

[54] **ANGULAR RATE BOMBING SYSTEM**
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[73] Assignee: The United States of America as represented by the Secretary of the Navy

2,819,461 1/1958 Bryan.....235/61.5 E
3,230,348 1/1966 Hammond, Jr.235/61.5 R
3,264,451 8/1966 Faxen et al.....235/61.5 R
2,825,055 2/1958 Chance.....235/61.5 D
2,995,984 8/1961 Helgeson et al.....89/1.5 E
3,288,984 11/1966 Bernier et al.235/61.5 R

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[21] Appl. No.: 87,519

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[52] U.S. Cl.....235/61.5 D, 89/1.5 E, 235/61.5 E
[51] Int. Cl.G06g 7/80
[58] Field of Search.....235/61.5 R, 61.5 D, 61.5 E,
235/61.5 G; 89/1.5 E

[56] References Cited

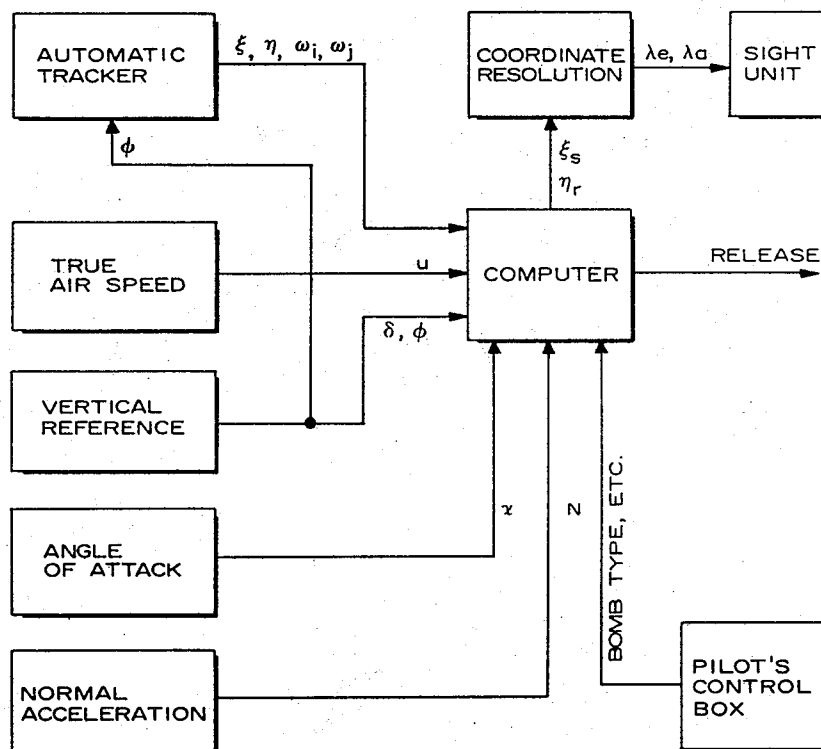
UNITED STATES PATENTS

3,033,084 5/1962 Wheeler et al.....89/1.5 E
2,737,652 3/1956 White et al.....235/61.5 E

[57] **ABSTRACT**

An angular rate bombing system (ARBS) which utilizes angular rate and angle measurements from an automatic target tracker, pitch and roll information from a standard vertical gyro, true airspeed data and angle of attack information to compute an azimuth lead angle to release ordnance at the correct elevation lead angle.

5 Claims, 14 Drawing Figures



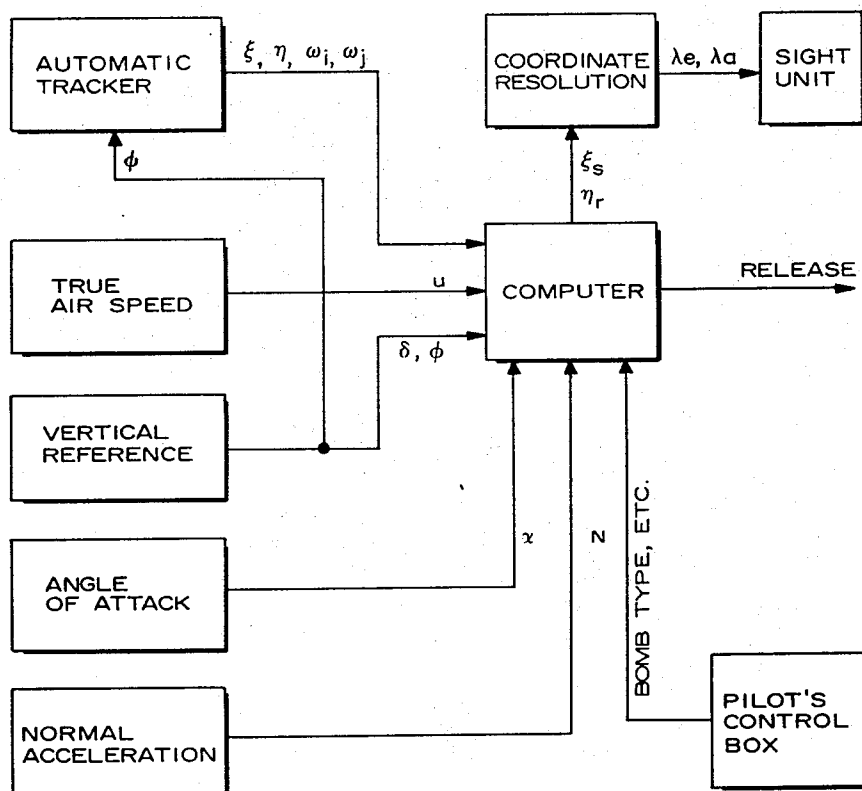


FIG. 1.

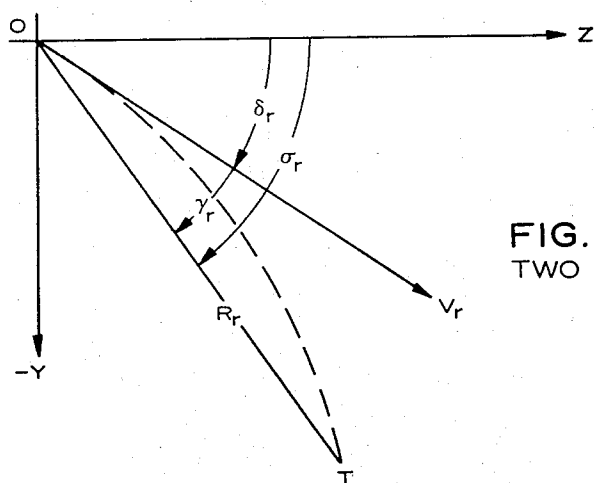
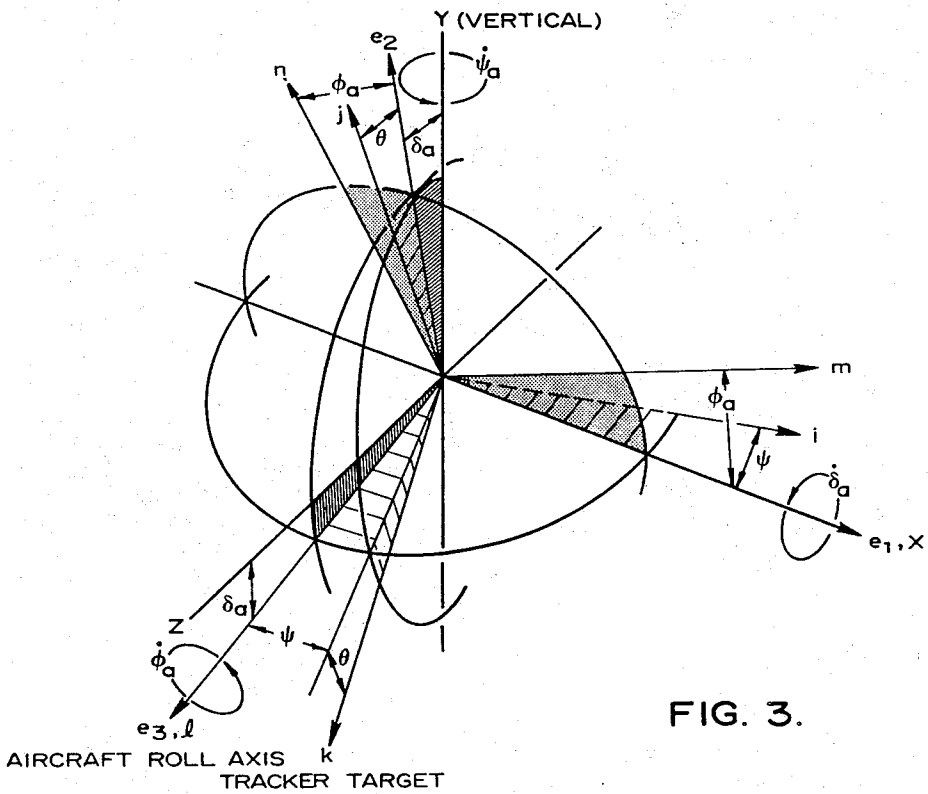
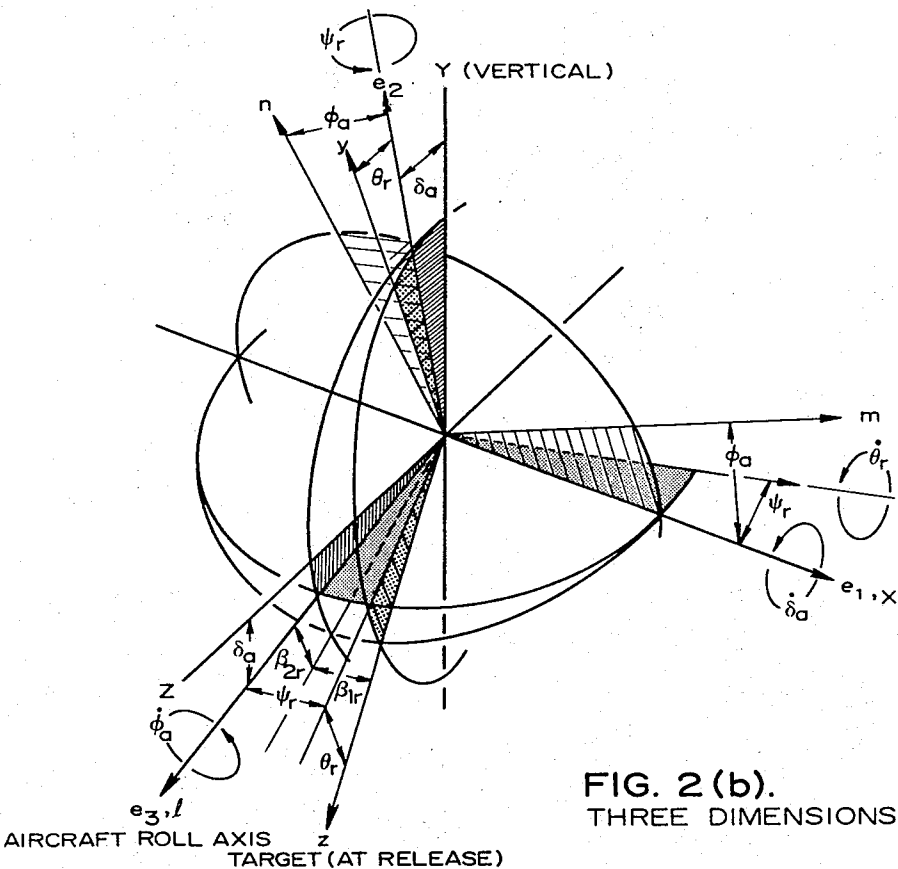


