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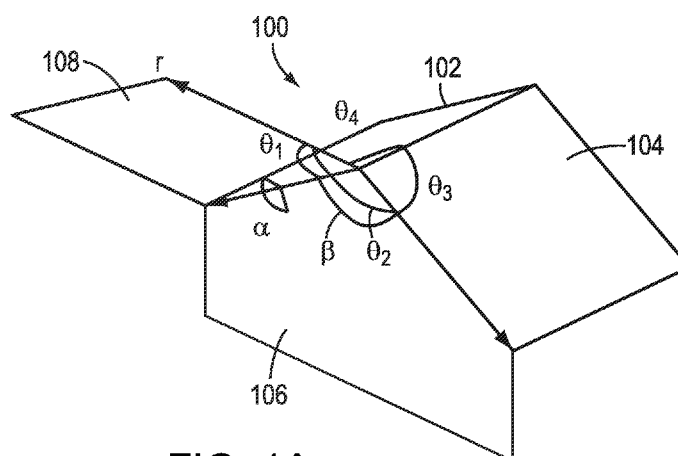


FIG. 1A

(57) Abstract: Developable quad surfaces are designed and constructed incrementally by examining and conforming to the conditions for developable growth at the boundary of an existing developable quad surface. The full design space may be explored at a boundary of an existing surface, individually computed new strips - chosen from the design space - are incrementally accumulated of into a larger developable surface.



**ADDITIVE DESIGN AND CONSTRUCTION
OF DEVELOPABLE QUADRILATERAL SURFACES**

CROSS-REFERENCE TO RELATED APPLICATION

[0001] This application claims priority to, and the benefit of, U.S. Provisional Patent Application No. 62/491,494, filed on April 28, 2017, the entire disclosure of which is hereby incorporated by reference.

FIELD OF THE INVENTION

[0002] Embodiments of the present invention generally relate to foldable structures, and in particular to the design and fabrication of structures involving developable quadrilateral (“quad”) surfaces.

BACKGROUND

[0003] Developable surfaces are surfaces that can be mapped locally to a planar domain without stretching or tearing. They represent the shapes obtainable with thin materials such as sheet metal or paper, which do not stretch. A developable surface can be constructed from a planar surface, and vice versa, without any creases or distortion. As a result, developables are relevant to the manufacturing of ship hulls, clothing, and sophisticated architectural designs such as those of Frank Gehry.

[0004] Despite their obvious importance, developable surfaces are difficult to generate with present-day CAD systems, and automated geometric modeling with developables is generally not available. Current approaches to design tend to assume some 3D topological constraints and rely on simple geometric methods, resulting in limited design freedom and restricted ability to design large, complex surfaces all at once.

SUMMARY

[0005] Embodiments of the present invention assess and exploit the possibilities for the incremental design of developable quad surfaces by examining the conditions for developable growth (i.e., growth that maintains the developable property of the surface) at the boundary of an existing developable quad surface. In contrast to prior approaches, embodiments of the invention allow for the exploration of the full design space possible at a boundary of an existing surface, and the incremental accumulation of individually computed new strips – chosen from the design

space – into a larger developable surface. Any regular developable quad surface (of which quad origami designs are a subset) can be designed in accordance herewith, and the incremental nature of the procedure affords convenient exploration of alternatives during the design process. In some embodiments, Miura-Ori-like origami, corresponding to a smoothly varying tessellation of unit cells, each of which is composed of a regular (e.g., 2×2) or irregular grid of quadrilaterals, is employed to closely approximate a curved surface by one that can be easily collapsed and deployed. The unit cells do not necessarily perfectly repeat, but do vary smoothly across the origami pattern.

[0006] Accordingly, in one aspect, the invention pertains to a method of fabricating a collapsible surface conforming to a target surface. In various embodiments, the method comprises the steps of providing a computational representation of the target surface; computationally generating a source shape approximating, with folds, a portion of the target surface, the source shape comprising a plurality of adjacent regular developable quadrilateral surfaces having a common boundary; extending the source shape from the common boundary by adding a quadrilateral strip to an edge of a first one of the surfaces along the common boundary in accordance with a design, wherein (i) the edge of the first surface terminates in at least one vertex shared with an adjacent second one of the surfaces, the first and second surfaces being adjacent along a ridge line extending from the vertex, (ii) first and second design angles θ_1, θ_2 between an edge of the quadrilateral strip projecting from the vertex and, respectively, first and second other edges of the vertex obey the relationship $\cos\theta_2 = \cos\theta_1 \cos\beta + \sin\theta_1 \sin\beta \cos\alpha$, where α is a flap angle between the quadrilateral strip and a plane passing through the vertex and perpendicular to the first and second other edges of the vertex and β is an angle between the two other edges, and (iii) $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 2\pi$, where θ_3, θ_4 are angles between the ridge line and, respectively, the first and second other edges of the vertex; repeating the extending step at least once; and fabricating the collapsible surface by (i) fabricating the quadrilateral surface and strips from a solid material and (ii) assembling the quadrilateral surface and strips into the extended source shape by foldably connecting them in accordance with the design.

[0007] In some embodiments, $\theta_1 + \theta_2(\theta_1) = k$, where $\theta_2(\theta_1) = \cos^{-1}(\cos\theta_1 \cos\beta + \sin\theta_1 \sin\beta \cos\alpha)$ and $k \in [\beta, 2\pi - \beta]$. Repeating the extending step may further comprise, for a new quadrilateral strip, selecting a new flap angle α for the new quadrilateral strip at a free edge of the extended shape, and constraining θ_1, θ_2 and lengths of all edges of the new quadrilateral strip to avoid intersection between any of the edges of the new quadrilateral strip any portion of the shape.

[0008] In some embodiments, the new quadrilateral strip has at least one free edge and at least one bound edge, and each bound edge is parallel to a bound edge of an adjacent quadrilateral strip. In general, angles around each interior vertex of the design sum to 2π . The fabrication step may comprise constructing the quadrilateral surface and strips from a solid material in a single large piece. Repeating the extending step at least once may produce Miura-Ori-like origami design conforming to the surface.

[0009] In some embodiments, the collapsible surface is flat-foldable without fracture. The design may have a plurality of nodes and may be flat-foldable at each of the nodes. The fabrication step may comprise constructing the design from a solid material in a single large piece.

[0010] As used herein, the terms “approximately” and “substantially” mean $\pm 10\%$, and in some embodiments, $\pm 5\%$. Reference throughout this specification to “one example,” “an example,” “one embodiment,” or “an embodiment” means that a particular feature, structure, or characteristic described in connection with the example is included in at least one example of the present technology. Thus, the occurrences of the phrases “in one example,” “in an example,” “one embodiment,” or “an embodiment” in various places throughout this specification are not necessarily all referring to the same example. Furthermore, the particular features, structures, routines, steps, or characteristics may be combined in any suitable manner in one or more examples of the technology. The headings provided herein are for convenience only and are not intended to limit or interpret the scope or meaning of the claimed technology.

BRIEF DESCRIPTION OF THE DRAWINGS

[0011] In the drawings, like reference characters generally refer to the same parts throughout the different views. Also, the drawings are not necessarily to scale, with an emphasis instead generally being placed upon illustrating the principles of the invention. In the following description, various embodiments of the present invention are described with reference to the following drawings, in which:

[0012] FIGS. 1A and 1B schematically illustrate developability and planarity of construction constraints in a single-vertex origami of degree four.

[0013] FIGS. 2A and 2B illustrate bounding of the sum of two angles in 3D and the projection of design angles onto the β plane in embodiments of the invention.

