An angulated element comprises two pivotally interconnected angulated rods (11, 12) which embrace a constant angle as the element is folded and expanded. Each rod has first and second portions, the length of the respective first portions of the rods being substantially the same, the length of the respective second portions of the rods being substantially the same, the angles between the first and second portions of the respective rods being different. Structures formed from the elements are also provided.
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EXPANDABLE/COLLAPSIBLE STRUCTURES

The present invention relates to an expandable/collapsible structure.

In US-A-5024031, there is disclosed a radial expansion/retraction truss structure consisting of pairs of identical angulated rods connected together by a single hinge in "scissor" pairs. Such pairs are referred to herein as "simple angulated elements". When the structure is expanded or retracted, certain critical angles are constant which allows the overall geometry of the structure to remain constant as it expands or collapses. A number of different deployable structures are disclosed. However, owing to mobility requirements, there must be some clearance within each pivot joint, which makes the structure undesirably flexible, especially when the number of elements is large. Furthermore, whilst there is disclosed in US-A-5024031 a foldable "iris-type" dome structure having a circular plan, the structure consisting of concentric rings connected by scissor hinges, in fact only a limited number of shapes for the structures can actually be constructed from the simple angulated elements disclosed in US-A-5024031.

According to a first aspect of the present invention, there is provided an angulated element, the element comprising two pivotally interconnected angulated rods which embrace a constant angle as the element is folded and expanded, in which each rod has first and second portions, the length of the respective first portions of the rods being substantially the same, the length of the respective second portions of the rods being substantially the same, the angles between the first and second portions of the respective rods being different.

According to a second aspect of the present invention, there is provided an expandable/collapsible structure, comprising a generalised angulated element which embraces a constant angle as the generalised angulated element is
folded and expanded, the generalised angulated element comprising a plurality of elements each consisting of two pivotally interconnected angulated rods, wherein the ends of rods of each element are pivotally connected to respective ends of rods of an adjacent element such that each closed loop formed by such connection is a parallelogram, with any unconnected portions of rods of any terminal elements forming isosceles triangles. Thus, where there are terminal elements, isosceles triangles are formed at the ends of the structure.

According to a third aspect of the present invention, there is provided an angulated element, the element comprising two interconnected angulated rods which embrace a constant angle as the element is folded and expanded, in which each rod has first and second portions, the ratio of the lengths of the respective first portions of the rods being substantially equal to the ratio of the lengths of the respective second portions of the rods, the angles between the first and second portions of the respective rods being substantially the same.

According to a fourth aspect of the present invention, there is provided an expandable/collapsible structure, comprising a generalised angulated element which embraces a constant angle as the generalised angulated element is folded and expanded, the generalised angulated element comprising a plurality of elements each consisting of two pivotally interconnected angulated rods, wherein the ends of rods of each element are pivotally connected to respective ends of rods of an adjacent element such that each closed loop formed by such connection is a parallelogram, with any unconnected portions of rods of any terminal elements forming similar triangles. Thus, where there are terminal elements, similar triangles are formed at the ends of the structure.

According to a fifth aspect of the present invention, there is provided an expandable/collapsible structure, the structure comprising a plurality of multi-angulated rods,
each rod being formed of at least three portions, the rods being pivotally connected to one another at at least one of the junctions between said portions such that one closed loop is a foldable base structure and any other closed loops are formed by parallelograms.

Structures of the fifth aspect of the present invention use multi-angulated rods, allowing a smaller number of parts and simpler pivot joints to be used whilst providing the same overall profile as disclosed in US-A-5024031. More general profiles can also be provided. The structure of the present invention retains the important features of the structure disclosed in US-A-5024031, but with the key advantages of higher stiffness and more general shapes allowing large and complex structures to be formed.

In one embodiment, for each multi-angulated rod, said portions are substantially the same length and the angles subtended at an origin by said portions of each rod are substantially the same with each successive portion of each rod being at an increasing distance from the origin.

The rods may be substantially identical.
All of the rods may be planar.
All of the rods may be non-planar.
Some of the rods may be planar and some may be non-planar.

In an alternative embodiment, for each multi-angulated rod, said portions are of substantially different length.

