

Feb. 25, 1958

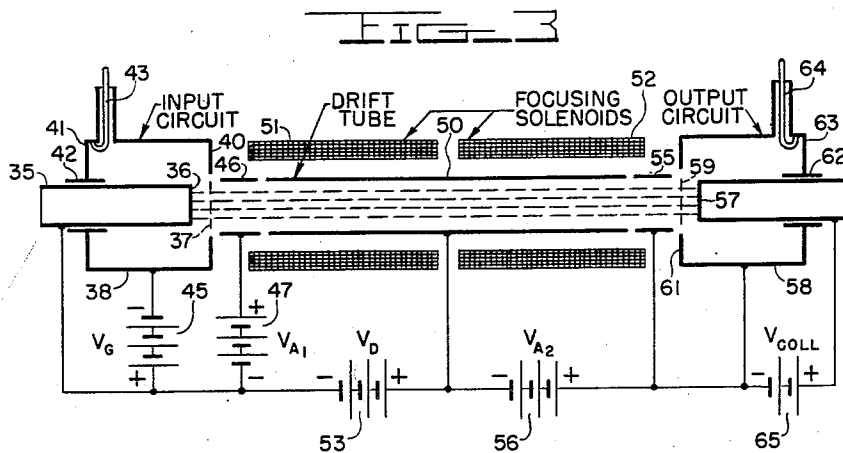
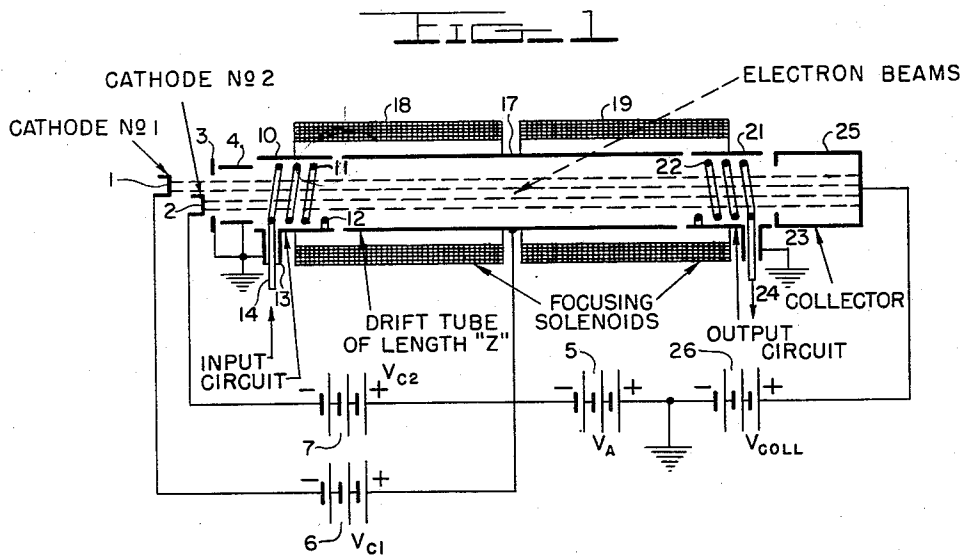
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2,824,997

ELECTRON WAVE TUBE

Filed Oct. 14, 1949

6 Sheets-Sheet 1



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ELECTRON WAVE TUBE

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FIG. 4

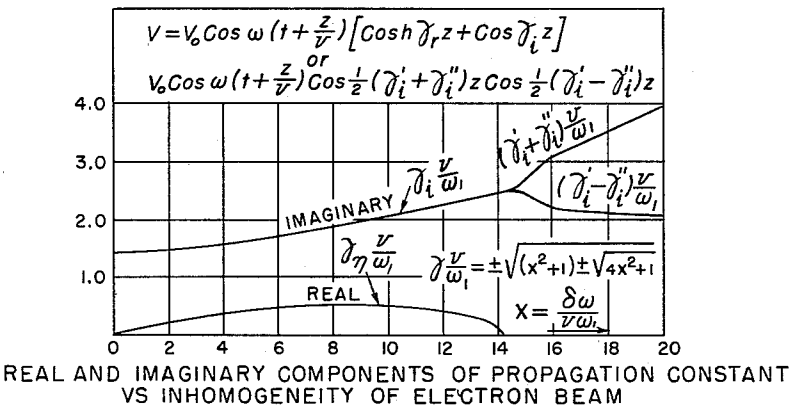


FIG. 5

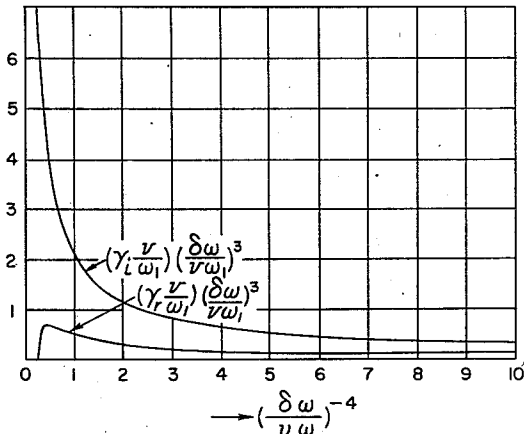
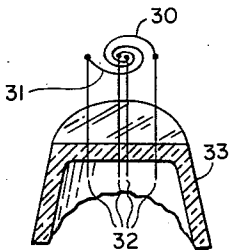