FIG. 2(a).
TWO DIMENSIONS

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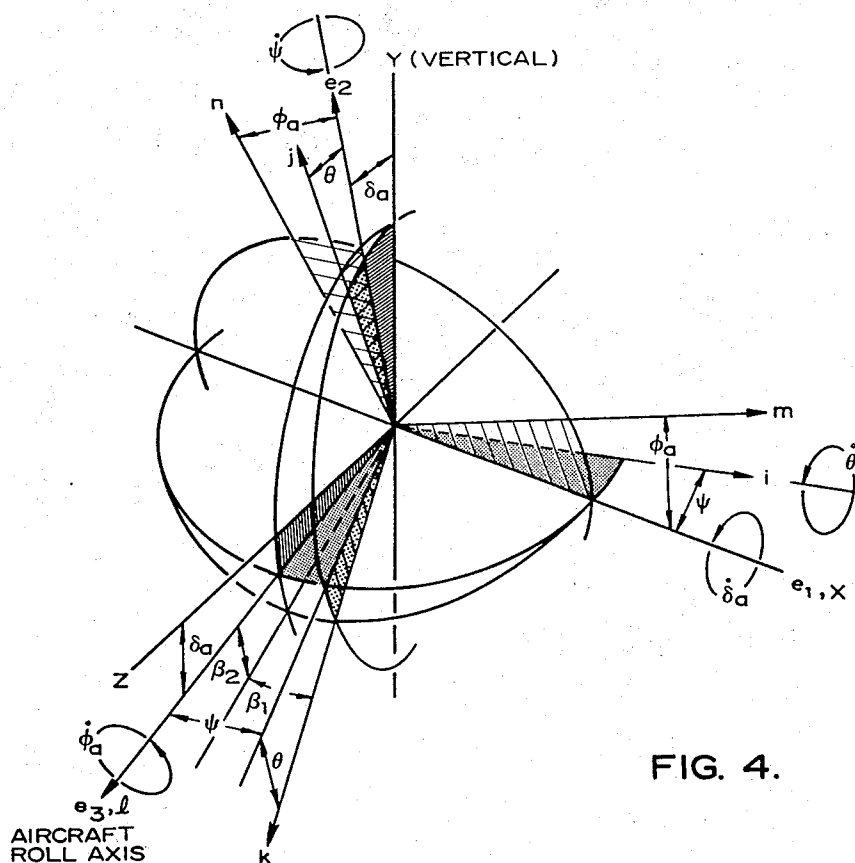


FIG. 4.

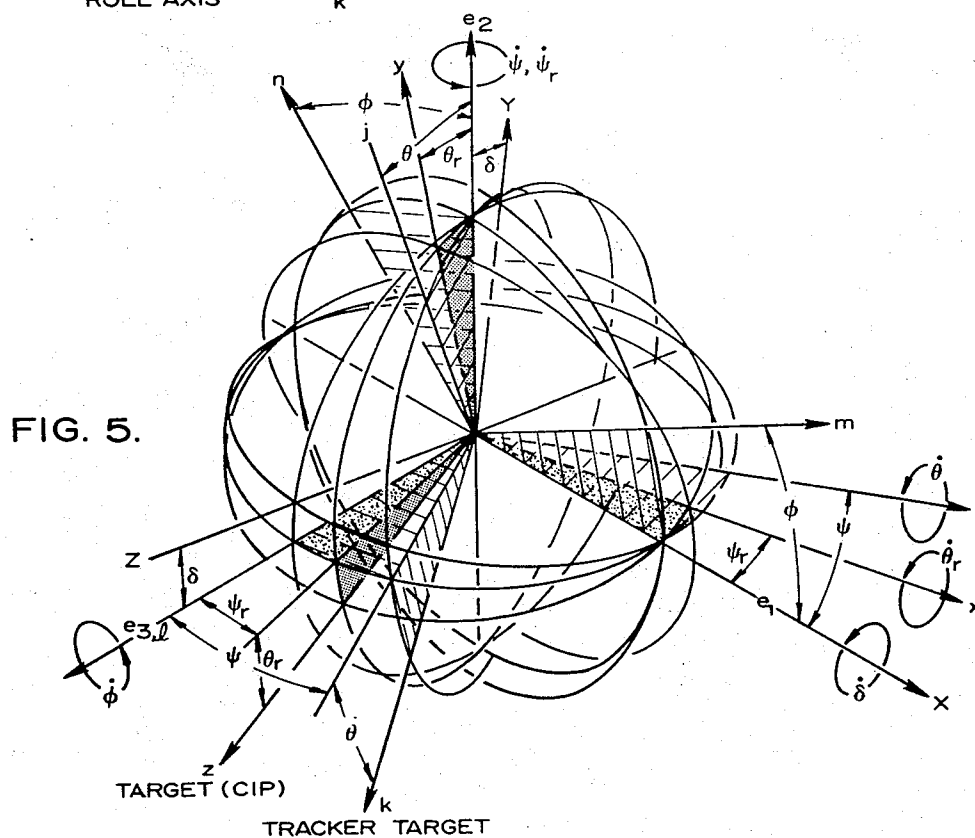


FIG. 5.

TARGET (CIP)

TRACKER TARGET

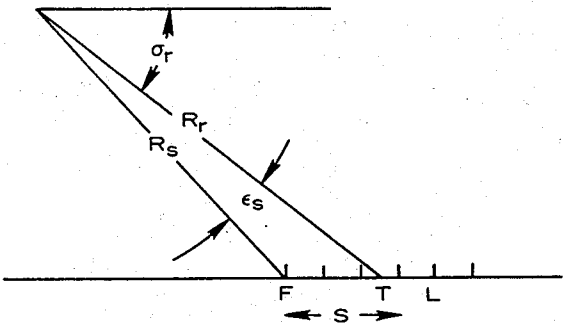


FIG. 6.

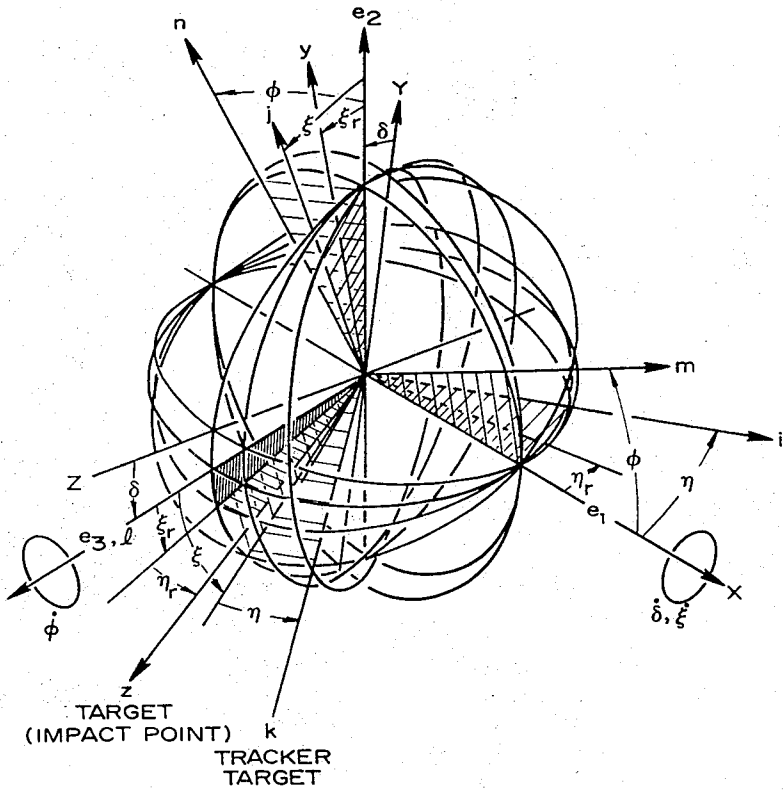


FIG. 7.

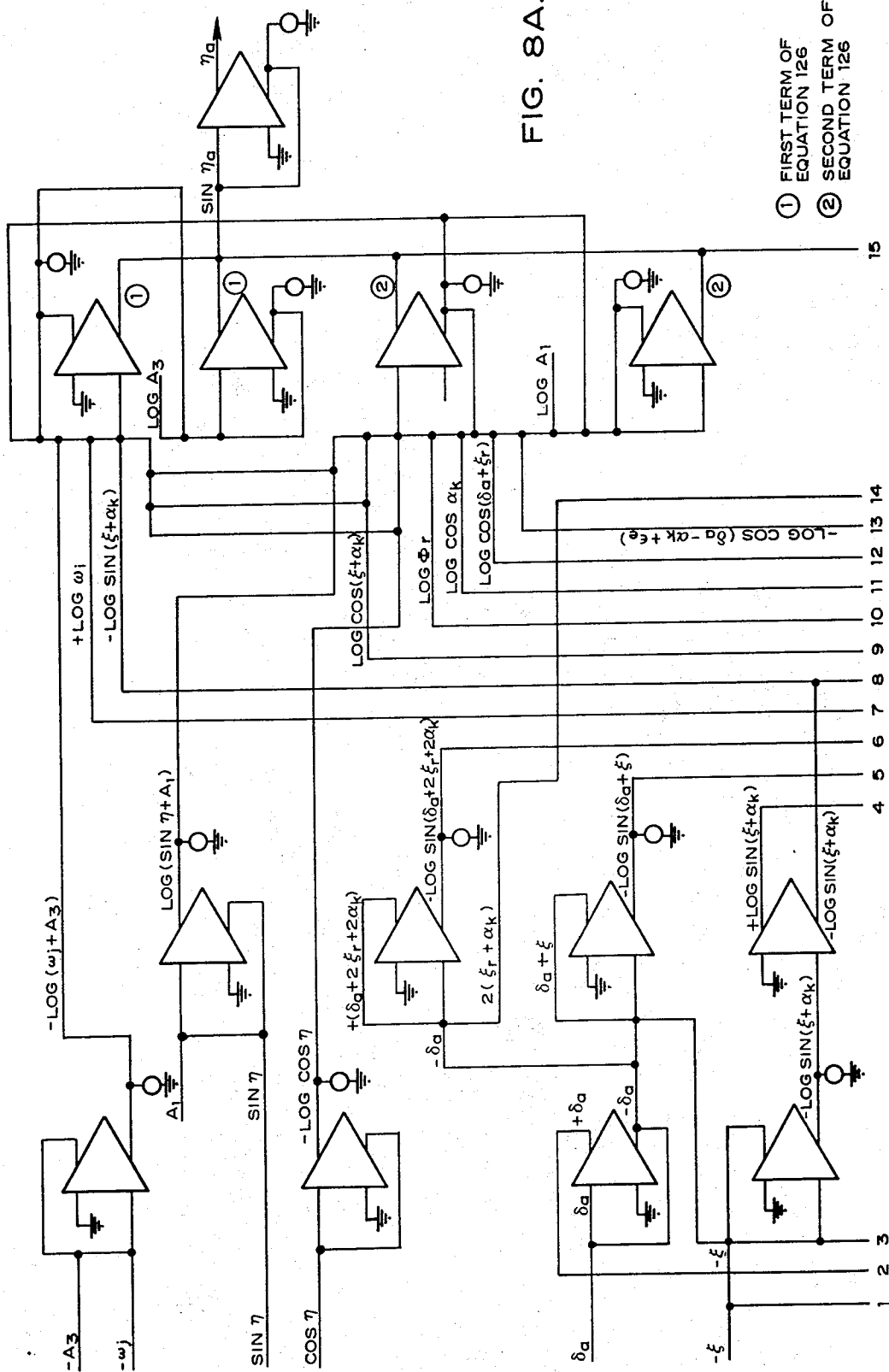


FIG. 8A.

