United States Patent

Venkatesan

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[45] Jan. 18, 1972

[54] PRESSURE VESSEL

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[52] U.S. Cl. **425/77,** 92/171, 100/269, 100/295, 264/314, 425/406

[51] Int. Cl. **B29c 3/00,** B30b 1/32

[58] Field of Search ..... 100/269, 295; 18/16.5, 16 R, 18/34 R, DIG. 26, DIG. 35; 264/314; 25/128 D; 92/169, 171; 192/88 B, 85 AT

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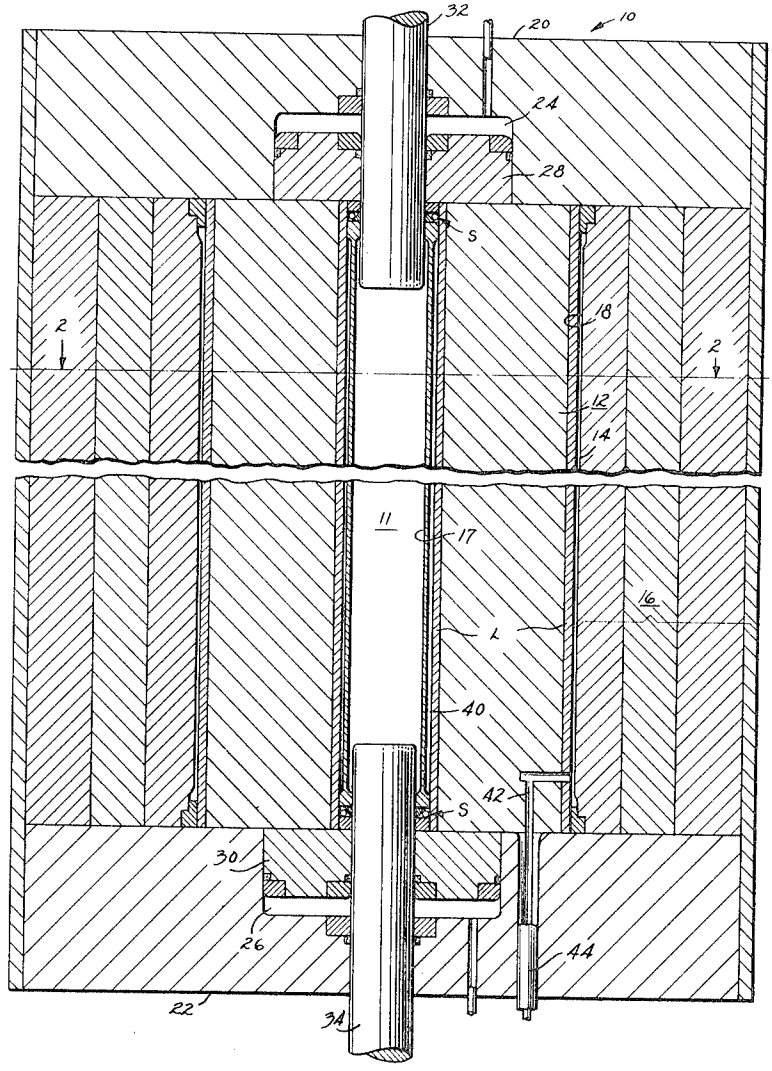
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[57] **ABSTRACT**

A pressure vessel providing more efficient use of constituent material for containing fluid pressurized to a predetermined operating level, and being provided with infinite cycle life. The vessel may include an array of radially disposed segmented blocks supplied with compressional axial and tangential support forces such that when the contained pressurized fluid is pressurized to its operating level, the working stress across the array of segmented blocks does not exceed the fatigue or endurance stress of the material of which the blocks are made, and/or the vessel may include a multiring assembly wherein the radial interferences between adjacent rings are chosen such that at each radius except the outermost, the working stresses are made substantially equal to each other and made equal to, or below, the fatigue or endurance stress of the material of which the rings are made.