FIG. 6



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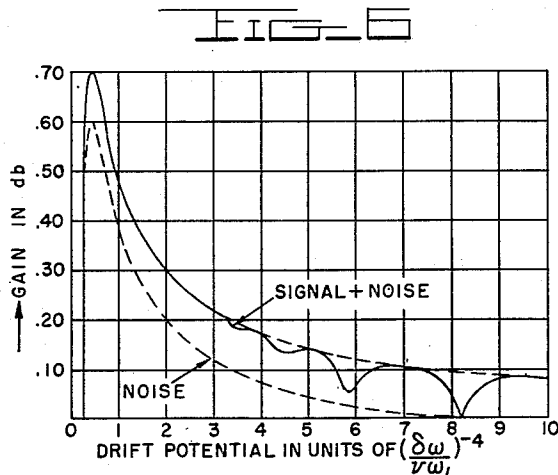
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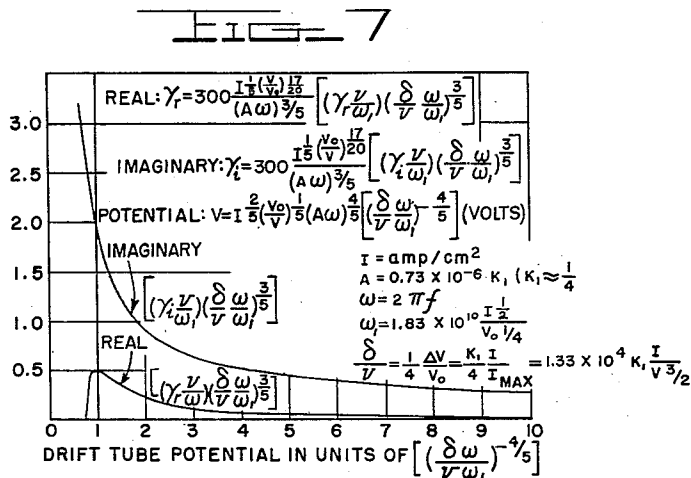
ELECTRON WAVE TUBE

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GAIN VS DRIFT TUBE POTENTIAL FOR THE ELECTRON WAVE TUBE AT S/N=10 db



REAL AND IMAGINARY COMPONENTS OF PROPAGATION CONSTANT VS DRIFT TUBE POTENTIAL FOR TETRODE ELECTRON WAVE TUBE

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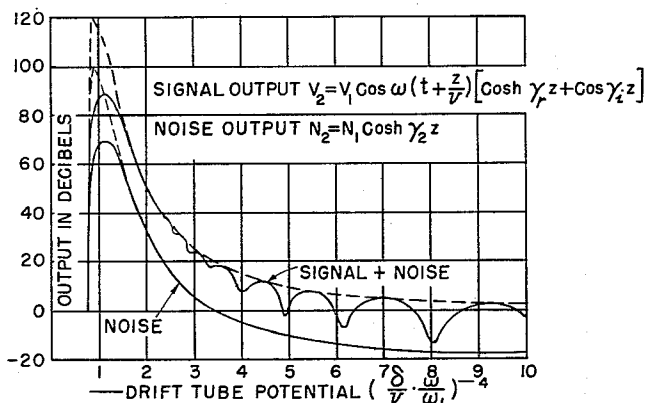
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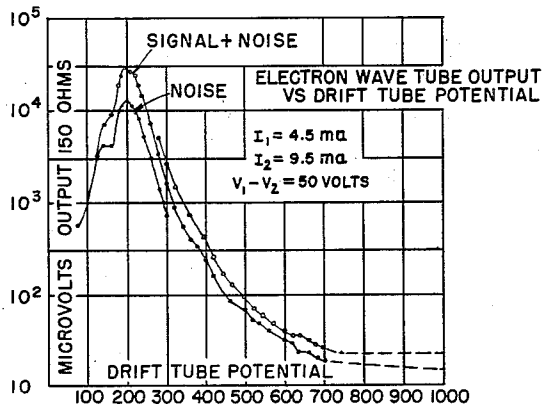
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FIG. 8



OUTPUT VS DRIFT TUBE POTENTIAL FOR TETRODE ELECTRON WAVE TUBE
($S_1 / N_1 = 20$ db)

FIG. 9



OUTPUT VS DRIFT TUBE POTENTIAL OF THE TWO-VELOCITY TYPE ELECTRON
WAVE TUBE
(BEAM CURRENTS: $I_1 = 4.5$ MA., $I_2 = 9.5$ MA., $V_1 - V_2 = 50$ VOLTS)

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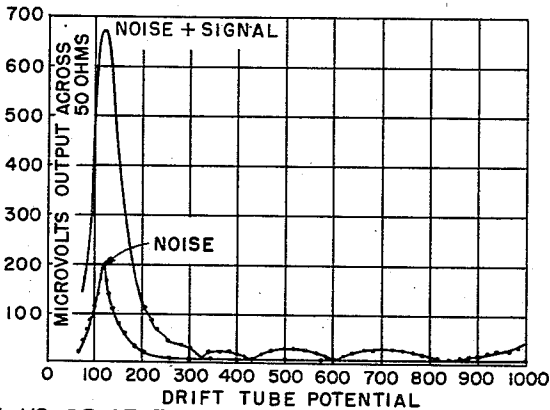
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ELECTRON WAVE TUBE