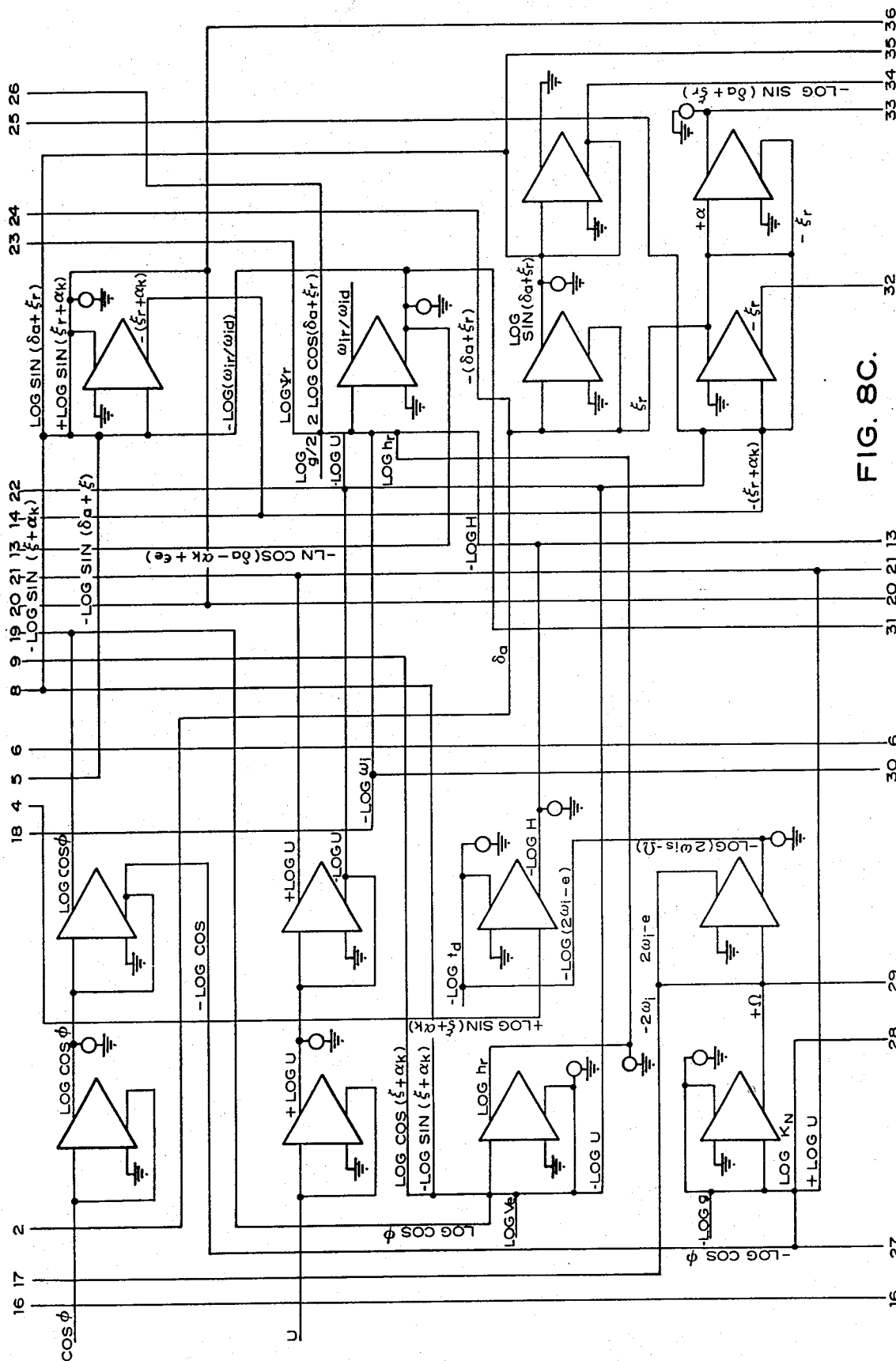


FIG. 8C.

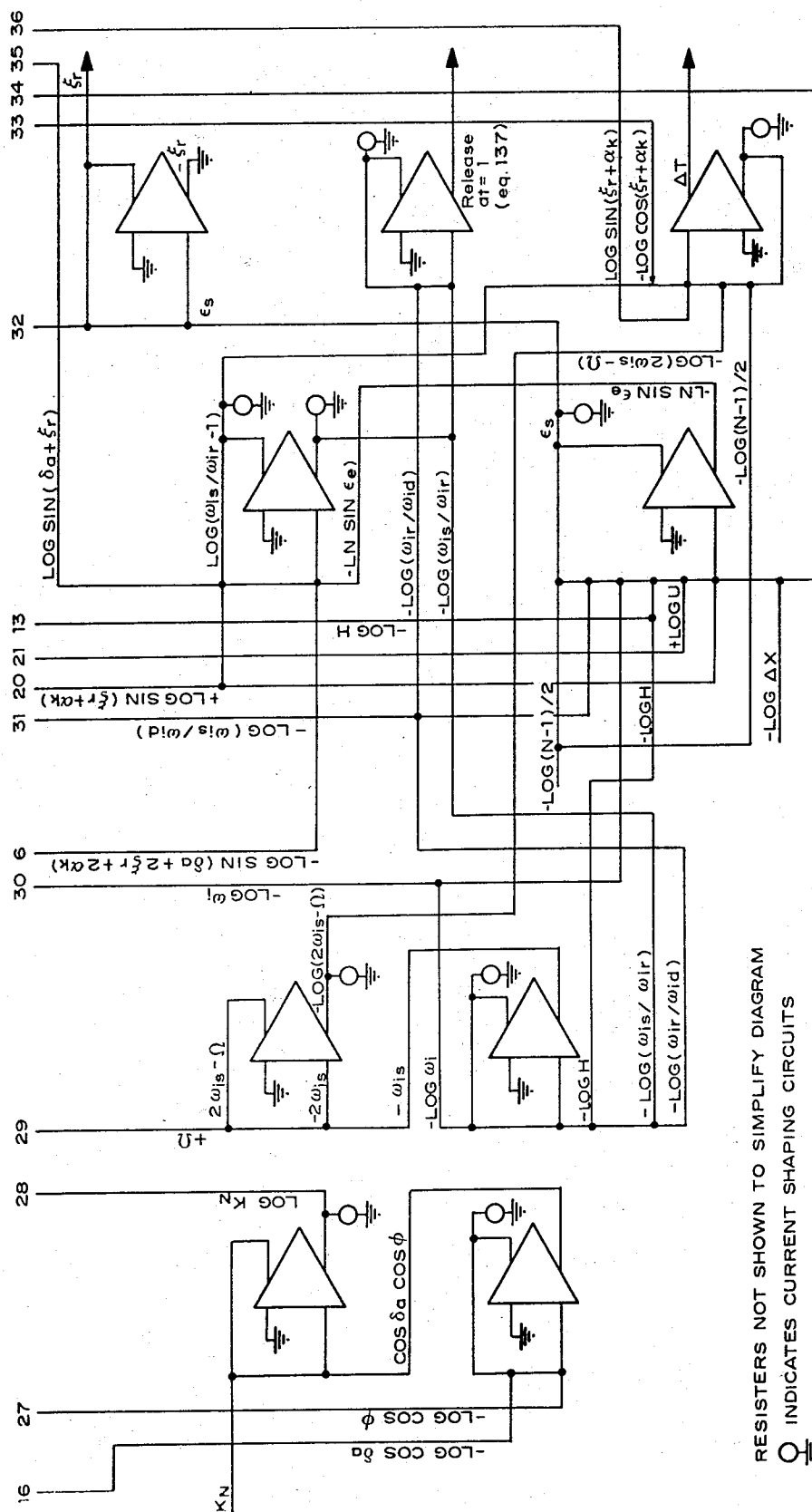


Fig. 8D.

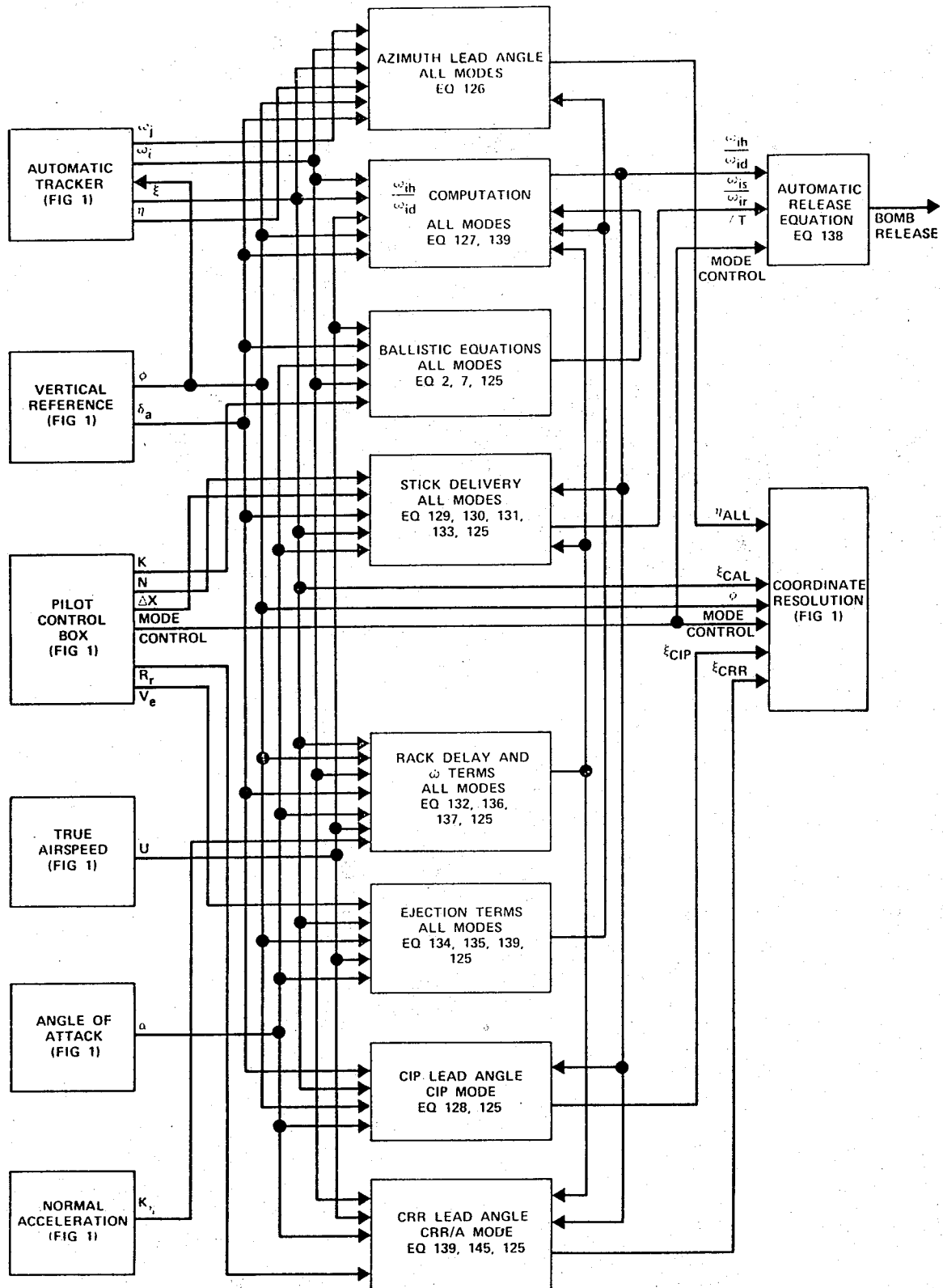


Fig. 9

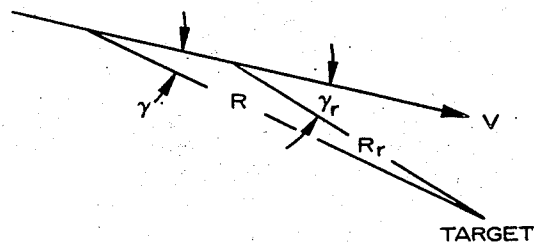


FIG.10.