[0014] FIGS. 3A and 3B illustrate addition of a quadrilateral strip to a folded structure in accordance with embodiments of the invention.

[0015] FIG. 4 illustrates allowable values for the flap angle of an added quadrilateral strip in accordance with embodiments of the invention.

[0016] FIG. 5 illustrates surface fitting in accordance with embodiments of the invention.

DETAILED DESCRIPTION

[0017] Refer first to FIG. 1A, which illustrates a single-vertex origami 100 of degree four. A single-vertex origami refers to a plane with straight-line rays called creases emanating from a fold vertex located in the interior or on the boundary of the plane. The structure 100 exists initially as a pair of quad surfaces 102, 104 and is defined by two pre-existing design angles θ_3 and θ_4 and two new design angles θ_1 and θ_2 incident to the vertex. Growth is desired along the vector r ; the angle between edges at the growth boundary is $\beta \in [0, \pi]$ and the oriented angle between the β plane 106 and the new face 108 along the vector r , which contains θ_1 , is the “flap angle” $\alpha \in [0, 2\pi)$. The new design angles θ_1 and θ_2 at the vertex must satisfy two equations:

$$\sum_i^4 \theta_i = 2\pi \quad \text{Eq. (1)}$$

$$\cos\theta_2 = \cos\theta_2 \cos\beta + \sin\theta_1 \sin\beta \cos\alpha \quad \text{Eq. (2).}$$

[0018] Eq. 1 guarantees the developability of the interior angles around the new interior node and Eq. 2 expresses compatibility between θ_1 , θ_2 , α , and β . As illustrated in FIG. 1B, solutions to Eq. 1 form curves specified by $\gamma(\beta, k)$, where $k = 2\pi - \theta_3 - \theta_4$. These curves are ellipses of spherical arcs and thus represent directions of r that use the exact amount of material required at the vertex to satisfy the developability criterion. The resulting simple loops enclose the lines containin the two growth front edges at the vertex, and sweeping α through $[0, 2\pi)$ uniquely selects a point on a given γ curve. Eq. 2 is the law of cosines from spherical trigonometry, with θ_1 , θ_2 , and β forming a spherical triangle with α opposite θ_1 .

[0019] The existing values of θ_3 and θ_4 determine the value of $\theta_1 + \theta_2 = 2\pi - (\theta_3 + \theta_4) = k$ by Eq. 1, hence

$$f(\theta_1) = \theta_1 + \theta_2(\theta_1) = k \quad \text{Eq. (3).}$$

where $\theta_2(\theta_1) = \cos^{-1}(\cos\theta_1 \cos\beta + \sin\theta_1 \sin\beta \cos\alpha)$. Note that $f(0) = \beta$, $f(\pi) = 2\pi - \beta$, and $f(\theta_1)$ is positive monotonic on $\theta_1 \in [0, \pi]$. Intuitively, $\theta_2(\theta_1)$ always decreases at a smaller rate than the rate of increase of θ_1 , except in the singular cases $\alpha = 0, \pi$ wherein the rates are equal and opposite, and $f(\theta_1)$ is constant. This implies that $f(\theta_1)$ is bounded by $[\beta, 2\pi - \beta]$ and positive monotonic for $\theta_1 \in [0, \pi]$. On the right-hand side of Eq. 3, it may be observed that $k \in [\beta, 2\pi - \beta]$. These constraints guarantee the existence and uniqueness of solutions to Eq. 3, and thereby to Eqs. 1 and 2.

[0020] With reference to FIGS. 2A and 2B, consider a spherical triangle consisting of the angles β , θ_3 and θ_4 . Let the angles θ_3' and θ_4' be the angles of the projections of θ_3 and θ_4 , respectively, onto the β plane. Let φ be the angle between the β plane and the edge incident to θ_3 and θ_4 . Now consider the sum $\theta_3 + \theta_4$ for all values of φ . At $\varphi = \pi$, $\theta_3 = \theta_4 = \pi/2$ and thus $\theta_3 + \theta_4 = \pi \in [\beta, 2\pi - \beta] \forall \beta(0, \pi)$. Also, at $\varphi = 0$, there are two possibilities: $\theta_3 + \theta_4 = \beta$ (the "inside" case) and $\theta_3 + \theta_4 = 2\pi - \beta$ (the "outside" case). By projecting $\theta_3 + \theta_4$ onto the β plane, we can construct two right spherical triangles, so that

$$\cos\theta_3 = \cos\theta_3' \cos\varphi + \sin\theta_3' \sin\varphi \cos\frac{\pi}{2} = \cos\theta_3' \cos\varphi$$

$$\cos\theta_4 = \cos\theta_4' \cos\varphi + \sin\theta_4' \sin\varphi \cos\frac{\pi}{2} = \cos\theta_4' \cos\varphi$$

and

$$\theta_3 + \theta_4 = \cos^{-1}(\cos\theta_3' \cos\varphi) + \cos^{-1}(\cos\theta_4' \cos\varphi)$$

[0021] $\theta_3 + \theta_4$ monotonically approaches π as σ goes from zero to $\frac{\pi}{2}$. The sign of $d/d\varphi(\theta_3 + \theta_4)$ is determined by which projected angle θ_3 or θ_4 deviates most from $\frac{\pi}{2}$ (i.e., if $|\frac{\pi}{2} - \theta_3'| > |\frac{\pi}{2} - \theta_4'|$, then $\text{sgn}[d/d\varphi(\theta_3 + \theta_4)] = \text{sgn}[\frac{\pi}{2} - \theta_3']$). This follows from the negative monotonicity and symmetry of inverse cosine about $\frac{\pi}{2}$. So $\theta_3 + \theta_4$ begins at an endpoint of the interval $[\beta, 2\pi - \beta]$ when $\varphi = 0$ and approaches π monotonically within the same interval as $\varphi \rightarrow 0$.

[0022] $f(\theta_1) = \theta_1 + \theta_2(\theta_1) = k$ is positive monotonic on the interval $\theta_1 \in [\beta, 2\pi - \beta]$.

This amounts to showing that $|d\theta_2(\theta_1)/d\theta_1| \leq 1$ or equivalently, $(d\theta_2(\theta_1)/d\theta_1)^2 \leq 1$. Recalling that $\theta_2(\theta_1) = \cos^{-1}(\cos\theta_1 \cos\beta + \sin\theta_1 \sin\beta \cos\alpha)$, it can be shown that

$$\begin{aligned} \frac{d\theta_2}{d\theta_1} &= \frac{\sin \theta_1 \cos \beta - \cos \theta_1 \sin \beta \cos \alpha}{\sqrt{1 - (\cos \theta_1 \cos \beta + \sin \theta_1 \sin \beta \cos \alpha)^2}} \\ \left(\frac{d\theta_2}{d\theta_1} \right)^2 &= \frac{(\sin \theta_1 \cos \beta - \cos \theta_1 \sin \beta \cos \alpha)^2}{1 - (\cos \theta_1 \cos \beta + \sin \theta_1 \sin \beta \cos \alpha)^2} \\ &= \frac{\cos^2 \beta + \sin^2 \beta \cos^2 \alpha - K^2}{1 - K^2} \leq 1 \end{aligned}$$

where $K = \cos \theta_1 \cos \beta + \sin \theta_1 \sin \beta \cos \alpha$ and equality occurs only for singular configurations. Thus $f(\theta_1)$ is positive monotonic on $\theta_1 \in [\beta, 2\pi - \beta]$ and solutions $f(\theta_1) = k$ are unique for all values $\alpha \neq \{0, \pi, 2\pi\}$ and $\beta \neq \{0, \pi\}$.

