Jan. 24, 1933. E. WILDHABER 1,895,360

GEARING

Filed Feb. 17, 1930 3 Sheets-Sheet 1

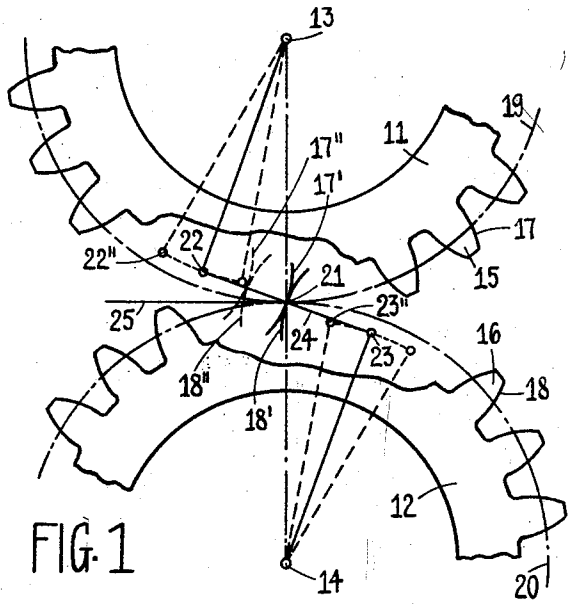


FIG. 1

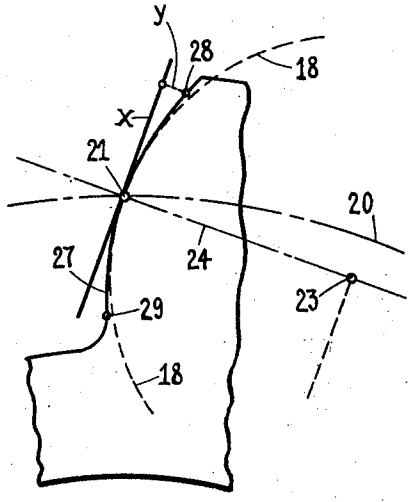


FIG. 2

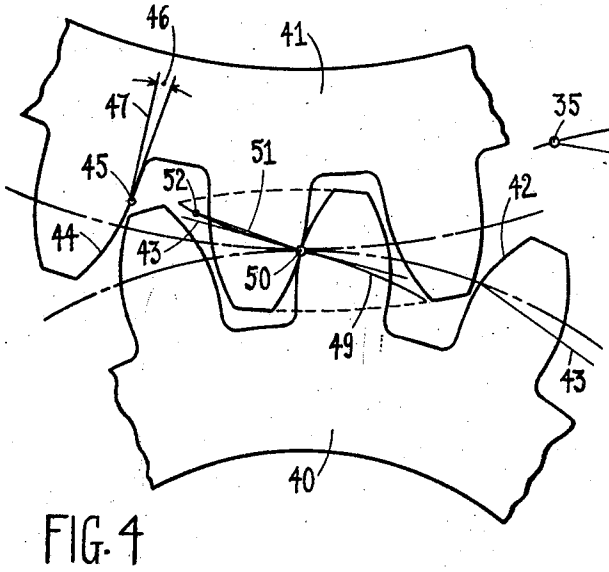


FIG. 4

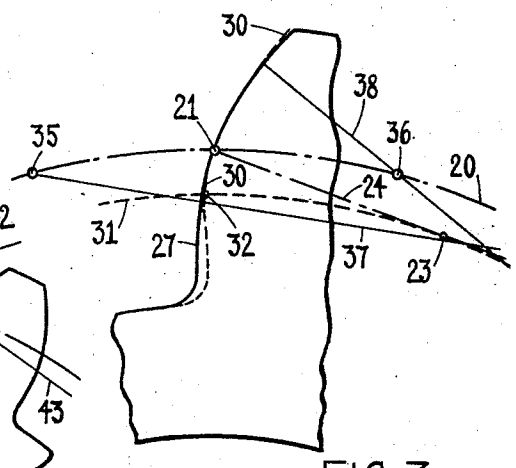


FIG. 3

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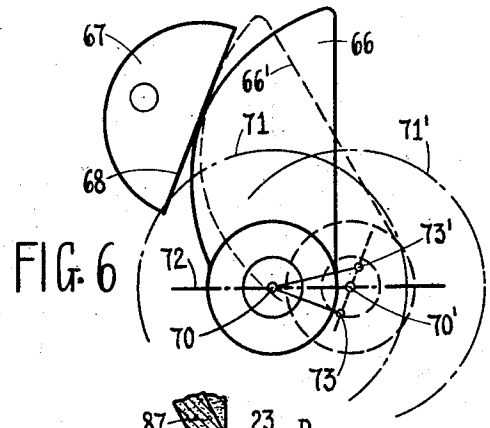