A three-dimensional structure may have a single layer the projection of which onto a plane is a structure according to the fifth aspect of the invention. Alternatively, it may have a base layer being a structure according to the fifth aspect of the invention, and a second layer being a structure according the fifth aspect of the invention, the projection of the second layer onto the base layer being substantially identical to said base layer.
The structures of all aspects of the present invention have many applications, including but not limited to portable shelters, roofs over stadia, swimming pools, and theatres, and garden furniture.

Examples of the present invention will now be described with reference to the accompanying drawings, in which:

Fig. 1 shows an angulated element used for a Type I General Angulated Element (GAE, see below);

Fig. 2 shows a general Type I GAE;

Fig. 3 shows an angulated element used for a Type II GAE;

Fig. 4 shows a general Type II GAE;

Fig. 5 is a diagram for explaining the geometry of a simple multi-angulated rod having the same kink angles;

Fig. 6 are plan views of a structure using multi-angulated rods;

Fig. 7 shows a further example of a structure using multi-angulated rods in which Figs. 7(a), (b) and (c) are respectively plan views of the structure in its fully deployed, partly deployed and fully collapsed configurations;

Fig. 8 shows a foldable closed loop structure which folds along a given polygon, the angulated element forming similar rhombuses;

Fig. 9 shows a foldable closed loop structure which folds along a given polygon, the angulated element forming similar parallelograms;

Fig. 10 shows two further examples of foldable closed loop structures which fold along a given shape;

Fig. 11 shows three foldable structures, Fig. 11(a) showing a base structure of several parallelograms, Fig. 11(b) showing an extended structure with additional hinged bars, and Fig. 11(c) showing a rigidly connected structure formed by multi-angulated rods;

Fig. 12 shows a foldable elliptical structure formed by multi-angulated rods;
Fig. 13(a) to (c) are respectively a perspective view of the components used in a hinge of the structure, a perspective view of the assembled components, and a plan view of the components; and,

Figs. 14(a) to (c) are respectively perspective views of an example of a double layer circular foldable structure in its fully deployed, partly deployed and fully collapsed configurations.

In the present description, reference will be made to a generalised angulated element (GAE). Such an element is a set of interconnected angulated rods that form a chain of any number of parallelograms with either isosceles triangles (referred to herein as “Type I GAE”) or similar triangles (referred to herein as “Type II GAE”) at either end. As will be shown, a generalised angulated element embraces a constant angle as the element is folded or expanded. Separate proofs that the angles of embrace of Type I and Type II GAEs are given next.

GAE’s without any parallelograms are considered first, for simplicity; it will be shown that the simple angulated element of US-A-5024031 is a special case of both Type I and Type II GAEs.

Type I GAE

Before discussing the general Type I GAE, consider first the angulated element 10 shown in Fig. 1 formed of angulated rods 11,12, which has

$$AE=DE, BE=CE$$ and in general $$\psi \neq \phi$$

(1)

From Fig. 1, the sum of the internal angles in the quadrangle OGEF is 360° and, since $$\angle OFE = \angle OGE = 90^\circ$$, the angle $$\alpha$$ can be expressed as

$$\alpha = 180^\circ - (\angle AEF + \beta + \angle BEG)$$

(2)

Because $$\triangle ADE$$ and $$\triangle BCE$$ are isosceles triangles,

$$\angle AEF = \frac{\phi - \beta}{2}$$ and $$\angle BEG = \frac{\psi - \beta}{2}$$

(3)

Hence, substituting Eq. 3 into Eq. 2:
\[ \alpha = 180^\circ - \frac{\phi + \psi}{2} = \text{constant} \] (4)

which shows that this element subtends a constant angle. Note that the simple element of US-A-5024031 is re-obtained, when \( \phi = \psi \).

A most interesting special case is obtained when either \( \phi = 180^\circ \), or \( \psi = 180^\circ \), which implies that one rod is angulated, while the other rod is straight.

More general Type I GAEs are made from two or more angulated elements. Fig. 2 shows an example with three elements 15,16,17, which satisfy the following conditions:

(i) each closed loop is a parallelogram, i.e.

\[ CE = BJ \text{ and } EB = CJ, HJ = TP \text{ and } TJ = HP \] (5)

(ii) \( \triangle AED \) and \( \triangle NPM \) are isosceles triangles, i.e.

\[ \frac{DE}{AE} = \frac{NP}{NP} = 1 \] (6)

Note that the structure shown in Fig. 2 can be regarded as being formed by "cutting" the element shown in Fig. 1 at the pivot point E and inserting parallelograms in between the triangles formed thereby.

