United States Patent

McKenna

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[54] POLYHEDRAL STRUCTURAL SYSTEM AND SPATIAL ENCLOSURE

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[52] U.S. Cl. 52/79, 52/81, 52/236, 52/604

[51] Int. Cl. E04b 1/348

[58] Field of Search 52/237, 79, 236, 604; 125/30; 46/25; 35/72, 46

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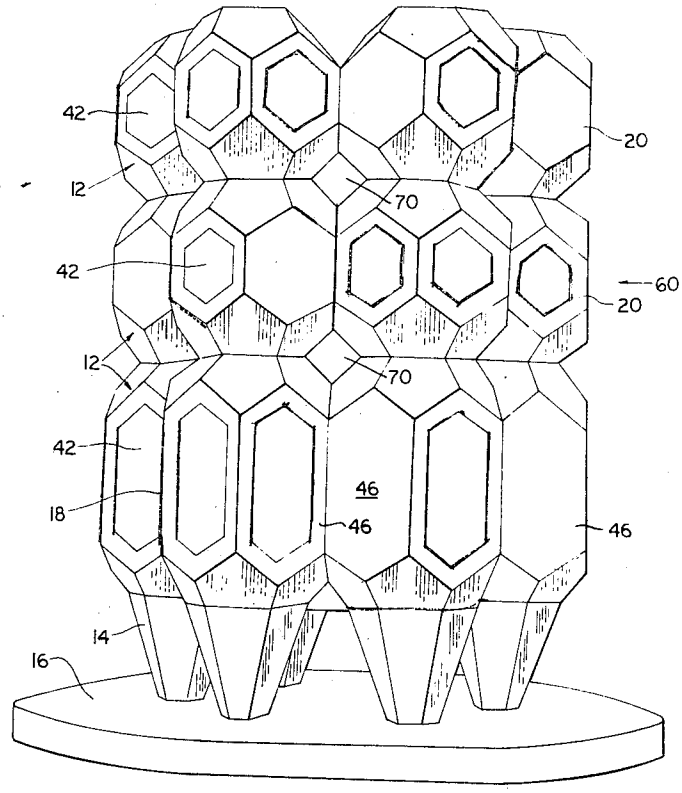
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[57] ABSTRACT

A structural system and spatial enclosure defined by a new polyhedron, the isoicosahedron, having 20 faces consisting of eight hexagons and 12 pentagons, 54 straight line elements, and 36 vertices, which, in a preferred form, lie on the surface of an imaginary sphere circumscribing the polyhedron. In the preferred form, the hexagons are regular and congruent while the pentagons though congruent are irregular having three sides equal in length to the side length of the hexagons and two sides equal to said length divided by the $\sqrt{2}$. In other forms, the 36 vertices of the polyhedron lie on the surface of an imaginary enclosing spheroid, oblate, elongate, or irregular. The isoicosahedra form modular units which can be reiterated in a building structure. The packed modular polyhedra define unique polyhedral spaces each having nine faces consisting of six pentagons and three squares, the endoenneahedron.