28 Claims, 5 Drawing Figures



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SHEET 1 OF 4

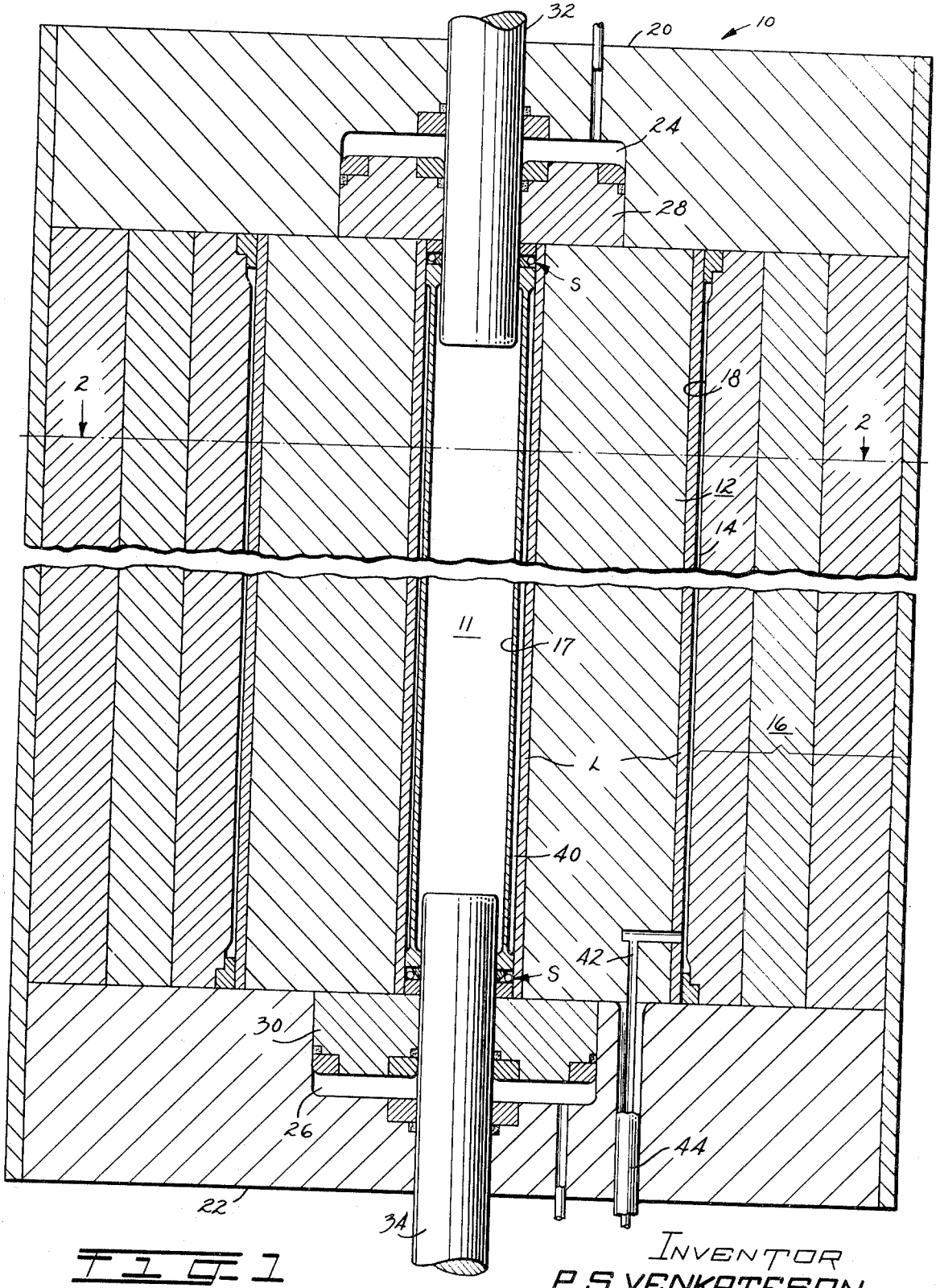


FIG. 1

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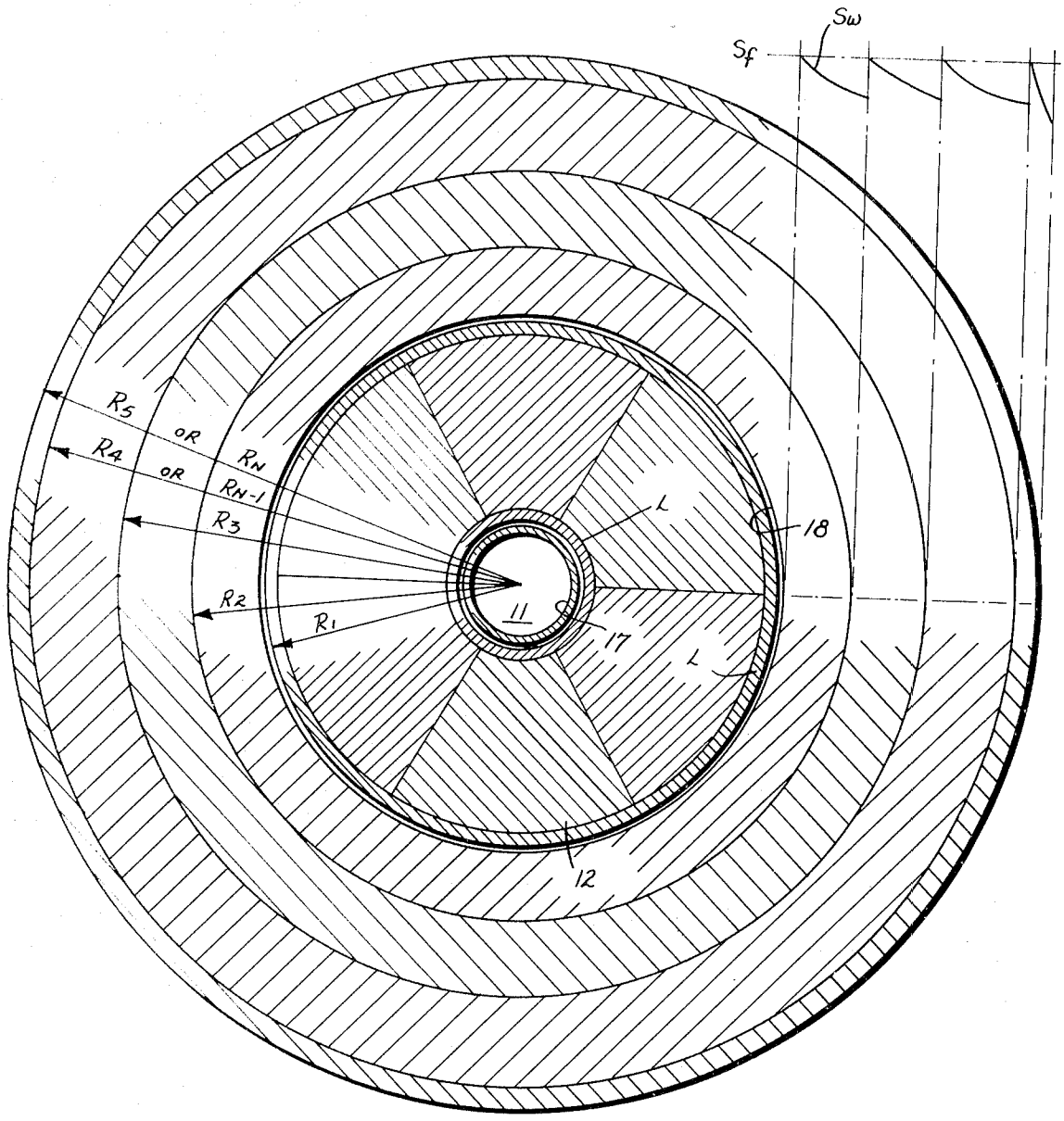


FIG. 2

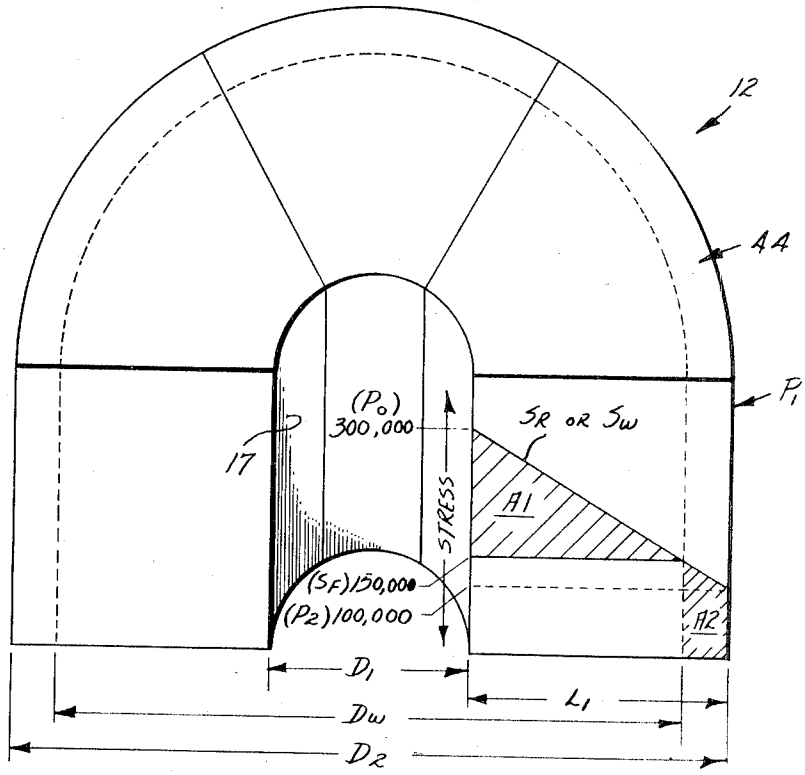

12
44
P_1
(P_o)
300,000
17
STRESS
S_R OR S_W
A1
A2
(S_F)150,000
(P_2)100,000
D_1
L_1
D_W
D_2

FIG 3

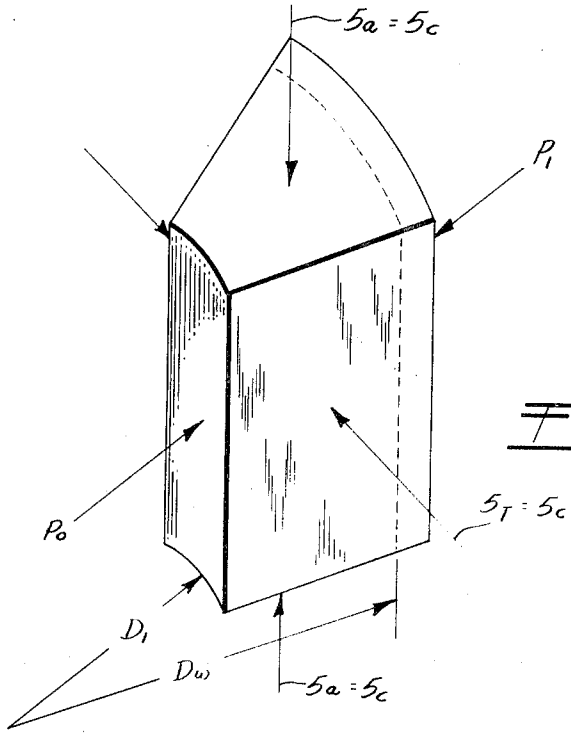

$S_a = S_c$
P_1
P_o
$S_T = S_c$
D_1
D_W
$S_a = S_c$

FIG 4

BORE PRESSURIZED

S_W = 150000.0

X = 0.4000

R 1 = 0.50000	S_{R_1} (P_1) = 150000.0	S_{T_1} = −0.0
R 2 = 0.70000	S_{R_2} = 113265.3	S_{T_2} = 56849.6
R 3 = 0.98000	S_{R_3} = 71604.5	S_{T_3} = 100779.6
R4 = 1.37200	S_{R_4} = 29388.0	S_{T_4} = 133131.1
R5 = 1.71722		

FIG 5

BORE DEPRESSURIZED

S_{T_1} NO LOAD = −177789.6

S_{T_2} NO LOAD = −40665.0

S_{T_3} NO LOAD = 44221.6

S_{T_4} NO LOAD = 97469.4

S_{R_2} NO LOAD = 43540.298519 S_{R_2} INTERF = 15175.833097 DELTA 2 = 0.002426

S_{R_3} NO LOAD = 42836.132563 S_{R_3} INTERF = 24483.619279 DELTA 3 = 0.004254

S_{R_4} NO LOAD = 21515.867466 S_{R_4} INTERF = 21515.867466 DELTA 4 = 0.006381

PRESSURE VESSEL

BACKGROUND

1. Field of the Invention

This invention relates to pressure vessels for containing pressurized fluids, in particular, highly pressurized fluids; and more particularly, to pressure vessels for containing fluids which are pressurized and depressurized repeatedly, or infinitely (infinitely in the sense that the crank shaft of an automobile is designed to have infinite cycle life). Further, this invention is directed toward providing a pressure vessel which makes a more efficient use of the constituent material for containing fluid pressurized to a given level, and in addition, this invention relates to pressure vessels for containing fluids which are pressurized to levels which are in excess of the fatigue or endurance stress of the "strongest" generally commercially available materials, i.e., not specialty materials.