FIG. 6

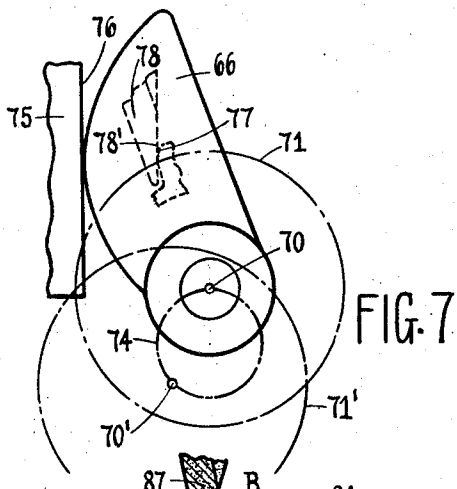


FIG. 7

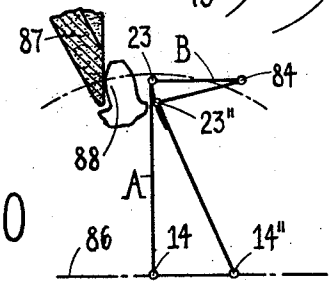


FIG. 10

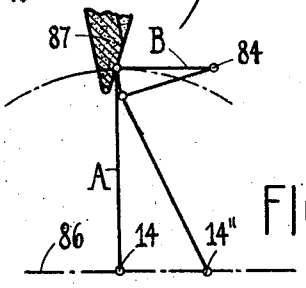


FIG. 11

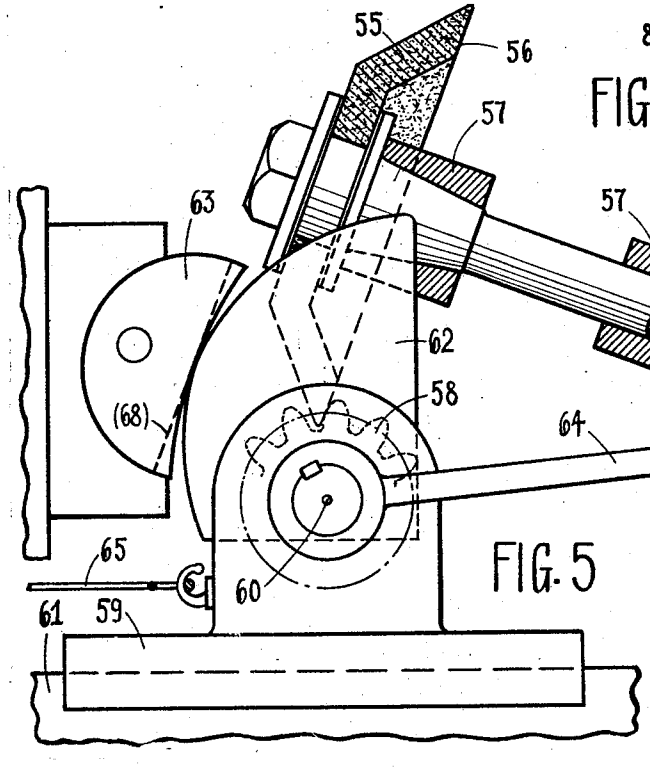


FIG. 5

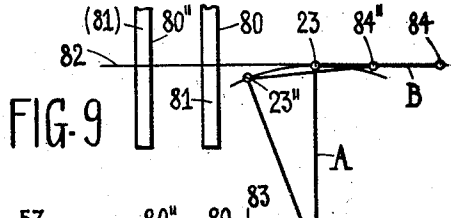


FIG. 9

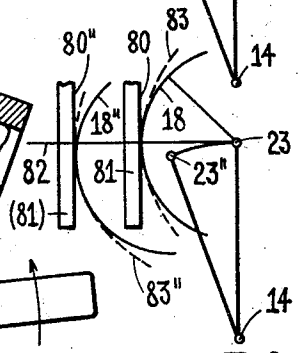


FIG. 8

INVENTOR

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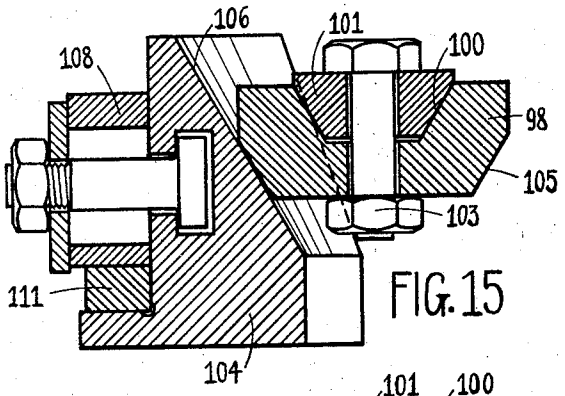


FIG. 15

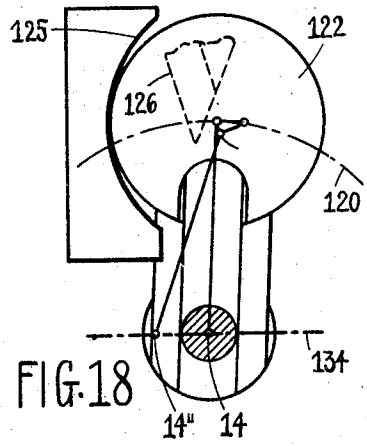


FIG. 18

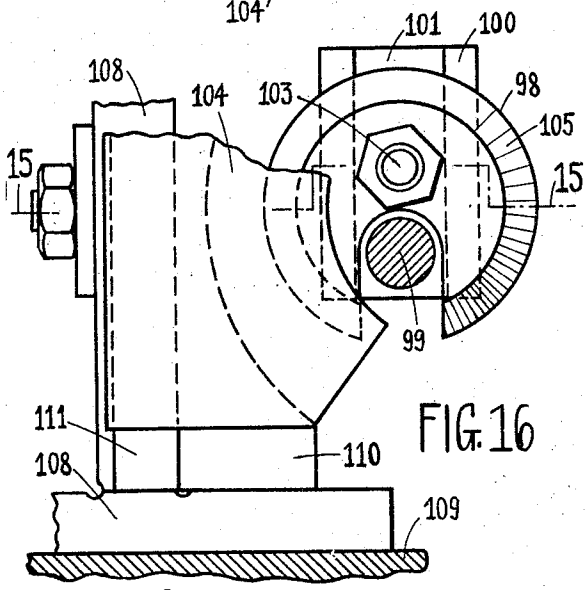


FIG. 16

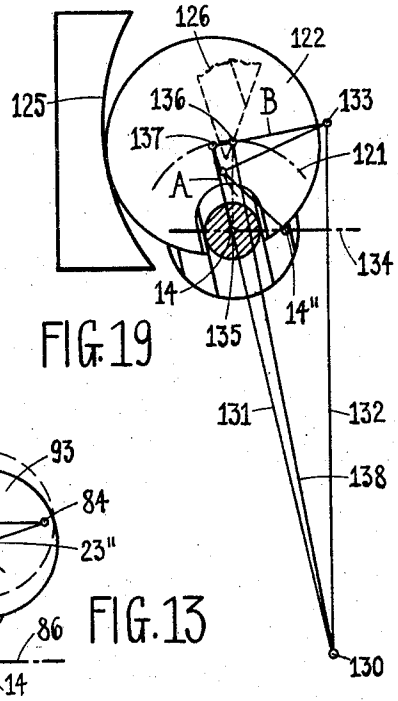


FIG. 19

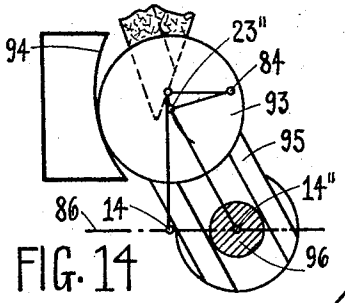


FIG. 14

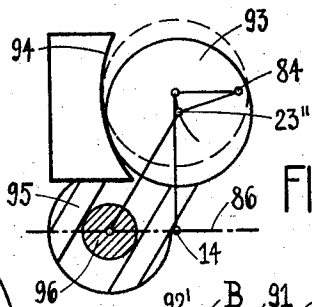


FIG. 13

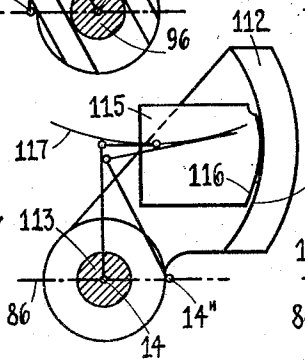


FIG. 17

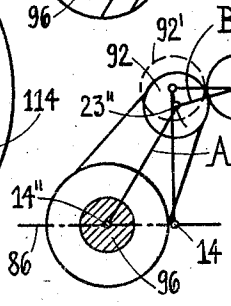


FIG. 12

INVENTOR

Ernest Wildhaber

Patented Jan. 24, 1933 1,895,360

UNITED STATES PATENT OFFICE

ERNEST WILDHABER, OF BROOKLYN, NEW YORK

GEARING

Application filed February 17, 1930. Serial No. 429,092.