ANGULAR RATE BOMBING SYSTEM

BACKGROUND OF THE INVENTION

Typical standard bombing systems require range-to-target and velocity (direction and magnitude) inputs. Bombing accuracy is basically a function of how accurate these inputs are. At first, airspeed and barometric altitude were used; next, these inputs were improved by using radar range; and then, Doppler-inertial velocity was used. It is a matter of history that each step in the improvement of bombing capability (accuracy, flexibility, or versatility) causes an escalation in system cost and complexity; and finally, the point is reached at which the increase in cost and complexity is unfavorably disproportionate to the increase in performance. This point appears to have been passed in some sophisticated systems.

Recent advances in air-to-ground tracking systems offer the possibility of approaching the practical limit of bombing accuracy while avoiding the problems of the standard bombing system. The mathematical sophistication inherent in a tracker-based system accounts for a decrease in hardware complexity and a concomitant increase in reliability through elimination of the need for inputs from inertial navigation and radar systems. The low level of complexity, plus today's microminiaturization techniques, allows a complete daylight bombing system to occupy a volume of less than 1 cu. ft.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a simplified block diagram of the invention; FIG. 2(a) is a graph illustrating bomb ballistic coordinates in two dimensions;

FIG. 2(b) illustrates the bomb ballistic coordinates in three dimensions;

FIG. 3 illustrates the coordinates for a three axis tracker;

FIG. 4 is another illustration of the coordinates for a three axis tracker;

FIG. 5 illustrates the coordinates for a CIP attack;

FIG. 6 illustrates stick bombing graphically;

FIG. 7 illustrates an alternate tracker coordinate system;

FIG. 8 (FIGS. 8A, 8B, 8C and 8D taken together) is a functional schematic of a computer used in the present invention;

FIG. 9 is a computational sub-section block diagram correlating FIG. 8 with the equations in the specification; and

FIG. 10 illustrates bomb ballistic coordinates in the vertical plane for CRR/A.

DESCRIPTION OF THE PREFERRED EMBODIMENT

The ability of an optical-TV tracker, operating in the visible spectrum, to lock on and automatically track targets having sufficient contrast to be detectable and recognizable by a pilot has been amply demonstrated. Such a target-tracking device, with proven ability to track tactical ground targets from moving aircraft, is applicable to an angular rate bombing system (ARBS) such as described here. The major benefits of such a system are as follows:

1. It is a relatively simple, small bombing system that has a high, inherent delivery accuracy.

2. Wind and target motion are both automatically accounted for in the solution without requiring an explicit measurement of target or aircraft ground velocity, thereby eliminating the need for ground velocity sensors. A range-to-target measurement is not required, thus eliminating the need for radar or laser air-to-ground ranging systems.

3. The development of laser (and beacon) target-designation schemes will give a semiblind bombing capability.

The ARBS is inherently a "mode-less" weapon-delivery system in that it will provide an automatic weapon-release signal whenever the measured (elevation) rate of the line-of-sight to the target equals the computed rate needed to obtain a hit. The system does not care how the aircraft reaches this condition; thus, the pilot can pursue any type of reasonable attack he chooses. The tactics include dive-toss, dive or glide, and medium- and low-altitude level deliveries. Stick bombing is available with any of these forms of delivery.

TYPICAL ATTACK (COMPUTED AZIMUTH LEAD, AUTOMATIC RELEASE)(CAL/A)

The sequence of events for a typical daylight attack is as follows. After the pilot visually detects and identifies the target, he dives the aircraft to put the (inertially stabilized) sighting pipper on the target. (The tracker axis is aligned with the sighting pipper.) When the pilot is sighting on the target, he pushes the lock-on button. The tracker then tracks the target automatically, and the computer generates the azimuth tracking commands and release signal. After lock-on, the movable reticle in the servoed sight is slaved to the tracking head so that sight-pip tracking in elevation is accomplished automatically. However, the reticle is displaced in azimuth in an amount sufficient to correct for cross-range wind and target motion, and the pilot must steer in azimuth to put the reticle on the target. Weapon release occurs automatically. From the lock-on point, the pilot may pursue any tactic he desires. For a glide attack, he may pull up slightly immediately after lock-on to establish a desired flight path. To convert to a toss attack, the pilot needs only to pull up at any point in the attack he chooses. Release will occur at the proper time.

TYPICAL ATTACK (CIP DELIVERY-MANUAL RELEASE)

Using the computed impact point (CIP) delivery, the sequence of events in an attack is as follows. After visually detecting and identifying the target, the pilot dives the aircraft such that the sighting pip is on or near the target. He pushes the lock-on button. The tracker tracks this point automatically, and the computer generates the azimuth and elevation lead angles. These angles, displayed on the sight unit, tell where the bomb would impact if it were dropped at that point. The pilot may fly any reasonable path to direct the sight pip (impact point) toward the target. When the pip is on the target, he pushes the manual bomb-release button.

Note that the aircraft may be in a banked turn. While the solution is acceptable for a rapid maneuver, the pilot reaction for a manual release is not fast enough for some pull-up rates. Thus an ideal CIP attack would be one similar to a present manual dive/glide attack.

(In CIP displays the pilot should not track the target but should move the sight pip through the target at a reasonable angular rate, as he does in present noncomputer attacks.)

TYPICAL ATTACK (CIP DELIVERY-AUTOMATIC RELEASE)

The response time problem as described above suggests another (and possibly the best) method of delivery. It is the automatic release with a CIP display. Here, the pilot initially locks the sight pip (tracker) on the target, and the display is the computed bomb impact point. The aircraft is flown so as to move the pip over the target. The release is automatic (based on elevation rate). (Note that the pilot would not have to "allow" an automatic release until just before the pip approaches the target.)

This latter form of operation will permit maneuvers (within the limit of the tracker gimbals) without the pilot response-time problems.

CHOSEN RELEASE RANGE, AUTOMATIC RELEASE (CRR/A)

Often it is desired to release weapons at a prechosen altitude or slant range. While this may be done for various reasons, generally it is aimed at improving accuracy by getting as close to the target before release as is safe (from enemy fire, ground collision, weapon effects). For example, retarded weapons, such as the Mk 82 with Mk 15 Snakeye fin retarded, for best results should be released at from 1,500 to 2,500 feet slant range (depending on speed). At shorter ranges the fins are not open long enough, and for longer ranges the time of flight is too long.

In this mode, after lock on, the pilot is to fly so as to put the sight pip on the target and track until release. This is the only delivery in which the pilot is asked to track the target with the sight pip. This CRR flight path is approximately a straight line or a slightly upward (positive g) curved path. Some maneuver is allowed.

ARBS USE WITH TARGET DESIGNATION SYSTEMS

For daylight operation the TV-ARBS weapon delivery system can be used basically as is. A target designation system (TDS) would furnish the TV tracker the direction to the target. The tracker would lock onto and track whatever optical contrast that is present at the indicated target position.

The presentation to the pilot would be dependent on the type of servo sight used.

a. If the sight unit in the aircraft has two movable controlled pips, one of these would indicate the target position. The other (standard CIP display) pip would indicate the bomb impact point as in a CIP delivery. The pilot flies so as to move the bomb impact point over the target indicator pip. Release is at this time. An alternate would be to use the TDS to indicate target position on the sight unit and have the tracker lock onto contrast at or near the target. Then use the standard ARBS CIP display with a manual release.

b. One movable pip is sufficient in actual use. The impact point (CIP) pip would be positioned relative to the sight's fixed pip (as if it were the target) and the

pilot flies to move this CIP pip over the fixed pip for release.

For day-night operations a, say, laser target designation system should have its own angular rate tracker and thus furnish an alternate source of angles and rates to the ARBS.

SYSTEM HARDWARE DESIGN

The keynote of the ARBS scheme is simplicity. This is carried out in the hardware design as indicated by the number of components (see FIG. 1). Measuring devices already utilized in standard aircraft flight equipment are used as much as is practical.

The heart of the ARBS is a gyro-stabilized optical-TV tracking head. Such trackers are described in U.S. Pat. Nos. 3,257,505 and 3,341,653. When locked on and tracking a target, the tracker provides the target line-of-sight rates and angles. These inputs, combined with air data and vertical information from standard aircraft equipment, are used to compute the instantaneous angular rate required for the weapon to impact the target. Bomb release occurs when this rate and the measured rate are the same.

The primary components and inputs to the system of FIG. 1 are:

a. Automatic Target Tracker. This is a TV contrast tracker similar to that used in the Walleye weapon. This input furnishes angular rate (ω_x, ω_y) and angles (ξ, η).

b. Vertical Reference. A standard aircraft vertical gyro is used to furnish pitch (δ) and roll (ϕ) information. In a highly maneuvering aircraft an improved erecting system may be desired, but is not necessary.

c. True Airspeed. A standard true airspeed measuring device is used to furnish magnitude of velocity (U) data.

Secondary or optional inputs may be also provided. However, these inputs are not absolutely necessary, but are desirable in that they increase stability and refine system accuracy. Also, they are normally already present in the aircraft.

These inputs are:

a. Angle of Attack (α). Angle of attack is used in weapon drag factors (ψ, Φ) calculation, in the azimuth lead angle, and to eliminate the need of a filter in the CIP elevation lead angle calculation. [A one degree error in angle of attack causes negligible impact error for most low-drag bomb deliveries.]

b. Pressure Altitude (H_p) or Static Pressure. Pressure altitude is used to compute density for air drag on the weapon. A hand set approximate target altitude could be used in its place. Altitude is not used in ARBS.

c. Normal Acceleration (K_n). Normal acceleration and angle of attack are inputs to a filter which is used to refine and stabilize the automatic tracker data. Only nominal accuracy is required for this use.