[0023] Showing the existence and uniqueness of solutions to the above system is tantamount to demonstrating that a new quad strip can always be added to a boundary of an existing surface.

[0024] Now consider a slightly more complex initial system: two incomplete single origami vertices along a boundary given by the intersections of three faces (i.e. the first and second faces intersect to give the first vertex, and the second and third faces intersect to give the second vertex, so both vertices are incident to the second face). We would like to make new origami vertices at both of these sites and expect that our design of the first vertex will influence that of the second. We can make two observations about single vertex systems that aid in understanding the design of adjacent vertices.

[0025] 1. Taken alone, solutions of Eq. 1 form an ellipse of spherical arcs with foci determined by intersections of the existing boundaries with a unit sphere centered at the vertex. Let this ellipse be called γ .

[0026] 2. Introducing Eq. 2 selects a unique point along γ , so α on the interval $[0, 2\pi)$ is bijective with the new design angles and solutions are symmetric about $\alpha = \pi$.

[0027] Now consider the single vertex p^i in relation to an adjacent vertex p^{i+1} along the growth front, ordered such that θ_2^i and θ_1^{i+1} belong to the same putative face in a new quad strip. Observing a convention that superscripted i indexes quantities along the growth strip and subscripted j indexes quantities around a single vertex, we denote design angles around the first vertex as θ_j^i and design angles around the second vertex as θ_j^{i+1} . Having designed θ_1^i and θ_2^i , the θ_1^{i+1} plane is determined and α^{i+1} of p_{i+1} is thus determined by the choice of α^i at p_i . If we denote this geometric transfer from the first flap angle to the second as the function $f^i(\alpha^i) = \alpha^{i+1}$, it can be seen that f^i is bijective with $\alpha^i, \alpha^{i+1} \in [0, 2\pi)$. Thus, choosing a unique flap angle at one vertex yields a unique flap angle at an adjacent vertex, and all possible values of flap

angles at the adjacent vertex are accessible by tuning the flap angle at the first. f^i can be written as an explicit function by labeling several angles in a system of adjacent vertices. Let α_1^i be the oriented flap angle between the β^i and θ_2^i planes, applying the spherical law of sines to the spherical triangle $(\beta^i, \theta_1^i, \theta_2^i)$ gives α_2^i .

$$\alpha_2^i = \sin^{-1} \left(\frac{\sin \theta_1^i}{\sin \theta_2^i} \sin \alpha_1^i \right)$$

[0028] Although α_2^i and α_1^{i+1} are measured relative to different planes β_1^i and β_1^{i+1} , these planes both contain a common edge along the growth front. So α_2^i and α_1^{i+1} are measured about a common axis and are thus related by a constant shift of the oriented angle between the β_1^i and β_1^{i+1} planes, which are denoted as φ^i . Then,

$$\alpha_1^{i+1} = \alpha_2^i - \varphi^i$$

[0029] This provides an explicit form of f^i , which describes how the choice of flap angle DOF at vertex i is transferred to the flap angle at vertex $i + 1$.

[0030] A suitable growth algorithm follows. Given an existing developable quad mesh with m quads along a boundary, m new quads can be added the boundary accordingly.

1. Choose the flap angle α at one new quad location
2. Given the single flap angle choice, solve all single-vertex systems from p^i to p^{m-1} to give all new interior design angles θ_1^i and θ_2^i and flap angles α_1^i for $i \in \{1, m - 1\}$
3. Use the solved design and flap angles to rotate the new compute interior edge directions along strip $(r_1$ through $r_{m-1})$ into place
4. Choose the design angle at each endpoint of the new strip and use it to compute r_0 and r_m
5. Compute any bounds on the new edge lengths by checking for edge intersections
6. Choose new edge lengths within intersection bounds

[0031] The foregoing provides a convenient parametrization the available degree of freedom (DOF) count obtained by observing that new strip of m quads has $3(m + 1)$ DOFs subject to m

planarity (Eq. 1) and $m - 1$ design angle (Eq. 2) constraints. Adding a new strip to a boundary with m quads produces a total of $m + 4$ DOFs to determine the geometry of the new strip: one flap angle to determine the interior design angles and edge directions ($\alpha \in [0, 2\pi]$), two boundary design angles at the endpoints of the strip ($\theta \in [0, \pi]$), and $m + 1$ edge lengths. First, choosing α at a given quad location sets up dependent instances of Eq. 3, which can be solved consecutively at each interior node along the new strip. Solving Eq. 3 at p_1 gives $\theta_{i,3}$ and $\theta_{i,4}$, which can be used to place the new edge direction of r_i . The new edge direction of r_i then determines α_{i+1} , thereby setting up another instance of Eq. 3, and so forth. This process is guaranteed not to fail by virtue of the single vertex analysis: no matter what flap angle a vertex along the strip inherits from a choice of α at a preceding location, a unique solution to Eqs. 1 and 2 will exist at that vertex. Second, the design angles at the ends of the strip can be chosen freely on the interval $(0, \pi)$. Finally, having chosen all new edge directions, we are left to choose the new edge lengths. These new lengths are bounded only by the observation that the new outward facing edges in each new quad cannot intersect each other, which occurs when the two interior angles of a new quad sum to less than π . So whereas previous origami design studies have relied on solving non-linear constraints indirectly by manipulating vertex positions, the present approach identifies design values directly, satisfying requisite constraints by construction, and then recovers the determined vertex positions by a simple layout routine in R^2 and R^3 .

[0032] This is illustrated in FIG. 3A, which shows existing strips of quads Q_i indicated at 302 and a putative new quad strip 304 to be attached along the boundary, defined by the points p_i . At each new interior vertex p_i , $i \in \{1, 2, 3, 4, 5\}$ let $\beta = \cos^{-1}((p_{i-1} - p_i)/\|p_{i-1} - p_i\| \cdot (p_{i+1} - p_i)/\|p_{i+1} - p_i\|)$ be the angle in space between the two boundary edges incident to p_i . Let γ_i be the set of possible directions satisfying Eq. 1 for new edge direction r_i at p_i . Choosing the plane of any quad (i.e. choosing α) along the putative strip uniquely determines the r_i , $i \in \{1, 2, 3, 4, 5\}$. FIG. 3B shows the development of the same system to the plane, where $\theta_{i,j}$ is now the j^{th} design angle at p_i . The values of $\theta_{i,j}$, $i \in \{1, 2, 3, 4, 5\}$ and $j \in \{1, 2\}$ are determined by the choice of α above, while the values $\theta_{0,2}$ and $\theta_{6,1}$ are unconstrained at the endpoints of the putative strip. Dashed lines illustrate the new edge directions, whose potential intersection points can be computed as set forth above.

[0033] One question remains, which is whether choosing different locations for the flap angle will yield different spaces of possible new designs. If this were true, it would complicate the search at each additive design step, requiring the design algorithm to explore design spaces yielded by varying α at each new quad and to collate the results before choosing a new design.

Fortunately, this is not the case: all choices of quad locations for the flap angle are equivalent, so exploring α values at any location along the growth front explores the full space of possible designs at that front.