2. Description of the Prior Art

Known to the prior art are many types of pressure vessels for containing a pressurized fluid in a central chamber. Some prior art vessels include an array of radially disposed, segmented blocks which provide radial support to the central chamber when the fluid contained thereby is pressurized to its operating level. Other prior art pressure vessels include the so-called multiring, or shrunk-ring, assembly wherein interferences between adjacent rings are such that the assembly is generally prestressed radially inwardly when the contained fluid is depressurized, so as to provide the multiring assembly with greater pressurized fluid containing capacity when loaded.

With specific regard to the prior art pressure vessel including an array of radially disposed, segmented blocks, such blocks are segmented to prevent the generation of unwanted or ruinous tangential stresses when the fluid contained therein is pressurized to its predetermined operating level. Also known is the technique of enhancing the pressure containing capacity of such a pressure vessel by providing radial support stress to the segmented blocks such that when the contained fluid is pressurized to its predetermined operating level, the radial stress experienced by the blocks is maintained within tolerable limits. Such radial support stress is typically provided by a surrounding concentric body of pressurized support fluid.

With specific regard to prior art multiring pressure vessels, the typical approach for enhancing the pressure containing capacity of a multiring pressure vessel has been to give attention to the tangential stress in the rings when the fluid contained thereby is pressurized to its operating level. More specifically, radial interference between rings has been provided to reduce the tangential stress present in the rings when the contained fluid is pressurized, from that value which would be present in a pressure vessel made of monolithic block, the contained fluid being similarly pressurized. Such prior art radial interferences are usually determined empirically, however, radial interference equations are available in the prior art for a two-ring pressure vessel whereby the tangential stress at the bore of the vessel can be maintained at or below a chosen value. For pressure vessels of more than two rings, such equations are not available and it has been found that the prior art pressure vessels of more than two rings have random, or even haphazard, tangential stress patterns across the constituent rings, depending upon the actual radial interferences present between the rings.

There are two primary operational considerations with regard to any pressure vessel, (1) the level to which the fluid contained by the vessel can be safely pressurized without fear of rupture, i.e., the upper pressure operating limit; and (2) the number of times the fluid contained by the vessel can be pressurized to the upper operating limit, and depressurized, before the pressure vessel fails from material fatigue, i.e., the cycle life of the vessel.

The aforementioned prior art pressure vessels generally satisfy both operational considerations while operation at the lower pressure levels (e.g., below 150,000 p.s.i.); and, as is known, by the use of highly excessive specialty materials and/or expensive and complex structural arrangements, such vessels can be designed to contain pressures up to, and in excess of, 500,000 p.s.i. However, in containing such elevated pressures, the vessels either have an extremely short cycle life (typically 10 cycles or less) which renders them commercially infeasible, or the vessels are inefficient in that they are so expensive, complex or massive vis-a-vis the commercial end to be achieved, that they are not commercially feasible.

With regard to cycle life, this consideration is quite highlighted by the fact that the "strongest" material presently commercially available (i.e., not specialty materials), for example a maraging steel, has a fatigue stress, or endurance stress, of only about 150,000 p.s.i. This means that a prior art pressure vessel as described above, constructed of maraging steel, could certainly not be safely operated above, or even slightly below, 150,000 p.s.i. without paying the price of an undesirably abbreviated cycle life. This limitation, obviously, places a commercial ceiling on the applications wherein pressurized fluid levels above 150,000 p.s.i. are required, or found to be highly desirable, for material forming or shaping; and more importantly, such limitation precludes such prior art pressure vessels from having an infinite cycle life (as noted above, infinite cycle life in the sense that the crankshaft of an automobile is designed to have infinite cycle life).

Accordingly, there exists in the pressure vessel art a need for a pressure vessel which makes a more efficient use of constituent material, and which can, with commercial feasibility, contain a pressurized working fluid which is pressurized to a lever which is in excess of the fatigue or endurance stress of the material of which the pressure vessel is made. There further exists a need for pressure vessels which have infinite cycle life in the sense noted above.

SUMMARY

The primary thrust of the present invention is directed toward providing a pressure vessel for containing fluids, in particular highly pressurized fluids, which has an infinite cycle life (as noted above) of pressurization and depressurization. Additionally, the present invention is directed toward providing a pressure vessel which makes a more efficient use of the constituent material for containing fluid pressurized to a given level. Further, the invention is related to a pressure vessel for containing pressurized fluids with generally commercially available materials whose fatigue or endurance stress is less than the pressure level to which the fluids contained therein are pressurized.

Referring now specifically to pressure vessel material infinite cycle life, it has been found that the cogent criterion is not any individual stress experienced by such material, i.e., the radial stress S_r, or the axial stress S_a, or the tangential (hoop) stress S_t, but rather the combined effect of these stresses, which was defined by Von Mise as the "working stress" S_w and expressed as follows:

$$2S_w^2=(S_a-S_r)^2+(S_r-S_t)^2+(S_t-S_a)^2. \quad (1)$$

The working stress S_w was equated to the yield strength by Von Mise, and generally is always so treated; however, in accordance with the teaching of the present invention, by equating the working stress S_w to the fatigue or endurance stress S_f of the material of which the pressure vessel is made, and by employing the material in accordance with the present invention such that Von Mise's equation is satisfied when the vessel is loaded to its predetermined operating level, the pressure vessel is provided with infinite cycle life, and the vessel can, with safety and commercial feasibility, contain fluid pressurized to a level in excess of the fatigue or endurance stress of the material of which the vessel is made.

The above-noted prior art problems are overcome by a pressure vessel embodying the present invention, which pressure vessel may include an array of radially disposed segmented blocks which are supplied with compressional axial and tangential support forces such that when the pressurized

fluid contained thereby is pressurized to its predetermined operating level, the working stress across the array of segmented blocks does not exceed the fatigue or endurance stress of the material of which the blocks are made. Additionally, such prior art problems are overcome by a pressure vessel embodying the present invention which may include a multiring assembly wherein the interferences between adjacent rings are chosen such that at each radius except the outermost, the working stresses are made substantially equal to each other, and also made substantially equal to or below the fatigue or endurance of the material of which the rings are made.

The primary advantages of pressure vessels embodying the present invention are that such pressure vessels provide a more efficient use of constituent material for containing fluid pressurized to a given level, and such vessels are provided with infinite cycle life. Further, the present invention permits pressure vessels to be made of commercially available materials having a fatigue or endurance stress which is less than the pressure level to which the fluid to be contained by the pressure vessel is to be pressurized.

FIG. 1 is a cross-sectional view of a pressure vessel embodying the present invention;

FIG. 2 is a cross-sectional view taken along the line 2—2 in FIG. 1;

FIGS. 3 and 4 are diagrammatic views illustrating one aspect of the present invention relating to an array of radially disposed segmented blocks; and

FIG. 5 is a tabulation of calculations of an example of one aspect of the present invention relating to a multiring assembly.