The present invention relates to gearing, namely to the tooth shape of gears mounted on parallel axes or on intersecting axes.

One object of the present invention is to provide an improved form of tooth for gear pairs having ratios between one to one and two to one and having comparatively small numbers of teeth.

A further aim of the present invention is to provide gear pairs of any ratio having teeth of great strength and operating with increased quietness. A further object of the present invention is to provide a tooth form which tends to wear more evenly than the conventional forms of tooth, and which is therefore more apt to conserve the correct tooth action and quiet operation.

Another object of the present invention is to devise an improved form of tooth which can be produced with high accuracy as well as expediency, and which may be finished after hardening as well as the conventional involute teeth.

A still further aim is to provide gear pairs with improved tooth shape of such character that each member of the pair is conjugate to the same third member or gear, and to provide a system of interchangeable gears, of improved tooth form.

More particularly an improved tooth form for spur and helical gears shall be provided, which may be produced with high accuracy and expediency, and in which both members contain curved tooth profiles.

The tooth shape of gears of the last said character hitherto has been based on the tooth form of a rack. The tooth form of the rack was ordinarily given the simplest profile of the gear system. Such rack has been generally referred to as a basic rack.

In accordance with the present invention I depart from this practice and embody the simplest tooth form of the gear system on a pinion, that is to say on an external gear with a comparatively small number of teeth. How this may be done, and what relations should be observed that this may be accomplished will be fully described hereafter.

A further object of the present invention is to provide a system of spur gears or helical gears derived from a basic pinion of simplest tooth form and of such character that each gear of a gear pair is conjugate to the same pinion. Moreover gear pairs having a ratio between one to one and two to one shall be provided, in which both gears of the pair contain wholly convex tooth profiles conjugate to the same basic pinion.

Another aim in view is to devise a pair of gears of which each is conjugate to the same basic pinion, which contains tooth profiles of symmetrical form with respect to a profile normal erected at a mean profile point, and further to devise a pinion of less than 25 teeth having a convex tooth profile whose curvature radii are a minimum at a point intermediate the profile ends and whose curvature radii increase at points disposed nearer to the tip end as well as at points disposed nearer to the root end of the teeth.

Further an improved system of spur and helical gearing shall be devised, which is specifically adapted to production with a gear shaper cutter. Another object is to establish an improved system of spur and helical gearing based on a pinion of the size of a gear shaper cutter, which has substantially symmetrical tooth profiles with a minimum curvature radius at a point intermediate the ends of a tooth profile.

Various further objects will appear in the course of the specification and from recital of the appended claims.

My invention is exemplified in the accompanying drawings, in which

Fig. 1 is a diagram explanatory of one of the principles underlying the present invention, and a partial view of a pair of gears containing tooth profiles of a character referred to in the specification.

Fig. 2 is a diagrammatic view of the profile of a pinion formed in accordance with the present invention and suited to constitute the basic tooth form of an improved pair of gears.

Fig. 3 is a diagrammatic view similar to Fig. 2 further explanatory of a preferred form of profile of a pinion or a basic pinion and affording a comparison between said

profile and the involute profile of a conventional pinion.

Fig. 4 is a partial view of a pair of spur gears constructed in accordance with the present invention.

Fig. 5 is a partial view, partly in section, of a device for finishing gears constructed in accordance with the present invention by abrasion. The novel feature resides in the interrelation between the translatory and the rotary motion of the blank and in the specific means for securing such novel interrelation, as will be fully explained hereafter.

Fig. 6 and Fig. 7 are diagrammatic views of two different embodiments for securing the proper interrelation between the translatory motion of the blank relatively to an abrasive wheel, and the angular motion about the axis of the blank.

Fig. 8 is a diagram useful in further explaining the relation of translatory and rotary motion between a gear blank and a tool.

Fig. 9 is a diagram explanatory of a simple way of obtaining a suitable interrelation of translatory and rotary motion between a blank and a tool, for accurately forming pinions constructed in accordance with the present invention.

Fig. 10 and Fig. 11 are diagrams similar to Fig. 9 and explanatory of a preferred procedure in which both a translatory motion and a rotary motion are performed by the blank.

Fig. 12 is a view of a circular cam cooperating with a convex stationary element, illustrative of one embodiment for effecting the interrelation between rotary motion and translatory motion of the blank, as indicated in diagrams Fig. 10 and Fig. 11.

Fig. 13 and Fig. 14 are views of different positions of a circular cam cooperating with a concave stationary element, illustrative of another and often preferred embodiment for effecting an interrelation between rotary motion and translatory motion of the blank, as indicated in diagrams Fig. 10 and Fig. 11.

Fig. 15 is partly a view of an adjustable cam and a stationary element cooperating therewith, and partly a section along lines 15—15 of Fig. 16.

Fig. 16 is a front elevational view, partly in section, of an adjustable cam of symmetrical form cooperating with an adjustable stationary element of concave profile. The cam and stationary element shown in Fig. 15 and Fig. 16 are adapted to effect the desired motions for forming contours shaped in accordance with the present invention.

Fig. 17 is a view of a hollow cam and a stationary element of convex involute profile, illustrative of a modified way of obtaining contours in accordance with the present invention.

Fig. 18 and Fig. 19 are views of adjustable circular cams cooperating with stationary elements, showing said cams and stationary elements in adjusted positions corresponding to the production of gears of different diameter and constructed in accordance with the present invention.

In Fig. 1 the numerals 11 and 12 denote two equal gears meshing with each other and rotatable on parallel axes 13 and 14 respectively. The gears contain identical teeth 15, 16 having tooth profiles 17, 18. The pitch circles 19, 20 are shown to contact with each other in a point 21 hereafter referred to as the pitch point. A pair of tooth profiles is shown enlarged in a position 17′ and 18′ respectively, in which contact is made at said pitch point 21.

I shall now introduce the present invention gradually and first assume that the profiles 17′, 18′ are single circular arcs, centered at points 22, 23. Points 22, 23 are also the projections of the gear centers 13, 14 to the joint profile normal 24 at pitch point 21.

Tooth profiles 17′, 18′ of exactly circular form will now be demonstrated to fall very little short of transmitting true uniform motion, and to be acceptable in many cases, especially when the number of teeth of the gears 11 and 12 is comparatively large. It will then further be shown that a slight modification of the circular profiles is sufficient to provide a correction which makes the gear profiles accurate, so that true uniform motion may be transmitted by the gears. This correction, it will be seen, will keep the tooth profiles in simple form, which can be easily reproduced, and which by reason of its simplicity is suited to constitute a basic form of tooth, as will be further described.

The circular tooth profiles are also shown in dotted lines 17″, 18″ in a position, which corresponds to a different angular position of gears 11, 12. In this latter position of the gears, the profile centers occupy positions 22″ and 23″ respectively.

