

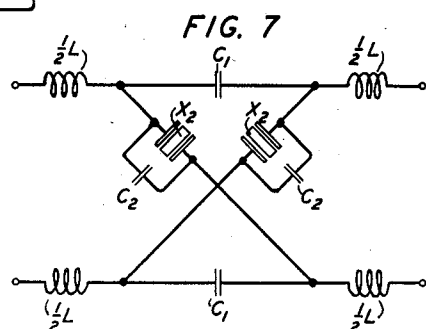
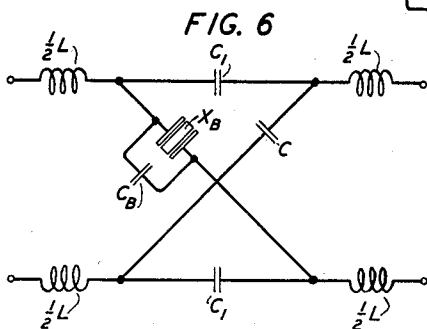
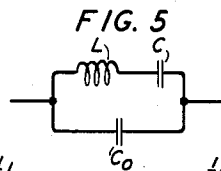
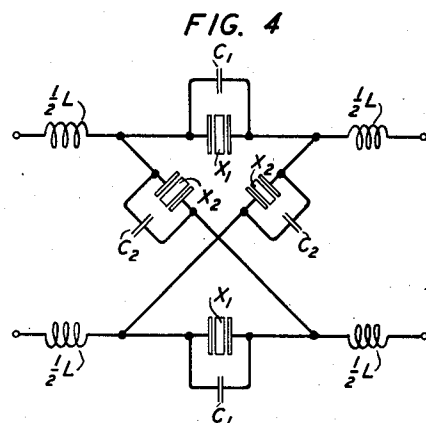
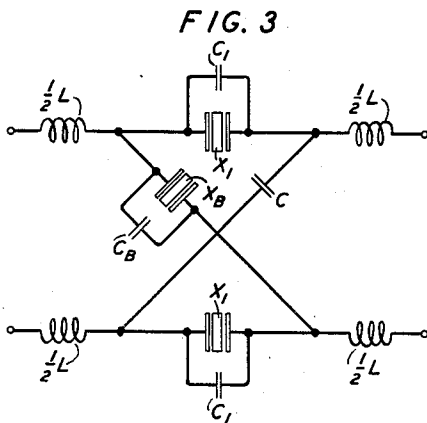
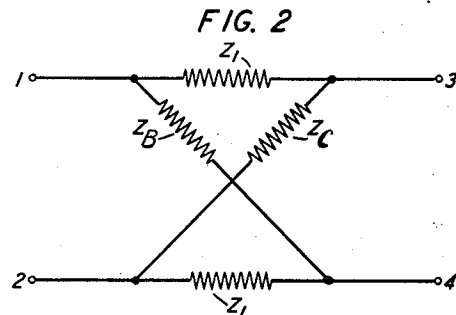
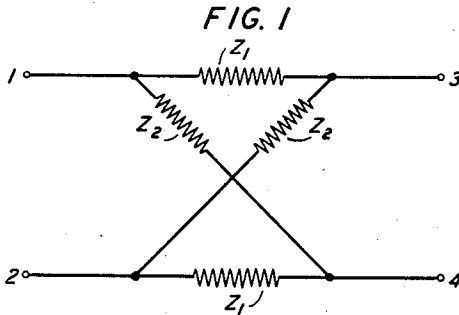
April 2, 1935.

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1,996,504

WAVE FILTER

Filed April 12, 1933



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1,996,504

WAVE FILTER

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Application April 12, 1933, Serial No. 665,685

4 Claims. (Cl. 178—44)

This invention relates to wave filters, particularly of the type using piezoelectric crystals as impedance elements, and has for its principal object reduction of the number of crystals required for the provision of a desired transmission characteristic.