GENERAL

FIGS. 1 and 2 show diagrammatically a pressure vessel **10** embodying the present invention. The pressure vessel includes an inner array of radially disposed segmented blocks **12** embodying the present invention and for supporting a body of pressurized operating fluid **11**, an intermediate cylindrical jacket of support fluid **14**, and an assembly of multirings **16** embodying the present invention for containing the support fluid **14** or an operating fluid such as fluid **11**. The operating fluid **11** is contained in a centrally provided chamber **17**, and the support fluid **14** is contained in an annular chamber **18** concentrically surrounding the array of segmented blocks **12**.

The pressure vessel is provided with end members **20** and **22** suitably constructed so as to accommodate and contain annular bodies of support fluid **24** and **26** respectively. Support fluids **24** and **26** pressure bias annular pistonlike members **28** and **30**, respectively, for providing inwardly directed, compressive axial support to the segmented blocks **12**, in a manner to be described more fully below. The end members **20** and **22** are suitably bored to slidingly accommodate opposed top and bottom pistons **32** and **34**, which pistons may be driven to pressurize the operating fluid **11** residing within the centrally provided pressure chamber **17**. Also residing within the pressure chamber **17**, in the manner known in the art, is a seal support cylinder **40** for maintaining suitable fluid seals S shown surrounding the pistons **32** and **34** in their appropriate sealing positions. Further, in the manner known in the art, the pressure vessel **10** is provided with suitable liners L, positioned on each side of the array of segmented blocks **12**, for preventing passage of pressurized fluid between the segmented blocks.

In general, the array of radially disposed segmented blocks **12** are supplied with compressional axial support stress from the members **28** and **30**, and with compressional tangential support stress or forces from the support fluid **14** such that when the operating fluid **11** is pressurized to its operating level, the working stress S_w across or experienced by the segmented blocks does not exceed, or substantially exceed, the fatigue or endurance stress S_f of the material of which the blocks are made; thus, the operating fluid **11** may be repeatedly pressurized and depressurized without the working stress across the array of segmented blocks **12** substantially exceeding the fatigue stress of the material of which the blocks are made, and hence, the array of segmented blocks is provided with infinite cycle life. Further generally, the interferences between adjacent component rings of the multiring assembly **16** are chosen such that, when the fluid supported thereby is pressurized to its predetermined operating level, (i) the working stress S_w across each ring does not exceed the fatigue or endurance stress S_f of the material of which the multirings are made, thus providing the multiring assembly with infinite cycle life, and such that (ii) the working stress S_w at the interfaces between the rings (or at the radii R3, R_2 and R4 of FIG. 2) are equal or at least substantially equal, thus providing a more efficient use of the multiring constituent material.

ARRAY OF SEGMENTED BLOCKS

The array of segmented blocks **12** of FIGS. **1** and **2** is segmented to eliminate the generation of tensional tangential stress which, vis-a-vis the material of which the blocks may be made, can quickly become ruinous, particularly if it is desired to contain an operating pressure P_o in the order of 300,000 p.s.i., and the blocks are made of generally commercially available materials.

As mentioned above, the "strongest" commercially available materials, e.g., a maraging steel, has a fatigue or endurance stress S_f of about 150,000 p.s.i. and it will be assumed that the blocks **12** are made of material having a fatigue or endurance stress of 150,000 p.s.i. Further, it will be assumed that the operating fluid **11** is pressurized to an operating pressure P_o of 300,000 p.s.i. The support fluid **14** will be pressurized to a pressure level P_1 which will be arbitrarily assumed or chosen to be 100,000 p.s.i., for purposes of illustration. However, it will be understood that the pressure level P_1 may be of other values in accordance with the present invention.

Accordingly, as noted above and on the basis of the foregoing assumed conditions, if the array **12** is to have infinite cycle life, the working stress S_w at each point across the array must not exceed, or substantially exceed, 150,000 p.s.i. the assumed fatigue or endurance stress S_f of the material of which the blocks **12** are made, when the fluid supported thereby is pressurized to its predetermined operating level.

Superimposed on the right-hand portion of the array **12** in FIG. **3** is a plot of radial stress S_r vs. the radial length L_1 of a representative segmented block **44** of the array **12**, the radial stress S_r being the radial stress experienced by the block **44** when the operating fluid **11** is pressurized to the assumed operating level P_o of 300,000 p.s.i., and the support fluid **14** is pressurized to the assumed level of 100,000 p.s.i. (It will be understood at this juncture, i.e., with no support stresses or forces yet applied, that the working stress S_w is equal to the radial stress S_r at each point across the radial length L_1 since the array of blocks **12** are only experiencing radial stress. Hence, the plot in FIG. **3** is also a plot of working stress S_w vs. the length L_1.) Thus, as shown, it will be noted that the radial stress S_r and the working stress S_w across the radial length L_1 of the block segment **44** will drop from approximately 300,000 p.s.i. at the bore D_1 of the pressure chamber **38**, to 100,000 p.s.i. at diameter D_2; 100,000 p.s.i. being the assumed value of the support fluid pressure P_1.

Since the support fluid P_1 of 100,000 p.s.i. is less than the allowable working stress S_w of 150,000 p.s.i. of the material of the block **44**, the radial stress S_r and working stress S_w across the radial length L_1 of the block **44** will drop to the value of the allowable working stress S_w at some diameter D_w which will be, as shown, intermediate the bore or chamber **17** diameter D_1 and the fluid support jacket diameter D_2. It will be appreciated that between diameter D_w and diameter D_2, represented by trapezoidal cross-hatched area A_2, the radial stress S_r and working stress S_w are less than the allowable working stress of 150,000 p.s.i., and hence attention need not be given to this area. However, for infinite cycle life, the working stress in excess of 150,000 p.s.i. between the chamber diameter D_1 and the diameter D_w (represented by triangular

cross-hatched area A_1) must be reduced and made substantially equal to, or less than, the allowable working stress S_w of 150,000 p.s.i.

It has been discovered that this reduction in working stress can be accomplished by the application to each block segment 44 of compressional axial support stress S_a and compressional tangential support stress S_t, as shown diagrammatically in FIG. 4, which will vary from (P_o-S_w) at the chamber diameter D_1, to zero (0) at the diameter D_w; it being understood that the hoop stress can be ignored due to the blocks 12 being segmented. By setting the compressional axial and tangential support stresses to be supplied equal to each other to eliminate the presence of two unknowns in one equation (and by assigning to each the common mathematical symbolic notation S_c), Von Mise's equation, equation (1) supra, can be written for the block segment material at the bore of the pressure chamber, i.e., at diameter D_1, and the value of the axial and tangential support stresses S_c at the diameter D_1 necessary to satisfy Von Mise's equation at diameter D_1 can be solved. Such equation can be shown to be:

$$2(P_o-S_c)^2=2S_w^2 \quad (2)$$

$$S_c=(P_o-S_w) \quad (3)$$

Since, in accordance with the teaching of the present invention, S_w is set equal to the endurance or fatigue stress of the material of the block segment 44, equation (3) can be rewritten as:

$$S_c=(P_o-S_f) \quad (4)$$

Further, since both the assumed values of the operating fluid 11 (300,000 p.s.i.) and the working stress S_w (150,000 p.s.i.) are known, the value of compressional axial and tangential support stresses S_c which must be supplied at diameter D_1 in order to reduce the working stress S_w in each block segment at the diameter D_1 to the endurance or fatigue stress S_f of the material of the block 44, can be determined.

In terms of specific apparatus, the axial compressive support stress S_a or S_c required will be provided by the pistonlike members 28 and 30 shown in FIG. 1. The tangential compressive support stress S_t or S_c required will be provided by the support fluid 14 shown in FIGS. 1 and 2.

Thus, it will be understood that the support fluid 14 must provide two quantities, viz, (1) radially inwardly directed radial force for countering the radially outwardly directed radial force from operating pressure 11 (FIGS. 1 and 2), i.e., the counterforce for maintaining the segmented block array 12 in equilibrium, and (2) radially inwardly directed stress or force for wedging or compressing the segmented blocks together with sufficient compressive stress so as to apply compressional tangential support stress S_t or S_c thereto which assists the compressional axial support stress S_a or S_c (supplied by pistons 32 and 34) in reducing the working stress S_w between diameters D_1 and D_w to substantially the value of the endurance or fatigue stress S_w of the block material thereby satisfying Von Mise's equation at each point between such diameters and thus providing the array of segmented blocks 12 with infinite cycle life.