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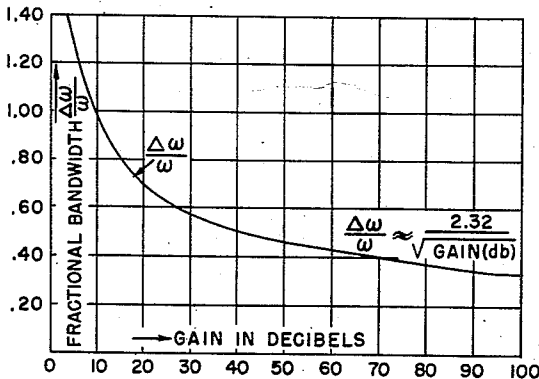
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FIG-10



OUTPUT VS DRIFT TUBE POTENTIAL OF THE TETRODE TYPE ELECTRON WAVE TUBE (BEAM CURRENT = 5 MA)

FIG-11



ELECTRONIC BANDWIDTH VS GAIN OF THE ELECTRON WAVE TUBE

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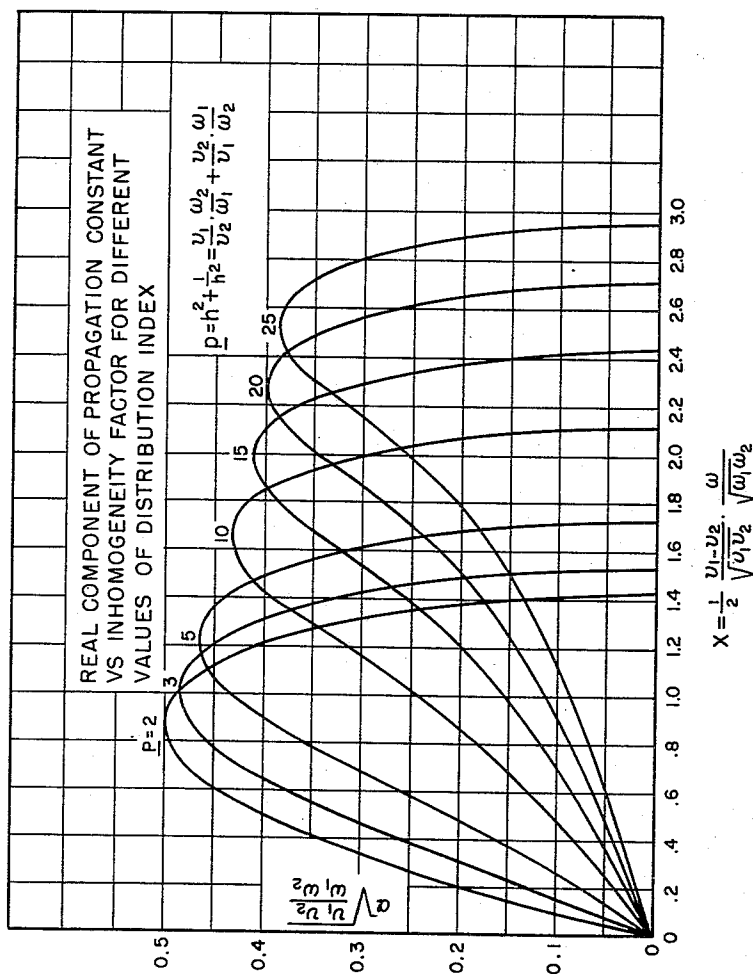
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FIG. 12



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2,824,997

ELECTRON WAVE TUBE

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Application October 14, 1949, Serial No. 121,435

2 Claims. (Cl. 315—5.39)

(Granted under Title 35, U. S. Code (1952), sec. 266)

This invention relates to the generation and amplification of electrical signals by the interaction of beams of charged particles carrying the signals as space charge or velocity variations and travelling at different average velocities. The invention is particularly applicable in the field of very high radio frequencies in which amplification and generation of oscillation is inefficient or impossible with previously known techniques and apparatus.

The operation of the present invention depends on energy transfer between beams of charged particles, travelling with different velocities, at least one of which carries a density or velocity modulation. The particles of the beams are in energy transferring relationship as they pass one another. This energy transfer amplifies a space charge variation carried by one of the beams during its passage past the other beam. The space charge modulation, as amplified, may then be removed from the beam by suitable coupling devices.

It is accordingly the object of the present invention to amplify and to generate electrical signals. More specific objects of the invention are to provide new apparatus for these purposes and novel methods of effecting these ends.

It would be preferred to describe the operation of the present invention in quantitative physical terms showing the transfer of the kinetic energy of the particles into the energy of the increasing space charge variation. Such computations, however, inherently are concerned with the behavior of a gas whose particles are subject to an inverse square law repulsion. Phenomena in such a gas produce results which are not negligible at remote points thereof, in critical distinction to the theory of gases operating under inverse cube and higher laws. The theory of inverse square law gases, largely because of the complexity referred to, has not been developed in classical physics.

Consequently, rigorous mathematical treatment of the present invention cannot be given at the present time. As will later appear, restricted solutions for a number of cases have been effected which provide explanations of a number of phenomena involved.