A type of piezoelectric crystal wave filter which has proven to be highly practical where relatively wide transmission bands are required, consists of a symmetrical lattice network the branches of which are equal in pairs and comprise only combinations of piezoelectric crystals and capacities, in combination with external inductances connected either in series or in shunt at each end of the lattice. The lattices in such filters may include crystals in all four branches or, in simpler cases, one pair of branches may comprise only capacities and the other pair a single crystal. In any case at least two crystals are required.

In accordance with this invention the practical construction of filters of this type is simplified by substituting a simple condenser for one of the lattice branches which includes a crystal and modifying the crystal of the other corresponding branch to compensate for the substitution. This results in an unsymmetrical lattice having one pair of equal branches which may or may not include piezoelectric crystals and a pair of unequal branches one of which consists of a simple capacity and the other of which includes a piezoelectric crystal. The filters of the invention include unsymmetrical lattice networks of this type in combination with external inductances connected either in series or in shunt at each end of the lattice.

The invention will be more fully understood from the following detailed description and by reference to the appended drawing of which:

Figs. 1 and 2 are schematic diagrams illustrating a principle involved in the invention;

Figs. 3 and 4 show respectively a filter in accordance with the invention and its symmetrical prototype;

Fig. 5 is a diagram representing the electrical properties of a crystal; and

Figs. 6 and 7 show respectively a modified network of the invention and its symmetrical prototype.

Referring to the drawing, Figures 1 and 2 show respectively general schematics of a symmetrical lattice and an unsymmetrical lattice of the type corresponding to the filters of the invention. In the former the line branch impedances are designated Z_1 and the lattice branch impedances Z_2 . In the latter the line branches also have the

impedance Z_1 but the lattice branches have unequal impedances Z_B and Z_C respectively. In both figures the input terminals are marked 1, 2, and the output terminals 3, 4.

The two networks may be made equivalent in respect of their image impedances and transfer constants provided the impedances Z_B and Z_C of Fig. 2 bear a certain required relationship to each other and to the impedances Z_1 and Z_2 . The required relationship may be determined from an examination of the expressions for the transmission parameters of the two networks in terms of the branch impedances. The products of the open circuit and short circuit impedances define the values of the image impedances and their ratios define the transfer constants. For the networks of Figs. 1 and 2 the respective image impedances K_1 and K_2 have the values

$$K_1^2 = Z_1 Z_2, \quad (1)$$

and

$$K_2^2 = \frac{Z_1 [Z_1 (Z_B + Z_C) + 2 Z_B Z_C]}{2 Z_1 + Z_C + Z_B} \quad (2)$$

The transfer constants θ_1 and θ_2 have the values

$$\tanh^2 \theta_1 = \frac{4 Z_1 Z_2}{(Z_1 + Z_2)^2}, \quad (3)$$

and

$$\tanh^2 \theta_2 = \frac{Z_1 [Z_1 (Z_B + Z_C) + 2 Z_B Z_C] (2 Z_1 + Z_C + Z_B)}{(Z_1 + Z_C)^2 (Z_1 + Z_B)^2}$$

The requirement that the two networks have the same image impedance gives

$$Z_2 = \frac{Z_1 (Z_B + Z_C) + 2 Z_B Z_C}{2 Z_1 + Z_C + Z_B} \quad (4)$$

from which

$$Z_B = \frac{2 Z_1 Z_2 - Z_C (Z_1 - Z_2)}{2 Z_C + (Z_1 - Z_2)} \quad (5)$$

If the impedance Z_0 is chosen arbitrarily Equation (4) permits a value of Z_B to be found such that the two networks will have the same image impedance.

It is also necessary for equivalence that the two transfer constants should be equal. It will be found, however, that the value of Z_2 given by Equation (3) is such as to fulfill this condition. This may be shown by substituting the given value of Z_2 in the expression for $\tanh \theta_1$, the resulting expression obtained being the same as

that for $\tanh \theta_2$. The relationship set forth in Equation (4) is therefore sufficient for the complete equivalence of the networks.