Further, it will be appreciated from FIG. 3 that the perfectly required axial and tangential compressive support stresses S_c required to be supplied would be varying stresses. It is difficult, however, to provide a varying stress in actual practice; accordingly, an average compressional support stress $S_{a\ avg}$, or $S_{t\ avg}$ or $S_{c\ avg}$, of:

$$S_{o\ avg}=\left[\frac{(D_w-D_1)(P_0-S_w)}{2}\right] \quad (5)$$

will be used to determine the diameter D_w; the determination of diameter D_w being an intermediate determination permitting the determination of diameter D_2, the outer diameter of the array 12. It will be understood by those skilled in the art that the use of an average support stress is a compromise for which at least some price must be paid with regard to the perfect attainment of infinite cycle life. However, even with the use of an average support stress, infinite cycle life is substantially obtained.

Accordingly, the value of diameter D_w can be determined as follows: diameter D_w will be that diameter which will provide an area, which area when multiplied by the allowable working stress S_w (or the fatigue or endurance stress of the material of the segmented blocks 12) will provide the radially inwardly directed force for (1) maintaining the array of blocks 12 in equilibrium, and (2) for supplying the average compressional tangential support stress $S_{t\ avg}$ or $S_{c\ avg}$ for assisting in reducing the working stress between diameters D_1 and D_w to substantially the value of the allowable working stress S_w for infinite cycle life. Such equation can be shown to be:

$$(D_w-D_1)(P_o-S_w/2)+P_oD_1=S_wD_w \quad (6)$$

By again setting the operating pressure P_o equal to the assumed value of 300,000 p.s.i., and by setting the working stress S_w equal to the assumed value of 150,000 p.s.i., the assumed value of the fatigue or endurance stress of the block material, and since D_1 is known (such diameter being the operating chamber 17 required for a given workpiece shaping operation), the diameter D_w can be determined. Once diameter D_w has been determined, the diameter D_2 can be determined as follows: diameter D_2 will be that diameter which will provide an area, which area when multiplied by the support fluid pressure P_1(arbitrarily and conveniently chosen above as 100,000 p.s.i.) will provide the radially inwardly directed force for (1) maintaining the array of segmented blocks 12 in equilibrium, and (2) for supplying the average compressive tangential support stress $S_{t\ avg}$ or $S_{c\ avg}$ for assisting in providing the reduced working stress S_w between the diameters D_1 and D_w for infinite cycle life, as taught above. Thus, by writing the radial force balance ro equilibrium equation between diameters D_1 and D_2, and providing such twofold radially inwardly directed force or stress, the diameter D_2 can be determined. Such equation can be shown to be:

$$(D_w-D_1)(P_o-S_w/2)+P_oD_1=P_1D_2 \quad (7)$$

Since in view of the foregoing assumptions and computations, the only unknown now is diameter D_2, diameter D_2 can be determined.

Accordingly, it will be understood that the pistons 28 and 30 will be structured in the manner so as to provide the array of segmented blocks 12 with the average compressional axial support stress $S_{a\ avg}$ or $S_{c\ avg}$ over the area between diameters D_1 and D_w; and the support fluid 14, the array being structured so as to be provided with diameter D_2 as determined above, will provide the array of segmented blocks 12 with the average tangential compressive support stress $S_{t\ avg}$ or $S_{c\ avg}$. Thus, when the fluid 11 is pressurized to its operating level, the actual working stress S_w across the array of segmented blocks 12 between diameters D_1 and D_w will be substantially reduced to the value of the fatigue or endurance stress of the material of which the array of blocks 12 are made, and hence at no point across the array will the working stress substantially exceed the fatigue or endurance stress of the material of which the blocks 12 are made. Accordingly, the array 12 will be substantially provided with infinite cycle life.

As noted above, in the development of the foregoing equations, the fatigue or endurance stress S_f of the material of the blocks 12 was substituted directly for the allowable working stress S_w and hence the average compressional support stresses $S_{c\ avg}$, were developed so as to reduce the working stress between diameters D_1 and D_w to substantially the value of the material fatigue or endurance stress. It will be understood that such working stress could be further reduced, or made any predetermined ratio of, the allowable working stress by substituting the term KS_f (instead of simply S_f) for the term S_w, where K is a fraction or number greater than unity. By such substitution, the working stress S_w can be made to be any predetermined ratio, i.e., fraction or multiple, of the fatigue or endurance stress of the material of which the blocks 12 are made.

Referring again to FIG. 1, support fluid 14 is supplied from a source (not shown) to the annular chamber 18 through a passageway 42 formed longitudinally in a block of the segmented array 12, and through a dependent pipe portion 44

formed integrally with the segmented block. This manner of supplying the pressurized support fluid **14** set forth in detail in a copending patent application of the assignee of the present invention, Ser. No. 652,112, filed July 10, 1967, now U.S. Pat. No. 3,490,344, in the names of John W. Archer and F. J. Fuchs, Jr.

With further reference to FIG. **1**, while the use of two pistonlike members **28** and **30** is particularly useful in supplying the compressional axial support stress S_a in accordance with the teaching of the present invention, such axial support stress may be provided by a single pistonlike member, such as pistonlike member **28**, and the complementary support stress may be provided by reactive forces from a monolithic block which would replace block **22** including the pistonlike member **30**.

MULTIRING ASSEMBLY

A. Bore Pressurized

Referring now to the present invention as embodied in the multiring assembly **16** of FIGS. **1** and **2**, the assembly is subjected to the internal pressure P_1 of the pressurized support fluid **14** (or as noted above, the multiring assembly could be utilized to directly support an operating fluid such as fluid **11**). As is known in the art, for a constant or uniform working stress S_w across the rings of a multiring pressure vessel, the rings would be of an infinite number with each ring being of an infinitesimally small thickness. Obviously, practical considerations negate such an approach. In accordance with the present invention, the ideal is approached, inter alia, by use of a design criterion $(1+X)$ where X is a dimensionless number which will be treated in detail infra. The design criterion $(1+X)$ is a variable design criterion and, except for the last or outermost ring, is made equal to the ratio of the outside radius to the inside radius of each ring in the assembly, viz:

$$R_{outside}/R_{inside}=(1+X) \tag{8}$$

Use of this design criterion makes it possible to relatively determine the radii of any ring in an assembly except the outside radius R_n as set forth below:

$R_1=R_1$
$R_2=(1+X)R_1$
$R_3=(1+X)^2R_1$
$R_4=(1+X)^3R_1$
. .
. .
. .
$R_{n-1}=(1+X)^{n-2}R_1$

It will be understood that the radius R_1 will be known, such dimension being determined typically by the diameter of the fluid chamber supported by the multiring assembly such as central chamber **17** or annular chamber **18**. Hence, once X is chosen all radii except the outermost will be known. And it will be understood by those skilled in the art that with regard to multiring assemblies comprised of tapered rings which are press fitted together, such radii are midpoint radii with regard to ring height of such tapered rings.