Accordingly, the invention will be described generally in physical terms as an introduction to the specific embodiments thereof, and the mathematical treatment postponed to the end of the discussion.

The amplification effects are produced by the interchange of energy between particles moving at different velocities, and the invention includes the cases where both beams are travelling in the same direction, particles in one overtaking the particles of the other, and also where the particles are travelling in opposite directions. While for convenience, two beams are referred to, more may be employed if desired.

In the operation of the invention energy is transferred from the faster to the slower particles during the transit of one particle past the other. Within this limitation, it has been found that space charge disturbances will be am-

2

plified with many types of experimental conditions so far as the average and relative beam velocities are concerned and so far as the space charge density and currents of the beams are concerned. It is not to be inferred, however, that any arbitrary arrangement will amplify energy at any desired frequency. Not only is the electronic bandwidth of the apparatus of the present invention limited in dependency on the space charge density of the beams and their velocity distribution but within the operating frequencies there exists a definite frequency for which the amplification is at its maximum.

In order to effect the desired energy transfer between the beams it is preferred that the beams traverse a common path. This offers little theoretical difficulties, since the statistical probability of collision within the beams of practical current density is substantially non-existent. Practically, interpenetrating electron beams are easily generated by two spiral filamentary cathodes or otherwise as will be apparent to those learned in the art.

The invention has been described by the inventor in the Physical Review, vol. 74, p. 1532, and in The Proceedings of the Institute of Radio Engineers, vol. 37, No. 1, pp. 4-10, and vol. 37, No. 7, p. 778. These publications may be referred to for additional information.

Further explanation of the invention will be had in connection with the exemplary embodiments thereof shown in the drawings, in which:

Fig. 1 shows schematically an embodiment of the invention,

Fig. 2 shows a cathode arrangement for the embodiment of Fig. 1,

Fig. 3 shows a second embodiment of the invention,

Fig. 4 is a graphical showing the solutions of an equation describing the operating conditions in the invention, Figs. 5 and 6 show the operation of the tube of Fig. 1, Figs. 7 and 8 show the operation of the tube of Fig. 3, Figs. 9 and 10 further show the operation of the tube of Fig. 1, and

Fig. 11 shows the electronic band-width of the tubes of the present invention.

The system shown in Fig. 1 comprises a two beam tube for the amplification of electrical signals. This tube comprises generally a pair of cathodes for generating a pair of interpenetrating beams of different velocities, a modulator for applying an electrical signal to the composite beam to provide effective variations of its space charge density, a drift tube wherein the space charge modulation is amplified during transit, a demodulator or output circuit for deriving the amplified energy from the composite beam, and a collector electrode for receiving the beam from the output circuit.

Initially the beam is generated by cathodes 1 and 2 whose heater elements are not shown in the drawings. As will be appreciated, the cathodes are insulated from one another in order that they may receive different operating potentials for establishing a beam having two components of velocity. The beam is initially electrostatically focused by the electrodes 3 and 4. These electrodes are returned to ground and the cathodes are supplied with negative potentials, cathode 1 being energized through voltage sources 5 and 6, and cathode 2 through sources 5 and 7.

The beam then enters a region where it receives a space charge modulation. Devices for modulating beams in this manner are known, and may operate by modulating either the current or the velocity of the beam. The embodiment shown includes a grounded cylindrical electrode 10 containing a conductive helix 11, connected thereto at 12, whose other end is led as through a collar 13 to provide an input terminal at 14 for receiving the modulating signals. Velocity modulation is thus effected.

Were the signal applied directly to the electrode 10

in the absence of helix 11, current or charge density modulation would be obtained. For this type of operation, of course, electrode 10 would not be grounded for the input signal frequency.

The modulated beam then enters a drift space whose potential is substantially uniform and is determined by that of cylindrical electrode 17. The operating potential of this electrode with respect to ground is provided by potential source 5. During passages of the composite beam through this tube, as well as through the modulator and demodulator, it is focused magnetically by solenoids 18 and 19.

During transit through the drift tube the energy interchange between the two components on the beam results in an amplification of the space charge variation imposed by the modulator. The signal level increases exponentially along the length of the drift tube. While the conditions setting a maximum limit to the gain have not been mathematically established, it is reasonable to presume that some equilibrium condition would eventually be reached in which the energy of the space charge distribution becomes constant and is not amplified during a further transit in the drift space.

The tube of Figure 1 is provided with a demodulator section 21 which receives the beam on its emergence from the drift space and abstracts the energy of the amplified space charge distribution. This comprises cylindrical electrode 21 enclosing a conductive helix 22 which surrounds the beam and is connected to 21 at one end, the other extending outwardly through collar 23 to form output terminal 24. Thus, the output circuit is inductively coupled to the beam.

A collector electrode 25 receives the beam from the demodulator and is itself energized at a positive potential with relation to the cathode by source 26.

The tube shown in Figure 1 is of generally circular section and of course is provided with an envelope for evacuation, to which solenoids 18 and 19 are preferably external.

A cathode arrangement for generating a composite beam is shown in Fig. 2, wherein two spiral filaments 30 and 31 are supported in the same plane by leads 32. As indicated, the leads may be positioned by press 33 of the tube envelope.

The operation of the invention as an oscillator is established by regenerative coupling between the output and input circuits. In the circuit of Fig. 1, manifestly such coupling would be provided from output lead 24 to input lead 14.