The angles 22—13—22″ and 23—14—23″ are seen to represent the turning angles of the gears, as compared with the position where contact is made at pitch point 21, and are assumed to be exactly alike. It is understood that corresponding turning angles of the gears 11 and 12 must be exactly alike, if the tooth profiles are mathematically accurate. On the other hand if the tooth profiles fall short of being mathematically accurate, and the gears are turned by exactly equal angles, then the tooth profiles will cease to touch each other in the new position of the two gears.

This indeed occurs when the assumed tooth profiles are true circular arcs, but it occurs in such small measure as will now be demonstrated. The two profiles 17″, 18″ are separated by a distance much too small to be illustrated, namely by a distance equal to the increase in length of distance 22″—23″ as com-

pared with distance 22—23, as will be readily understood by anyone skilled in the art or skilled in mathematics. In other words the two profiles 17″, 18″ are separated by a distance equal to the difference of distance 22″—23″ minus distance 22—23.

For convenience let symbol a denote the angle between normal 24 and a tangent 25 to the pitch circles at pitch point 21. Angle a is sometimes referred to as the pressure angle of the gear pair. Let further R denote the radius of the pitch circles 19, 20, and u the turning angle of the gears. Angle u is seen to be the same as angle 22—13—22″ and angle 23—14—23″ above referred to.

Radius 13—22, and also radius 14—23 are equal to R·cos a, and distance 22—23 equals 2R·sin a. It can be readily demonstrated that the projection of distance 22″—23″ to normal 24 is equal to distance 22—23 and therefore equal to 2R·sin a.

Further it is seen that the normal distance of center 23″ and of center 22″ from normal 24 or its elongation equals each R·cos a·(1− cos u), and that distance 22″—23″ can be resolved into a distance 2R·sin a and a distance 2R·cos a·(1−cos u) which are perpendicular to each other. Hence it is obvious that distance 22″—23″ may be expressed as the square root of the squares of said two distances which are perpendicular to each other. In mathematical terms:

Distance 22″—23″

$$=\sqrt{(2R\cdot\sin a)^2+[2R\cdot\cos a\cdot(1-\cos u)]^2}$$

$$=2R\cdot\sin a\cdot\sqrt{1+[\operatorname{cotan} a\cdot(1-\cos u)]^2}.$$

Total error E

= distance 22″—23″ − distance 22—23

$$=2R\cdot\sin a[\sqrt{1+[\operatorname{cotan} a\cdot(1-\cos u)]^2}\,[-1].$$

For turning angles u which are comparatively small, and not much exceeding say ten or fifteen degrees (10° or 15°) the expression within parentheses may be transformed to the closely approximate and simpler expression

$$\frac{(1-\cos u)^2}{2\tan^2 a}$$

so that the total error amounts to approximately:

$$E=R\frac{\cos a}{\tan a}\cdot(1-\cos u)^2$$

Herein the factor

$$\frac{\cos a}{\tan a}$$

is used in place of the equal factor

$$\frac{\sin a}{\tan^2 a},$$

which is directly obtained above.

Error E is understood to be the total error composed of the added errors of both profiles.

In an example $R=1.500''$; $a=20$ degrees; and turning angle $u=10$ degrees, the total error E may be computed as $E=.00089''$ or an error per gear of .00045″. A maximum error of the said character and magnitude is acceptable in many instances. The error is zero at zero turning angle and increases to the above maximum error at a turning angle of 10 degrees in either direction from zero turning angle. Truly circular profiles of the character and dimensions referred to are therefore suited to turn the gears 11 and 12 through an arc of 20 degrees with a maximum error as given above.

Furthermore a pair of gears, of which each member is conjugate to a pinion having circular tooth profiles as described, will contain errors of the above said character, although the tooth profiles of said gears usually are different from circular arcs.

It is therefore in many instances possible to base a system of gearing on a pinion having circular tooth profiles, and to make each gear of a pair of gears conjugate to said pinion.

Moreover, in accordance with the preferred embodiments of the present invention a correction is provided, which reduces or eliminates the small error existing on gears based on a pinion having circular tooth profiles of the character described, and which makes it possible to obtain a larger arc of action with less error or with no error. Advantages already existing are thereby amplified, as will be further explained hereafter.

Reference is made to Fig. 2, where a circular tooth profile 18 is indicated in dotted lines, and in a larger scale than was used in Fig. 1. The center 23 of said circular profile is again situated on the projection of the gear center to profile normal 24 at pitch point 21. Circular tooth profiles (18) provided on a pair of equal gears cease to contact with each other, when said gears are turned through equal angles, as has been explained with reference to Fig. 1. To maintain tooth contact in various angular positions of such gears, tooth profiles may be provided, which extend outside of said circular arc, on both sides of pitch point 21. A tooth profile of this character is denoted with numeral 27 in Fig. 2. This profile has a minimum curvature radius at pitch point 21 and gradually increasing curvature radii at profile points disposed on each side of pitch point 21. A curvature radius is understood to be the radius of a circle, which of all circles approximates the profile most closely adjacent a given point. This definition is in harmony with the common meaning of the term curvature radius or radius of curvature.

In a profile for transmitting true uniform motion the curvature radius at the pitch point 21 equals the radius of the aforesaid circular

profiles, and with the symbols above introduced equals ($R \cdot \sin a$). Point 23 is the curvature center of the profile adjacent pitch point 21.

The curvature radii of profile 27 at all points outside the pitch point are larger than the above said curvature radius of ($R \cdot \sin a$) and they increase at a rate which is faster, the larger the distance of the considered profile point is from pitch point 21. At a point 28 at the extreme tip of the tooth, and at a point 29 situated at the same distance from pitch point 21 adjacent the root of the tooth, the curvature radius is usually more than one and one half times the curvature radius ($R \cdot \sin a$) at pitch point 21.

Preferably profile 27 is made a curve symmetrical with respect to normal 24 at a point (21) intermediate the profile ends, usually with respect to the profile normal 24 at the pitch point.

There exists, of course, one curve which is symmetrical with respect to normal 24 at pitch point 21, and which transmits mathematically true uniform motion. Such curve might be described in relation with a coordinate system X, Y, of which the X axis is identical with the profile tangent at point 21, and of which the Y axis is identical with normal 24. A pair of coordinates x, y corresponding to such a system are indicated in Fig. 2.

However a large number of curves of the character described will have the same effect in practice, as the curve which is mathematically accurate. For it is understood that all that is necessary to fulfill the most exacting practical needs is to provide a little modification of a circular arc, a modifiction which amounts to a few thousandths of an inch at the ends of the active portion of the curve.

It is therefore sufficient to describe a few more characteristics of the curves which may be used.

Curves tangent to the X axis at the origin (21) of a coordinate system X, Y can be mathematically described as follows:

$$y = C_2 \cdot x^2 + C_3 \cdot x^3 + C_4 \cdot x^4 + C_5 \cdot x^5 + \cdots$$

in which C_2, C_3, C_4, C_5 and so on are constants.