For the optimum pressure vessel design, i.e., the most efficient use of the material of which the rings are made, the working stresses S_w at radii R_1, R_2,...R_{n-1}, when the contained fluid is pressurized to its operating level, are required to be equal or substantially equal; and for infinite cycle life as noted above, the working stress S_w across each ring must not exceed the fatigue or endurance stress S_f of the material of which the rings are made. With the multiring assembly **16** of FIG. **2** subjected to the internal pressure P_1, and since no axial stress S_A is applied to the assembly **16**, the only stresses present at radii R_1, R_2,...R_{n-1} are the radial compressive stresses S_R and the tangential, tensile stresses S_T. For a known or desired working stress S_w and a known internal or bore pressure P_1, the tangential stress S_{T1} acting at radius R_1 can be obtained by solving the following quadratic equation obtained from equation (1) above, viz:

$$2S_w^2=S_{T1}^2+S_{R1}^2+(S_{T1}+S_{R1})^2 \tag{9}$$

since $S_{R1}=P_1$, i.e., the radial stress at R_1 is equal to the pressure of fluid **14** or P_1, equation (9) can be rewritten as:

$$2S_w^2=S_{T1}^2+P_1^2+(S_{T1}+P_1)^2 \tag{10}$$

For a known ring material, S_w will be known being made equal, as above, to the fatigue or endurance stress S_f of the material of which the rings **16** are made; and P_1 will be known being the pressure of the support fluid **14**, now assumed to be, for example, 150,000 p.s.i. Substitution of the knowns S_w and P_1 into equation (9) will give the tangential stress S_{T1} acting at radius R_1 in terms of a quadratic equation. Solution of the quadratic equation will give S_{T1} in terms of two roots, one value will yield a compressive tangential stress and the other value will yield a tensile tangential stress including the value zero. The latter root is chosen over the former root because it represents a lower ring interference at radius R_2 and a lower maximum in tangential stress (typically compressive) at radius R_1 when the pressure P_1 is removed. Once the tangential stress S_{T1} is determined, the value of the radial compressive stress S_{R2}, acting at radius R_2, provided by the outer rings and the pressure P_1 in order that the working stress at radius R_1 be equal to or substantially equal to the allowable working stress S_w (i.e., equal to S_f), is obtained by use of the following equation:

$$S_{R2}=\frac{P_1(R_1^2+R_2^2)-S_{T1}(R_2^2-R_1^2)}{2R_2^2} \tag{11}$$

Since P_1 is known (assumed or chosen value), since S_{T1} is known from equation (9), since radius R_1 will be known as noted above, and since R_2 is known in terms of R_1 due to the design criterion $(1+X)$, the only unknown is S_{R2} which can be determined.

It will be noted from above that radial stress S_{R2} is the sum of the radial stress at radius R_2 due to fluid pressure P_1 and that due to the total interference present at R_2 after the entire multiring **16** assembly is assembled. Using the foregoing teaching with regard to the determination of tangential stress S_{T1} and radial stress S_{R2}, the tangential stresses S_{T2} to S_{Tn-1} acting at radii R_2 to R_{n-1}, and the radial stresses S_{R3} to S_{Rn-1}, may be determined; it being understood that such determination will be discontinued upon the radial stress S_R equaling zero or upon its sign becoming negative. Since the exterior of the outer ring is exposed to the atmosphere, the radial stress at radius R_n will be zero, thus, the outside radius R_n of the last ring may then be determined from the following equation

$$S_{Tn-1}=\frac{S_{Rn-1}(R_{n-1}^2+R_n^2)}{R_n^2-R_{n-1}^2} \tag{12}$$

which may be rewritten as:

$$R_n^2=\frac{S_{Rn-1}R_{n-1}^2+S_{Tn-1}R_{n-1}^2}{S_{Tn-1}-S_{Rn-1}} \tag{13}$$

The tables of FIG. **5** list values of the radii R_1 ...R_n (in inches) and the radial and tangential stresses (in p.s.i.) acting at radii R_1 to R_{n-1} when the supported fluid pressure P_1 (and hence S_{R1}) =150,000 p.s.i., X=0.4 and S_w=150,000 p.s.i.

B. Bore Depressurized

The ring interferences necessary at the various radii in order to have the working stresses S_w at the various radii, except the last, equal to each other and equal to, or substantially equal to the fatigue or endurance stress S_f of the ring material when the supported fluid P_1 is pressurized (e.g., fluids **11** or **14**), are obtained by first determining the no-load radial stresses present at the various radii when the supported fluid P_1 is depressurized, i.e., the radial stress at the various radii due only to the ring interferences, for example, the radii stress present (with the supported fluid P_1depressurized) at a particular radius due to the interference at the particular radius plus the radial stress at the particular radius due to the interferences present at any outer radii. The no-load radial stresses are obtained by subtracting the radial stresses acting at R_1, R_2,...R_{n-1} due to the supported fluid P_1 when pressurized, from the above-determined total radial stress present at these radii due to both the pressurized fluid P_1 and the total ring interferences

present. The no-load radial stresses are given by the following expressions, the first term being the total radial at each radii, except the last, when the supported fluid P_1 is pressurized, and the second term being the radial stress existing at such radii due solely to the fluid pressure P_1.

$$S_{R1\text{ no load}}=S_{R1}-\left[\frac{S_{R1}R_1^2}{R_n^2-R_1^2}\left(\frac{1-R_n^2}{R_1^2}\right)\right]=0$$

$$S_{R2\text{ no load}}=S_{R2}-\left[\frac{S_{R1}R_1^2}{R_n^2-R_1^2}\left(\frac{1-R_n^2}{R_2^2}\right)\right]$$

$$S_{R3\text{ no load}}=S_{R3}-\left[\frac{S_{R1}R_1^2}{R_n^2-R_1^2}\left(\frac{1-R_n^2}{R_3^2}\right)\right]$$

$$\vdots \qquad\qquad \vdots$$

$$S_{Rn-1\text{ no load}}=S_{Rn-1}-\left[\frac{S_{R1}R_1^2}{R_n^2-R_1^2}\left(\frac{1-R_n^2}{R_{n-1}^2}\right)\right] \quad (14)$$

As will be seen infra, the no-load radial stress will be utilized to determine the actual interferences at the radii.

While not required to determine the physical characteristics of the rings, it is desirable to determine the tangential stresses present in the rings when the supported fluid P_1 is depressurized in order that the suitability of a given material for pressure vessel fabrication in accordance with the present invention may be more fully evaluated.

The no-load tangential stresses may be obtained as follows:

$$S_{T1\text{ no load}}=S_{T1}-\left[\frac{S_{R1}R_1^2}{R_n^2-R_1^2}\left(\frac{1+R_n^2}{R_1^2}\right)\right]$$

$$S_{T2\text{ no load}}=S_{T2}-\left[\frac{S_{R1}R_1^2}{R_n^2-R_1^2}\left(\frac{1+R_n^2}{R_2^2}\right)\right]$$

$$\vdots \qquad\qquad \vdots$$

$$S_{Tn-1\text{ no load}}=S_{Tn-1}-\left[\frac{S_{R1}R_1^2}{R_n^2-R_1^2}\left(\frac{1+R_n^2}{R_{n-1}^2}\right)\right] \quad (15)$$

The actual interferences between rings are determined from what is termed in accordance with the present invention, the interference radial stresses. The interference radial stresses acting at R_2, R_3 and R_4 are denoted by $S_{R2\ interf}$, $S_{R3\ interf}$, and $S_{R4\ interf}$, respectively, and $S_{R2\ interf}$, for example, is defined as the interference radial stress acting at radius R_2 after the first two rings are assembled, i.e., the radial stress acting at radius R_2 due solely to the interference between the first two rings. More generally, the interference radial stress at particular inner radius is obtained by subtracting from the no-load radial stress at the particular inner radius, the component of the no-load radial stress at the next adjacent outer radius acting back at the inner radius. More specifically, and by way of example, the interference radial stress $S_{R2\ interf}$ at radius R_2 is obtained by subtracting from the above-determined no-load radial stress ($S_{R2\ no\ load}$) at radius R_2, the component of the no-load radial stress ($S_{R3\ no\ load}$) at radius R_3 acting at radius R_2, i.e.,

$$S_{R2\text{ interf}}=S_{R2\text{ no load}}-\left[\frac{S_{R3\text{ no load}}R_3^2}{R_3^2-R_1^2}\left(\frac{1-R_1^2}{R_2^2}\right)\right]$$

$$S_{R3\text{ interf}}=S_{R3\text{ no load}}-\left[\frac{S_{R4\text{ no load}}R_4}{R_4^2-R_1^2}\left(\frac{1-R_1^2}{R_3^2}\right)\right] \quad (16)$$

and so similarly for $S_{R4\ interf}$. The interference radial stresses for radii R_2–R_4 are tabulated in FIG. 5, and such stresses in accordance with the present invention are converted into actual ring radial interference values referred to as "Deltas," by use of the following relations:

$$\text{Delta}_2=\frac{2S_{R2\text{ interf}}R_2^2(R_2^2-R_1^2)}{E(R_2^2-R_1^2)(R_3^2-R_2^2)}$$

$$\text{Delta}_{n-1}=\frac{2S_{Rn-1\text{ interf}}R_{n-1}^2(R_n^2-R_1^2)}{E(R_{n-1}^2-R_1^2)(R_n^2-R_{n-1}^2)} \quad (17)$$

where E is the elastic modulus of the material of which the rings are made, and Deltan$_{-1}$ denotes the radial interference at R_{n-1} after $n-2$ rings are assembled. Referring again to FIG. 5, there is shown a tabulation of actual ring interferences or Deltas (in inches) at radii R_2, R_4 and R_3, where, as noted above, P_1=150,000 p.s.i., X=0.4 , and S_w or S_f=150,000 p.s.i.