2 Claims, 14 Drawing Figures



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SHEET 1 OF 5

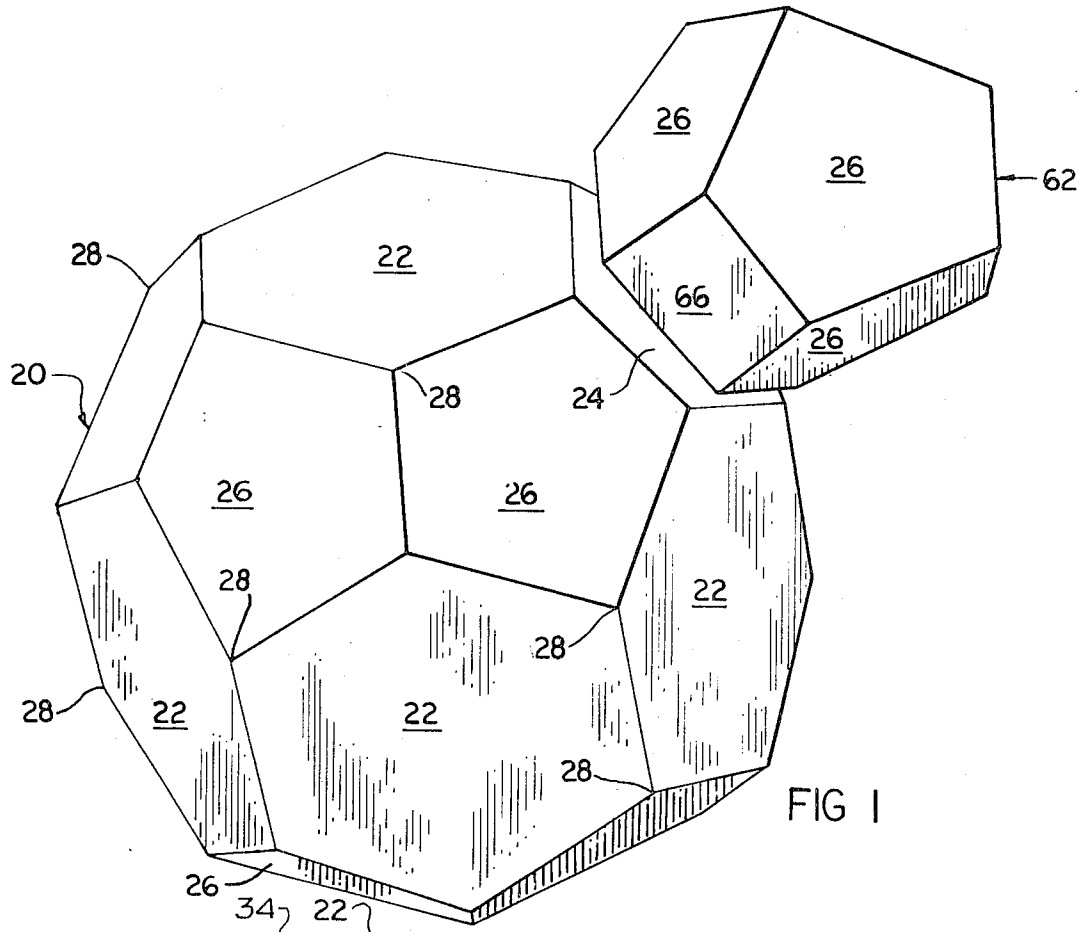


FIG 1

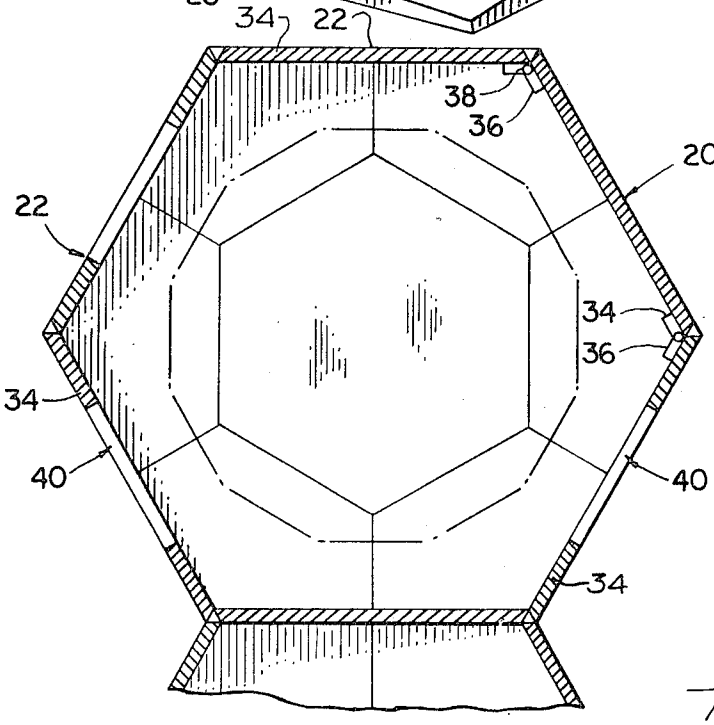


FIG 10

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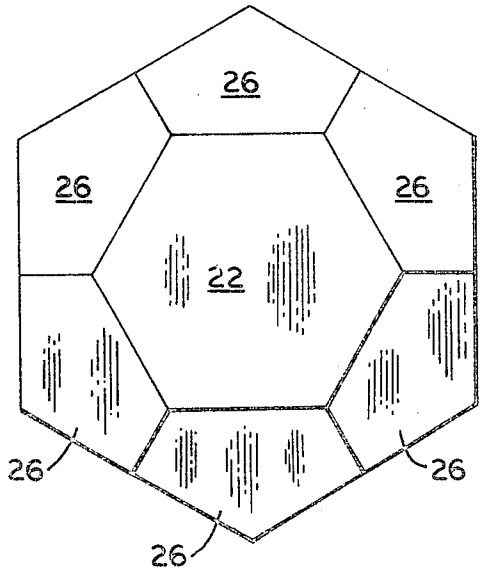


