measure of the deformation and the resulting bending stresses in the element in a corresponding zone. The invention has among its objects the application of such principles for the purposes heretofore set forth.

If the reflective surface is not exactly plane, either because of surface irregularities or an overall curvature under no-load condition, the reflected no-load optical pattern will deviate somewhat from the transmitted optical pattern. However, a further change in the reflected optical pattern will result upon loading of the element and it is within the scope of the present invention to compare the no-load reflected optical pattern with the optical pattern obtained under loaded conditions to analyze the shape or bending moments in the element under consideration.

A general object of the invention is to provide a method and apparatus for the relatively rapid and exact analysis of stresses in complex structures by photographic or optical means.

Further objects and advantages of the invention will be evident to those skilled in the art from the following description, taken from the drawing, which illustrates and describes the use of the invention in solving a relatively simple problem, yet which will exemplify and guide those skilled in the art to the use of the invention in solving the more complex problems for which it is particularly adapted.

Referring to the drawing:

Fig. 1 is an isometric view of a simple arrange-

ment of equipment contemplated by the invention;

Fig. 2 is a side view of the equipment of Fig. 1, with the patterned screen shown in section;

Fig. 3 is a simplified view of one type of patterned screen or optical pattern that may be employed; and

Fig. 4 is a diagrammatic view of the type of distorted optical image produced under load conditions with the equipment shown in Fig. 1.

Referring particularly to Fig. 1, the element to be analyzed is indicated by the numeral 10, shown as a part of a model 11 of a building structure, although it may be an element of an actual building structure. By way of example, the element 10 will be considered as an element of the model 11, built to the same relative stiffness as a building structure in which the corresponding element is a monolithic floor slab.

As suggested in Fig. 1, the element 10 is a flat element having a front reflective or mirrorized surface 13 and a back surface 14. The element 10 is supported at its opposite sides by side walls 15 and 16 and at its ends by end walls 17 and 18, corresponding respectively to side walls and end walls of a rectangular building structure. The walls 15-18 are respectively secured to and supported by a thicker foundation wall 20, corresponding to the foundation of the building structure. For purpose of illustration, a single column 23 extends between and is secured to the surface 14 of the element 10 and the foundation wall 20.

The model 11 is constructed of structural elements of the same relative stiffness as the building structure. It is also a model of the building structure in the sense of being proportional in shape and placement of supports and loads. In an actual situation, the supports may include a large number of irregularly positioned columns, beams, etc., placed in a model or building or other structure of regular or irregular shape.

The model 11 may be made of any suitable material but the element 10 should be of a material which can be made reflective so as to present a smooth reflective surface hereinbefore designated as the reflective surface 13. While metals can be used for the element 10, they are not the best materials, particularly if they present somewhat porous surfaces. Glass can be used but the most convenient and satisfactory materials are the so-called plastic materials. The material known as "Lucite," for example, is excellently suited, being of low porosity and being capable of being highly polished to receive a vapor-deposited coating to form the reflective surface 13. The element 10 of the model 11 should preferably be of a material having the following characteristics: (1) It should have a relatively high ultimate tensile strength as compared with its Young's modulus. (2) It should be capable of being adequately polished to be made effective. (3) It should be capable of being joined into a monolithic structure. (4) Its stress-strain curve should be close to a straight line within the range of working stresses. (5) It should have a reasonably flat optical surface in the as-purchased condition or be capable of optical finishing. (6) It should be reasonably stable as concerns shrinkage, plastic flow and temperature effects.

The walls 15-18 and 20 may be formed of the same material as the element 10, and the same may be true as to the column 23 illustrated. In practice, the complete model 11 may be made

from sheets or rods of transparent plastic material adhered together by a suitable solvent or adhesive to provide a hollow model having an inner space 24 bounded and closed by the surface 14 of the element 10 and by the inner surfaces of the walls 15-18 and 20.

If a uniform transverse load is to be applied to the element 10, it is one of the desirable features of the invention that the model 11 or the inner space 24 should be made air tight so that a pressure difference can be established inside and outside the model. This is a simple and extremely accurate way of applying a uniform load, although it should be clear that the invention is not limited thereto and that uniform loads can be applied to the element 10 by other means, such as by partially filling the inner space 24 with shot or liquid and permitting the weight thereof to deform the element 10 between its supports; also, air-tight bags may be inserted in such a manner as to give a uniform load over only part of the element 10.