Typical operating voltages for the tube of Fig. 1, relative to ground, for a drift tube 10 inches long, at 3000 mc./s. are as follows:

Cathode 1	—1400 v., I_1 : 5 ma.
Cathode 2	—1350 v., I_2 : 10 ma.
Electrodes 3, 4 and 10	0.
Drift tube 17	—1200.
Electrode 21	0.
Collector 25	+100.

The tube of Fig. 3 illustrates two main departures from that of Fig. 1, which may be employed separately as well as together. These features comprise the use of a single cathode to generate the beam, and the use of a screen to establish tetrode operating conditions in the tube.

In Fig. 3 the indirectly heated cathode consists of a cylinder 35 with an emitting end face 36. Current modulation is obtained by potential variations of grid 37, caused by operation of a resonant input circuit formed by cathode member 35 and a cylinder 38 coaxial therewith. Annulus 40 connects the grid to one end of cylinder 38, whose other end is coupled to the center conductor by annulus 41 and coupling ring 42. The resonant structure is excited by an input circuit 43. The desired biasing potential is provided by voltage source 45. The voltage variation at the signal frequency between the

cathode 36 and grid 37 modulates the beam current in the usual fashion.

The modulated beam is focused by electrode 46 energized at a positive potential by voltage source 47. The beam is then passed through drift tube 50 under the focusing action of solenoids 51 and 52. The drift tube potential is established by voltage source 53.

From the drift tube, the beam passes through another focusing electrode 55 which is maintained at the potential of voltage source 56.

The output energy is derived from the beam by a resonant circuit similar to the input circuit. This comprises a central collector electrode 57 and outer cylinder 58. The beam enters the output circuit through grid 59 connected to the outer cylinder by annulus 61. The other end of the cylinder is capacitatively coupled to electrode 57 at ring 62 through annulus 63. The potential of the collector relative to the grid is determined by voltage source 65. The output energy is delivered through circuit 64.

The shielding action of grid 59 improves the efficiency of the tube by reducing the effective transit time of electrons received by the collector 57; and consequently should be positioned near the latter, at a distance short relative to the physical wave-length of the beam modulation. Since the signal is amplified in the drift tube before it arrives at the screen, the partition noise caused by the partial interception of current by the screen grid is negligible. In this respect the signal-to-noise ratio approaches that of the conventional triode.

As previously mentioned, the tube may form an oscillator where regenerative coupling is provided from the output to the input.

The provision of a single cathode in the construction of Fig. 3 causes the formation of what is usually considered a single beam. The electron velocities of such a beam, however, are not uniform, since the potential interiorly of the beam section progressively decreases as the center thereof is approached, as a result of space charge fields. Components of different velocity are therefore present for the exchange of energy between faster and slower electrons.

The theory of operation of this tube is not fully understood at present, however, and the above factors may not constitute the entire explanation. The amplification is obtained at higher average current levels than the double beam tube employs, and these levels approach half or more of the saturation current that can be passed through the drift tube under the operating conditions. Such saturation currents for various conditions are described by the inventor in The Proceedings of the Institute of Radio Engineers, September 1939, pp. 586-602.

Typical operating voltages for the tube of Fig. 3 with a 20 cm. drift tube, 5 ma. beam current, at 3000 mc./s., would be a negative grid bias of 5 volts, first focusing electrode 200 volts, drift tube 120 volts, second focusing electrode and screen grid 200 volts, and the collector electrode 150 volts, all relative to cathode.

The structural differences between Figs. 1 and 3 should not obscure the fact that their methods of operation may be varied as desired. Thus, the tube of Fig. 1 may be operated as a single cathode tube where the two cathodes are held at the same potential. The tube of Fig. 3 may be employed as a double cathode tube where the grid 37 is biased positive of the cathode 36 so as to draw current and be heated to emission temperatures. Secondary emission from the positive grid also may be employed for a second source of electrons. While the structure of Fig. 1 would be preferred for the latter type of operation, the main principles involved are substantially the same.

Having described the exemplary embodiments of the invention, the operating theory will be considered in such detail as is known.

When a high temperature body is placed close to a low

temperature body the heat energy flows in the direction which tends to equalize the temperatures of the two bodies in contact. This is one case of a general law of statistics that the entropy tends to increase. If an electron stream of high energy is projected in the proximity of another electron stream of lower energy, then, by the mechanism of elastic collisions between fast and slow electrons, there will be a gradual interchange of energy between the two streams with the result that on the average, the fast electrons are slowed down and the slow ones are speeded up. The nature of this process is similar to elastic scattering of high energy particles by other particles which are either stationary or moving slowly. In the case of electron scattering, the forces acting between the electrons obey Coulomb's law. As a consequence the mechanism of scattering of electrons by electrons can be described in terms of the familiar concepts of electromagnetic theory.

Consider a uniform stream of electrons of charge density p_1 moving in the Z-direction with the velocity v_1 . Any disturbance of the stream will produce electron waves in the stream. By electron waves is meant either space-charge density fluctuations with associated electric field fluctuations, or velocity fluctuations of electrons in the stream. The phase velocity of these waves depends upon the space-charge density. Two waves are usually associated with any disturbance. One of these waves has its phase velocity somewhat higher than the velocity of translation of electrons in the stream and the other wave has velocity lower than the translational velocity.