The network shown schematically in Fig. 3 represents a piezoelectric crystal band pass filter in accordance with the invention which may be derived in the manner outlined above from the symmetrical prototype network shown in Fig. 4. In Fig. 3 the line branches of the lattice, corresponding to impedances Z_1 , comprise piezoelectric crystals X_1 shunted by capacities C_1 . The lattice branch corresponding to the impedance Z_c of Fig. 2 consists of a simple capacity C and the branch corresponding to impedance Z_B comprises a piezoelectric crystal X_B shunted by a capacity C_B . External inductances of value designated by $L/2$ are added in series in each line at both ends of the lattice. These cooperate with the crystals in determining the width of the band in the manner described in the copending application of W. P. Mason, Serial No. 653,622, filed January 26, 1933.

The symmetrical prototype network of Fig. 4 is similar in its structure except that the two lattice arms comprise similar crystals X_2 shunted by capacities C_2 .

The crystal branches in each case comprise crystals shunted by capacities. The crystals are preferably of quartz, cut and mounted in the manner described in the above identified application and the shunting condensers are preferably small well insulated variable air condensers. The combination of the crystal and the shunt capacity corresponds very accurately in its impedance characteristics to an electrical impedance of the type illustrated schematically in Fig. 5, comprising the combination of an inductance L and capacity C connected in series with a shunt capacity C_0 . The inductance L and the capacity C represent the piezoelectric properties of the crystal and the capacity C_0 represents the electrostatic capacity of the crystal electrodes plus the external shunting capacity. A characteristic property of the combination is that the effective shunting capacity C_0 is very large in comparison with the piezoelectric capacity C , its minimum value being at least as great as 125 C .

On account of this large value of the capacity ratio the impedance of the crystal is characterized by a very sharp transition from a resonance condition to an anti-resonance condition as the frequency is varied. The resonance and the an-

ti-resonance frequencies differ by a maximum of 0.4 per cent, the resonance frequency being the lower. Mathematically the impedance may be expressed by the formula

$$Z = \frac{1}{j\omega C_0} \frac{\omega^2 - \omega_a^2}{\omega^2 - \omega_b^2} \quad (5)$$

where Z denotes the impedance, ω denotes 2π times frequency, and the subscripts a and b correspond to resonance and anti-resonance respectively.

The design of a filter of the type shown in Fig. 3 is most readily accomplished by derivation from its symmetrical prototype shown in Fig. 4 by the general procedure outlined above. The transmission characteristics and the design computation of the prototype network are fully explained in the above mentioned copending application of W. P. Mason. Briefly the design procedure is as follows: The inductances added external to the lattice are equivalent to inductances of value L added in series with each branch. The combination of an inductance in series with a capacity shunted crystal gives a reactance characterized by resonances at two frequencies and an intermediate anti-resonance. The separation of these critical frequencies may be made substantially uniform by proportioning the inductance to resonate with the total shunt capacity of the crystal at the crystal resonance frequency and the amount of the separation may be controlled by varying the value of the shunt capacity. By giving the crystals appropriate natural frequencies the location of the critical frequencies of the branches may be controlled and by so allocating these frequencies that the lower resonance and the anti-resonance frequencies of one pair of branches coincide respectively with the anti-resonance and upper resonance frequencies of the other pair a single transmission band is obtained.

The derivation of the network of Fig. 3 from its prototype proceeds as follows: The line branch impedances, Z_1 , of the lattice and of its prototype can be expressed in terms of the critical frequencies by the formula

$$Z_1 = \frac{1}{j\omega C_{01}} \frac{\omega^2 - \omega_{1a}^2}{\omega^2 - \omega_{1b}^2} \quad (6)$$