As shown in Fig. 1, a difference in pressure can be established inside and outside the model by forcing air into or evacuating air from the inner space 24 through a pipe 25, the pressure difference between the interior and exterior of the model being shown by a manometer 27. In practice, an internal pressure of a few inches of water can be maintained inside the model to cause the element 10 to bow outwardly between its supports. The resulting distorted reflected optical pattern can be used to determine the curvature of or stress in the element 10 just as effectively as if the inner space 24 is at subatmospheric pressure as the degree and location of the flexure of the element 10 will be the same, except reversed, in the two instances.

In the simplest practice of the invention, the model 11 is placed in front of a source of an optical pattern shown as a patterned screen 30 having an opening 31 therein. The screen 30 is shown as having a front surface 32 facing the model 11 and carrying a pattern 33 of known design. In the preferred practice of the invention, the pattern is a regular arrangement of parallel lines, usually with crossing lines to form a grid, as suggested in Fig. 3. Use of the invention is made simpler if the longitudinal and transverse lines are equally spaced. However, any optical pattern of known design, preferably a two-dimensional pattern, can be transmitted to the reflective surface 13 without departing from the spirit of the invention. This pattern may even be irregular or it may consist merely of a large number of dots or lines arranged in a known pattern. The use of a pattern involving straight lines is preferred and the use of the invention is facilitated if the direction of the lines corresponds to the plane or planes of the model 11 or the building structure in which the stresses are to be analyzed.

While a back-lighted translucent screen 30 can be used, I prefer to use an opaque screen which is front-lighted by a plurality of reflectors 35 carrying light sources and designed equally to illuminate the front surface 32 of the screen 30. In any event, an optical pattern of known design is transmitted to the reflective surface 13 and reflected therefrom as a reflected optical pattern which can be viewed through the opening 31 of the screen. In the preferred practice of the invention, the reflected optical pattern is recorded photographically and for this purpose, I prefer to dispose a camera 40 to the

5

rear of the screen 30 and having a lens 41 receiving the reflected optical pattern to focus same on a photographic film or plate 42 at the rear of the camera. In other instances, a lens 41 may be mounted in the opening 31, the screen 30 forming one wall of a dark room and focusing the image on a photo-sensitive or other surface of the dark room on the opposite side from the screen 30. Other systems, sometimes employing auxiliary mirrors, can be used if it is not desired that the opening 31 be employed in the screen to receive the reflected image.

If the reflective surface 13 is plane and if the element 10 is not loaded, it will be apparent that an optical pattern of known design will be transmitted to the reflective surface 13 and will be reflected therefrom as an optical pattern of the same design, this pattern being accurately recorded by the camera 40. If the reflective surface 13 has surface irregularities, the optical pattern reflected from the unloaded element 10 will deviate from the transmitted pattern in minute degree, depending upon the surface irregularities. Such a deviating pattern will be recorded by the camera 40 and the resulting photographic image or an enlargement thereof can be used for purpose of comparison with the load-distorted pattern later recorded.

If the element 10 is loaded, as by applying a superatmospheric pressure inside the model, the resulting curvature of the reflective surface 13 will distort the reflected optical pattern, the distorted pattern being recorded by the camera 40. By comparing the distorted photographic image, further enlarged if desired, with the transmitted pattern or the reflected no-load pattern, the degree of flexure of the element 10 in any particular zone or area can be observed and the stresses or bending moments in such zone determined.

By way of example, if the transmitted optical pattern represents a grid of two series of equally-spaced lines as suggested in Fig. 3 (the actual number of lines being much greater than that exemplified), the reflected distorted optical pattern will show curved or differently spaced lines in the stressed zones of the element 10 (the word "stressed" is usually herein used in a broad sense as including stress or strain forces). In any particular zone, measurement of the difference in spacing as between the lines of the transmitted and reflected patterns will give a basis for calculation of the actual bending moment in this zone. By determining the bending moments throughout the element 10 or along selected lines therein, the stress pattern of any portion or all of the element can be plotted. From this, one can determine, in the case of reinforced concrete, for example, the reinforcing required to resist the bending stresses and can design the number, placement, length and shape of the reinforcing rods or members required for adequate support of the transverse loading.