When the curve is fully symmetrical with respect to the Y axis, which is identical to normal 24, the constants C_3, C_5, C_7 etc. are zero. In accordance with the present invention, at least constant C_3 is zero, so that the above equation can be written:

$$y = C_2 \cdot x^2 + C_4 \cdot x^4 + \cdots$$

Moreover it is found that constant C_4 should be negative and larger than C_2, to fulfill exacting requirements. Another way of expressing this mathematical statement is by saying that profile 27 should extend not only outside of its curvature circle 18 but also outside of the common parabola which contains the same curvature circle. The equation of the common parabola is known to be $y = C_2 \cdot x^2$, whereas the ordinates y of profile 27 are smaller, on account of the negative sign of constant C_4.

Fig. 3 further illustrates profile 27 by affording a comparison with an involute profile indicated by dotted lines 30. It is clearly seen how profile 27 extends outside of the involute profile on the tooth flank, and that it provides greater strength of the tooth. The involute proper ends at its base circle 31, and contains gradually reduced radii of curvature, the nearer a considered point of the involute lies to point 32, where the involute reaches the base circle 31. The curvature radius of the involute, at point 32, is known to be zero.

Small radii of curvature are known to be a drawback by causing much stress of the tooth surface when the teeth carry a given load. Moreover they precipitate wear.

Large radii of curvature, as afforded by the present invention, reduce the stress of the contacting tooth surfaces. They give therefore longer wear.

Wear depends also on relative sliding. In accordance with the present invention the curvature radii of the basic tooth profiles increase with increasing distance of the profile points from the pitch point. Sliding also increases with increasing distance of contacting points from the pitch point. By providing larger radii of curvature and smaller surface stresses at points which contain increased sliding, the present invention also evens up wear, that is to say it tends to make the wear more uniform and to conserve the exact tooth profiles.

A practical way of obtaining an excellent tooth profile 27 is to first assume a suitable symmetrical curve of the character described, which extends outside of curvature circle 18 (see Fig. 2), and which can be formed with simple mechanical means. The latter requirement is most important. Thereafter the profile is checked for errors. If the errors are not small enough, one may try another curve, for there is a large choice of curves fulfilling to the extent desired the kinematical requirements.

One such way of procedure will be further described hereafter.

A check which may be applied to a curve is indicated in Fig. 3.

In gear pairs, of which each member is conjugate to the same basic pinion, as is the case in the examples here described at length, the profile 27 of the basic pinion is subject to the following requirement:

Take on pitch circle 20 any two points 35, 36, which are equidistant from pitch point 21. Draw normals 37, 38 to the tooth profile 27 through said points 35, 36. If the

assumed profile answers the kinematical requirement with mathematical accuracy, then any two normals 37, 38 will intersect the pitch circle at equal angles. On the other hand the difference of the angles referred to is a measure of the departure from the theoretical profile.

Fig. 4 illustrates a pair of gears 40, 41 of which both are conjugate to the same basic pinion. The latter contains tooth profiles of the character described with reference to Fig. 2 and Fig. 3. In the instance illustrated gear 40 is identical with said basic pinion and contains tooth profiles 42, which are symmetrical with respect to the profile normal 43 at the pitch point. As explained, the curvature radii of said profiles are a minimum at said pitch point. The tooth profiles of gear 41 differ from the tooth profiles of gear 40, and may be determined, if need be, in the conventional manner from the profiles of the pinion. Both gears may be generated with a gear shaper cutter corresponding to said basic pinion.

Gear pair 40, 41 has a ratio between one to one and two to one, such as is frequently used for instance in transmissions of automotive vehicles. It is there desirable that the tooth shape permits grinding with a flat faced grinding wheel. This is indeed possible with gears formed in accordance with the present invention, as will be further described hereafter, for also the tooth profiles 44 of the larger gear 41 are wholly convex. They also contain a minimum curvature radius at a point intermediate their ends. A tooth profile 44 is however not symmetrical with respect to the profile normal at its pitch point. Profile 44 ends adjacent the root with a straight line element. In other words the end portion of tooth profile 44, namely the portion which engages the tip of the teeth of gear 40, is not curved at all, but contains a curvature radius of infinity. A profile 44 contains adjacent its root end curvature radii which gradually increase and approach infinity at point 45. The straight line element is disposed at an angle 46 with respect to a radius 47 drawn between the gear center and point 45.

Mesh between gears 40, 41 takes place along a line of action 49, which extends in polar symmetry with respect to pitch point 50.

Numeral 51 denotes a radius vector drawn from pitch point 50 to a point 52 of the line of action. Let symbol e denote the angle between said radius vector and the tooth normal at pitch point 50. Let further u denote the turning angle through which the basic pinion would have to turn to move the tooth contact from pitch point 50 to point 52 of the line of action, and let a denote the pressure angle as defined above. Angle e can then be expressed as follows, as will be understood by those familiar with mathematics:

$$e = E_2 \cdot u^2 + E_4 \cdot u^4 + \text{members of higher order.}$$

In this equation the angles e and u are to be introduced in radians, as is common in mathematics, and E_2 and E_4 are constants.

In accordance with the present invention preferably constant E_2 is kept between $\frac{1}{2}$ cotan a and $\frac{1}{4}$ cotan a, and constant E_4 is negative and larger than constant E_2. The basic pinion preferably contains a number of teeth not exceeding thirty.

Gears of the character described and formed in accordance with the present invention, and particularly gears with narrow face may be finished after hardening with a flat faced abrading wheel, as will now be described with reference to Fig. 5. Numeral 55 denotes an abrading wheel having a flat or plane working surface 56. Abrading wheel 55 is mounted on two bearings 57 and may be rotated in any suitable known manner. A gear blank 58 is mounted on a slide 59, which is movable in a direction perpendicular to axis 60 of gear blank 58, along guides provided on a bed 61. Slide 59 is continuously drawn in one direction by means of a wire or rope 65, to which a suitable weight is attached. The gear blank is given a combined rotary motion on its axis 60 and translatory motion perpendicular to its axis, as is quite usual in gear grinding. The desired interrelation between said rotary motion and said translatory motion of the blank is effected by a cam device consisting of a cam 62 cooperating with a stationary element 63, which is angularly as well as linearly adjustable. During a grinding pass, cam 62 is maintained in constant relation to gear blank 58. The connection between the cam and the gear blank is however disrupted for indexing, as readily understood. Gear blank 58 may be oscillated, and the cam device then imparts a corresponding translatory motion to it, so that the gear blank is moved past abrading wheel 55 in a manner that a tooth surface is successively engaged by said abrading wheel. In diagrammatic Fig. 5 a hand lever 64 is indicated as simplified means for effecting oscillation of the gear blank.

The only novel feature of the device indicated in Fig. 5 resides in the interrelation between the oscillatory motion and translatory motion of the blank, and in the cam device for effecting it. For this reason it was deemed sufficient to outline a very simple abrading device, and to enlarge now more on said motions and the cam device.