If another stream of electrons of density p_2 moving with velocity v_2 is injected near or into the space occupied by the first electron stream, the electron waves interact so that the phase velocities of electron waves in the two streams are modified and an interchange of energy between the two streams takes place. It is most convenient for the purposes of analysis to consider the two streams of electrons as components of one inhomogeneous stream. The space-charge densities p , current densities i , and velocities v , can be expressed as:

$$\begin{aligned} p_1' &= p_1 + \bar{p}_1 \\ p_2' &= p_2 + \bar{p}_2 \end{aligned} \quad (1)$$

$$\begin{aligned} v_1' &= v_1 + \bar{v}_1 \\ v_2' &= v_2 + \bar{v}_2 \end{aligned} \quad (2)$$

$$\begin{aligned} i_1' &= i_1 + \bar{i}_1 = i_1 + \bar{p}_1 v_1 + \bar{v}_1 p_1 \\ i_2' &= i_2 + \bar{i}_2 = i_2 + \bar{p}_2 v_2 + \bar{v}_2 p_2 \end{aligned} \quad (3)$$

In these expressions the first symbol represents average or D. C. quantities and the second symbol represents first order quantities which can be assumed to vary with time and distance in an exponential manner similar to variations of the voltage V :

$$V = V_0 e^{-\Gamma Z + i\omega t} \quad (4)$$

where Γ is the propagation constant, ω is the frequency of the disturbance, or signal, having an initial value V_0 at $Z=0$.

The fundamental equations which are required in the analysis are: (a) Poisson's equation:

$$\nabla \cdot D = p, \text{ or } e \frac{\partial^2 V}{\partial Z^2} = p_1' + p_2'$$

or

$$e \Gamma^2 V = \bar{p}_1 + \bar{p}_2 \quad (5)$$

(b) The equation expressing conservation of electric charge:

$$\frac{\partial p}{\partial t} + \frac{\partial i}{\partial Z} = 0$$

or

$$\begin{aligned} j\omega \bar{p}_1 &= \Gamma \bar{i}_1 \\ j\omega \bar{p}_2 &= \Gamma \bar{i}_2 \end{aligned} \quad (6)$$

5 and (c) the force equation:

$$m \frac{dv}{dt} = e \frac{\partial V}{\partial Z}$$

10 where

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial Z} \frac{\partial Z}{\partial t} = \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial Z}$$

$$j\omega \bar{v}_1 - v_1 \Gamma \bar{v}_1 = -\frac{e}{m} \Gamma V$$

$$j\omega \bar{v}_2 - v_2 \Gamma \bar{v}_2 = -\frac{e}{m} \Gamma V \quad (7)$$

By using Relations 3 in the Formula 6 we obtain:

$$\begin{aligned} j\omega \bar{p}_1 &= \Gamma (\bar{p}_1 v_1 + \bar{v}_1 p_1) \\ j\omega \bar{p}_2 &= \Gamma (\bar{p}_2 v_2 + \bar{v}_2 p_2) \end{aligned} \quad (8)$$

Solving for $\bar{v}_1$ and $\bar{v}_2$, we get:

$$\begin{aligned} \bar{v}_1 &= \frac{\bar{p}_1}{\Gamma p_1} (j\omega - \Gamma v_1) \\ \bar{v}_2 &= \frac{\bar{p}_2}{\Gamma p_2} (j\omega - \Gamma v_2) \end{aligned} \quad (9)$$

Substituting (9) into (7):

$$\begin{aligned} \frac{\bar{p}_1}{\Gamma p_1} (j\omega - v_1)^2 &= -\frac{e}{m} \Gamma V \\ \frac{\bar{p}_2}{\Gamma p_2} (j\omega - v_2)^2 &= -\frac{e}{m} \Gamma V \end{aligned} \quad (10)$$

35 Solving for $\bar{p}_1$ and $\bar{p}_2$:

$$\begin{aligned} \bar{p}_1 &= \frac{\frac{e}{m} p_1}{(\omega + j\Gamma v_1)^2} \Gamma^2 V \\ \bar{p}_2 &= \frac{\frac{e}{m} p_2}{(\omega + j\Gamma v_2)^2} \Gamma^2 V \end{aligned} \quad (11)$$

Finally, the substitution (11) into (5) results in the following expression which defines the propagation constant Γ in terms of the frequency ω , velocities v_1 and v_2 of the two electron streams, and the corresponding space-charge densities p_1 and p_2 :

$$\Gamma^2 V = \left[\frac{\frac{e}{m} p_1}{(\omega + j\Gamma v_1)^2} + \frac{\frac{e}{m} p_2}{(\omega + j\Gamma v_2)^2} \right] \Gamma^2 V \quad (12)$$

Excluding trivial solutions corresponding to conditions that either $\Gamma=0$ or $V=0$ and replacing space-charge densities by electron plasma frequencies ω_1 and ω_2 from defining expressions:

$$\frac{e}{m} \frac{p_1}{e} = \omega_1^2$$

60 and

$$\frac{e}{m} \frac{p_2}{e} = \omega_2^2 \quad (13)$$

we obtain the equation from which the propagation constant of the inhomogeneous stream can be obtained:

$$1 = \frac{\omega_1^2}{(\omega + j\Gamma v_1)^2} + \frac{\omega_2^2}{(\omega + j\Gamma v_2)^2} \quad (14)$$