With regard to the design criterion (1+X), X may be any value, however, it will be appreciated that as X decreases, the number of rings in the assembly will increase thereby causing an increase in manufacturing costs. Thus, it will be appreciated that for a given set of other conditions, an optimized value of X may be chosen so as to reasonably limit the number of rings to a reasonable minimum overall vessel size. It has been found that for multiring assemblies for supporting fluid pressures between 70,000 p.s.i. and 150,000 p.s.i., values of X between 0.2 and 1.0 are particularly advantageous and useful.

As noted above, in the development of the immediately foregoing equations, the fatigue or endurance stress S_f of the material of which the rings of the assembly 16 are made, was substituted directly for the allowable working stress S_w and hence the actual ring interferences or Deltas were developed so as to reduce the working stresses at the inner radii to the value of, or substantially the value of, the material fatigue or endurance stress, and hence equal to each other. It will be understood, as with regard to the array of segmented blocks 12, such working stress could be further reduced, or made any predetermined ratio of, the allowable working stress by substituting the term KS_f (instead of simply S_f) for the term S_w, where K is a fraction or number greater than unity. By such substitution, the working stress S_w S_w can be made to be any predetermined ratio, i.e., fraction or multiple, of the fatigue or endurance stress of the material of which the rings of the assembly 16 are made.

With further regard to the radial interferences or Deltas, once such radial interferences are known and with the radii R_1 ...R_n, being determined as taught above, the rings of the multiring assembly may then be suitably constructed and assembled by being press fitted or shrunk fitted together in the manner known in the art.

Referring again to FIG. 2, there is shown, superimposed on the multiring assembly, a graph of the working stress S_w across the rings of the assembly when the fluid 14 is pressurized to its predetermined level and the multiring assembly 16 has been constructed in accordance with the teaching of the present invention. In the graph shown, it will be noted that the working stresses at all inner radii are equal, or substantially equal to, the fatigue stress S_f of the material of which the rings are made. The working stress S_w drops slightly across each inner ring and then drops slightly further across the other rings, however, the average of the graph would be a substantially uniform or constant working stress across the multiring assembly for increased efficiency of use of ring material.

Additionally, it will be understood from the foregoing, that the height of the segmented blocks 12 and rings of the assembly 16 is immaterial to the teaching of the present invention. Further, that such height may be of any reasonable value.

PRESSURE VESSEL FIGS. 1 and 2

In summary, the pressure vessel 10, combining the above-described array of segmented blocks 12 and the multiring assembly 16, provides a more efficient use of the material of which the segmented blocks and rings are made, and with commercial feasibility, permits the vessel to contain fluids

pressurized to levels which are in excess of the fatigue or endurance stress of the "strongest" generally commercially available materials. Further, the working stress S_w across the entire vessel, when the fluids **11** and **14** are pressurized to their predetermined levels, can be made equal to, or substantially equal to the fatigue or endurance stress of the material of which the segmented blocks and rings are made; or more generally, the working stress S_w can be made a predetermined ratio of such fatigue or endurance stress.

With regard to the term "multiring assembly," it will be understood that such term in the foregoing specification and following claims defines an assembly including two or more rings.

Many modifications and variations of the present invention are possible in the light of the above teachings. It is therefore to be understood that the invention may be practiced otherwise than as specifically described.

What is claimed is:

1. A pressure vessel for containing a fluid to be pressurized to a predetermined level, comprising:

an array of radially disposed segmented blocks for supporting said fluid; and

means for supplying compressional axial and tangential support stresses to said array such that when said fluid is pressurized to said predetermined level the working stress across at least a portion of said array is a predetermined ratio of the fatigue stress of the material of which said blocks are made.

2. A pressure vessel according to claim **1** wherein said predetermined ratio is such that said working stress is substantially equal to the fatigue stress of the material of which said blocks are made.

3. A pressure vessel according to claim **1** wherein said predetermined ratio is such that said working stress is less than the fatigue stress of the material of which said segmented blocks are made.

4. A pressure vessel for containing a fluid to be pressurized to a predetermined level, comprising:

an array of radially disposed segmented blocks for supporting said fluid;

means for supplying compressional axial support stress to said array; and

means for supplying compressional tangential support stress to said array, said applied support stresses being of predetermined values such that when said fluid is pressurized to said predetermined level, the working stress across said array does not exceed the fatigue stress of the material of which said blocks are made.

5. A pressure vessel for containing a fluid to be pressurized to a predetermined operating level, comprising:

an array of radially disposed segmented blocks for supporting said fluid and which experience working stress upon said fluid being pressurized to said predetermined operating level;

means for applying compressional axial support stress to said array; and

means for applying compressional tangential support stress to said array, said support stresses being of predetermined values for making said working stress across at least a portion of said array a predetermined ratio of the fatigue stress of the material of which said segmented blocks are made.

6. A pressure vessel according to claim **5** wherein said predetermined ratio is substantially equal to the fatigue stress of the material of which said segmented blocks are made.

7. A pressure vessel according to claim **5** wherein said predetermined ratio is such that said working stress is less than the fatigue stress of the material of which said segmented blocks are made.

8. A pressure vessel according to claim **5** wherein said compressional axial support stress is provided by opposed pistons engageable with at least a portion of said array.

9. A pressure vessel according to claim **5** wherein said compressional tangential support stress is provided by a body of pressurized support fluid to be contained in an annular pressure chamber concentrically surrounding said array.

10. A pressure vessel according to claim **5** wherein said array is surrounded by a chamber for receiving a body of pressurized support fluid for providing said compressional tangential support stress.

11. In a pressure vessel including a central chamber for containing a body of operating fluid to be pressurized to a predetermined operating level P_o, the improvement comprising:

an array of radially disposed segmented blocks surrounding and supporting the chamber, said blocks experiencing working stress S_w upon said operating fluid being pressurized to said operating level P_o; and

means for supplying compressional axial S_a and tangential S_t support stresses to said array such that when said operating fluid is pressurized to said predetermined operating level P_o, the working stress S_w across at least a portion of said array is a predetermined ratio of the fatigue stress of the material of which said segmented blocks are made.

12. In a pressure vessel the combination according to claim **11** wherein said compressional axial S_a and tangential S_t support stresses at the bore of said central chamber are substantially equal to the operating fluid pressure P_o of said operating fluid minus the fatigue stress S_f of the material of which said segmented blocks are made.

13. In a pressure vessel the combination according to claim **11** wherein said compressional axial and tangential support stresses applied to said array are average compressional support stresses substantially equal to $[(D_w-D_1)(P_o-S_w/2)]$.

14. A pressure vessel for containing a body of fluid to be pressurized to a predetermined level; comprising:

a multiring assembly including a plurality of rings for supporting said fluid,

said rings being provided with radial interferences such that when said fluid is pressurized to said predetermined level the working stress at each radius, except the outermost, is a predetermined ratio of the fatigue stress of the material of which the rings are made.

15. A pressure vessel according to claim **14** wherein said predetermined ratio is such that said working stress is equal to the fatigue stress of the material of which said rings are made.

16. A pressure vessel according to claim **14** wherein said predetermined ratio is such that said working stress is less than the fatigue stress of the material of which rings are made.