BALLISTIC EQUATIONS

ELEVATION LEAD ANGLE

In solving for the bomb trajectory, it is assumed that the only forces acting on the bomb after release are gravity and air drag along the direction of the air-mass velocity. Here, the effect of wind on the trajectory is restricted to the vertical plane containing the air-mass

velocity vector; crosswind is considered later. The wind structure is assumed to be uniform and equal to the wind velocity at release. These assumptions give the following ballistic (For vacuum conditions, the standard equation is $\sin \gamma_r = (gR_r \cos^2 \sigma_r / 2V_r^2 \cos \delta_r)$) equations where:

$$\sin \gamma_r = (gR_r \psi_r \cos^2 \sigma_r / 2V_r^2 \cos \delta_r) \quad (1)$$

$$\psi_r = \psi[(k_1 U^2 \sin \gamma_r \sec \sigma_r - k_2)f(W)] \quad (2)$$

where k_1 and k_2 are constants, U is the true airspeed, and W is the range wing.

From FIGS. 2a and b note that

$$\beta_{2r} = \gamma_r + \alpha_v + \epsilon_e \quad (3)$$

$$\sigma_r = \delta_a + \beta_{2r} \quad (4)$$

where ϵ_e accounts for release ejection effects, α_v is the angle 1 between the forward axis of the aircraft and the velocity direction, and δ_r is δ_a corrected for ϵ and α_v .

$$\delta_r = \sigma_r - \gamma_r = \delta_a + \alpha_v + \epsilon_e \quad (5)$$

Similarly, the time of flight can be shown to be

$$t_f = \left(\frac{R_r \cos \sigma_r}{V_r \cos \delta_r} \right) \Phi_r \quad (6)$$

where

$$\Phi_r = \Phi[(k_1 U^2 \sin \gamma_r \sec \sigma_r - k_2)f(W)] \quad (7)$$

For low drag bombs

$$\Phi_r = \psi_r \quad (8)$$

Thus, the azimuth lead angle β_{1r} (see FIG. 2) is

$$\sin \beta_{1r} = \frac{V_{cf} V_c \cos \sigma_r \Phi_r}{R_r V_r \cos \delta_r} \quad (9)$$

where V_c is the azimuth (crosswind) component of velocity.

COORDINATE SYSTEMS

There are several coordinate systems used in describing the present invention. All, except the air-mass system, are translating with the attacking aircraft's velocity (V) relative to the tracked point (target). FIG. 3 is useful in discussing four of the systems used in deriving the equations used in ARBS. An alternate tracker coordinate system is illustrated in FIG. 1 and will be referred to later in the discussion. The coordinates of FIG. 1 were used in the computer block diagram of FIG. 8.

Tracking Coordinates (i, j, k)

These are defined as i, j, k with the unit vector k along the tracker forward axis (ideally toward the (tracker) target); i along the inner gimbal; and j given by $k \times i$. This is the coordinate system in which the rate measurement is made.

The coordinate origin is moving with the attacking aircraft velocity.

Aircraft Coordinates (m, n, l Axes)

These are used here as l along the forward aircraft axis, m along left wing (l to l), and n given by $l \times m$. Thus, when the aircraft is horizontal, n is vertical.

Vertical Plane Reference Coordinate (e_1, e_2, e_3)

This is a translating (with V) nonrotating nonaccelerating system in which two of the axes are in a vertical plane that contains the aircraft l axis. The coordinate system is defined only at the time of weapon release. The e_3 axis is along l and the e_2 axis is perpendicular to e_3 in the above vertical plane. The e_1 direction is such that $e_2 = e_3 \times e_1$. These axes are the aircraft axes (m, n, l) if the roll angle, ϕ_a , is zero.

This coordinate system is selected for use in the fire control system (a three-axis tracker) to compute necessary lead angles.

AIR-MASS COORDINATES

The bomb drag is computed in air-mass coordinates. This reference system differs from the e_1, e_2, e_3 coordinates because of wind.

COORDINATE TRANSFORMATIONS

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33} \end{pmatrix} \begin{pmatrix} i \\ j \\ k \end{pmatrix} \quad (10)$$

where

$$a_{11} = \cos \psi \quad (11)$$

$$a_{12} = \sin \theta \sin \psi \quad (12)$$

$$a_{13} = \cos \theta \sin \psi \quad (13)$$

$$a_{21} = 0 \quad (14)$$

$$a_{22} = \cos \theta \quad (15)$$

$$a_{23} = -\sin \theta \quad (16)$$

$$a_{31} = \sin \psi \quad (17)$$

$$a_{32} = \sin \theta \cos \psi \quad (18)$$

$$a_{33} = \cos \theta \cos \psi \quad (19)$$

where ψ and θ are measured at the tracker gimbals.

$$\begin{pmatrix} m \\ n \\ l \end{pmatrix} = \begin{pmatrix} \cos \phi \sin \phi & 0 \\ -\sin \phi \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

where the a subscript is dropped from ϕ_a , which is the measured aircraft roll angle.

AIRCRAFT-TARGET VELOCITY

The relative velocity ($\vec{V}$) of the target in the nonaccelerating tracker coordinates (see Tracking Coordinates) is given by the equation

$$\vec{V} = \vec{v} + \vec{\omega} \times \vec{r} \quad (20)$$

where

$\vec{v}$, $\vec{\omega}$, and $\vec{r}$ are measured in the rotating tracker coordinates.

$\vec{v}$ is the apparent velocity

$\vec{\omega}$ is the angular rate of coordinate system

$\vec{r}$ is the radius vector to the target

Thus, if one assumes the tracker is tracking the target perfectly,

$$\vec{r} = rk \quad (21)$$

$$\vec{v} = vk \quad (22)$$

ω_i, ω_j components (The value of ω_k , while it can be measured by a separate gyro on the platform, is not directly determined by the target tracking device. In the case of a two-axis tracker this ω_k would be nearly the roll rate (ϕ). For the coordinates such as in FIG. 2, the magnitude of ω_k is generally small in a bombing attack. The actual

value of ω_k can be determined using the dive angle (δ_a) and the target angles (ψ and θ). of $\vec{\omega}$ are the rate (ideally) measured in the tracker gyros. Thus, Equation 20 becomes

$$\vec{V} = \omega_j r i - \omega_l r j + \dot{r} k \quad (23)$$

This velocity in the e_1, e_2, e_3 coordinates is

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} \cos \psi & \sin \theta \sin \psi & \cos \theta \sin \psi \\ 0 & \cos \theta & -\sin \theta \\ -\sin \psi & \sin \theta \cos \psi & \cos \theta \cos \psi \end{pmatrix} \begin{pmatrix} \omega_j r \\ -\omega_l r \\ \dot{r} \end{pmatrix} \quad (24)$$

or

$$V_1 = \omega_j r \cos \psi - \omega_l r \sin \theta \sin \psi + \dot{r} \cos \theta \sin \psi \quad (25)$$

$$V_2 = -\omega_l r \cos \theta - \dot{r} \sin \theta \quad (26)$$

$$V_3 = -\omega_j r \sin \theta \cos \psi - \omega_l r \sin \theta \cos \psi - \dot{r} \cos \theta \cos \psi \quad (27)$$

Thus,

$$\sin \theta + (V_2/\dot{r}) = -(\omega_l r \cos \theta / \dot{r}) \quad (28)$$

Using equations 26 and 27

$$V_3 = -\omega_j r \sin \theta + \dot{r} (\sin \psi + V_2/\dot{r}) \tan \theta \cos \psi + \dot{r} \cos \theta \cos \psi \quad (29)$$

or

$$V_3 = -\omega_j r \sin \psi + \dot{r} \cos \psi [1 + (V_2/\dot{r}) \sin \theta] \sec \theta \quad (30)$$

or

$$\frac{\dot{r}}{\cos \theta} = \frac{V_3 \left(1 + \frac{\omega_j r \sin \psi}{V_3}\right)}{\cos \psi [1 + (V_2/\dot{r}) \sin \theta]} \quad (31)$$

Thus,

$$\sin \theta + \frac{V_2}{\dot{r}} = -\frac{\omega_j r \cos \psi [1 + (V_2/\dot{r}) \sin \theta]}{(V_3 + \omega_j r \sin \psi)} \quad (32)$$

or for $V_2 < \dot{r}$,

$$\sin(\theta + \alpha_K) = -\left(\frac{\omega_j r \cos \psi}{V_3}\right) \left(\frac{1 + \alpha_K \tan \theta}{1 + \frac{\omega_j r \sin \psi}{V_3}}\right) \quad (33)$$

where

$$\alpha_K = V_2 \cos \theta / \dot{r} \quad (34)$$

From equations 25, 26, and 31

$$V_1 = (\omega_j r + V_3 \sin \psi) \sec \psi \quad (35)$$

or

$$V_1 = \left[\frac{V_3}{\cos \psi} \right]$$

$$\left[-\left(\frac{\omega_j}{\omega_l}\right) \left(\frac{\sin(\theta + \alpha_K)}{\cos \psi}\right) \left(\frac{1 + \frac{\omega_j r \sin \psi}{V_3}}{1 + \alpha_K \tan \theta}\right) + \sin \psi \right] \quad (36)$$

Equations 33 and 36 are used with the bomb lead angle (equations 1 and 9) to make the bombing system equation.

COMMENTS, ASSUMPTIONS, AND APPROXIMATIONS

The following comments apply primarily to the computed lead angle mode of operation.

The angle ψ is small, usually less than 100 milliradians. Thus, in many places the cosine is used as 1, and the sine or tangent can be used as ψ . However, ψ may be either plus or minus. (ψ is limited to the tracking gimbal limits, e.g., $\pm 30^\circ$, but is small when the pilot is near a release condition.)

The angle θ will be positive since one needs to fly above the target (and tracked point) to measure an angular rate, which is required for correct release lead angle. The value of θ_r to the target will be fairly small since the pilot must be able to see the target over the aircraft nose. The value of θ to the tracked point must be within the tracker gimbal limits at all times, and thus will probably be limited to somewhere between 45° and 60° , depending on the design of the trackers.

The velocity (V_1, V_2, V_3) is the negative of the ground velocity. Thus, a head wind causes an increase in V_3 , a down wind increases V_2 , and a wind from the left side increases V_1 .

To get an indication of the magnitude of these components, note that

V_1 is approximately the crosswind

V_2 is approximately $\alpha_a U$

$-V_3$ is approximately U

where U is the aircraft true airspeed, and α_a is the aircraft angle of attack.

Define

$$V_{23} = (V_2^2 + V_3^2)^{1/2} \quad (37)$$

$$V_{23} = V_3 \sqrt{1 + (V_2/V_3)^2} \quad (38)$$

or

$$V_{23} = V_3 \sec \alpha_v \quad (39)$$

where

$$\tan \alpha_v = V_2/V_3 \quad (40)$$

Thus, by comparing the sections on ballistic equations and aircraft-target velocity (Technically $\sqrt{V_r^2 + V_e^2} = -V_{23}$, where v_e is the ejection velocity, but for reasonable ejection, the v_e can be neglected here; it is included in the effective direction of bomb initial velocity by use of v_e/v_r as an angle.)

$$V_r = -V_{23} = -V_3 \sec \alpha_v \quad (41)$$

and

$$V_e = -V_1 \quad (42)$$

Consider the definition of α_K , α_v , and α_a . For practical purposes these are all the same for a nonrolled aircraft (unless slow airspeed and large, upward wing velocities are present). Thus,

$$\alpha_K = \alpha_v = -\alpha_a \cos \phi \quad (43)$$

The angle α_a is a small angle, particularly when the aircraft is not pulling several g acceleration. The subscript is dropped for ease in writing.

