Aug. 30, 1949. H. WALLMAN 2,480,205

STAGGER DAMPED TUNED AMPLIFIER

Filed Sept. 14, 1945

Fig. 1.

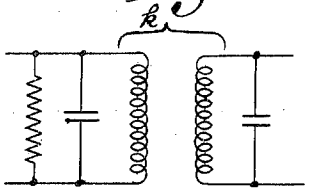


Fig. 2.

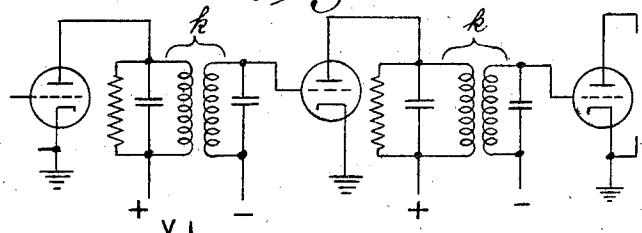


FIG. 3

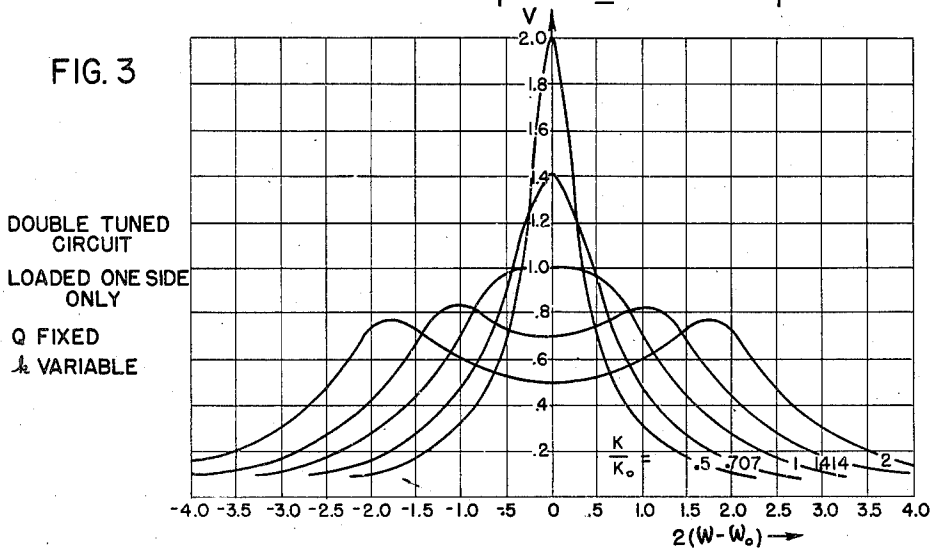


FIG. 4

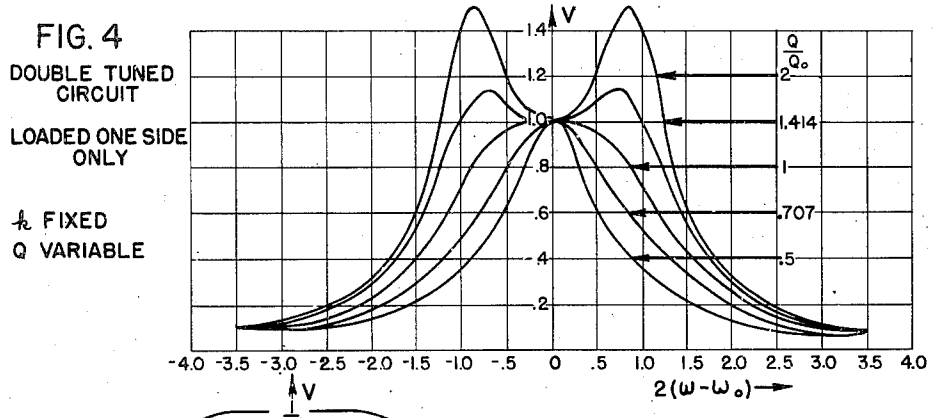
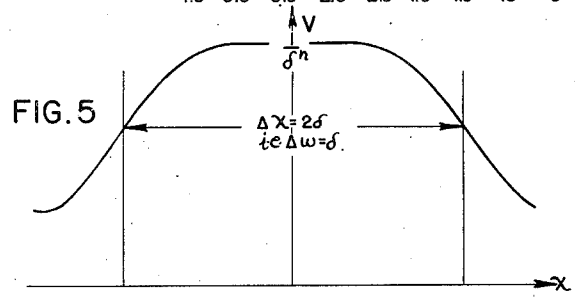


FIG. 5



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UNITED STATES PATENT OFFICE

2,480,205

STAGGER DAMPED TUNED AMPLIFIER

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Application September 14, 1945, Serial No. 616,371

5 Claims. (Cl. 179—171)

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This invention relates in general to electrical circuits and more particularly to amplifiers containing a plurality of stages.