17. A pressure vessel for containing a body of fluid to be pressurized to a predetermined level, comprising:

a multiring assembly including a plurality of rings for supporting said fluid;

said rings fitted together and having radial interferences at the inner radii thereof; and

said interferences being such that, when said fluid is pressurized to said predetermined level, said radial interferences are such that the working stress at said inner radii are equal to each other and equal to the fatigue stress of the material of which said rings are made.

18. A pressure vessel for containing a body of fluid to be pressurized to a predetermined level, comprising:

a multiring assembly including a plurality of rings for supporting said fluid;

said rings being press fitted together and having radial interferences at the inner radii thereof; and

said interferences being such that, when said fluid is pressurized to said predetermined level, said stress at said inner radii are substantially equal to each other and less than the fatigue stress of the material of which said rings are made.

19. A pressure vessel for containing a body of fluid to be pressurized to a predetermined level, comprising:

a multiring assembly including a plurality of rings for supporting said fluid;

said rings being press fitted together and having radial interferences at the inner radii thereof; and

said interferences being such that when said fluid is pressurized to said predetermined level, the stresses experienced by said assembly at said inner radii are defined by:

$$\text{Delta}_{n-1}=\frac{2S_{\text{Rn-1 interf}}\,R_{n-1}^{3}(R_n^2-R_1^2)}{E(R_{n-1}^2-R_1^2)(R_n^2-R_{n-1}^2)}$$

20. A pressure vessel, comprising:

an array of radially disposed segmented blocks for supporting a first body of fluid to be pressurized to a predetermined level,

means for supplying a compressional axial and tangential support stresses to said array such that when said fluid is pressurized to said predetermined level the working stress across at least a portion of said array is a predetermined ratio of the fatigue stress of the material of which said blocks are made;

a multiring assembly including a plurality of press fitted rings for supporting a second body of fluid to be pressurized to a predetermined level,

said rings having radial interferences at the inner radii thereof such that when said second body of fluid is pressurized to said predetermined level, the working stresses at said inner radii are substantially equal to each other and substantially equal to the fatigue stress of the material of which said rings are made.

21. A pressure vessel, comprising:

an array of radially disposed segmented blocks for supporting a body of operating fluid to be contained within such vessel and to be pressurized to a predetermined level;

a multiring assembly including a plurality of press fitted rings for supporting a body of support fluid to be received within said vessel and to be pressurized to a predetermined level, said support fluid for supplying compressional tangential support stress to said array; and

means for supplying compressional axial support stress to said array;

said axial and tangential support stresses being of predetermined magnitudes such that when said operating fluid is pressurized to said predetermined level the working stress across at least a portion of said array is a predetermined ratio of the fatigue stress of the material of which said blocks are made;

said plurality of press fitted rings having radial interferences between the inner radii thereof of predetermined magnitudes such that when said body of support fluid is pressurized to said predetermined level the working stresses at said inner radii are substantially equal to each other and substantially equal to the fatigue stress of the material of which said rings are made.

22. A pressure vessel, comprising:

a central chamber for receiving a body of operating fluid to be pressurized to a predetermined level;

an array of radially disposed segmented blocks for supporting said central chamber;

means for supplying compressional axial support stress to at least a portion of said array;

an outer annular chamber for receiving a body of support fluid to be pressurized to a predetermined level;

when said operating fluid and said support fluid are pressurized to their respective predetermined levels, said support fluid supplies compressional tangential support stress to said array and cooperates with said axial support means to maintain the working stress across at least a portion of said array at a level substantially equal to the fatigue stress of the material of which said segmented blocks are made; and

a multiring assembly including a plurality of press fitted rings for supporting said annular chamber, said rings having radial interferences at the inner radii thereof such that, when said support fluid is pressurized to its predetermined level, the working stresses at the inner radii are substantially equal and substantially equal to the fatigue stress of the material of which said rings are made.

23. A pressure vessel for containing a fluid to be pressurized to a predetermined operating level, comprising:

an array of radially disposed segmented blocks for supporting said fluid and which experience working stress upon said fluid being pressurized to said predetermined operating level;

means for applying variable compressional axial support stress to said array;

means for applying variable compressional tangential support stress to said array;

said support stresses being of predetermined values for making said working stress across at least a portion of said array a predetermined ratio of the fatigue stress of the material of which said segmented blocks are made; and

said means for applying variable compressional axial support stress being operable independently of said means for applying compressional tangential support stress.

24. A pressure vessel according to claim 23 wherein said predetermined ratio is substantially equal to the fatigue stress of the material of which said segmented blocks are made.

25. A pressure vessel according to claim 23 wherein said predetermined ratio is such that said working stress is less than the fatigue stress of the material of which said segmented blocks are made.

26. A pressure vessel according to claim 23 wherein said compressional axial support stress is provided by opposed pistons engageable with at least a portion of said array.

27. A pressure vessel according to claim 23 wherein said compressional tangential support stress is provided by a body of pressurized support fluid to be contained in an annular pressure chamber concentrically surrounding said array.

28. A pressure vessel according to claim 23 wherein said array is surrounded by a chamber for receiving a body of pressurized support fluid for providing said compressional tangential support stress.

* * * * *

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UNITED STATES PATENT OFFICE
CERTIFICATE OF CORRECTION

Patent No. 3,635,616 Dated January 18, 1972

Inventor(s) PERUVEMBA SWAMINATHA VENKATESAN

It is certified that error appears in the above-identified patent and that said Letters Patent are hereby corrected as shown below:

No.[72] Inventor: "Swaminathan" should read --Swaminatha--.
Column 1, line 53, after "of" insert --a--.
Column 2, line 31, "lever" should read --level--.
Column 6, line 31, "ro" should read --or--.
Column 8, Equation 12, should read

$$--S_{Tn-1} = \frac{S_{Rn-1}(R_{n-1}^2 + R_n^2)}{R_n^2 - R_{n-1}^2} --;$$

Equation 13, should read

$$--R_n^2 = \frac{S_{Rn-1}^2 R_{n-1}^2 + S_{Tn-1} R_{n-1}^2}{S_{Tn-1} - SR_{n-1}} --$$

Column 8, line 67, "radii" should read --radial--.
Column 9, Equation 14, lines 19 and 20, should read

$$--S_{Rn-1 \text{ no load}} = S_{Rn-1} - \left[\frac{S_{R1} R_1^2}{R_n^2 - R_1^2}\left(\frac{1 - R_n^2}{R_{n-1}^2}\right)\right] --$$

Column 9, Equation 15, lines 43 and 44, should read

$$--S_{Tn-1 \text{ no load}} = \left[S_{Tn-1} - \frac{S_{R1} R_1^2}{R_n^2 - R_1^2}\left(\frac{1 + R_n^2}{R_{n-1}^2}\right)\right]$$

(continued next page)

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UNITED STATES PATENT OFFICE
CERTIFICATE OF CORRECTION

Patent No. 3,635,616 Dated January 18, 1972

Inventor(s) PERUVEMBA SWAMINATHA VENKATESAN

It is certified that error appears in the above-identified patent and that said Letters Patent are hereby corrected as shown below:

Column 10, Equation 17, lines 7 and 8, should read

$$--\text{Delta}_{n-1} = \frac{2S_{Rn-1\ interf}\ R_{n-1}^{3}\ (R_n^2 - R_1^2)}{E\ (R_{n-1}^2 - R_1^2)\ (R_n^2 - R_{n-1}^2)}--$$

Column 10, line 11, "Deltan$_{-1}$" should read --Delta$_{n-1}$--.

Column 13, claim 19, lines 7 and 8, should read

$$--\text{Delta}_{n-1} = \frac{2S_{Rn-1\ interf}\ R_{n-1}^{3}\ (R_n^2 - R_1^2)}{E(R_{n-1}^2 - R_1^2)\ (R_n^2 - R_{n-1}^2)}--$$

Column 13, claim 20, line 14, delete "a" .

Signed and sealed this 27th day of June 1972.

(SEAL)
Attest:

EDWARD M.FLETCHER,JR.
Attesting Officer

ROBERT GOTTSCHALK
Commissioner of Patents