The last factor in equation 33 is approximately 1, and thus the equation can be used as

$$\sin(\theta + \alpha_K) \doteq -(\omega_l r \cos \psi / V_3) \quad (44)$$

when used in second order terms of the bombing system equation.

Using the above, for equations 33 and 36

$$\sin(\theta - \alpha \cos \phi) = \left(\frac{\omega_i r \cos \psi}{V_r \cos(\alpha \cos \phi)} \right)$$

$$\left(\frac{1 - \alpha \cos \phi \tan \theta}{1 - (\omega_i/\omega_1) \tan \psi \sin(\theta - \alpha \cos \phi)} \right) \quad (45) \quad 5$$

and

$$V_o = \left[\frac{V_r \cos(\alpha \cos \phi)}{\cos \psi} \right] \quad (46) \quad 10$$

$$\left[- \left(\frac{\omega_i \sin(\theta - \alpha \cos \phi)}{\omega_1 \cos \psi} \right) \right]$$

$$\left(\frac{1 - (\omega_i/\omega_1) \sin \psi \sin(\theta - \alpha \cos \phi)}{1 - \alpha \cos \phi \tan \theta} \right) + \sin \psi \quad (46) \quad 15$$

The component of range (r) to tracked point in the e_2, e_3 plane is (see FIGS. 4 and 5)

$$r_{23} = r \cos \beta_1 \quad (47) \quad 20$$

Thus,

$$\sin(\theta - \alpha \cos \phi) = \left(\frac{\omega_i r_{23} \cos \psi}{V_r \cos \beta_1 \cos(\alpha \cos \phi)} \right)$$

$$\left(\frac{1 - \alpha \cos \phi \tan \theta}{1 - (\omega_i/\omega_1) \tan \psi \sin(\theta - \alpha \cos \phi)} \right) \quad (48) \quad 25$$

Using the spherical trigonometric equation

$$\sin \theta = \sin \beta_2 \cos \beta_1$$

and the fact that α and ϵ_e are small and θ is not too large,

$$\sin(\beta_2 - \alpha \cos \phi) = \frac{\omega_i H r_{23} \cos \psi}{V_r \cos(\alpha \cos \phi)} \quad \text{results} \quad (49) \quad 30$$

results where

$$H = \frac{(1 - \alpha \cos \phi \tan \theta) \sec^2 \beta_1}{1 - (\omega_i/\omega_1) \sin \psi \sin(\theta - \alpha \cos \phi)} \quad (50) \quad 35$$

BOMBING SYSTEM EQUATIONS

In a bombing attack, note that the proper time to release a bomb is when

$$\beta_{1rc} \text{ (computed)} = \beta_{1rm} \text{ (measured)} \quad (51)$$

$$\left. \begin{array}{l} \beta_{1rc} \text{ (computed)} = \beta_{1rm} \text{ (measured)} \\ R_{rc} \text{ (computed)} = R_{rm} \text{ (measured)} \end{array} \right\} \text{for weapon release} \quad (52) \quad 40$$

where β_{1rm} is target angle as in FIGS. 4 and 5; β_{1rc} is computed by equations 7 and 46; R_{rc} is from equation 1; and R_{rm} is the e_2, e_3 component of range to target.

Thus, to compute an azimuth lead angle equations 9 and 46 are used:

$$\sin \beta_{1r} = \left(\frac{\omega_i \sin(\theta - \alpha \cos \phi)}{\omega_1 H \cos \psi} + \sin \psi \right)$$

$$\frac{\cos(\alpha \cos \phi) \cos \alpha_r \Phi_r}{\cos \psi \cos \delta_r} \quad (53) \quad 55$$

Note that this is the required azimuth lead even if the tracker is not tracking the target.

For the elevation lead angle consider equation 49. Here, if the target was being tracked, the equation gives

a relationship of the variables at the release time (A subscript r is used to denote this.) Thus,

$$\sin(\beta_{2r} - \alpha \cos \phi) = \frac{\omega_{ir} H_r R_r \cos \psi_r}{V_r \cos(\alpha \cos \phi)} \quad (54)$$

Combining this equation with equations 1 and 3 gives

$$\omega_{ir} = \frac{g h_r \psi_r \cos^2 \sigma_r \cos(\alpha \cos \phi)}{2 H_r V_r \cos \delta_r \cos \psi_r} \quad (55)$$

where α and ϵ_e are small and

$$h_r = \left(1 - \frac{\epsilon_e}{\tan(\beta_{2r} - \alpha \cos \phi)} \right)^{-1}$$

$$= \left(1 - \frac{\epsilon_e \cos \psi_r}{\tan(\theta_r - \alpha \cos \phi)} \right)^{-1} \quad (56)$$

Compare equation 54 with the actual measured quantities when some point near the target is being tracked.

$$\frac{\sin(\beta_{2r} - \alpha \cos \phi)}{\sin(\beta_2 - \alpha \cos \phi)} = \frac{\omega_{ir} R_r H_r \cos \psi_r}{\omega_i r_{23} H \cos \psi} \quad (57) \quad 25$$

Assume the target and the tracked point are at the same altitude (Assume that these are close together, i.e., one is not dealing with large offset bombing.) Then

$$\frac{R_r}{r_{23}} = \frac{\sin(\delta_a + \beta_2)}{\sin(\delta_a + \beta_{2r})} \quad (58) \quad 30$$

or

$$\frac{\sin(\beta_{2r} - \alpha \cos \phi)}{\sin(\beta_2 - \alpha \cos \phi)} = \frac{H_r \omega_{ir} \cos \psi_r}{H \omega_i \cos \psi} \frac{\sin(\delta_a + \beta_2)}{\sin(\delta_a + \beta_{2r})} \quad (59) \quad 35$$

Using equation 55 gives

$$\frac{\sin(\beta_{2r} - \alpha \cos \phi)}{\sin(\beta_2 - \alpha \cos \phi)} \frac{\sin(\delta_a + \beta_2)}{\sin(\delta_a + \beta_{2r})} = \frac{g \Psi_r h_r \cos^2 \sigma_r \cos(\alpha \cos \phi)}{2 V_r H \omega_i \cos \delta_r \cos \psi} \quad (60) \quad 40$$

Rewriting (The left side of equation 61 can be written as $\beta_{2r} = \beta_2 + \Delta\beta$)

$$\frac{[1 + \tan \Delta\beta \cotn(\beta_2 - \alpha \cos \phi)][1 + \tan \Delta\beta \cotn(\beta_2 + \delta_a)]}{[1 - \tan \Delta\beta \tan(\delta_a + \beta_2)]^2} \left[\frac{\cos(\delta_a + \alpha \cos \phi + \epsilon_e)}{\cos^2(\delta_a + \beta_2)} \right] h_r$$

equation 60 gives

$$\frac{\sin(\beta_{2r} - \alpha \cos \phi)}{\sin(\beta_2 - \alpha \cos \phi)} \frac{\sin(\delta_a + \beta_{2r})}{\sin(\delta_a + \beta_2)} \frac{\cos(\delta_a + \alpha \cos \phi + \epsilon_e)}{\cos^2(\delta_a + \beta_{2r})}$$

$$\left(1 - \frac{\epsilon_e}{\tan(\beta_{2r} - \alpha \cos \phi)} \right) = \frac{g \Psi_r \cos(\alpha \cos \phi)}{2 V_r H \omega_i \cos \psi} \quad (61) \quad 50$$

where

$$H = \frac{(1 - \alpha \cos \phi \tan \theta) \sec^2 \beta_1}{1 - (\omega_i/\omega_1) \sin \psi \sin(\theta - \alpha \cos \phi)} \quad (\text{Equation 50})$$

These equations give the elevation lead angle β_{2r} since all the other quantities in the equations are measurable.

Rewriting equation 53 for the azimuth lead angle gives

$$\sin \beta_{1r} = \left(-\frac{\omega_j \sin (\theta - \alpha \cos \phi)}{\omega_i H \cos \psi} + \sin \psi \right) \frac{\cos (\alpha \cos \phi) \cos (\delta_a + \beta_{2r}) \Phi_r}{\cos \psi \cos (\delta_a + \alpha \cos \phi + \epsilon_s)} \quad (62)$$

Generally, the lead angles are given in terms of Ψ_r and θ_r . Using the spherical trigonometric relations

$$\sin \theta_r = \sin \beta_{2r} \cos \beta_{1r} \quad (63)$$

and

$$\tan \psi_r = \tan \beta_{1r} \sec \beta_{2r} \quad (64)$$

or

$$\tan \theta_r = \tan \beta_{2r} \cos \psi_r \quad (65)$$

and

$$\sin \psi_r = \sin \beta_{1r} \sec \theta_r \quad (66)$$

and in aircraft coordinates the lead angles λ_e and λ_a are

$$\sin \lambda_e = +\sin \theta_r \cos \phi + \sin \psi_r \cos \theta_r \sin \phi \quad (67)$$

and

$$\sin \lambda_a = (-\sin \theta_r \sin \phi + \sin \psi_r \cos \theta_r \cos \phi) \sec \lambda_e \quad (68)$$

Thus, the computed lead angles for the CIP are obtained from equations 61, 62, 63, 64, 67, and 68, along with the added relations, equations 2, 3, 4, 5, 7, and 50.

If time of flight is explicitly needed, it is available by combining equations 6, 54, 55, and 56 (or directly from ballistic equations)

$$t_f = \frac{21' \sin (\beta_{2r} - \alpha \cos \phi - \epsilon_s)}{g \Psi_r^{1/4} \cos (\delta_a + \beta_{2r})}$$

It can be shown that the angle of fall at impact (τ_i) is given by

$$\tan \tau_i = \tan$$

$$(\delta_a + \alpha \cos \phi + \epsilon_s) + \frac{2 \Psi_r^{1/2} \sin (\beta_{2r} - \alpha \cos \phi - \epsilon_s)}{\cos (\delta_a + \beta_{2r}) \cos (\delta_a + \alpha \cos \phi + \epsilon_s)} \quad (69)$$

STICK BOMBING

Using FIG. 6 where F is the desired first bomb impact, L is the last bomb impact (six bombs in the stick) and T represents the target,

$$\sin \epsilon_s = \frac{s \sin \sigma_r}{R_s} = \frac{s \sin (\sigma_r + \epsilon_s)}{R_r} \quad (69)$$

and

$$s = \frac{(N-1)}{2} \Delta X \quad (70)$$

$$\sin \epsilon_s = \frac{(N-1) \Delta X}{2} \frac{\sin (\sigma_r + \epsilon_s)}{R_r} \quad (71)$$

where ϵ_s is the change in elevation lead angle (μ_2) for the stick bombing case, ΔX is the bomb spacing, and N is the number of bombs.

Rewriting equation 54 results in

$$R_r = \frac{V_r \cos (\alpha \cos \phi) \sin (\beta_{2r} - \alpha \cos \phi)}{\omega_{ir} H_r \cos \psi_r} \quad (72)$$

5

or

$$\sin \epsilon_s = \frac{(N-1) \Delta X}{2} \frac{H_r \omega_{ir} \sin (\sigma_r + \epsilon_s) \cos \psi}{V_r \cos (\alpha \cos \phi) \sin (\beta_{2r} - \alpha \cos \phi)} \quad (73)$$

Using equation 55 for ω_{ir} gives

$$\sin \epsilon_s = \frac{gh \Psi_r (N-1) \Delta X \sin (\sigma_r + \epsilon_s) \cos^2 \sigma_r}{4 V_r^2 \cos \delta_r \sin (\beta_{2r} - \alpha \cos \phi)} \quad (74)$$

For practical purposes the ϵ_s in $\sin (\sigma_r + \epsilon_s)$ will be neglected. Thus, for elevation lead angle β_{2s}

$$\beta_{2s} = \beta_{2r} + \epsilon_s \quad (75)$$

The azimuth lead is the same as for a single release since the center of the stick should be correct in azimuth.