A simple cam device, as may be used in an abrading device of the character indicated in Fig. 5, will now be separately described with reference to Fig. 6. Cam 66 is maintained in constant relation with a blank during the pass of the blank past an abrading

wheel. Cam 66 cooperates with a stationary member 67 having a straight profile 68, which may be adjusted to any suitable angularity, and maintained stationary during the abrasive process. Let it be supposed at first that the working surface of the abrading wheel is a plane parallel to line 68. It is then evident to those familiar with mathematics that to produce a desired tooth contour, the contour of cam 66 must be a curve equidistant to said tooth contour, that is to say a curve obtainable from the tooth contour by plotting a constant distance on the normals of said tooth contour and by connecting the points so determined.

In Fig. 6 a gear blank having a center 70 is indicated by its pitch circle 71. In another position of the gear blank, where its center is at point 70′ and its pitch circle in the position denoted with numeral 71′, the cam has position 66′ indicated in dotted lines.

If so desired, the gear center may be given in addition a movement parallel to straight line 68, so that the gear center moves from a position 70 to a position 73 or to a position 73′. Such additional motion will not affect the angular position of the cam, and it only shortens or lengthens the distance covered by the gear center.

In the embodiment illustrated in Fig. 6 the gear center is made to move along a straight line (72). Fig. 7 illustrates an embodiment, in which the gear center 70 is made to move on a circle 74, in a closed path. Cam 66 engages a stationary element 75 having a straight profile 76. The profile of cam 66 is of the nature described with respect to Fig. 6, namely a line extending equidistant to the tooth profile 77 of a gear blank, which is indicated by a tooth profile and by its pitch circle 71. Tooth profile 77 is engaged by an abrading wheel 78, having a flat or plane working surface 78′ parallel to straight line 76. Another position of the cam and gear blank is indicated by position 70′ of the gear center and by a position 71′ of the pitch circle. In the position last referred to, the gear teeth are out of the reach of abrading wheel 78. In such a position the gear blank is disconnected from cam 66 and indexed. The gear center (70, 70′) is moved at a constant rate on its closed circular path 74, and abrading starts on a different tooth after the gear has come within reach of abrading wheel 78.

Gear forming processes so far described depend very much on the accuracy of reproduction of a computed form of tooth, or of a cam computed in accordance therewith.

Thus far it has been disclosed how theoretically improved gears of high accuracy may be obtained. I shall now proceed to show how the improved features can be actually and accurately effected in practice. In other words it will now be explained and shown, how tooth forms in accordance with the present invention, and how suitable cams useful in their production by abrasion, may be obtained with very simple mechanical means, which permit accurate reproduction at any time.

The first and main necessity is to be able to obtain the basic form of tooth with simple mechanical means. The production of a preferred basic form of tooth, or of a corresponding cam profile will now be explained.

In Fig. 8 numeral 14 denotes the center of an element or gear containing a circular profile 18 centered at point 23. Profile 18 contacts with the straight side 80 of an element 81, which is movable in the direction of line 82. In another position 18″ the circular profile contacts with position 80″ of the straight side of element 81.

It is evident that a plane working surface of an abrasive wheel forms a circular profile (18), when said abrasive wheel is moved in the same manner as element 81 with respect to a blank centered at point 14.

In order to form a profile of the character described and as indicated in dotted lines 83 and 83″, it is necessary to provide a slight correction of the translatory motion of the abrasive wheel or of element 81 which may represent it. Profile 83″ is seen to intersect side 80″ of element 81. It is evident that element 81 should be moved a little further, to make contact with profile 83″.

If on the other hand element 81 is moved further in such manner as to contact continuously with profile 83, or 83″, then a flat faced abrasive wheel substituted for element 81 will form the desired profile 83.

A suitable way of obtaining a correction of the motion of element 81 is indicated in Fig. 9. Element 81 is maintained in constant relation with point 84, which is connected by a link B with an arm A of a blank. Link B and arm A are connected with each other by a joint disposed at point 23. In the position 23″ of point 23, point 84 has moved to a position 84″, and side 80 of element 81 has moved to a position 80″, which is slightly advanced in the direction of motion as compared with the corresponding position in Fig. 8.

Let u be the turning angle 23—14—23″, and let w denote the angle between line 82 and link B, whose center line equals line 23″—84″.

With the above symbols the displacement of point 84 and of member 81 can be mathematically expressed as follows, as can be readily checked by anyone skilled in mathematics:

$$A \sin u + B (1 - \cos w).$$

The displacement obtained with circular

profile 18 in accordance with Fig. 8 can be expressed with the same symbols as:

$$A \cdot \sin u.$$

The difference of the said two displacements is therefore given by:

$$B \cdot (1 - \cos w).$$

Now angle w is a small angle defined by the relation:

$$\sin w = \frac{A \cdot (1 - \cos u)}{B},$$

as may be derived from Fig. 9 by employing the customary procedures. On account of angle w being small, the factor $(1 - \cos w)$ can be substituted by the expression $\frac{1}{2} \sin^2 w$.

With this substitution the term $B \cdot (1 - \cos w)$ becomes:

$$\frac{1}{2} \cdot \frac{A^2}{B} \cdot (1 - \cos u)^2.$$

This expression characterizes the difference of motion of element 81, in the motion defined by Fig. 9 and in the motion defined by Fig. 8, as effected by a circular profile.

The expression is also a measure of the difference between a circular profile and a profile as obtainable with a flat faced abrading wheel in motions corresponding to Fig. 9.

It should be noted that the above expression is of the same character as the expression derived for the total error E of truly circular tooth profiles, the variable u appearing in the same form $(1 - \cos u)^2$ in both cases. It is therefore understood that the profile obtainable with motions as indicated in Fig. 9 differs from a true circular profile by amounts, which may be selected to just make up for the said computed error of the circular profile.

One feature should be particularly noted, namely the change of the ratio between the rotary motion of arm A and the translatory motion of point 84 and element 81. It can be readily demonstrated that in any position the instantaneous ratio between the translatory motion and the rotary motion is given by a changing leverage (L), which in Fig. 9 is equal to the distance between point 14 and the intersection point between line 14—23 and line 23″—84″. In other words an element of the translatory motion equals an element of the rotary motion multiplied by said leverage (L).

In the central position of the device, Fig. 9, the said leverage (L) is equal to the length or arm A. In any angular position of arm A adjacent said central position the said leverage (L) is smaller than the length of arm A, as is readily seen. Translation is therefore effected at a changing rate, which is a maximum in the central position shown in Fig. 9.

Similarly, tooth profiles of the character described may be formed in a process in which the blank performs a translatory and a rotary motion, whereby the translatory motion contains a maximum ratio as compared with the rotary motion, when an intermediate point of the tooth profile is being formed.

In principle it does not make any difference which motion is performed by a tool and which motion is performed by the blank, as long as the relative motions are the same. In Fig. 10, and in the following figures, it is supposed that a translatory motion in direction of line 86 as well as a rotary motion on center 14 are both imparted to the blank, which is maintained in unison with arm A during the abrasive passes. A flat faced abrasive wheel 87 is shown in engagement with the pitch portion of the tooth 88 of a gear blank. The plane working surface of the abrading wheel is here perpendicular to the direction of motion of the blank, that is to say to line 86.