[0034] Let α_i be the flap angle at the chosen location along strip (dihedral angle between θ_1 face and β face), α'_i be the exterior dihedral angle between the θ_2 face and the β face and α_{i+1} be the flap angle at next quad, as shown in FIG. 4. The solution curve forms an ellipse of arcs on the sphere ($\theta_1 + \theta_2 = k$), values of which are unique for a choice of α_i . The foci of this ellipse are given by the edges that define the β face, i.e. the boundary edges of the current growth front. Now it can be observed that α_i and α'_i are the counter-clockwise angles about the foci between the β face and their respective new quad faces (θ_1 face and θ_2 face). Any choice of α_i yields a unique α'_i , which is related to α_{i+1} by a ± 1 prefactor and a constant phase shift given by the dihedral angle between the current and next β planes. Thus sweeping α_i through the interval $[0, 2\pi]$ also sweeps α_{i+1} uniquely through the same interval. This establishes a bijective mapping between two consecutive flap angles, which in turn determines unique interior angles along the entire strip geometry. So all choices of which quad along the strip to pick as the flap angle location are arbitrary: any sweeping of the flap angle at any quad location along the strip will explore the full space of possible new interior strip geometries uniquely.

[0035] As noted above, edge lengths are bounded primarily by the need to avoid intersection with other portions of the surface. For each new quad M with interior angles θ_2^i and θ_1^{i+1} , if these angles sum to less than π they will intersect at some distance in the plane of M . These intersection points can be detected by parameterizing the two new ray directions and minimizing the distance between points on each ray. Let the new unit ray directions at points p_i and p_{i+1} be $\hat{r}_i$ and $\hat{r}_{i+1}$, respectively. The new rays can be parameterized as $x_i(t) = p_i + t\hat{r}_i$ and $x_{i+1}(s) = p_{i+1} + s\hat{r}_{i+1}$. The distance D between any two points on the rays is

$$D(t, s) = \|x_i(t) - x_{i+1}(s)\|.$$

[0036] This formulation is valid for skew rays in R^3 , but because $\hat{r}_i$ and $\hat{r}_{i+1}$ are coplanar if $\theta_2^i + \theta_1^{i+1} < \pi$, it is expected that t and s will have values such that $D(t, s) = 0$. For simplicity, this can be accomplished by minimizing D^2 instead:

$$\nabla D^2(t, s) = \left\langle \frac{\partial}{\partial t}, \frac{\partial}{\partial s} \right\rangle D^2 = 0,$$

producing a system of two linear equations in t and s , which can be inverted analytically.

$$\frac{l}{1-(\hat{r}_i \cdot \hat{r}_{i+1})^2} \begin{bmatrix} 1 & \hat{r}_i \cdot \hat{r}_{i+1} \\ \hat{r}_i \cdot \hat{r}_{i+1} & 1 \end{bmatrix} \begin{bmatrix} (p_{i+1} - p_i) \cdot \hat{r}_i \\ (p_i - p_{i+1}) \cdot \hat{r}_i \end{bmatrix} = \begin{bmatrix} t \\ s \end{bmatrix}$$

[0037] Plugging the solved values of t and s back into $x_i(t)$ and back into $x_{i+1}(s)$ gives the point of intersection of the new ray directions in R^3 , which becomes an upper bound on the choice of new edge lengths. Each interior edge direction can be bounded by its intersection with two edge directions, r_{i-1} preceding and r_{i+1} following, so the final edge length bound L_i is chosen as the smaller of the two bounds given by the two potential intersections. A given edge direction does not necessarily intersect either of its neighbors, however, in which case its selection has no upper bound.

[0038] A numerical solver can implement the above computations to resolve single-vertex systems and generate a geometric layout that advances a system along the growth front. Choosing the DOFs available at each stage is highly application-dependent, and three different strategies are detailed below.

[0039] The foregoing growth algorithm offers the designer $m + 4$ opportunities to interact with the geometry of the newly added quad strip. Perhaps the simplest way to select the degrees of freedom in a new strip geometry is to sample them from a uniform probability distribution. For example, the flap angle could be chosen as $\alpha = \mathcal{U}(0, 2\pi)$, the boundary design angles as $\theta_{0,2} = \mathcal{U}(0, 2\pi)$ and $\theta_{m,1} = \mathcal{U}(0, \pi)$, and the edge lengths as $l_i = \mathcal{U}(0, l_i)$ (ignoring the possibility of $l_i = \infty$). Uniformly selected α values tend on average to produce extreme design values locally, however, and even regular local design angles can propagate to extreme design angles downstream on the new strip.

[0040] Another way to choose the DOFs is to define a cost function on the candidate strip geometry and then to minimize this function with respect to the DOFs, ultimately choosing the optimal value of each parameter. This approach is particularly well-suited for engineering applications as a flexible, modular-geometry cost function can optimize intrinsic geometric quantities connected to desirable surface properties (i.e., flat- and rigid-foldability) and can link the surface geometry to a variety of external controls such as a pair of sandwich-structure enveloping surfaces. The most critical parameter to optimize is α , which dictates the new edge directions along the entire growth front, followed by the edge lengths. A suitable cost function will seek new edge directions parallel to the existing edges on the surface, i.e., ruling

continuation. With existing edges in the surface incident to but not defining the boundary defined as $\bar{r}$, the ruling continuation cost function is given by

$$C(\alpha) = \sum_{i=1}^{m-1} 1 - \bar{r} \cdot r_i(\alpha)$$

which is expected to attain a single global minimum of zero on the interval $\alpha \in [0, 2\pi)$, corresponding to ruling continuation directions. Numerically, the global minimum can be found by an initial broad sampling of $C(\alpha)$ followed by standard gradient-based methods to produce an accurate optimal value of α . Because $r(\alpha)$ may be a complicated function that depends on the in-situ layout of edge directions for each choice of α , numerical estimates of $dC/d\alpha$ may be employed to optimize α , e.g., implemented using Matlab's **fmincon** function with the option to provide constraints on α values based on resultant strip geometry.

[0041] If selection of the DOF values is left to the designer, the flap angle, boundary design angles and edge directions may be chosen directly via a visual interface. The α and boundary design angles may, for example, be explored interactively via sliders while the edge lengths can be manipulated graphically one-by-one or as a group via a freeform spine to determine the boundary. Each new candidate DOF value initiates a re-computation and re-rendering of the strip geometry for visual consideration by the designer.

[0042] One application of the approach described herein is design and fabrication of origami tessellations. Origami is an art form that probably originated with the invention of paper in China, but was refined in Japan. The ability to create complex origami structures depends on folding thin sheets along creases, a natural consequence of the large-scale separation between the thickness and the size of the sheet. This allows origami patterns to be scaled; the same pattern can be used at an architectural level or at a nanometric level. Much of the complexity of the folding patterns arises from the possibilities associated with the basic origami fold — the unit cell associated with a four-coordinated mountain-valley structure that forms the heart of the simplest origami tessellation

[0043] The well-known periodic Miura-ori pattern is formed by tiling the plane with a unit cell of four parallelograms tiles and creasing along tile edges. The suitability of the Miura-ori for engineering deployable or foldable structures is due to its high degree of symmetry embodied in its periodicity, and four important geometric properties: it can be rigidly folded (that is, it can be continuously and isometrically deformed from its flat, planar state to a folded state); it has only one isometric degree of freedom, with the shape of the entire structure determined by the folding

angle of any single crease; it exhibits negative Poisson's ratio (folding the Miura-ori decreases its projected extent in both planar directions); and it is flat-foldable (that is, when the Miura-ori has been maximally folded along its one degree of freedom, all faces of the pattern are coplanar).