The above provides the first bomb release condition. Now consider a time-based release of the rest of the stick. The approach taken here is also aimed at being compatible with the automatic release operation. Using equation 54 gives

$$\frac{\sin (\beta_{2t} - \alpha \cos \phi + \epsilon_s)}{\sin (\beta_{2r} - \alpha \cos \phi)} = \frac{\omega_{is}}{\omega_{it}} \frac{R_s}{R_t} \quad (76)$$

where subscript t refers to the target and where some small quantities are neglected. From FIG. 6

35

$$\frac{R_s}{R_t} = \frac{\sin \sigma_t}{\sin (\sigma_t + \epsilon_s)}$$

combining the last two equations, and remembering that ϵ_s is small

$$\omega_{is}/\omega_{it} \doteq 1 + \epsilon_s [\cot (\beta_{2t} - \alpha \cos \phi) + \cot \sigma_t] \quad (77)$$

or

$$\frac{\omega_{is}}{\omega_{it}} \doteq 1 + \epsilon_s \left[\frac{\sin (\sigma_t + \beta_{2t} - \alpha \cos \phi)}{\sin \sigma_t \sin (\beta_{2t} - \alpha \cos \phi)} \right] \quad (78)$$

Combining this equation 74 gives approximately

$$\frac{\omega_{is}}{\omega_{ir}} \doteq 1 + \frac{gh \Psi_r (N-1) \Delta X \cos^2 \sigma_r \sin (\sigma_r + \beta_{2r} - \alpha \cos \phi)}{4 V_r^2 \sin^2 (\beta_{2r} - \alpha \cos \phi) \cos \delta_r} \quad (79)$$

50

49 (or 54), but neglecting factors near 1,

$$\omega_i = (V/R) \sin \xi \quad (80)$$

where

$$\xi = \beta_2 - \alpha \cos \phi = \sigma - \eta \quad (81)$$

60 and

$$\eta = \delta_a + \alpha \cos \phi \quad (82)$$

With the assumption that V does not change much in magnitude during the stick release,

65

$$\omega_i \doteq \frac{\omega_i (2\omega_i - \dot{\eta})}{\tan \xi} = \frac{\omega_i (\omega_i + \dot{\xi})}{\tan \xi} \quad (83)$$

where $\dot{\eta}$ is the flight path angular rate in the vertical plane (e_1, e_3).

During the time of a stick release one can assume $\dot{\eta}$ and ξ are constant in equation 83. This provides two solutions for ω_i : one for $\dot{\eta} = 0$ and one for $\dot{\eta} \neq 0$.

$$\omega_{is} = \omega_i \left(1 - \frac{\dot{\omega}_{is} t}{\omega_{is}} \right) \text{ for } \dot{\eta} = 0 \quad (84)$$

$$\omega_i = \frac{\dot{\eta} \omega_{is}^2}{2\omega_{is}^2 - \dot{\omega}_{is} \tan \xi \exp(\dot{\eta} t / \tan \xi)} \text{ for } \dot{\eta} \neq 0 \quad (85)$$

The first of these can be written as

$$\omega_i - \omega_{is} = (\omega_i / \omega_{is}) \dot{\omega}_{is} t \quad (86)$$

Using equation 83, the second ω_i equation can be written as

$$\omega_{is} \omega_i \dot{\eta} - \omega_i \dot{\omega}_{is} \tan \xi [\exp(\dot{\eta} t / \tan \xi) - 1] = \dot{\eta} \omega_{is}^2 \quad (87)$$

or

$$(\omega_i - \omega_{is}) \omega_{is} \dot{\eta} = \omega_i \dot{\omega}_{is} \tan \xi [\exp(\dot{\eta} t / \tan \xi) - 1] \quad (88)$$

or

$$\omega_i - \omega_{is} = \frac{\omega_i \dot{\omega}_{is} \tan \xi}{\omega_{is} \dot{\eta}} \left(\frac{\dot{\eta} t}{\tan \xi} + \dots \right) \quad (89)$$

Thus, approximately (first order)

$$\omega_i - \omega_{is} = (\omega_i / \omega_{is}) \dot{\omega}_{is} t \quad (90)$$

Thus, one can use the following equation for either $\dot{\eta} = 0 \neq \dot{\eta}$ for the time (ΔT) between the bomb releases of the stick:

$$\frac{(N-1)\Delta T}{2} = \left(\frac{\omega_{is}}{\omega_{ir}} \right) \left(\frac{\omega_{ir} - \omega_{is}}{\dot{\omega}_{is}} \right) \quad (91)$$

This may be used in the form

$$\begin{aligned} \frac{(N-1)\Delta T}{2} &= \left(\frac{\omega_{is}}{\omega_{ir}} \right) \left(1 - \frac{\omega_{is}}{\omega_{ir}} \right) \\ &= \left(\frac{\tan \beta_{2r} - \alpha \cos \phi}{2\omega_{is} - \dot{\eta}} \right) \left(1 - \frac{\omega_{is}}{\omega_{ir}} \right) \end{aligned} \quad (92)$$

The stick release equations are the normal single-bomb case plus equations 74, 79, and 92.

BOMB RELEASE DELAY AND EJECTION RELEASES

The release point must be anticipated by an amount of time, t_d , in order to account for the delay in bomb release. To do this, (The other quantities that are affected by this small delay have a minor effect on the results.) one uses in place of the measured angular rate ω_i , the estimated rate ω_{id} . That is, ω_i is replaced by

$$\omega_{id} = \omega_i + \dot{\omega}_i t_d \quad (93)$$

in the elevation lead angle equation 61 and where $\dot{\omega}_i$ is computed from ω_i or by the equation

$$\dot{\omega}_i = \frac{\omega_i (2\omega_i - \dot{\eta})}{\tan(\beta_2 - \alpha \cos \phi)} \quad (94)$$

The effect of ejection velocity of the bomb has been included in the elevation computation by ϵ_e . This is the effective change in release angle due to ejection. It is

$$\epsilon_e = (V_e / V_r) \cos \phi \quad (95)$$

where V_e is the ejection velocity.

The azimuth lead should include the azimuth component of this velocity. That is, equation 46 needs the term (This assumes neglecting the difference in sideways ejection in such bomb racks as the MER/TER. These racks are more useful in stick releases.)

$$V_e \sin \phi \quad (96)$$

added to it. Thus, to correct all the azimuth lead equation the quantity $\sin \psi$ would be replaced by

$$\sin \psi + \frac{V_e \cos \psi \sin \phi}{\cos(\alpha \cos \phi)} \quad (97)$$

or, for practical purposes replace $\sin \psi$ by

$$\sin \psi + V_e \sin \phi \quad (98)$$

In actual use the $V_e \sin \phi$ term should be used in the azimuth lead equation with a delay-smoothing function operating on the $\sin \phi$ factor. This term has a destabilizing or noisy effect on the azimuth signal and thus presents a tracking problem for the pilot.

AUTOMATIC RELEASE (CIP DISPLAY)

Thus far, it has been assumed that the tracker tracks either a point near the true target or the target itself. Since the bombing system is not necessarily aware of the location of the target, a manual release by the pilot is necessary.

Consider the case where the pilot locks the tracker on the target; (Or, technically, to either side of the target, since only accurate elevation information is needed.) now an automatic release is practical. This release may be given by requiring the measured elevation rate (ω_i) to equal the computed rate (ω_{is}), or, at release,

$$T = \left(\frac{\omega_{ir}}{\omega_i} \right) \left(\frac{\omega_i}{\omega_{id}} \right) \left(\frac{\omega_{is}}{\omega_{ir}} \right) = 1 \quad (99)$$

where ω_{ir} is given by equation 55.

An alternate method of computing release is used when the measured angle (β_2) is the value computed (β_{2s}), or, at release,

$$\beta_2 = \beta_{2s} \quad (100)$$

EQUATION SUMMARY

The basic equations for a CIP delivery are equations 2, 50, 56, 61, 62, 63, 64, 67, 68, 74, 75, 79, 92, and 93.

$$\begin{aligned} \sin \beta_{1r} &= \left[- \left(\frac{\omega_i \sin(\theta - \alpha \cos \phi)}{\omega_i H \cos \psi} \right) \right. \\ &\quad \left. + \sin \psi + \epsilon_e \sin \phi \right] \frac{\cos(\alpha \cos \phi) \cos(\delta_a + \beta_{2r}) \Phi_r}{\cos \psi \cos(\delta_a + \alpha \cos \phi + \epsilon_e)} \end{aligned} \quad (101)$$

$$\begin{aligned} \frac{\sin(\beta_{2r} - \alpha \cos \phi) \sin(\delta_a + \beta_{2r}) \cos(\delta_a + \alpha \cos \phi + \epsilon_e)}{\sin(\beta_2 - \alpha \cos \phi) \sin(\delta_a + \beta_2) \cos^2(\delta_a + \beta_{2r})} h_{r-1} \\ = \frac{g \psi_r \cos(\alpha \cos \phi)}{2V_r H \omega_{id} \cos \psi} \end{aligned} \quad (102)$$

$$H = \frac{(1 - \alpha \cos \phi \tan \theta) \sec^2 \beta_1}{1 - (\omega_i / \omega_i) \sin \psi \sin(\theta - \alpha \cos \phi)} \quad (103)$$

$$\begin{aligned} \sin \epsilon_a &= \frac{(N-1)\Delta X}{2} \\ &\quad \frac{g h_r \psi_r \sin(\delta_a + \beta_{2r}) \cos^2(\delta_a + \beta_{2r})}{2V_r^2 \cos(\delta_a + \alpha \cos \phi + \epsilon_e) \sin(\beta_{2r} - \alpha \cos \phi)} \end{aligned} \quad (104)$$

$$\frac{\omega_{is}}{\omega_{ir}} = 1 + \frac{\sin \epsilon_a \sin(\delta_a + 2\beta_{2r} - 2\alpha \cos \phi)}{\sin(\delta_a + \beta_{2r}) \sin(\beta_{2r} - \alpha \cos \phi)} \quad (105)$$

$$\frac{(N-1)\Delta T}{2} = \left(\frac{\omega_{is}}{\omega_{ir}} \right) \left(1 - \frac{\omega_{is}}{\omega_{ir}} \right) \quad (106)$$