It can be easily shown that in a general case of a composite stream of electrons having many components of plasma frequencies ω_i and velocities v_i the formula takes the form:

$$1 = \sum_i \frac{\omega_i^2}{(\omega + j\Gamma v_i)^2} \quad (14')$$

Again, in the case of a continuous distribution of velocities between the limits of v_1 and v_2 the Formula 14 can be written as follows:

$$1 = \int_{v_1}^{v_2} \frac{dw_1^2}{dv} \cdot dv \quad (14'')$$

In the case of a homogeneous electron beam when $w_2=0$ the Formula 14 reduces to a familiar form:

$$(w + j\Gamma v_1)^2 = w_1^2 \quad (15)$$

The solution of Equation 15 immediately gives the familiar expression for the propagation constant:

$$\Gamma_1 = j \left(\frac{w}{v_1} \pm \frac{w_1}{v_1} \right) \quad (16)$$

The Expression 16 shows that two electron waves propagate along the beam with the velocities somewhat higher and somewhat lower than the average electron velocity.

These waves produce interference effects in the beam which manifest themselves by the fact that along the beam periodic conversion of kinetic energy of electrons into space-charge field energy takes place without amplification of the original disturbance.

In order to solve the Equation 14 in the case when the velocities are not equal we can assume that:

$$\begin{aligned} v_1 &= v + \delta \\ v_2 &= v - \delta \end{aligned} \quad (17)$$

where v is the average velocity of electrons in the stream, and 2δ represents the difference in velocity of the two streams, i. e., $2\delta = v_1 - v_2$.

By putting:

$$\Gamma = j \frac{w}{v} + j\gamma \quad (18)$$

we obtain:

$$\begin{aligned} w + j\Gamma v_1 &= -\frac{\delta}{v} w - v\gamma \\ w + j\Gamma v_2 &= +\frac{\delta}{v} w - v\gamma \end{aligned} \quad (19)$$

In terms of δ and γ , the Formula 14 can be written as:

$$1 = \frac{w_1^2}{\left(\frac{\delta}{v} w + \gamma v\right)^2} + \frac{w_2^2}{\left(\frac{\delta}{v} w - \gamma v\right)^2} \quad (20)$$

This equation can be solved explicitly for a case when $w_1 = w_2$. In this case the equation reduces to the following form:

$$1 = \frac{1}{\left(\frac{\delta w}{v w_1} + \gamma v\right)^2} + \frac{1}{\left(\frac{\delta w}{v w_1} - \gamma v\right)^2} \quad (21)$$

It can be easily shown that Equation 21 has the following solution:

$$\gamma \frac{v}{w_1} = \pm \sqrt{\left(\frac{\delta w}{v w_1}\right)^2 + 1} \pm \sqrt{4 \left(\frac{\delta w}{v w_1}\right)^2 + 1} \quad (22)$$

The factor $\delta w / v w_1$ is dimensionless and expresses the degree of inhomogeneity of the electron stream; the quantity $\gamma v / w_1$ is proportional to the looked-for component of the propagation constant, and the average velocity, v , of the electrons and is inversely proportional to the electron plasma frequency, w_1 . The real and imaginary components of the normalized propagation constant $j(\gamma v / w_1)$ are shown plotted in Figure 4 as a function of the inhomogeneity $\delta w / v w_1$ of the electron stream. The real component has finite value only over a limited range of the inhomogeneity factor, from 0 to a value of $\sqrt{2}$. It

is only over this range that effective amplification of the disturbances can take place. The maximum value of the real component is equal to one-half when the inhomogeneity is equal to the value of $\sqrt{3}/2$. The formula expressing the amplitude of the disturbance in this case is as follows:

$$V = V_0 \cos w \left(t + \frac{Z}{v} \right) [\cosh \gamma_r Z + \cos \gamma_i Z] \quad (23)$$

where γ_r is the real, and γ_i is the imaginary component of the propagation constant.

The value of the imaginary component of the propagation constant increases with the degree of inhomogeneity.

When the inhomogeneity exceeds the value of $\sqrt{2}$, four interfering waves are present in the beam without any amplification effects. In this case the magnitude of the disturbance along the beam can be expressed as follows:

$$V = V_0 \cos w \left(t + \frac{Z}{v} \right) \cos \frac{1}{2} (\gamma_i' + \gamma_i'') Z \cos \frac{1}{2} (\gamma_i' - \gamma_i'') Z \quad (24)$$

Since the inhomogeneity of the beam is proportional to the signal frequency, the curves of Figure 4 also represent variations of the real and imaginary components of the propagation constant with frequency. The ratio of the square of the amplitude of the disturbance at a distance Z along the beam to the magnitude of the disturbance at its origin at $Z=0$, represents a gain in energy of the disturbance. For large value of r^2 the gain increases exponentially with the length Z .

The general solution to Equation 14 has been obtained by standard methods. The solutions defining conditions for amplification are shown in Fig. 12, where the normalized real component of the propagation constant, $\alpha(v_1 v_2 / w_1 w_2)^{1/2}$, is plotted as a function of the inhomogeneity factor x for different values of a distribution factor p . The general inhomogeneity factor corresponds to that mentioned above if the geometric mean values of velocity and plasma frequency are used in the expression defining the homogeneity factor. h is $(v_1 w_2 / v_2 w_1)^{1/2}$.