The same linkage A and B as explained with respect to Fig. 9 connects the gear blank with a point 84, which is maintained in constant relation to the working face of abrasive wheel 87, and which is here stationary. The center of the blank is shown in a further position 14″, to which corresponds a position 23″ of the joint between arm A and link B.

It can be readily demonstrated, that in the arrangement illustrated by Fig. 10 the following relations exist, when the above defined symbols are kept:

$$A = R \cdot \cos a.$$

Moreover,

$$B = A \cdot \tan a,$$

or between $(A \cdot \tan a)$ and $2(A \cdot \tan a)$, depending on the effect desired. It is often desirable to modify the tooth profiles slightly adjacent their ends, as is well known.

In the arrangement indicated in Fig. 11, the plane working face of abrasive wheel 87 is set to an angle such as the above defined angle a. In this case arm A equals radius R. To obtain the same effect as in the arrangement shown in Fig. 10, length B is selected in the same proportion to length A as there.

B is preferably selected between $A \cdot \tan a$ and $2A \cdot \tan a$.

The same relations between A and B may also be used for forming master cams corresponding to the basic form of tooth, as the profile of such cams is simply a line equidistant to a tooth profile, which requires the same generating motions or forming motions as the tooth profile.

The motions described with reference to Figures 9 to 11 can be obtained by providing actual linkage, or preferably they are obtained with eccentrics or cams of circular form, as will now be described.

In Fig. 12 a linkage of the character just

described is indicated by auxiliary lines A and B. The turning center of a blank is shown in a position 14″, and the joint which connects arm A with link B in a position 23″. The means used for effecting the position prescribed by the linkage are a stationary circular element 91, or roller 91, and a cam or eccentric 92. Element 91 has a fixed center 84, which coincides with the stationary point of the aforesaid linkage.

The circular profile of cam 92 is centered at point 23″. On account of the circular profiles centered at points 84 and 23″ respectively, it is evident that point 23″ is kept at a constant distance from point 84, provided only that the said circular profiles are maintained in contact with each other. The said distance is equal to the sum of the profile radii, as is readily understood. Furthermore, point 23″ is kept at a constant distance from turning center 14″, as said center also forms the turning center of cam 92.

It is therefore seen that the circular cam 92 and the stationary element 91 constitute very simple means which may accurately replace the aforesaid linkage.

The central position 92′ of cam 92 is indicated in dotted lines in Fig. 12.

Fig. 13 and Fig. 14 illustrate a slightly modified embodiment for obtaining the desired motions by means of circular cams cooperating with circular elements. The linkage which prescribes the motions is precisely the same as in Fig. 12. The means used for effecting the motions differ not only in the use of an adjustable cam 93, but also in the use of a concave profile 94 on the stationary element, which cooperates with the circular profile of cam 93. Profile 94 is a circular arc centered at point 84. Cam 93 is adjustable readily along straight guides 95 of an arm rigidly secured to shaft 96, to which also the blank is secured. It is noted that the provision of a concave profile 94 permits to obtain cam surfaces of large diameter and to reduce surface stresses and wear through the cam contact. The fact that the profiles used on cam 93 and on the stationary element are circular, and broadly are symmetrical curves, facilitates very much their accurate production. If so desired, a circular roller centered at point 23″ may be used in place of circular cam 93. In the following and in the claims rollers are included in the broad term cam.

Fig. 15 and Fig. 16 illustrate an adjustable cam engaging an adjustable stationary element, such as is useful for finishing by abrasion gears constructed in accordance with the present invention.

Cam 98 may be adjusted radially of shaft 99 along tapered sides 100 of an arm 101, which is integral with shaft 99. A screw 103 serves to clamp cam 98 to arm 101 in any suitable position. Cam 98 contains a circular profile and engages a hollow stationary element 104. The cam surface 105 is a conical surface engaging a hollow conical surface 106 of element 104. Element 104 is secured to a holder 108, which may be adjusted along a base 109. Element 104 may furthermore be adjusted upwardly and downwardly on holder 108, and fixed in any desired position after introducing a spacing block 110 of suitable thickness. Element 104 may further be adjusted on holder 108 in the direction of shaft 99. A suitable spacer 111 is used to obtain a desired position lengthwise of shaft 99. The last said adjustment, together with an adjustment of holder 108, serves to change the relation between the parallel center lines of the conical surfaces 105 and 106. In other words it serves to change the aforesaid distance B.

A cam device, having adjustments as indicated in Fig. 15 and Fig. 16 may be used in forming by abrasion gears formed in accordance with the present invention, as well as other gears and cams.

Fig. 17 points out a cam arrangement for obtaining basic tooth forms and master cams in a modified way. Cam 112 is secured to shaft 113, to which also a blank is secured. Cam 112 contains a hollow circular profile 114, which engages a convex stationary element 115. In the instance illustrated the convex profile 116 of element 115 is an involute having a base circle 117.

Once the basic tooth form is mechanically determined, other tooth forms may be obtained for instance through generation with a pinion shaped cutter containing said basic tooth form. In this operation the cutter is reciprocated and the cutter and blank roll upon each other in the manner of two intermeshing gears. It is of course understood that a pinion to be used as a cutting tool should be provided with suitable relief, so that only the cutting edges engage a blank. Such relief may be provided in known manner.

Tooth forms of gear pairs of comparatively small ratios may furthermore be directly obtained with an abrasive device of otherwise known character, by introducing there an adjustable cam device of the character described with respect to Figures 14 to 16, and by suitably adjusting said cam device.

Fig. 18 illustrates a cam device set for the generation of a gear of larger diameter than a basic pinion of the character described in detail; and Fig. 19 illustrates a device set for the generation of a gear of smaller diameter than the basic pinion. The pitch circles of the respective gears are denoted with numerals 120, 121. In Fig. 18 and Fig. 19 a circular cam is denoted with numeral 122, and a hollow stationary member engaged by said cam is denoted with numeral 125. An abrasive wheel is indicated in dotted lines 126. Its plane working surface is inclined

to the aforesaid angle a and in the indicated position of the gear blank, contact between the abrasive wheel and said gear blank is made at the pitch point of the tooth profile of the gear blank.

To form gears constructed in accordance with a preferred embodiment of the present invention, the ratio between the translatory motion of the gear blank and its rotation should be a maximum when abrasive contact is made at the pitch point of a tooth profile of the gear blank, as hereinbefore explained. In other words the leverage (L) which characterizes said ratio should be a maximum when grinding contact is made at the pitch point of a tooth profile, and it should be reduced in adjacent positions of the gear blank, at a rate which will now be described.