Amplifiers that are used to amplify pulses so as to maintain the same shape in the output voltage as in the applied voltage must amplify a wide band of frequencies with very nearly uniform gain throughout the band. When amplifiers must have high gain characteristics it is difficult to maintain wide bandwidth characteristics as the two functions are usually related in such a manner that if one is increased, the other is decreased. Furthermore, it can be shown that the problem becomes even more difficult when multiple stages are cascaded. The gain of a multistage amplifier consisting of n identical stages is the gain of one stage to the nth power. It can be also shown that the overall bandwidth of n stages of a single-tuned amplifier is

$$\sqrt{2^{\frac{1}{n}}-1}$$

times the bandwith of one stage. Whereas, the overall bandwith of n stages of a double-tuned amplifier is

$$\sqrt[4]{2^{\frac{1}{n}}-1}$$

times the bandwidth of one stage. These formulas indicate that the bandwidth of two cascaded stages of a single-tuned I. F. amplifier would have only .643 times the bandwidth of one stage and that two cascaded stages of a double-tuned I. F. amplifier would have .803 times the bandwidth of one stage.

In certain types of radio object locating devices, it is necessary to employ receivers that will faithfully detect and amplify pulses having a duration of the order of one-tenth microsecond and the amplification must be extremely large. It is common practice under these conditions to use a receiver having a large number of I. F. stages. There is thus presented the problem of obtaining high-gain characteristics simultaneously with wide band characteristics.

It is therefore an object of this invention to provide a means for obtaining wide bandwidth in an amplifier and simultaneously maintaining high gain per stage and overall amplification. It is further an object of this invention to provide a method of choosing the values of the elements

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employed in the circuit so that the gain and bandwidth desired may be realized with a minimum amount of experimentation and adjustment.

Other objects, features, and advantages of this invention will suggest themselves to those skilled in the art and will become apparent from the following description of the invention taken in connection with the accompanying drawings in which:

Fig. 1 is a diagram of a double-tuned circuit loaded on one side only;

Fig. 2 is a diagram of a double-tuned circuit illustrating its use as a coupling network between successive vacuum tubes;

Fig. 3 is a graph showing certain relative selectivity curves for a double-tuned circuit loaded on one side only;

Fig. 4 is a graph showing other relative selectivity curves having a double-tuned circuit loaded on one side only; and

Fig. 5 is a graph of a typical selectivity curve.

Referring now more particularly to Fig. 1 which shows a double-tuned circuit loaded on one side only, resistor R_p represents the equivalent parallel resistance of the primary circuit. Capacitor C_p represents the equivalent parallel capacitance of the primary circuit. Inductance L_p represents the equivalent parallel inductance of the primary circuit and k is the coefficient of coupling between the primary circuit and the secondary circuit. Inductance L_s represents the equivalent parallel inductance of the secondary circuit and capacitor C_s represents the equivalent parallel capacitance of the secondary circuit.

It can be shown that the voltage across the secondary capacitor (C_s) divided by the voltage applied in series with a primary circuit may be expressed as

$$\frac{-1}{\gamma^2}\sqrt{\frac{L_s}{L_p}}\frac{k}{\left[k^2+\frac{1}{Q_pQ_s}-\left(1-\frac{1}{\gamma^2}\right)^2+j\left(1-\frac{1}{\gamma^2}\right)\left(\frac{1}{Q_p}+\frac{1}{Q_s}\right)\right]} \quad (A)$$

where

$\omega = 2\pi$ times frequency

$\omega_0 = 2\pi$ times resonant frequency

$\gamma = \frac{\omega}{\omega_0} = \frac{\text{actual frequency}}{\text{resonant frequency}}$

$Q_p = \frac{\omega L_p}{R_p}$ for primary circuit

$Q_s = \frac{\omega L_s}{R_s}$ for secondary circuit

$k = \frac{M}{\sqrt{L_pL_s}}$ = coefficient of coupling

L_p=total inductance of primary circuit

L_s=total inductance of secondary circuit

Equation A is a basic equation for a double-tuned circuit and appears on page 155 of the "Radio Engineer's Handbook" by Frederick E. Terman, published by McGraw-Hill Book Co., Inc., in 1943.