Where C_{01} is the total shunt capacity of the impedance and ω_{1a} and ω_{1b} correspond to the resonance and anti-resonance frequencies respectively. The lattice branch impedance Z_2 of the prototype may be expressed by the similar formula

$$Z_2 = \frac{1}{j\omega C_{20}} \frac{\omega^2 - \omega_{2a}^2}{\omega^2 - \omega_{2b}^2} \quad (7)$$

in which the quantities C_{20} , ω_{2a} , and ω_{2b} correspond to the like quantities in Equation (7). The value of Z_c which in Fig. 3 is a simple capacity is given by

$$Z_c = \frac{1}{j\omega C} \quad (8)$$

Substituting the values given above in Equation (4) an expression is obtained for the value of the impedance Z_B necessary to make the network of Fig. 3 equivalent to its prototype. This expression is as follows:

$$Z_B = \frac{1}{j\omega} \frac{2C(\omega^2 - \omega_{1a}^2)(\omega^2 - \omega_{2a}^2) - C_{20}(\omega^2 - \omega_{1a}^2)(\omega^2 - \omega_{2b}^2) + C_{10}(\omega^2 - \omega_{1b}^2)(\omega^2 - \omega_{2a}^2)}{2C_{10}C_{20}(\omega^2 - \omega_{1b}^2)(\omega^2 - \omega_{2b}^2) + CC_{20}(\omega^2 - \omega_{1a}^2)(\omega^2 - \omega_{2b}^2) - CC_1(\omega^2 - \omega_{1b}^2)(\omega^2 - \omega_{2a}^2)} \quad (9)$$

The numerator and the denominator of this expression are of the fourth degree in ω , but involve no odd degree terms. The expression therefore represents in general an impedance having two resonance and two anti-resonance frequencies such as might be obtained by connecting two crystals in series. I have found, however, that, by giving the capacity C a particular value, a common factor can be removed from the numerator and the denominator leaving a simplified expression of the second degree in ω which represents an impedance that can be realized by a capacity shunted crystal.

If the numerator and the denominator of Equation (9) be expanded and the terms collected at

will be found that the terms of zero degree are respectively

$$\omega_{2a}^2(C\omega_{1a}^2 + C_{1o}\omega_{1b}^2) + \omega_{1a}^2(C\omega_{2a}^2 - C_{2o}\omega_{2b}^2) \quad (10)$$

for the numerator, and

$$C_{2o}\omega_{2b}^2(C\omega_{1a}^2 + C_{1o}\omega_{1b}^2) - C_{1o}\omega_{1b}^2(C\omega_{2a}^2 - C_{2o}\omega_{2b}^2) \quad (11)$$

for the denominator.

If the numerator and the denominator have a common factor of the type $(\omega^2 - A^2)$, where A is a constant, then, from ordinary mathematical principles, it follows that A^2 will be a common factor of the zero degree terms of the two expressions. As written above the expressions 10 and 11 do not have a common factor but by choosing the capacity C such that

$$\frac{C\omega_{2a}^2 - C_{2o}\omega_{2b}^2}{C - C_{2o}} = \frac{C\omega_{1a}^2 + C_{1o}\omega_{1b}^2}{C + C_{1o}} \quad (12)$$

Expressions 10 and 11 may be transformed respectively to

$$(C\omega_{2a}^2 - C_{2o}\omega_{2b}^2)(\omega_{2a}^2 \frac{C + C_{1o}}{C - C_{2o}} + \omega_{1a}^2) \quad (13)$$

and

$$(C\omega_{2a}^2 - C_{2o}\omega_{2b}^2)(C_{2o}\omega_{2b}^2 \frac{C + C_{1o}}{C - C_{2o}} - C_{1o}\omega_{1b}^2) \quad (14)$$

A simple trial shows that the factor

$$\omega^2 - (C\omega_{2a}^2 - C_{2o}\omega_{2b}^2)$$

is common to both numerator and denominator and its removal therefrom leaves the expression for Z_B in the form

$$Z_b = \frac{1}{j\omega C_e} \frac{\omega^2 - \omega_c^2}{\omega^2 - \omega_b^2} \quad (15)$$

where

$$C_e = \frac{2C_{1o}C_{2o} + C(C_{2o} - C_{1o})}{2C + C_{1o} - C_{2o}} \quad (16)$$

$$\omega_c^2 = \frac{\omega_{1a}^2(C - C_{2o}) + \omega_{2a}^2(C + C_{1o})}{2C + C_{1o} - C_{2o}} \quad (17)$$

and

$$\omega_b^2 = \frac{\omega_{2b}^2 C_{2o}(C + C_{1o}) - \omega_{1b}^2 C_{1o}(C - C_{2o})}{2C_{1o}C_{2o} + C(C_{2o} - C_{1o})} \quad (18)$$