For convenience, let R′ denote the pitch radius of a gear constructed in accordance with a preferred embodiment of the present invention, and let u' denote the turning angle of said gear as compared with the angular position of the gear in which abrasive contact is made at the pitch point of a tooth profile. Further let as herein before R denote the pitch radius of a basic pinion characterized by having a minimum radius of curvature at the pitch point of its tooth profile; and let u denote the turning angle of said basic pinion corresponding to turning angle u' of the aforesaid gear. In other words let u be defined as follows:

$$u=u'\cdot\frac{R'}{R}$$

It can be demonstrated with the known methods of mathematics that to obtain the tooth characteristics explained, the leverage L should be reduced with increasing angle u' or u as follows, u being introduced in radians (as the length of an arc measured on a circle of a radius one):

$$L=L_0(1-\tfrac{1}{2}u^2)+\ \ldots$$

wherein the dots indicate members of a higher order in u, for instance members containing the factor u^3, or u^4, or u^5 etc.

Attention should be focussed especially on the coefficient ½ of the member containing u^2, which characterizes the rate at which the leverage L is reduced at small angles u and u', and also in the average. Moreover it is found that leverage L is reduced more rapidly when grinding the flank portion of a tooth profile than when grinding its tip portion. Further characteristics may be computed with the known methods of mathematics with the directions given.

In Fig. 19 numeral 130 denotes the intersection point between the elongation 131 of arm A and a line 132, which is drawn through stationary point 133 perpendicular to the direction 134 of motion of center 135 of the gear blank. Point 136 is the intersection between line 137—133 and a line drawn through gear center 135 perpendicular to line 134. Line 138 is drawn through point 136 perpendicular to line 137—133. It should be noted that line 138 passes through the aforesaid point 130. It can be demonstrated mathematically that this characteristic expresses the requirement that the ratio between the translatory motion and the rotary motion of the gear blank is a maximum in the position of the cam device as indicated in full lines.

Other positions are indicated in Fig. 18 and Fig. 19 by the position of the linkage only.

It is found that gear pairs of comparatively small ratio may be formed very accurately with the use of a simple cam device as described.

While in the examples chosen for explanation spur gears were particularly referred to, it is understood that the invention is not confined to spur gears, but is equally applicable to gears mounted on parallel axes and to gears mounted on intersecting axes, that is to say to gears whose pitch surfaces roll on each other without sliding, and also to other gears. The teeth may be straight or curved and extend along generatrices of the pitch surfaces or at any angles thereto. Moreover the invention is not confined to gears, but is applicable broadly to gear shaped articles, including particularly gear shaper cutters.

Various modifications and changes may be made in my invention without departing from its spirit, by simply applying the customary practice and established knowledge of the art.

For definition of the scope of the invention, it is relied on the annexed claims.

What I claim is:

1. A gear shaped article, containing a plurality of equal tooth like projections, the side profiles of said projections being wholly convex and having a minimum radius of curvature at a point intermediate the ends of a profile, said profile being substantially symmetrical with respect to the profile normal at said point.

2. A gear shaped article, containing a plurality of equal tooth like projections, the side surfaces of said projections being so shaped as to contact along a line with a plane tangent to a side surface, the profile of a side surface being wholly convex and having a minimum radius of curvature at a point intermediate the ends of a profile, said profile being substantially symmetrical with respect to the profile normal at said point.

3. A gear shaped article, containing less than twenty five equal tooth like projections, the side profiles of said projections being wholly convex and having a minimum radius of curvature at a point intermediate the ends

of a profile, said profile being symmetrical with respect to the profile normal at said point.

4. A gear shaped article, containing a plurality of equal tooth like projections having convex side profiles in a plane perpendicular to the axis of said article, said profiles containing a minimum curvature radius at a mean profile point, the curvature radius at the root ends of said profiles being at least fifty percent larger than said minimum curvature radius.

5. A gear having a convex tooth profile with a minimum curvature radius at the pitch point of said profile, the curvature radius at the root end of a profile exceeding said minimum curvature radius by at least fifty percent.

6. A gear of less than twenty five teeth, having convex active tooth profiles, said profiles being symmetrical with respect to the profile normal at the pitch point.

7. A gear of less than twenty five teeth, having convex active tooth profiles, said active tooth profiles having a minimum curvature radius at their pitch points.

8. A gear of less than twenty five teeth, having convex active tooth profiles, said active tooth profiles having a minimum curvature radius at a point intermediate their ends, the curvature radius at the root end of an active tooth profile being at least fifty percent larger than said minimum curvature radius.

9. A pair of gears whose pitch surfaces roll on each other without sliding, containing a line of action of continuously and gradually changing curvature, said line of action having a point of inflection adjacent its pitch point and corresponding to an equation $e=E_2 \cdot u^2+E_3 \cdot u^3+E_4 \cdot u^4+$ members of higher order in u, wherein e and u are angles measured in radians, e denoting the angle included by the tangent to the line of action at said point of inflection with a radius drawn from said point of inflection to a point of the line of action, u referring to the smaller gear of said pair and denoting its turning angle which corresponds to the gear action between said point of inflection and said point of the line of action, and wherein E_2, E_3, E_4 are constants, E_2 being larger than one.

10. A pair of gears having straight or helical teeth, containing a line of action of continuously and gradually changing curvature on its whole length, said line of action extending in polar symmetry with respect to its pitch point in such manner as to furnish a minimum pressure angle at said pitch point.

11. A pair of gears having straight or helical teeth, containing a line of action of continuously and gradually changing curvature, said line of action having a point of inflection at its pitch point and extending in polar symmetry with respect to said pitch point, so as to correspond to an equation

$$e=E_2 \cdot u^2+E_4 \cdot u^4+$$

members of higher order in u wherein e and u are angles measured in radians, e denoting the angle included by the tangent to the line of action at said pitch point with a radius drawn from said pitch point to a point of the line of action, u referring to the smaller gear of said pair and denoting its turning angle which corresponds to the gear action between said pitch point and said point of the line of action, and wherein E_2, E_4 are constants, of which E_4 is negative and more than twice as large as E_2.

12. A pair of gears of which each member is conjugate to the same basic pinion, said basic pinion having a tooth profile which is symmetrical with respect to the profile normal at the pitch point of said profile.

13. In a pair of gears of a ratio between two to one and one to one, a larger gear having convex active tooth profiles ending with a straight line element adjacent the roots of the teeth, said straight line element being disposed at an angle to a radius drawn through the gear center, and a smaller gear having convex active tooth profiles containing a minimum radius of curvature at a point intermediate the ends of said profiles.

14. A pair of gears mounted on parallel axes, each gear of said pair being conjugate to the same basic pinion, said basic pinion having a convex tooth profile with a minimum curvature radius at its pitch point, and said profile being symmetrical with respect to the profile normal at said pitch point.

15. A pair of gears mounted on parallel axes, each gear of said pair being conjugate to a basic pinion, said pinion having a convex tooth profile with a minimum curvature radius at a point intermediate the profile ends, and said profile being symmetrical with respect to the profile normal at said point.

ERNEST WILDHABER.