To simplify mathematical computation the circuit of Fig. 1 is considered when $C_p=1$, $L_p=1$, $C_s=1$, and $L_s=1$,

When

k=coefficient of coupling

then

$$\omega_o=\frac{1}{\sqrt{L_pC_p}}=\frac{1}{\sqrt{L_sC_s}}=1$$

$Q_p=Q_{\text{primary}}=\omega_oR_pC_p=R_p$

$Q_s=Q_{\text{secondary}}=\infty$

Assuming Q_p to be large and that one ampere is applied through the primary L_p, and also making the approximation that

$$1-\frac{1}{\gamma^2}=1-\frac{1}{\omega^2}=\frac{\omega^2-1}{\omega^2}=\frac{(\omega-1)(\omega+1)}{\omega^2}=2(\omega-1)$$

Equation A may be reduced to

$$E(x)=\frac{k}{k^2+\frac{jx}{Q_p}+(jx)^2} \quad \text{where } x=2(\omega-\omega_o) \qquad (B)$$

It is observed that the secondary voltage at the bandwidth center is entirely independent of Q_p and depends only on the coefficient of coupling k. The absolute value of Equation B is

$$|E(x)|=\frac{k}{\sqrt{k^4+x^2\left(\frac{1}{Q_p^2}-2k^2\right)+x^4}} \qquad (C)$$

When

$$k=\frac{1}{(Q_p\sqrt{2})}$$

the stages are transitionally-coupled, i. e., the curve is flat-topped, and

$$|E(x)|=\frac{k}{\sqrt{k^4+x^4}} \qquad (D)$$

Considering the bandwidth to be the distance between the half-power points, or the points 3 decibels less than the maximum,

$$\Delta x=2k, \text{ or } \Delta\omega=k, \text{ i. e., } k=\frac{\Delta\omega}{\omega_0}, \text{ or}$$

coefficient of coupling=

$$\frac{\text{bandwidth at transitional coupling}}{\text{center frequency}}$$

The voltage at the center frequency is

$$\frac{1}{k}$$

hence, the product of voltage times $\Delta\omega=1$ for transitional coupling. But the total capacity of the circuit is 2; this leads to the well-known result that the gain times bandwidth product of a double-tuned circuit, loaded on one side only, at transitional coupling is twice that of a single-tuned circuit.

For fixed Q, transitional coupling is illustrated by

$$k=k_0=\frac{1}{Q\sqrt{2}}$$

Fig. 3 shows relative selectivity curves for k_o=.5, .707, 1, 1.414, 2. The ordinate value 1 denotes a voltage of $Q\sqrt{2}$ and the abscissa value 1 denotes a value for x of

$$\frac{1}{(Q\sqrt{2})}$$

This condition is in contrast with the familiar behavior of equal-Q double-tuned circuits. It is seen that maximum secondary voltage does not occur at transitional coupling, nor does the height of the peaks remain constant upon overcoupling.

For fixed k, transitional coupling is yielded by

$$Q=Q_0=\frac{1}{(k\sqrt{2})}$$

Fig. 4 shows relative selectivity curves for

$$\frac{Q}{Q_0}=.5, .707, 1, 1.414, 2$$

The ordinate value 1 denotes a voltage of

$$\frac{1}{k}$$

and the abscissa value 1 denotes a value for x of k.

It is also noted that with k fixed and Q variable the voltage at center-frequency is constant; this is indeed evident from Equation B.

The basic idea of this invention is to have successive coupling stages of an I. F. amplifier identical except for their damping resistors so that some will be overcoupled and some undercoupled. This method of arranging a plurality of amplifying stages is here termed "stagger-damping."

It is often thought that getting the maximally flat selectivity curve shown in Fig. 5 having the form

$$\frac{\delta^n}{\sqrt{\delta^{4n}+x^{4n}}}$$

is desirable, i. e., there is something inherently desirable in a flat selectivity curve. One intrinsic value of a curve of this form is its relatively simple mathematical character. It is here adopted as a means to an end, the end being the high stage gain times overall bandwidth.