The curves rise to a maximum value of .5 for $p=2$, ($h=1$); the maxima for the other curves are lower, but the maxima do not fall off particularly rapidly as p increases. Consequently, the tubes of the present invention will produce amplification over a wider range of inhomogeneity factor as p increases.

The tube of Fig. 1 utilizes an inhomogeneous electron stream for amplification of high frequency energy. The electron stream originates at two separate cathodes, 1 and 2, placed near the input end of the tube. Accelerator and focusing electrodes placed in front of the cathodes, project the electron streams through a long, hollow metal cylinder, usually called the drift tube. At the opposite end of the tube the electrons are collected by a collector electrode. The divergence of the electron stream is prevented by the use of magnetic field parallel to the velocity of electrons in the stream. This field is created by focusing solenoids as shown in the figure. The input signal is brought to the input end of the tube by means of a transmission line which terminates in a short helix surrounding the electron stream. In this manner the signal voltage produces initial disturbance in the beam. This disturbance is amplified in an exponential manner along the length of the drift tube. Greatly amplified fluctuations in the electron stream induce voltages in the output circuit. This circuit can be similar to the input circuit as is shown in the figure.

In order to obtain maximum gain in the tube, the potentials between cathode 1, cathode 2, and the drift tube are adjusted so that the inhomogeneity factor approaches the value of $3/2$. Since this factor involves electron plasma frequency, the optimum adjustment depends upon the value of current density in the beam. Since the maximum value of the normalized real component of the

propagation constant is equal to 0.5, the maximum gain of the electron wave tube can be expressed as:

$$\text{Gain (db)} = 20 \log_{10} e^{\gamma r^2} = 8.68 \left(\frac{\gamma r^2}{w_1} \right) \frac{w_1}{v} Z = 1330 Z \sqrt{\frac{I}{V^{3/2}}} \quad (25)$$

$$\frac{\text{cm. (amp./cm.}^2)^{1/2}}{\text{volts } 3/4}$$

Instead of adjustment of the difference of potential between the two cathodes, the drift tube potential can be adjusted for optimum gain. For a constant current in the beam the variation of the real and imaginary components of the propagation constant with the potential of the drift tube expressed in dimensionless units is shown in Figure 5. The real component reaches its maximum value at the normalized drift tube voltage of approximately 0.4. The variation of the output, or gain of the tube with the drift tube potential can be easily derived from the generalized curves of Figure 4. Such a diagram is represented in Figure 6 where a rapid increase in gain as the voltage is lowered can be readily seen. The effect of interference of the component waves is apparent in the fluctuations of the output as the drift voltage is varied.

In the tube of Fig. 3, only one electron beam is used. In this case the inhomogeneity of the beam is the result of the space-charge reduction of the potential along the axis of the beam so that outer electrons travel at a velocity higher than the velocity of electrons along the axis. The variation of the real and imaginary components of the propagation constant with the drift tube voltage for this "space-charge" type of electron wave tube, is shown in Figure 7. In general, the shape of the curves in this figure resemble very much those of Figure 5. The variation of the output with the drift tube voltage for space-charge type tube is shown in Figure 8.

Experimental curves of Figures 9 and 10 show variation of output with the drift tube potential. A close resemblance, in most respects, to the theoretical curves of Figure 8 is apparent. These experimental data were obtained for the drift tube 20 centimeters in length.

The generalized curves of Figure 4 make it possible to estimate the electronic bandwidth of the electron wave tube. The variation of the fractional bandwidth with the gain of the tube is shown in Figure 11. It is interesting to observe that at gains as high as 100 decibels, the electronic bandwidth is still greater than 30 percent.

It will be understood that the embodiments described above are not to be taken as limiting, but as exemplary preferred constructions and techniques for illustrating the practice of the present invention.

The invention described herein may be manufactured and used by or for the Government of the United States

of America for governmental purposes without the payment of any royalties thereon or therefor.

What is claimed is:

1. A signal amplifying device comprising shield means establishing a beam transmitting path of uniform potential along an extended portion of its length, a single cathode gun means positioned to inject an inhomogeneous beam of charged particles at different velocities into said shield means, said gun means having a single emitting surface and adapted to produce a single beam of said particles having a relatively large cross-sectional area, beam modulating means positioned between said gun means and said shield means operative to modulate beam particles at a high signal frequency, said gun means being adjustable to supply an average beam current which is a fraction not less than about half of the maximum beam current transmissible through the shield means transmitting path, and beam modulation responsive means operative at the signal frequency positioned to receive the modulated beam particles after passage through the beam transmitting path.

2. A signal amplifying device comprising shield means establishing a beam transmitting path of uniform potential along an extended portion of its length, a single cathode gun means positioned to inject an inhomogeneous beam of charged particles at different velocities into said shield means, said gun means having a single emitting surface and adapted to produce a single beam of said particles having a relatively large cross-sectional area, resonant chamber modulating means encompassing at least said emitting surface of said particle gun means and operative to modulate beam particles at a high signal frequency, said gun means being adjustable to supply an average beam current which is a fraction not less than about half of the maximum beam current transmissible through the shield means transmitting path, and beam modulation responsive means operative at the signal frequency positioned to receive the modulated beam particles after passage through the beam transmitting path.

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