For example, the flexure of the element 10 shown in Fig. 1 and when an internal superatmospheric pressure is applied to the model 11, will produce a distorted optical pattern of the type suggested very diagrammatically in Fig. 4. In the zones of highest stress, the parallel lines of the transmitted optical pattern will show the greatest deviation in spacing, and measurement of this deviation along any selected line A-A, as compared with the lines of the transmitted image, will permit calculation of the actual bending moments along this line. The

6

bending moment at any position or zone can be determined from the following formulae:

$$\frac{1}{\rho_x} = C \left(1 - \frac{J_x}{K_x} \right)$$

and

$$M_y = \frac{EI}{1 - \mu^2} \left(\frac{1}{\rho_x} + \mu \frac{1}{\rho_y} \right)$$

where ρ_x is the radius of curvature in the x direction and ρ_y in the y direction; C is a constant depending on the geometry of the equipment; J_x is the distance in the x direction between grid lines on the photographic image in the zone in question when the element is not stressed; K_x is the distance in the x direction between such lines when the element is stressed; M_y is the bending moment in the y direction; μ is Poissons ratio; E is the Young's modulus; and I is the moment of inertia. The ratio

$$\frac{J_x}{K_x}$$

is thus a measure of the extent and direction of the bending moment. When this ratio is greater than one a negative moment is present and when less than one a positive moment is present. Stated in other words and with an internally-pressured model, as the lines become farther apart in the strained condition a negative moment is indicated and as the lines become closer together a positive moment is indicated, the opposite being true if the element of the model is stressed by a reduction in internal pressure. At a zone where the above ratio is unity, i. e., with no image distortion, a condition of zero moment is indicated.

If an extremely accurate grid of lines is present on the screen 30 and the unstressed reflective surface 13 is plane, the photographic image of the distorted pattern can be correspondingly enlarged and the line spacings measured along a line of the model or structure where the moment measurement is to be taken. If the grid of lines is less accurate or if there are small irregularities in the unstressed reflective surface 13, a photographic image of the pressurally-undistorted or no-load reflected image can be made, enlarged and compared, along any line where moments are to be determined, with the similarly enlarged photographic image showing the lines as spaced when the element 10 is under load. This can be done by means of a comparison microscope.

In either event, the calculated bending moments along any such line can be plotted, with positive moments above the X-axis and negative moments below such axis, to produce an initial bending moment diagram which may be sufficiently accurate for some design purposes. If desired, a similar bending moment diagram can be plotted from the unstrained image and employed to correct the initial bending moment diagram.

In determining the amount and placement of steel reinforcing needed in a reinforced concrete slab, represented by the element 10 of the model, such moment curves are of great value. The steel in the top and bottom of the slab, required to resist negative and positive bending moments, respectively, will be shown by the portions of the curve below and above the X-axis thereof. Alternatively, a contour-like plot of bending moments can be made and used for

design of the amount, placement and length of the reinforcing members.

While the invention has been described with reference to slab design, it is also applicable to the stress analysis in roofs, flat or pitched, or other flat elements and can sometimes be applied to other elements of a non-flat nature. Likewise, while a particular means has been described for creating and photographing the optical pattern, it should be clear that the optical pattern of known design can be transmitted to or received by the reflective surface from various sources and in various ways and that the reflected pattern or its pattern can be viewed, photographed or compared with the pressurally-undistorted pattern in various ways without departing from the spirit of the invention.

I claim as my invention:

1. A method of analyzing the bending stresses in a substantially flat element of a structure, said substantially flat element being supported at predetermined points and being subject to bending due to the application of transverse loading, which method includes the steps of: building a model of said structure, said model having a correspondingly-shaped, correspondingly supported substantially flat element having an exposed substantially flat surface; making said exposed substantially flat surface reflective; transmitting to said reflective surface to be reflected therefrom an optical pattern of predetermined design, the reflected optical pattern deviating from the transmitted optical pattern upon deformation of said reflective surface; applying a transverse load to said substantially flat element of said model perpendicular to and to deform said reflective surface in degree and location determined by its supports and by such transverse load; and measuring the deviations of the resulting reflected optical patterns as a measure of the deformation of and the resulting bending stresses in said substantially flat element of said structure when subjected to a corresponding transverse load.