[0044] For example, Miura-ori and other origami designs may closely approximate a curved surface yet can be easily collapsed and deployed. The unit cells do not necessarily perfectly repeat, but do vary smoothly across the origami pattern. For generalized cylinders, constructed patterns are rigid-foldable and flat-foldable, and thus can be easily adapted to thick origami. For doubly curved surfaces, the approach described herein facilitates design of physically realizable tessellations. When the Miura-ori tessellations are not flat-foldable, a mechanical model of these surfaces may be used to quantify the strains and energetics associated with snap-through as the pattern moves from the flat to folded configuration. Flat-foldable structures that may be fabricated using the principles described herein include solar sails (i.e., arrays of photovoltaic material that may be launched in a compact form into space and there deployed into an expanded configuration with large surface area), shelters (such as tents), foldable batteries, antennas, containers and similar structures, "sandwich cores" (i.e., folded, curved origami-cores used to provide structural stability/rigidity to a surface structure with non-negligible thickness, where the folded origami connects two smooth surfaces — for example, in a helmet or an airfoil — or supports the weight of a single surface — for example, as an architectural facade, or forming the deployable/collapsible base of a "quarter-pipe" or "half-pipe" in extreme sports), "pop-up" cards based on origami (the origami-surface can be collapsed into the folded card, and deploys by unfolding into a target shape with the opening of the card), high-end decorative items (e.g., light fixtures, lamps, coffee-table supports, sculpture), decorative or functional packaging (e.g., providing an aesthetically distinctive, flexible protective shell that wraps around an item such as a wine bottle), and passive humidifiers.

[0045] The representation of a target surface is a regular, orientable quad mesh (all interior nodes have valence four and the normals of the quads are orientable), herein referred to as the "base mesh." The base mesh can be obtained by discretizing the two families of curves formed by a parameterization of the target surface and may form the basis for an initial structure guess provided to the expansion algorithm described above. The initial guess for the positions of all nodes in a Miura-ori structure may be obtained by populating each individual quad with nine nodes (four at corners, four at the edges and one central node); displacing the edge and central nodes to construct a Miura-ori unit cell guess at each quad according to chosen orientations and local length scales; and merging nodes at interior edges by averaging their positions. The

positions of the four undisplaced corner nodes in each “unit cell” are required to remain fixed throughout expansion. This ensures that the solved structure closely approximates the target surface and further flexibility in designing patterns.

[0046] The above-described approach can be used to fit tessellations to target surfaces. As illustrated in FIG. 5, given a smooth target surface, two surfaces displaced in the normal direction from the target surface (upper and lower) can be defined and a simple singly-corrugated strip defined running along the length of the middle of the target patch in between the upper and lower surfaces, with one side of the strip lying on the upper surface and one side on the lower surface. Then, applying the additive algorithm described above, strips are added to either side of the seed (and continuing on the growing patch) that effectively reflect the origami surface back and forth from upper to lower surfaces and induce an additional corrugation in a transverse direction to that of the corrugation in the seed. The optimal DOF framework may be employed to choose the flap angle value, with a cost function that avoids extreme design angles:

$$C(\alpha) = \sum_{i,j} \left(\theta_{i,j} - \frac{\pi}{2} \right)^2, i \in \{1, m-1\}, j \in \{1, 2\}$$

[0047] Boundary design angles are chosen such that the boundary rays are parallel to the next interior ray and edge lengths are chosen such that the next row of points lies exactly on the other corrugating surface, either lower or upper, via a numerical projection routine that computes intersections between rays and a surface. FIG. 5 illustrates a hypar pattern grown additively at a resolution (40 faces per strip) that would be unfeasible to compute in accordance with other inverse design techniques. Additionally, all nodes on the upper side of the origami surface fall exactly on the upper corrugating surface and all nodes on the lower side of the origami patch fall exactly on the lower corrugating surface; accordingly, this approach is well-suited to design origami sandwich structures that reside interstitially between smooth surfaces. The additive design approach facilitates seamlessly selection among tessellations, curved folds, crumpled patterns and singly-corrugated structures, largely ignoring qualitative and quantitative differences between these systems that are generally considered limiting assumptions in previous origami design studies.

[0048] Generating origami designs in accordance with the foregoing techniques may be achieved using a computer programmed to read a file describing a desired surface and to execute the iterative expansion steps set forth above for creating the origami design conforming approximately to the target surface. The resulting design may be physically fabricated in solid

form using any suitable material for the cells, and assembling them in accordance with the design. In some embodiments, the computer returns a computational (e.g., CAD) description of the cells, which may be fabricated using a 3D printer, laser cutter or other automated fabrication equipment capable of using the computer-generated design. The cells may be assembled into the finished structure manually or by automated means that include stamping or forging.

[0049] Design generation as described above may be implemented by computer-executable instructions, such as program modules, that are executed by a conventional computer. Generally, program modules include routines, programs, objects, components, data structures, etc. that performs particular tasks or implement particular abstract data types. Those skilled in the art will appreciate that the invention may be practiced with various computer system configurations, including multiprocessor systems, microprocessor-based or programmable consumer electronics, minicomputers, mainframe computers, and the like. The invention may also be practiced in distributed computing environments where tasks are performed by remote processing devices that are linked through a communications network. In a distributed computing environment, program modules may be located in both local and remote computer-storage media including memory storage devices.

[0050] The general-purpose computer may include a processing unit, a system memory, and a system bus that couples various system components including the system memory to the processing unit. Computers typically include a variety of computer-readable media that can form part of the system memory and be read by the processing unit. By way of example, and not limitation, computer readable media may comprise computer storage media and communication media. The system memory may include computer storage media in the form of volatile and/or nonvolatile memory such as read only memory (ROM) and random access memory (RAM). A basic input/output system (BIOS), containing the basic routines that help to transfer information between elements, such as during start-up, is typically stored in ROM. RAM typically contains data and/or program modules that are immediately accessible to and/or presently being operated on by processing unit. The data or program modules may include an operating system, application programs, other program modules, and program data. The operating system may be or include a variety of operating systems such as Microsoft WINDOWS operating system, the Unix operating system, the Linux operating system, the Xenix operating system, the IBM AIX operating system, the Hewlett Packard UX operating system, the Novell NETWARE operating system, the Sun Microsystems SOLARIS operating system, the OS/2 operating system, the BeOS

operating system, the MACINTOSH operating system, the APACHE operating system, an OPENSTEP operating system or another operating system of platform.

[0051] Any suitable programming language may be used to implement without undue experimentation the analytical functions described within. Illustratively, the programming language used may include assembly language, Ada, APL, Basic, C, C++, C*, COBOL, dBase, Forth, FORTRAN, Java, Modula-2, Pascal, Prolog, Python, REXX, and/or JavaScript for example. Further, it is not necessary that a single type of instruction or programming language be utilized in conjunction with the operation of the system and method of the invention. Rather, any number of different programming languages may be utilized as is necessary or desirable.

[0052] The computing environment may also include other removable/nonremovable, volatile/nonvolatile computer storage media. For example, a hard disk drive may read or write to nonremovable, nonvolatile magnetic media. A magnetic disk drive may read from or writes to a removable, nonvolatile magnetic disk, and an optical disk drive may read from or write to a removable, nonvolatile optical disk such as a CD-ROM or other optical media. Other removable/nonremovable, volatile/nonvolatile computer storage media that can be used in the exemplary operating environment include, but are not limited to, magnetic tape cassettes, flash memory cards, digital versatile disks, digital video tape, solid state RAM, solid state ROM, and the like. The storage media are typically connected to the system bus through a removable or non-removable memory interface.

[0053] The terms and expressions employed herein are used as terms and expressions of description and not of limitation, and there is no intention, in the use of such terms and expressions, of excluding any equivalents of the features shown and described or portions thereof. In addition, having described certain embodiments of the invention, it will be apparent to those of ordinary skill in the art that other embodiments incorporating the concepts disclosed herein may be used without departing from the spirit and scope of the invention. Accordingly, the described embodiments are to be considered in all respects as only illustrative and not restrictive.