Next, it will be shown that the angle \( \alpha \) embraced by this element has constant magnitude. From Fig. 2, it can be seen that

\[ \alpha = \angle DON = \alpha_1 + \alpha_2 + \alpha_3 \] (7)

where

\[ \alpha_1 = 180^\circ - (\angle AEF + \beta_1 + \angle BEG) \]
\[ \alpha_2 = 180^\circ - (\angle BJK + \beta_2 + \angle HJL) \]
\[ \alpha_3 = 180^\circ - (\angle HPQ + \beta_3 + \angle MPR) \] (8)

Condition (i) implies

\[ \angle BEC = \angle BJC \text{ and } \angle HIJ = \angle HPI \] (9)

and also

\[ \angle BEG + \angle BJK = \angle BEC \text{ and } \angle HJL + \angle HPQ = \angle HJI \] (10)

Substituting Eqs. 8, 9 into Eq. 7 gives
\[
\alpha = 3 \times 180^\circ - (\angle AEF + \psi_1 + \psi_2 + \beta_2 + \angle MPN) \quad (11)
\]

From condition (ii), we know that
\[
\angle AEF = \frac{\angle AED}{2} = \frac{\phi_1 - \beta_1}{2} \quad \text{and} \quad \angle MPN = \frac{\angle MPN}{2} = \frac{\psi_3 - \beta_3}{2} \quad (12)
\]

Note that Eq. 9 can be rewritten as
\[
\psi_1 - \beta_1 = \phi_2 - \beta_2 \quad \text{and} \quad \psi_2 - \beta_2 = \phi_3 - \beta_3 \quad (13)
\]

and adding up these two equations gives
\[
\psi_1 + \psi_2 - \beta_1 = \phi_1 + \phi_2 - \beta_2 \quad (14)
\]

Adding \( \phi_1 + \psi_3 \) to both sides of Eq. 14 and tidying up gives
\[
\phi_1 - \beta_1 = \Sigma \phi - \Sigma \psi + \psi_3 - \beta_3 \quad (15)
\]

where
\[
\Sigma \phi = \phi_1 + \phi_2 + \phi_3 \quad \text{and} \quad \Sigma \psi = \psi_1 + \psi_2 + \psi_3 \quad (16)
\]

Substituting Eq. 15 into Eq. 12, and the result into Eq. 11 gives
\[
\alpha = 3 \times 180^\circ - \frac{\sum \phi + \sum \psi}{2} = \text{constant} \quad (17)
\]

i.e. the angle of embrace for a Type I GAE is a constant.

**Type II GAE**

Consider first the angulated element 20 shown in Fig. 3 formed of angulated rods 21,22, which has
\[
\frac{AE}{DE} = \frac{CE}{BE} \quad \text{and} \quad \psi = \phi \quad (18)
\]

To show that the angle of embrace \( \alpha \) is constant in this case, it is noted that Eq. 2 is still valid.

Because \( \Delta AED \) and \( \Delta BEC \) are similar
\[
\angle BEG = \angle DEF \quad (19)
\]

and, substituting Eq. 19 into Eq. 2
\[
\alpha = 180^\circ - \phi \quad (20)
\]

Note that the simple element of US-A-5024031 is re-obtained when
\[ \frac{AE}{DE} = \frac{CE}{BE} = 1 \]  

(21)

A more general Type II GAE with three angulated elements 25, 26, 27 is shown in Fig. 4. This element satisfies the following conditions:

(i) each closed loop is a parallelogram

(ii) the triangles on the sides, \( \triangle AED \) and \( \triangle NPM \), are similar, i.e.

\[ \frac{DE}{PM} = \frac{AE}{NF} \text{ and } \angle AED = \angle MPN \]  

(22)

Note that the structure shown in Fig. 4 can be regarded as being formed by "cutting" the element shown in Fig. 3 at the pivot point E and inserting parallelograms in between the triangles formed thereby.