2. A method as defined in claim 1, in which said transverse load applied to said substantially flat element of said model is produced by developing a pressure differential on opposite sides of said substantially flat element of said model.

3. A method as defined in claim 1, in which said model is hollow and includes an inner space closed by one surface of said substantially flat element of said model, the other surface of such substantially flat element being said reflective surface, and in which said transverse load is applied to said substantially flat element of said model by developing a pressure differential between said surfaces of said substantially flat element of said model.

4. A method as defined in claim 3, in which said inner space is substantially air-tight and said reflective surface is exposed to atmospheric pressure, and in which said pressure differential is developed by changing the pressure in said inner space relative to atmospheric pressure.

5. A method of determining the distribution and magnitude of bending stresses in a structure, comprising the steps of: providing a test member with a specular reflective surface of substantial area, supporting spaced portions of said substantial area of said test member against movement in a direction perpendicular to said surface, loading said test member by applying pressure thereto in a direction perpendicular to said surface and between said supported portions to distort said

reflective surface, reflecting a single unitary image of a fixed pattern of geometrically related spaced indicia from said substantial area of said distorted surface, and measuring the relative displacement of a plurality of said indicia in said reflected image as an indication of the distribution and magnitude of the stresses produced in corresponding portions of said substantial area by said loading.

6. A method as defined in claim 5 including the step of reflecting a first image of said pattern from said surface before applying pressure thereto to establish a subjective datum image, and wherein said measuring step comprises measuring the differences in spacing between corresponding indicia on said datum image and the image reflected from said distorted surface as a measure of the distortions in corresponding portions of said surface caused by said loading.

7. A method as defined in claim 5 including the step of photographically recording said reflected image and wherein said measuring step comprises determining the difference in spacing between corresponding indicia on said recorded image and said fixed pattern.

8. A method of determining the distribution and magnitude of bending stresses in a structure, comprising the steps of: providing a test member with a specular reflective surface of substantial area, supporting spaced portions of said substantial area of said test member against movement in a direction perpendicular to said surface, reflecting a single unitary image of a fixed pattern of geometrically related spaced indicia from said surface, making a first permanent datum record of said reflected image, loading said test member by applying pressure thereto in a direction perpendicular to said surface and between said supported portions to distort said reflective surface, reflecting a second unitary image of said pattern from said distorted surface, making a second permanent record of said second image, and determining the difference in spacing between a plurality of corresponding indicia on said first and second records as an indication of the distribution and magnitude of stresses in that area of said test member from which said indicia were reflected.

9. In a system for determining the distribution and magnitude of bending stresses in a structure, the arrangement comprising: a member having a specular reflective surface of substantial area, spaced supports engaging said member and holding spaced portions of said area against movement in a direction perpendicular to said reflective surface, means defining a fixed pattern of geometrically related spaced indicia, an image receiving means fixedly supported in position to receive an image reflected from said surface, means holding said member and supports in position to reflect a unitary image of said pattern from said reflective surface to said image receiving means, and loading means arranged to apply a loading force to said member in a direction perpendicular to said reflective surface between said supports to distort said surface and thereby cause reflection of a distorted image of said pattern to said image receiving means.

10. Apparatus as defined in claim 9, wherein said means defining said pattern comprises a patterned screen substantially parallel to said reflective surface, said screen having an opening therethrough, said image receiving means being positioned to receive a reflected image of said pattern through said opening.

11. Apparatus as defined in claim 9, wherein said member comprises a wall of a hollow body and wherein said loading means comprises means for producing different fluid pressures within said hollow body.

Number
1,590,532
2,014,688
2,380,501
5 2,459,418

OLIVER GERALD BOWEN.

References Cited in the file of this patent

UNITED STATES PATENTS

Number	Name	Date
1,457,209	Chanier	May 29, 1923
1,552,450	Roach	Sept. 8, 1925

Name	Date
Lenouvel	June 29, 1926
Maddoux	Sept. 17, 1935
Christian et al.	July 31, 1945
Ellis	Jan. 18, 1949

FOREIGN PATENTS

Number	Country	Date
573,656	Germany	Apr. 5, 1933
10 816,087	France	Apr. 26, 1937

OTHER REFERENCES

Scientific American, October 1946, page 178.
(Copy in U. S. Patent Office Library.)