CLAIMS

What is claimed is:

1. A method of fabricating a collapsible surface conforming to a target surface, the method comprising the steps of:

providing a computational representation of the target surface;

computationally generating a source shape approximating, with folds, a portion of the target surface, the source shape comprising a plurality of adjacent regular developable quadrilateral surfaces having a common boundary;

extending the source shape from the common boundary by adding a quadrilateral strip to an edge of a first one of the surfaces along the common boundary in accordance with a design, wherein:

- (a) the edge of the first surface terminates in at least one vertex shared with an adjacent second one of the surfaces, the first and second surfaces being adjacent along a ridge line extending from the vertex,
- (b) first and second design angles θ_1, θ_2 between an edge of the quadrilateral strip projecting from the vertex and, respectively, first and second other edges of the vertex obey the relationship $\cos\theta_2 = \cos\theta_1 \cos\beta + \sin\theta_1 \sin\beta \cos\alpha$, where α is a flap angle between the quadrilateral strip and a plane passing through the vertex and perpendicular to the first and second other edges of the vertex and β is an angle between the two other edges, and
- (c) $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 2\pi$, where θ_3, θ_4 are angles between the ridge line and, respectively, the first and second other edges of the vertex;

repeating the extending step at least once; and

fabricating the collapsible surface by (i) fabricating the quadrilateral surface and strips from a solid material and (ii) assembling the quadrilateral surface and strips into the extended source shape by foldably connecting them in accordance with the design.

2. The method of claim 1, wherein $\theta_1 + \theta_2(\theta_1) = k$, where $\theta_2(\theta_1) = \cos^{-1}(\cos\theta_1 \cos\beta + \sin\theta_1 \sin\beta \cos\alpha)$ and $k \in [\beta, 2\pi - \beta]$.

3. The method of claim 1, wherein repeating the extending step further comprises, for a new quadrilateral strip:

selecting a new flap angle α for the new quadrilateral strip at a free edge of the extended shape; and

constraining θ_1 , θ_2 and lengths of all edges of the new quadrilateral strip to avoid intersection between any of the edges of the new quadrilateral strip any portion of the shape.

4. The method of claim 3, wherein the new quadrilateral strip has at least one free edge and at least one bound edge, each bound edge being parallel to a bound edge of an adjacent quadrilateral strip.

5. The method of claim 1, wherein angles around each interior vertex of the design sum to 2π .

6. The method of claim 1, wherein the fabrication step comprises constructing the quadrilateral surface and strips from a solid material in a single large piece.

7. The method of claim 1, wherein repeating the extending step at least once produces Miura-Ori-like origami design conforming to the surface.

8. The method of claim 1, wherein the collapsible surface is flat-foldable without fracture.

9. The method of claim 1, wherein the design has a plurality of nodes and is flat-foldable at each of the nodes.

10. The method of claim 1, wherein the fabrication step comprises constructing the design from a solid material in a single large piece.