To show that the angle \( \alpha \) is constant, we note that Eq. 11 is also valid for Type II elements. Also, because of condition (ii)

\[ \angle AEF = \angle NPR \]  

(23)

and hence Eq. 11 is equivalent to

\[ \alpha = 3 \times 180^\circ - \sum \psi = \text{constant} \]  

(24)

It is interesting to note that, since

\[ \angle AED = \phi_1 - \beta_1 \text{ and } \angle MPN = \psi_3 - \beta_3 \]  

(25)

and since these angles are equal, Eq. 13, which is also valid for Type II elements, is equivalent to

\[ \sum \psi = \sum \phi \]  

(26)

which shows that the sum of the kink angles of the two sets of angulated rods that make up a Type II GAE is constant.

For angulated elements consisting of two angulated rods only, Eq. 26 becomes \( \phi = \psi \), which agrees with Eq. 18.

Multi-Angulated Rods
Next, it will be shown that in circular foldable structures made from identical, symmetric angulated elements, contiguous angulated rods can be connected rigidly to one another, to form multi-angulated rods.

Consider two identical angulated rods 30, 31 (A₀ to A₂ and A₂ to A₄) of semi-length \( \ell \), lying in neighbouring sectors subtending equal angles \( \alpha \) as shown in Fig. 5. No three of the nodes A₀, A₁, A₂, A₃ lie in a straight line. Let node A₂ be the connection point of the two elements 30, 31.

It will be shown that the angle between the two rods, \( \angle A₂A₁A₃ \), has constant magnitude.

Considering the first angulated rod 30, which lies between the lines OP₀ and OP₂, the distance of hinge A₂ from point O is

\[
OA₂ = \frac{A₂C₁}{\sin \alpha /2} = \frac{\ell \cos (\angle A₂A₁C₁)}{\sin \alpha /2} \tag{27}
\]

where

\[
\Delta A₁A₂C₁ = 90° - \angle A₂A₁C₁ = 90° - (180° - \angle A₂A₁O) \tag{28}
\]

Because

\[
\angle A₂A₁O = \angle S₁A₁O + \angle A₂A₁S₁ = \angle S₁A₁O + \frac{180° - \alpha}{2} \tag{29}
\]

Eq. 28 becomes

\[
\angle A₁A₂C₁ = \angle S₁A₁O - \alpha /2 \tag{30}
\]

Substituting Eq. 30 into Eq. 27

\[
OA₂ = \frac{\ell \cos (\angle S₁A₁O - \alpha /2)}{\sin \alpha /2} \tag{31}
\]

Also

\[
\angle A₂A₁O = 180° - \frac{\alpha}{2} - \angle A₂A₁O = 90° - \angle S₁A₁O \tag{32}
\]

Considering the second angulated rod 31, the distance of hinge A₂ from point O is

\[
\angle A₂A₃C₃ = \angle S₃A₂C₃ + \alpha /2 = \angle S₃A₂O + \alpha /2 \tag{34}
\]
10

\[ \overline{OA_2} = \frac{\overline{A_2C_3}}{\sin \alpha /2} = \frac{\cos (\angle A_2A_3C_3)}{\sin \alpha /2} \]  \hspace{1cm} (33)

Hence

\[ \overline{OA_2} = \frac{\cos (\angle S_3A_3O + \alpha /2)}{\sin \alpha /2} \]  \hspace{1cm} (35)

Comparing Eq. 31 with Eq. 35,

\[ \angle S_1A_1O - \alpha /2 = \angle S_2A_2O + \alpha /2 \]  \hspace{1cm} (36)

Also

\[ \angle A_3A_2O = \angle A_3A_2C_3 + 90^\circ - \alpha /2 = 90^\circ + \angle S_3A_3O \]  \hspace{1cm} (37)

The angle between the two angulated elements can be calculated from Eq. 32 and Eqs. 36-37

\[ \angle A_1A_2A_3 = \angle A_1A_2O + \angle A_2A_3O \]  \hspace{1cm} (38)

This proof can be extended to any number of contiguous rods of equal semi-length \( \ell \), provided that they are at an increasing distance from the centre: when they start to turn back towards the centre, the angle \( \angle S_1A_1O \) becomes negative and hence the above proof is no longer valid. Subject to this condition, the rods can be rigidly linked together to form a multi-angulated rod with a kink angle of \( 180^\circ - \alpha \) = constant.

Fig. 6(a) shows a circular foldable structure 40 containing 48 five-segment multi-angulated rods 41. This structure has

\[ \alpha = \frac{360^\circ \times 2}{48} = 15^\circ \]  \hspace{1cm} (39)

Fig. 6(b) shows that modest shape changes can be made by varying the number of segments in some rods.