The expression in Equation (15) will correspond to an impedance that can be realized by a capacity shunted crystal only if the frequencies ω_a and ω_b are separated by an amount less than 0.4 per cent. I have found, however, from the computation of a sufficient number of designs that the frequency separation is within the required range so long as the prototype filter branches include only crystals as illustrated by Fig. 3. The value of the capacity C in the capacity arm of the filter is obtainable from Equation 12. This, upon simplification, is a quadratic equation, which gives two values of C one of which is always positive and the other always negative. The positive value must of course be chosen.

A simpler form of the invention is illustrated schematically in Fig. 6, the symmetrical prototype being shown in Fig. 7. In these networks the line branches of the lattices, i. e. the Z_1 impedances, consist of simple capacities C_1 instead of capacity shunted crystals. The impedances of the other branches of Fig. 6 correspond to the respective branches of Fig. 3 and those of Fig. 7 to the respective branches of Fig. 4.

The prototype network is a band pass filter which will pass a single band of frequencies provided the inductance L resonates with the line branch capacity C_1 at the anti-resonance of crystal capacity combination X_2C_2 of the lattice branches. Preferably also the inductance L should be such as to resonate with the total capacity of the shunted crystals at some frequency close to the crystal resonance. This assures a high attenuation level both above and below the band. The design of the unsymmetrical filter of the invention from the constants of the prototype network follows a similar procedure to that discussed in connection with Fig. 3. The impedance Z_1 of the line branches of the prototype lattice, and also of the lattice of Fig. 6, has the value

$$Z_1 = \frac{1}{j\omega C_1},$$

and the lattice branch impedance Z_2 of the prototype has the value given by Equation (7). The substitution of these values in Equation (4) gives directly for the impedance Z_b an expression of the same form as Equation (15) in which the effective capacity and the critical frequencies have the values

$$C_e = \frac{2C_1C_{2o} + C(C_{2o} - C_1)}{C_1 + 2C - C_{2o}}, \quad (19)$$

$$\omega_a^2 = \frac{\omega_{2a}^2(C_1 + 2C) - \omega_{2b}^2C_{2o}}{C_1 + 2C - C_{2o}}, \quad (20)$$

and

$$\omega_b^2 = \frac{\omega_{2b}^2C_{2o}(C_1 + 2C) - \omega_{2a}^2C_1C}{2C_1C_{2o} + C(C_{2o} - C_1)}. \quad (21)$$

In this case the capacity C may have a range of values limited only by the requirement that the critical frequencies ω_a and ω_b must be sufficiently close together to correspond to the frequencies of a capacity shunted crystal.

What is claimed is:

1. A broad band wave filter comprising a lattice network having line branches of equal impedance, a lattice branch consisting of a simple capacity, a second lattice branch comprising a piezoelectric crystal, and an inductance connected at each end of the lattice external thereto, the reactance of said inductance and the impedances of the branches of said lattice network being proportioned with respect to each other to provide a single transmission band between two preassigned frequencies.

2. A broad band filter in accordance with claim 1 in which the impedances of the equal line branches of the lattice are constituted by simple capacities.

3. A broad band filter in accordance with claim 1 in which the impedances of the equal line branches are constituted by piezoelectric crystals and capacities in shunt thereto.

4. A wave filter comprising a lattice network interconnecting a pair of input terminals and a pair of output terminals, similar piezoelectric crystals included as impedance elements in the line branches of said lattice, a lattice branch including only a simple capacity, and a second lattice branch including a piezoelectric crystal, the impedances of said branches being proportioned with respect to each other to provide a single pass band between preassigned frequencies.

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