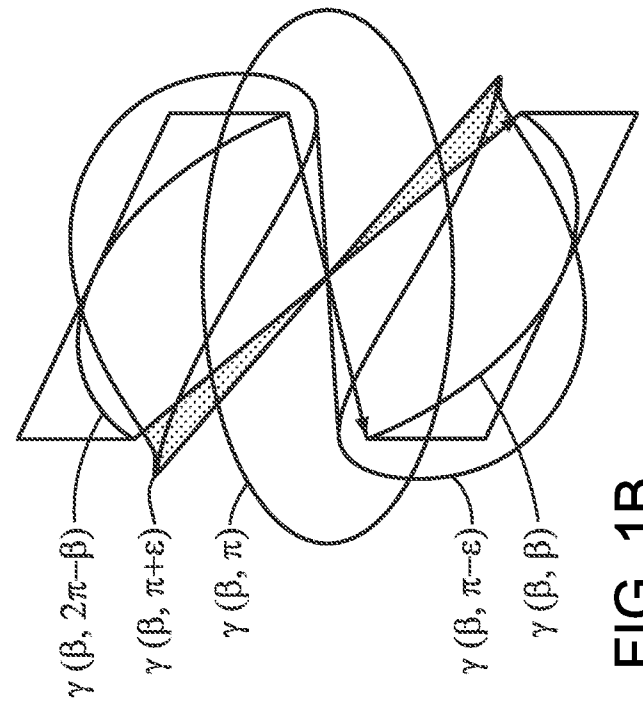


FIG. 1B

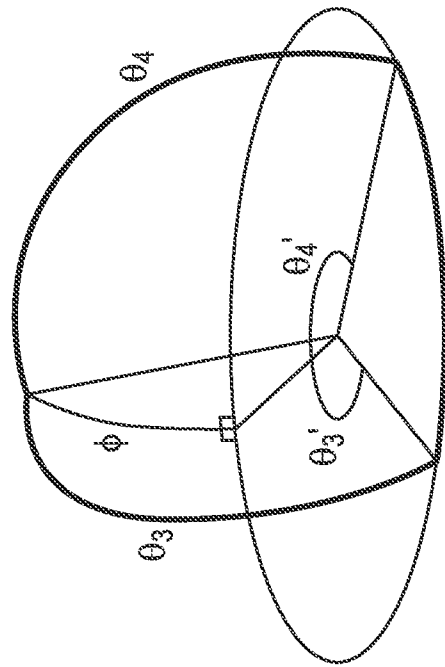


FIG. 2B

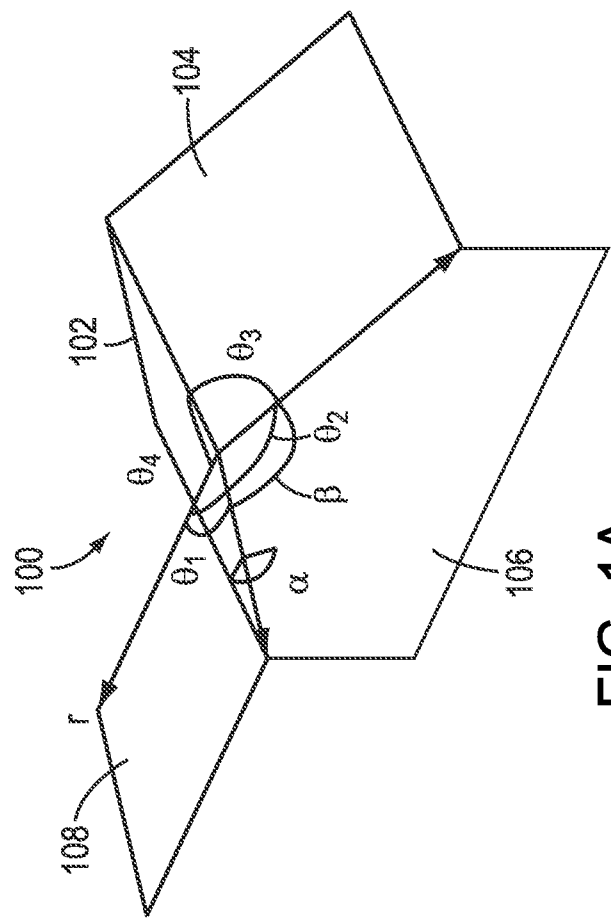


FIG. 1A

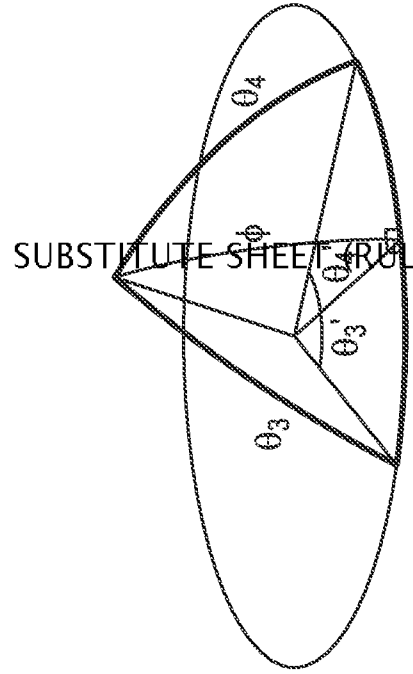


FIG. 2A

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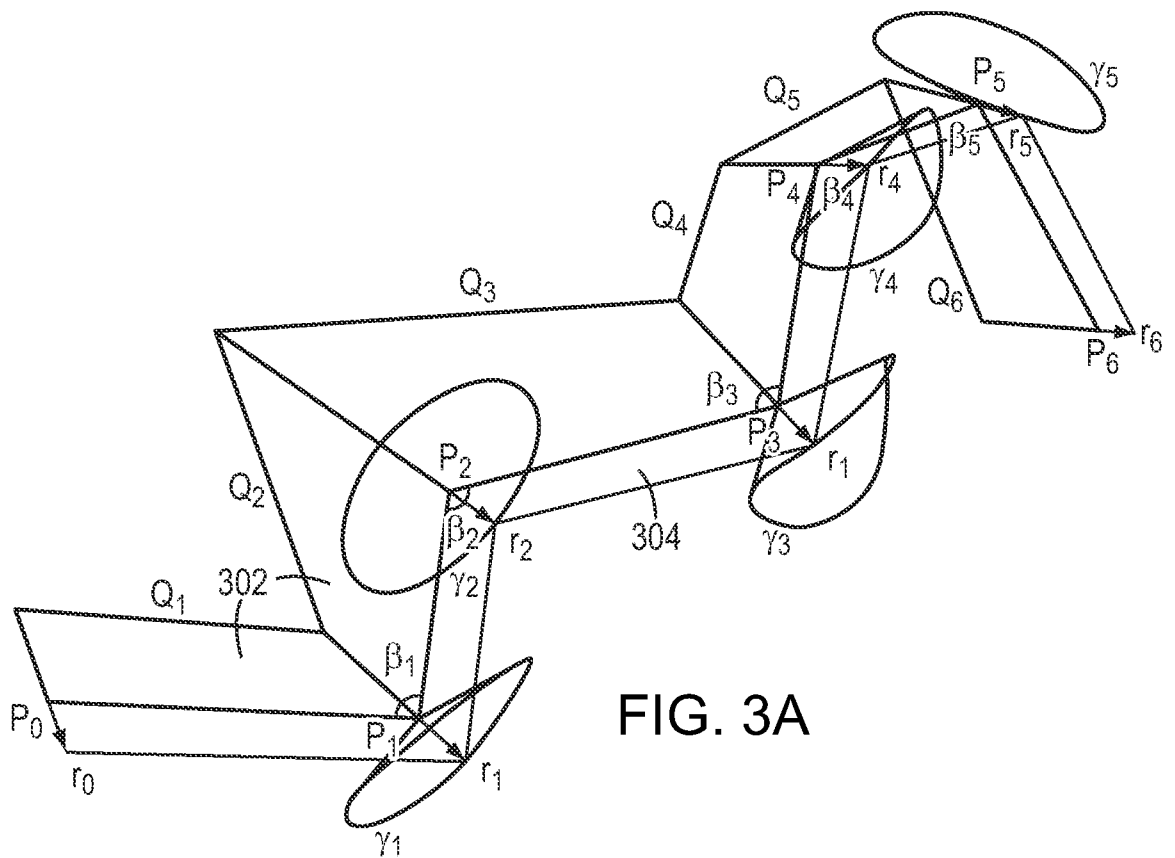