Fig. 7 shows photographs of a model structure 50 with

\[ \alpha = \frac{360^\circ \times 2}{24} = 30^\circ \]  \hspace{1cm} (40)

whose 24 identical multi-angulated rods 51 each has a kink angle of \( 30^\circ \) and consists of three segments of length \( \ell = \)
11

100mm. The fully deployed and fully folded or collapsed configurations of this model are shown in Fig. 7(a) and Fig. 7(c), respectively. Note that the rods 51 cannot fully overlap because of the physical size of the joints.

5

FOLDABLE STRUCTURES OF GENERAL SHAPE

It might be expected that two-dimensional foldable structures with many different shapes might be made by a straightforward extension of the ideas introduced above. Indeed, an obvious way of doing this would be to divide any given boundary shape into straight segments and circular arcs, and then assemble together straight-edged, trellis-type structures of suitable length, and simple angulated elements as disclosed in US-A-5024031 with an appropriate angle of embrace. Unfortunately, a rod structure of this type is not foldable. The problem is that, although it is possible to vary the semi-length of the simple angulated elements that make up a circular sector, so that the hinges connecting this sector to its neighbouring trellis-type structure are equally or proportionally spaced in the radial direction, this can be done only for a particular configuration. The scissor hinges do not remain equally spaced when the configuration is varied. Hence, a circular sector cannot be connected to a structure consisting of straight rods, whose scissor hinges are always equally, or proportionally spaced.

To obtain the layout of a two-dimensional foldable structure with a boundary of prescribed shape one must begin by finding a foldable base structure, i.e. a structure consisting of angulated rods whose hinges lie on the prescribed boundary. Once a suitable layout for the base structure has been selected, extra members can be connected to it by means of scissor hinges, until the required shape and overall dimensions are obtained. It will be shown that such a structure is foldable and, subject to certain conditions, it remains foldable if
contiguous rods are firmly connected into multi-angulated rods.

Finding a base structure that meets all the shape and folding requirements of a given application is the key to a successful overall solution. The method will be explained by describing the procedure which has been followed for a series of representative examples. All of the examples are of the same basic type, continuous loop structures with a central hole of variable size. Such structures are suitable for foldable roofs for, e.g. sports stadia and tennis courts. Open loop structures are subject to fewer restrictions, and hence much easier to configure using any combination of GAEs.

Fig. 8 illustrates a simple technique as disclosed in US-A-5024031 to construct a single-loop foldable rod structure of any shape. Fig. 8(a) shows an illustrative, general polygon 60 which may be constructed from a series of simple angulated elements 61₁ to 61₆ as disclosed in US-A-5024031 whose internal hinges coincide with the vertices of the polygon. The semi-length of each angulated rod 62₁, 62₁’ is equal to half the length of each respective side of the polygon 60 and the two rods 62₁, 62₁’ in each respective element 61₁ form equal kink angles, which are equal to the corresponding internal angle of the polygon 60. Hence, in the fully folded configuration as shown in Fig. 8(b), the elements 61₁ overlap with the sides of the polygon 60. Note that half of the angulated rods 62₁, 62₁’ are hidden by the other rods 62₁, 62₁’. In general, of course, these angulated elements 61₁ are not symmetric and hence a radial mismatch develops as the structure is folded. However, the overall mismatch adds up to zero as shown in Fig. 8(c) because in this case the angulated elements form a chain of similar rhombuses whose diagonals are reduced in length by proportional amounts and also remain at constant angles during folding.