By analysis, it can be shown that

$$\frac{\delta^n}{\sqrt{\delta^{4n}-x^{4n}}}$$

is the absolute value of the product

$$\frac{\delta}{\delta^2+jx2\delta\sin\frac{\pi}{4n}+(jx)^2}\cdot\frac{\delta}{\delta^2+jx2\delta\sin\frac{3\pi}{4n}+(jx)^2}\cdots\frac{\delta}{\delta^2+jx2\delta\sin\frac{(2n-1)\pi}{4n}+(jx)^2} \qquad (E)$$

Each factor of Equation E has the form of Equation B, thus the first factor of Equation E, for example, may be realized by a double-tuned circuit loaded on one side only and satisfying

$$k=\delta$$

$$Q=\frac{1}{\left(2\delta\sin\frac{\pi}{4n}\right)}$$

Therefore when considering each successive stage of a stagger-damped amplifier having n serially connected stages, the Q for each successive stage should satisfy the relationship

$$Q=\frac{1}{2k\sin\frac{y\pi}{4n}}$$

where y for each successive stage equals a term in an arithmetic series of odd integers beginning with 1 for the first stage and ending with $(2n-1)$ in the nth stage.

Observe the following two very convenient features of this stagger-damping scheme:

(*a*) All the transformers have the same coefficient of coupling, which means that only one transformer type is needed throughout and the only circuit difference from one stage to another is in the value of the shunting resistor.

(*b*) The coefficient of coupling is the ratio of desired overall bandwidth to center frequency.

While there has been described hereinabove what is at present considered to be a preferred embodiment of this invention, it will be obvious to those skilled in the art that various changes and modifications may be made therein without departing from the scope of the invention as set forth in the appended claims.

The invention claimed is:

1. In an electronic amplifier a plurality of n serially connected stages, each of said stages being tuned to a resonant frequency by means of two tuned coupled resonant circuits one of which is loaded, each of said stages having a coefficient of coupling k between said resonant circuits, said loading of each stage having the value such that the quality factor, Q, which is equal to $\omega_0 RC$, is approximately equal to

$$\frac{1}{2k \sin \frac{y\pi}{4n}}$$

with y equal to an odd integer, beginning with one and ending with $(2n-1)$, ω_0 is equal to 2π times the resonant frequency of said circuits, R is the equivalent shunt resistance in said loaded circuit and C is equal to the equivalent shunt capacitance of said loaded circuit.

2. An electron amplifier including n cascaded stages each of said stages being tuned to a resonant frequency f_0 by means of two resonant circuits, one of which is loaded, the coefficient of coupling k for each of these stages being equal to δ where δ is the ratio of the overall bandwidth of said amplifier in cycles per second to f_0 in cycles per second, and the quality factor of each of said stages is equal to

$$\frac{1}{2k \sin \frac{y\pi}{4n}}$$

where y is a different value for each stage, said values being a series of successive odd integers, beginning with one and ending with $(2n-1)$,

said amplifier having an overall selectivity voltage curve of the form

$$\frac{\delta^n}{\sqrt{\delta^{4n}+X^{4n}}}$$

where $X=2(\omega-\omega_0)$, $\omega=2\pi$ times the frequency of the input signal to said amplifier in cycles per second and $\omega_0=2\pi f_0$.

3. In an electronic amplifier a plurality of n serially connected stages, each of said stages being tuned to a resonant frequency by means of two tuned coupled resonant circuits, each of said stages having a coefficient of coupling k between said resonant circuits, said loading of each stage having a value such that the ratio of the quality factor for that stage to the quality factor that would give transitional coupling is

$$\frac{\sqrt{2}}{2 \sin \frac{y\pi}{4n}}$$

where y is a series of successive odd integers, beginning with one and ending with $(2n-1)$.

4. In an electronic amplifier a plurality of serially connected double tuned amplifier stages, the coupling between successive stages being a constant determined by the ratio of the desired bandwidth at transitional coupling to the desired center frequency, one of the resonant circuits in each of said stages being loaded so that the damping of each stage has a different value, the damping of some of said stages being greater than the value of damping necessary to give transitional coupling and the damping of the remainder of said stages being less than the value of damping necessary to give transitional coupling.

5. An amplifier in accordance with claim 4 wherein the damping of each of said stages is a function of the number of stages in said amplifier and the damping of each successive stage differs by the reciprocal of the sine function of the odd integers of an arithmetic series.

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