FIG. 3A

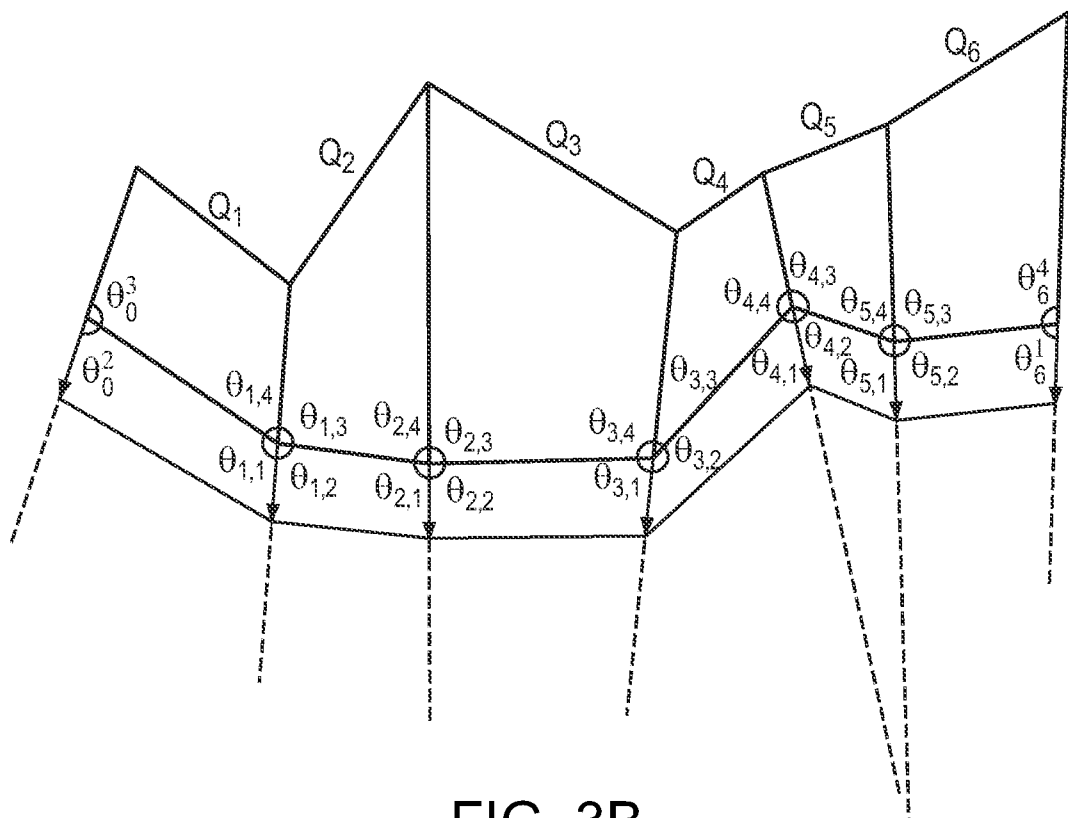


FIG. 3B

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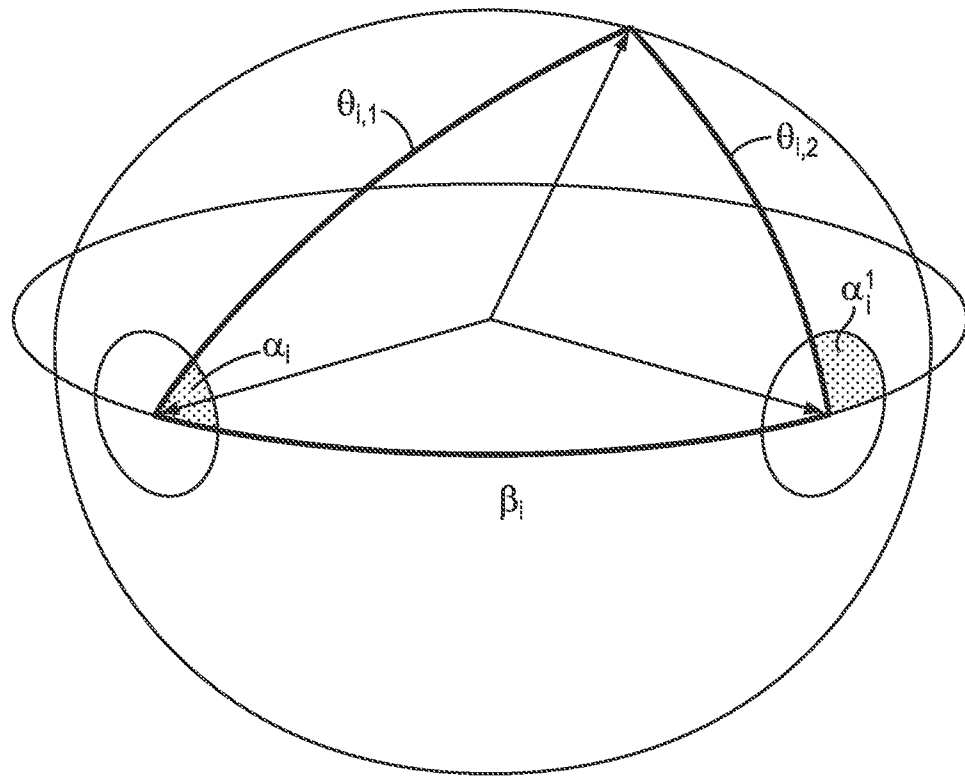


FIG. 4

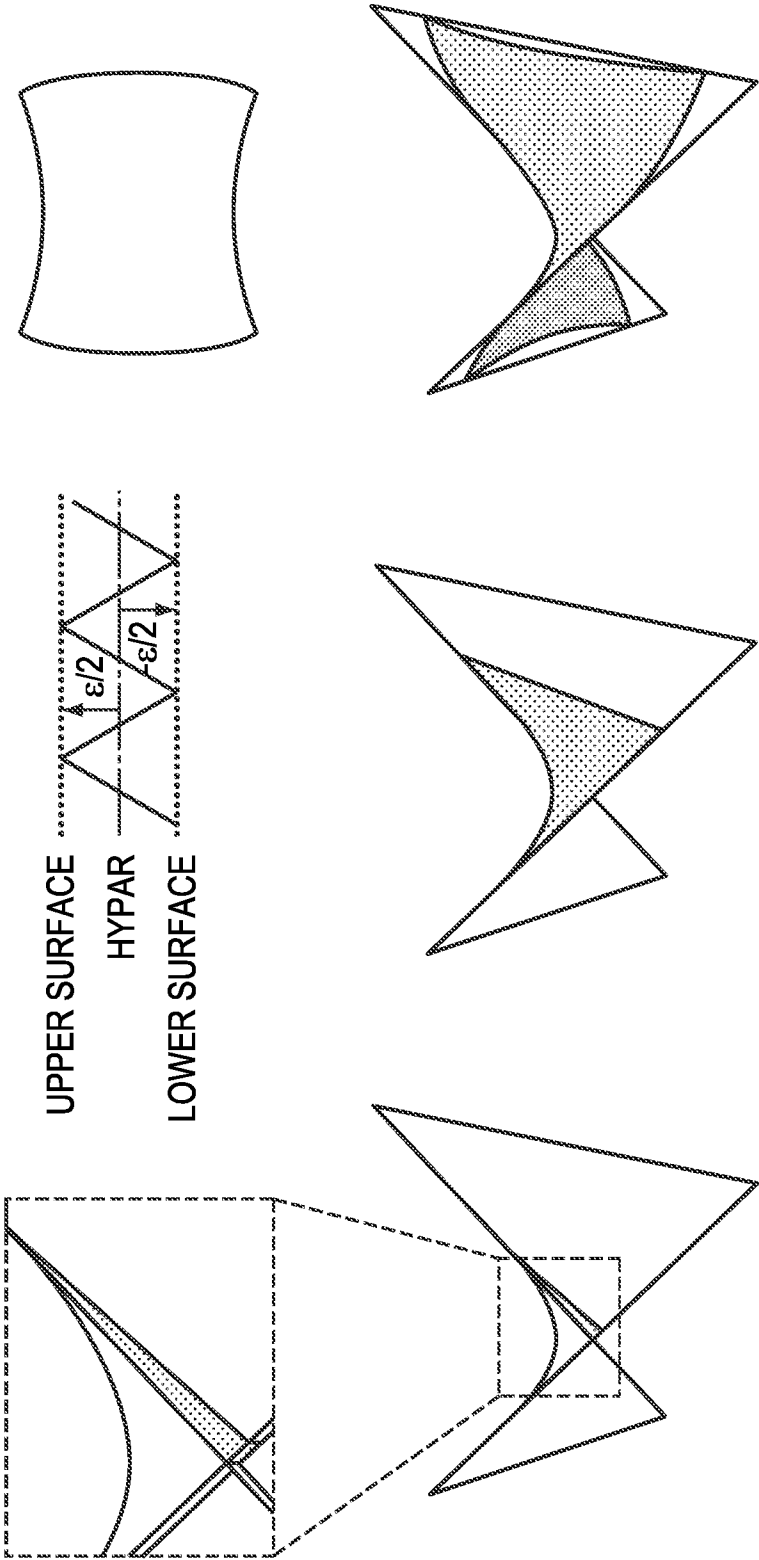


FIG. 5

INTERNATIONAL SEARCH REPORT

International application No.

PCT/US 18/29773

A. CLASSIFICATION OF SUBJECT MATTER
 IPC(8) - B31D 5/04, G06T 17/00 (2018.01)
 CPC - B31D 5/04, B65D 5/48034, G06T 17/30

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

See Search History Document

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

See Search History Document

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

See Search History Document

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	WO 2016/141264 A1 (DUDTE et al.), 09 September 2016 (09.09.2016), entire document, especially Abstract; para [0007], [0019], [0033], [0063]-[0066]	1-10
A	Tachi "Freeform Rigid-Foldable Structure using Bidirectionally Flat-Foldable Planar Quadrilateral Mesh." In: Advances in Architectural Geometry 2010. Springer, Vienna [online] [retrieved on 28 June 2018 (28.06.2018)] Retrieved from the Internet <URL: https://link.springer.com/chapter/10.1007/978-3-7091-0309-8_6#citeas >, entire document, especially Abstract; page 4-6	1-10
A	US 2015/0342050 A1 (ARIZONA BOARD OF REGENTS ACTING FOR AND ON BEHALF OF ARIZONA STATE UNIVERSITY), 26 November 2015 (26.11.2015), entire document, especially Abstract; para [0006], [0058]-[0060]	1-10
A	Chudoba et al. "Modelling framework for design and manufacturing of folded shell Structures." In: Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2013, 23-27 September 2013, [online] [retrieved on 28 June 2018 (28.06.2018)] Retrieved from the Internet <URL: https://www.ingentaconnect.com/content/iass/piass/2013/00002013/00000013/art00007 >, entire document	1-10
A	US 2008/0225043 A1 (Rosel), 18 September 2008 (18.09.2008), entire document	1-10
A	US 2014/0135195 A1 (Kling), 15 May 2014 (15.05.2014), entire document	1-10
A	US 2004/0099636 A1 (Scipioni), 27 May 2004 (27.05.2004), entire document	1-10

☐ Further documents are listed in the continuation of Box C.

☐ See patent family annex.

* Special categories of cited documents:

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"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search

29 June 2018

Date of mailing of the international search report

11 JUL 2018

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