Fig. 9 shows a more general type of closed loop structure 70, whose internal hinges also coincide with the
vertices of the polygon 60 of Fig. 8(a). Here, the
angulated rods 72i, 72i′ making up each element 71i are no
longer identical, but still have a kink angle equal to the
internal angle of the polygon 60 and form a chain of
similar parallelograms 73. This property implies that the
loop structure 70 is foldable, because the sides of the
polygon vary by proportional amounts and hence no geometric
mismatch builds up during folding. Note that the angulated
elements used in this solution are simple Type II GAEs,
i.e. without any parallelograms as in Fig. 3.
In addition to the above solutions for base structures
forming closed loops of any shape, greater freedom is
available in the case of loops with one or more axes of
symmetry. Basically, any GAE can be used to form the basic
repeating unit and since, by symmetry, all units behave in
the same way, geometric compatibility in all configurations
is automatically satisfied. Fig. 10 shows two loop
structures whose innermost hinges lie on a rectangle with
rounded corners. The base structure shown in Fig. 10(a)
consists of identical rhombuses, and hence there is no need
to invoke symmetry to prove that this structure is
foldable. The base structure shown in Fig. 10(b), however,
is based on a symmetric arrangement of GAEs of Type I or
Type II. This can be seen by means of the central line of
symmetry 85 which divides two opposite rhombuses 80,81 into
similar isosceles triangles. Note that in the base
structure shown as an example in Fig. 10, the
quadrilaterals 80,81 around the line of symmetry need not
be parallelograms, but may be general symmetrical
quadrilaterals.
Any base structure can be extended by the addition of
a pair of rods of any length, connected to one another and
to the base structure by scissor hinges. The resulting
structure will be foldable, like the original base
structure. Repeating the same argument it can be shown
that any number of pairs of rods connected by hinges to the
base structure will leave its mobility unchanged.
Fig. 11(a) shows a general, small part of a rod structure consisting of angulated elements. Additional members are connected to its outer hinges as shown in Fig. 11(b), such that the quadrangles $A_2A_3B_1B_2$, etc. are parallelograms. This extended structure is foldable because all additional members are free to rotate with respect to the base structure but, in fact, no relative rotation between consecutive rods occurs as the structure is folded, i.e. $\angle A_1A_2A_3$, $\angle B_1B_2B_3$, etc. remain constant.

Consider, for example, $\angle A_1A_2A_3$. Because $A_1A_2$ and $A_2A_3$ remain parallel to $B_1B_2$ and $B_2B_3$, respectively,

$$\angle A_1A_2A_3 = \angle B_2B_3B_2 = \text{constant} \quad (41)$$

since $\angle B_2B_3B_2$ is the kink angle of an angulated rod, which is fixed.

In conclusion, this foldable structure can be made from multi-angulated rods similar to those described above as shown in Fig. 11(c), but note that the kink angles along these multi-angulated rods are not the same. The same procedure is valid for all other closed loop base structures discussed in this section, as for any open loop base structure. Fig. 12 shows a foldable structure whose internal boundary has an elliptical shape which has not been achievable previously.

The two-dimensional solutions derived above easily extend to curved structures, such as domes, by projecting any two-dimensional solution onto a surface with the required shape. Thus, each multi-angulated rod can be curved out of plane. Of course, all connectors between multi-angulated rods should be perpendicular to the plane of projection.

The folding angle may be restricted if the rods are not allowed to overlap during folding. This problem can be solved by a proper design of the connections. An example of a suitable connector between two rods is shown in Figs. 13(a) to (c). In the drawings, 90 is one of the rods whilst the other rod is in two parts 91, 92. One part 92 of the other rod has a circular cross-section cylindrical post
93 of height H and the other part 91 of the other rod has
a cap 94 which can be securely fitted onto the post 93. At
the pivot point of the first rod 90 there is provided an
open ring 95 of height h which is less than the height H of
the post 93. The post 93 is inserted into the ring 95 and
the cap 94 fitted over the part of the post 93 which
projects through the ring 95. The parts 91, 92 of the other
rod can then be rigidly fixed to each other by some
suitable means such as screws 96 so that the parts 91, 92 of
the other rod effectively act as one long rod. As the
height H of the post 93 is greater than the height h of the
ring 95, the two rods can rotate with respect to each
other. Of course, other connectors will be suitable for
allowing the rods to rotate with respect to each other.

Fig. 14 shows a double layer model structure whose
curved top layer is connected to the flat bottom layer by
long bolts. The bottom layer is identical to the model
shown in Fig. 7 and the orthogonal projection of the top
layer onto the plane of the bottom layer is also identical
to it. This model folds until the outer rods overlap
fully, and thus demonstrates that the interference between
rods connected to the same hinge has been successfully
eliminated. Note that bracing elements could be added
between the upper and lower cords, to increase the
stiffness of the structure, if desired.