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$$\omega_{id} = \omega_i + \dot{\omega}_i t_d \quad (107)$$

$$\Psi_r = \Psi[k_1 U^2 \sin(\beta_{2r} - \alpha \cos \phi - \epsilon_e) \sec(\delta_a + \beta_{2r}) - k_2] \quad (108)$$

$$\Phi_r = \Phi[k_1 U^2 \sin(\beta_{2r} - \alpha \cos \phi - \epsilon_e) \sec(\delta_a + \beta_{2r}) - k_2] \quad (109)$$

$$\sin \phi_r = \sin \beta_{2r} \cos \beta_{1r} \quad (110)$$

$$\sin \lambda_e = \sin \beta_{2r} \cos \beta_{1r} \cos \phi + \sin \beta_{1r} \sin \phi \quad (111)$$

$$\sin \lambda_a = (-\sin \beta_{2r} \cos \beta_{1r} \sin \phi + \sin \beta_{1r} \cos \phi) \sec \lambda_e \quad (112)$$

$$\beta_{2r} = \alpha_{2r} - \epsilon_e \quad (113)$$

$$h_r^{-1} = 1 - \frac{\epsilon_e}{\tan(\beta_{2r} - \alpha \cos \phi)} \quad (114)$$

$$\epsilon_e = (V_e/V_r) \cos \phi \quad (115)$$

For automatic release (tracker locked on target):

$$\frac{\omega_{ir}}{\omega_{id}} = \frac{gh_r \psi_r \cos^2(\delta_a + \beta_{2r}) \cos(\alpha \cos \phi)}{2\omega_{id} V_r H \cos \psi \cos(\delta_a + \alpha \cos \phi + \epsilon_e)} \quad (116)$$

release at

$$T = \frac{\omega_{ir} \omega_{is}}{\omega_{id} \omega_{is}} = 1 \quad (117)$$

or at

$$\beta_2 = \beta_{2r} \quad (118)$$

ALTERNATE TRACKER COORDINATES

Instead of using the coordinates of the section on aircraft-target velocity, the tracker coordinates shown in FIG. 7 may be used. Here the first rotation is about the horizontal axis and the second rotation is about the (tracker) vertical axis. This coordinate system is used in the functional block diagram of the computer of FIG. 8. Thus the transformation between i, j, k and e_1, e_2, e_3 is

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos \eta & 0 & \sin \eta \\ \sin \xi \sin \eta & \cos \xi & -\sin \xi \cos \eta \\ -\cos \xi \sin \eta & \sin \xi & \cos \xi \cos \eta \end{pmatrix} \begin{pmatrix} i \\ j \\ k \end{pmatrix} \quad (119)$$

Therefore, the velocity components in the e coordinates are

$$V_1 = \omega_{ir} \cos \eta + \dot{r} \sin \eta \quad (120)$$

$$V_2 = \omega_{ir} \sin \xi \sin \eta - \omega_{ir} \cos \xi - \dot{r} \sin \xi \cos \eta \quad (121)$$

$$V_3 = -\omega_{ir} \cos \xi \sin \eta - \omega_{ir} \sin \xi + \dot{r} \cos \xi \cos \eta \quad (122)$$

Combining these as in the section on aircraft-target velocity gives

$$V_1 \quad V_3 \left[\frac{\omega_j}{\omega_i} \sec \eta \sin(\xi + \alpha_k) + \sec \eta \sin \eta \cos(\xi + \alpha_k) \right] \quad (123)$$

and

$$\omega_1 = -\left(\frac{V_3}{r}\right) \sin(\xi + \alpha_k) \quad (124)$$

where

$$\alpha_k \doteq V_2/V_3 \doteq -\alpha \cos \phi \quad (125)$$

These relations are the basic equations for an angular rate bombing system.

Using methods set forth previously gives the complete system equations as follows.

$$\sin \eta_i \left[\frac{\omega_i}{\omega_1} \sin(\xi + \alpha_k) + \sin \eta \cos(\xi + \alpha_k) + \epsilon_e \sin \phi \cos \eta \right] \frac{\Phi_r \cos \alpha_k \cos(\delta_a + \xi_r)}{\cos \eta \cos(\delta_a - \alpha_k + \epsilon_e)} \quad (126)$$

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$$\frac{\omega_{ir}}{\omega_{id}} = \frac{gh_r \psi_r \cos^2(\delta_a + \xi_r)}{2\omega_{id} V_r \cos(\delta_a - \alpha_k + \epsilon_e)} \quad (127)$$

$$\frac{\sin(\xi_r + \alpha_k) \sin(\delta_a + \xi_r)}{\sin(\xi + \alpha_k) \sin(\delta_a + \xi)} = \frac{\omega_{ir}}{\omega_{id}} \quad (128)$$

$$\sin \epsilon_a = \frac{(N-1) \Delta X}{2} \frac{\omega_{ir} \sin(\delta_a + \xi_r)}{\sin(\xi_r + \alpha_k)} \quad (129)$$

$$\frac{\omega_{is}}{\omega_{ir}} = 1 + \frac{\sin \epsilon_a \sin(\delta_a + 2\xi_r + 2\alpha_k)}{\sin(\delta_a + \xi_r) \sin(\xi_r + \alpha_k)} \quad (130)$$

$$\frac{(N-1) \Delta T}{2} = \left(\frac{\omega_{is}}{\omega_{ir}}\right) \left(1 - \frac{\omega_{is}}{\omega_{ir}}\right) \quad (131)$$

$$\omega_{id} = \omega_i + \dot{\omega}_i t_d \left[\text{rewritten as } \omega_{id} \right]$$

$= \omega_i h_r$, (combine Equations 132 and 136) where H

$$= 1 + \frac{(2\omega_i - \Omega) t_d}{\tan \xi + \alpha_k} \quad (132)$$

$$\xi_s = \xi_r - \epsilon_s \quad (133)$$

$$h_r^{-1} = 1 - \epsilon_e / \tan(\xi_r + \alpha_k) \quad (134)$$

$$\epsilon_e = (V_e/V_r) \cos \phi \quad (135)$$

$$\dot{\omega} = \frac{\omega_i (2\omega_i - \Omega)}{\tan(\xi + \alpha_k)} \quad (136)$$

where Ω is the flight angular rate and

$$\Omega = -(g/V_r)(K_n - \cos \delta_a \cos \phi) \cos \phi \quad (137)$$

(Equation 137 is the same as equation 83 transformed into the alternate tracker coordinates.) For automatic release:

$$T = \frac{\omega_{ir} \omega_{is}}{\omega_{id} \omega_{ir}} = 1 \quad (138)$$

The sight display (CIP mode) uses the angles ξ_s and

η_r . Also

$$U = V_r \quad (139)$$

Equations 2 and 7 may be written as follows:

$$\psi_r = f_1[KU^2 \sin(\xi_r + \alpha_k) \sec(\delta_a + \xi_r)] \quad (2')$$

$$\Phi_r = f_2[KU^2 \sin(\xi_r + \alpha_k) \sec(\delta_a + \xi_r)] \quad (7')$$

CHOSEN RELEASE RANGE (CRR/A)

The following paragraphs describe how to include in an angular rate bombing system an operational mode that will arrange for a release at a (pilot) prechosen slant range. Alternatively, in other delivery modes it will indicate what range (or modified to provide altitude) from target the release will occur.

When high accuracy in retarded bomb delivery (This delivery will also produce good results using low drag bombs, of course.) is necessary, the ground fire is at an acceptable level to allow the pilot some tracking time. This tracking time need not be long unless the pilot locks on and tracks from a long way out—which is unnecessary.

Assume a straight-line path from shortly after lock-on until release. (This to be modified to a possible upward curved course later.) Then

$$\sin \gamma = (R_r/R) \sin \gamma_r \quad (140)$$

where the quantities are defined in FIG. 10. Using the alternate coordinate system:

$$\omega_i = (V/R) \sin \gamma \quad (141)$$

where

$$\gamma = \xi + \alpha_k \quad (142)$$

At the proper release time

$$\omega_r = (V/R) \sin \gamma_r \quad (143)$$

Thus these four equations give

$$\sin \gamma = (\sqrt{\omega_i \omega_r}) (R_r/V) \quad (144)$$

The elevational lead angle is the value of ξ (or θ in the main body of the report) computed from γ .

The azimuth lead angle computation is the same as in all the other modes.

The (CRR/A) mode requires that the target be tracked in elevation and an automatic release is necessary.

If a positive g curved course is desired, the equation is written as

$$\sin \gamma = \omega_i (\omega_r/\omega_i)^n (R_r/V) \quad (145)$$

where $n > 0.5$ for a curved course, and usually $n < 1$ for desirable courses.

For a digital computer one could use to obtain a similar result

$$\sin \gamma = \omega_i \left[n + (1-n) \left(\frac{\omega_r}{\omega_i} \right) \right] \left[\frac{R_r}{V} \right] \quad (146)$$

where $0.5 \leq n < 1$.

COMPUTED RANGE AT RELEASE

In other delivery modes whenever the target is tracked (in elevation) equation 142 above can be used to compute the range at which release will occur if the pilot continues on in the same manner.

$$R_r = (V \sin \gamma / \sqrt{\omega_i \omega_r}) \quad (147)$$

FIG. 8 (FIGS. 8A, 8B, 8C and 8D taken together) and FIG. 9 correlate the computational process carried out in the computer with the various inputs thereto. FIG. 8 is a schematic diagram illustrating the interconnection between operational amplifiers and shaping circuits with inputs thereto and resultant outputs therefrom. The symbology used on the input and output lines corresponds to that used in the equations supra. The symbol(s) on the input lines to a particular circuit element represent the signals coupled thereto while the symbol(s) on the output line(s) represent(s) the input signal(s) after being operated on or transformed by the particular circuit element.

FIG. 9 is a computational sub-section block diagram derived from FIG. 8 which sets forth wherein the computer various equations are treated.

What is claimed is:

1. In an angular rate bombing system for use with an aircraft for providing an automatic weapon-release signal without a range to target measurement, inertial

velocity-of-aircraft measurement or true-altitude-over-target measurement whenever the measured (elevation) rate of the line-of-sight to the target equals the computed rate necessary to obtain a hit;

5 a tracker carried by said aircraft and providing outputs corresponding to angular rates ω_i , ω_r and angles ξ , η ;

vertical reference means carried by said aircraft and furnishing outputs corresponding to pitch δ and roll ϕ ;

10 true airspeed measuring means on said aircraft furnishing an output corresponding to magnitude of velocity (U);

15 true airspeed measuring means on said aircraft furnishing an output corresponding to magnitude of velocity (U);

computer means carried by said aircraft operatively receiving the outputs from the automatic tracker, vertical reference means and true airspeed means and for calculating outputs corresponding to the azimuth and

elevation lead angles necessary to obtain a hit and to cause automatic weapon release.

2. The system as set forth in claim 1 and further including;

sight unit means carried by said aircraft;

said sight unit operatively receiving the outputs from said computer means corresponding to azimuth and elevation lead angles and displaying them on said sight unit as a computed impact point.

3. The system as set forth in claim 1 further including;

angle of attack measuring means carried by said aircraft and furnishing an output signal corresponding to aircraft angle of attack $\alpha_k = V_2/V_3 = -\alpha \cos \phi$ which signal is also inputted to said computer means and used to compute $\sin \eta_r$, $(\omega_{ir}/\omega_{id})$, $\sin \epsilon_s$, $(\omega_{ig}/\omega_{ir})$, h_r^{-1} , ω_i , ψ_r , ϕ and τ as set forth in equations 126-130, 134, 136, 2', 7' and 142 respectively.

4. The system as set forth in claim 3 and further including;

pressure altitude measuring means for measuring barometric pressure and outputting a signal corresponding thereto;

said output signal being inputted to said computer means for computing air drag on a released weapon.

5. The system as set forth in claim 3 and further including;

normal acceleration measuring means for measuring aircraft acceleration and outputting a signal corresponding to k_n

said signal being inputted to said computer means and used to compute Ω as set forth in equation 137.

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