In summary, a general method for the design of
two-dimensional foldable structures has been introduced.
The new method extends and generalises the familiar
trellis-type structures, based on a tiling of
parallelograms whose sides are collinear, to structures
based on any tiling of parallelograms. It has been shown
that a rod structure of this type is (i) foldable and (ii)
can be made from multi-angulated, rigid rods connected by
scissor hinges. This result affords much greater freedom
in the range of shapes that can be achieved, and of
boundary conditions that can be met. This approach can be
easily extended to three-dimensional dome structures.
Also, a family of elements for foldable structures has been introduced. These consist of angulated rods connected by scissor hinges. It has been shown that any element bounded by either isosceles triangles or similar triangles, with any number or parallelograms in between, maintains a constant angle of embrace.

Finally, a method for the design of structures consisting of multi-angulated rods that fold along their perimeter has been described and there is practically no limit to the range of perimeter shapes that can be achieved.
CLAIMS

1. An angulated element, the element comprising two pivotally interconnected angulated rods which embrace a constant angle as the element is folded and expanded, in which each rod has first and second portions, the length of the respective first portions of the rods being substantially the same, the length of the respective second portions of the rods being substantially the same, the angles between the first and second portions of the respective rods being different.

2. An expandable/collapsible structure, comprising a generalised angulated element which embraces a constant angle as the generalised angulated element is folded and expanded, the generalised angulated element comprising a plurality of elements each consisting of two pivotally interconnected angulated rods, wherein the ends of rods of each element are pivotally connected to respective ends of rods of an adjacent element such that each closed loop formed by such connection is a parallelogram, with any unconnected portions of rods of any terminal elements forming isosceles triangles.

3. An angulated element, the element comprising two interconnected angulated rods which embrace a constant angle as the element is folded and expanded, in which each rod has first and second portions, the ratio of the lengths of the respective first portions of the rods being substantially equal to the ratio of the lengths of the respective second portions of the rods, the angles between the first and second portions of the respective rods being substantially the same.

4. An expandable/collapsible structure, comprising a generalised angulated element which embraces a constant angle as the generalised angulated element is folded and
expanded, the generalised angulated element comprising a plurality of elements each consisting of two pivotally interconnected angulated rods, wherein the ends of rods of each element are pivotally connected to respective ends of rods of an adjacent element such that each closed loop formed by such connection is a parallelogram, with any unconnected portions of rods of any terminal elements forming similar triangles.

5. An expandable/collapsible structure, the structure comprising a plurality of multi-angulated rods, each rod being formed of at least three portions, the rods being pivotally connected to one another at at least one of the junctions between said portions such that one closed loop is a foldable base structure and any other closed loops are formed by parallelograms.

6. A structure according to claim 5, wherein, for each multi-angulated rod, said portions are substantially the same length and the angles subtended at an origin by said portions of each rod are substantially the same with each successive portion of each rod being at an increasing distance from the origin.

7. A structure according to claim 5 or claim 6, wherein the rods are substantially identical.

8. A structure according to any of claims 5 to 7, wherein all of the rods are planar.

9. A structure according to any of claims 5 to 7, wherein all of the rods are non-planar.

10. A structure according to any of claims 5 to 7, wherein some of the rods are planar and some are non-planar.
11. A structure according to any of claims 5 to 10, wherein for each multi-angulated rod, said portions are of substantially different length.

12. A three-dimensional structure having a single layer the projection of which onto a plane is a structure according to any of claims 5 to 11.

13. A three-dimensional structure having a base layer being a structure according to any of claims 5 to 8 and a second layer being a structure according to any of claims 5 to 7, the projection of the second layer onto the base layer being substantially identical to said base layer.

14. An element according to claim 1 or claim 3, or a structure according to any of claims 2, 4 or 5 to 13, in which the pivotal interconnection between pairs of rods is formed by a post on one of the rods engaging in a ring on the other of said rods.
Figure 9
Figure 11
Figure 13
**INTERNATIONAL SEARCH REPORT**

### A. CLASSIFICATION OF SUBJECT MATTER

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According to International Patent Classification (IPC) or to both national classification and IPC

### B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

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Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

### C. DOCUMENTS CONSIDERED TO BE RELEVANT

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Further documents are listed in the continuation of box C.

Patent family members are listed in annex.

Date of the actual completion of the international search: 1 April 1997

Date of mailing of the international search report: 11.04.97

Name and mailing address of the ISA:
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**Information on patent family members**

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