FIG 2

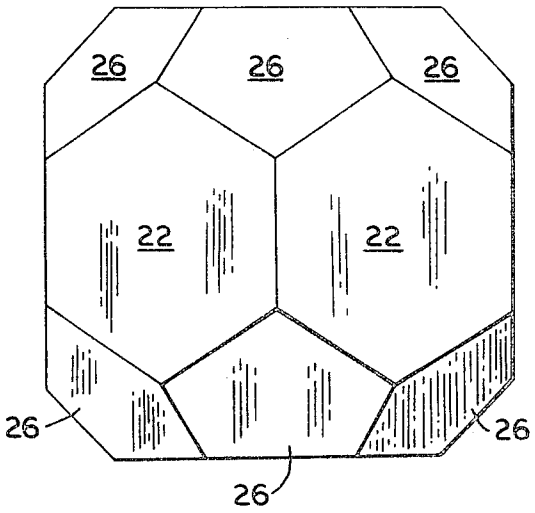


FIG 4

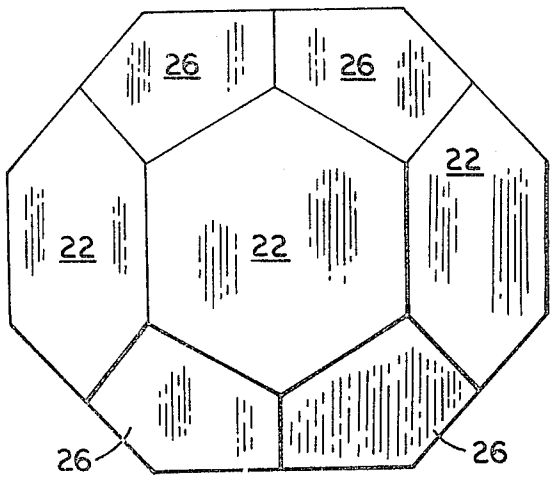


FIG 3

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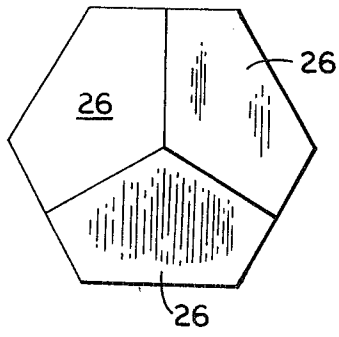


FIG 5

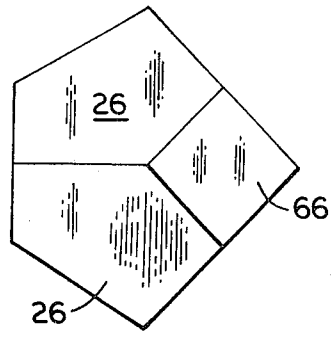


FIG 6

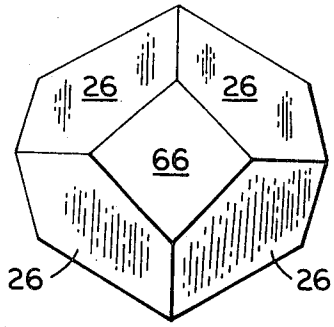


FIG 7

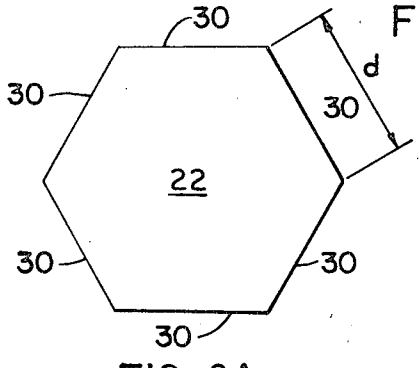


FIG 8A

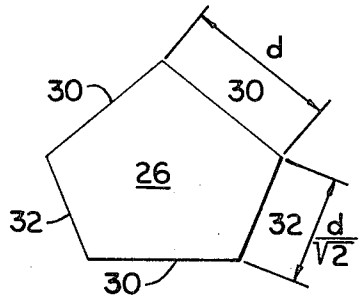


FIG 8B

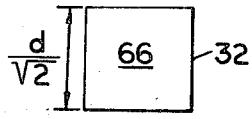


FIG 8C

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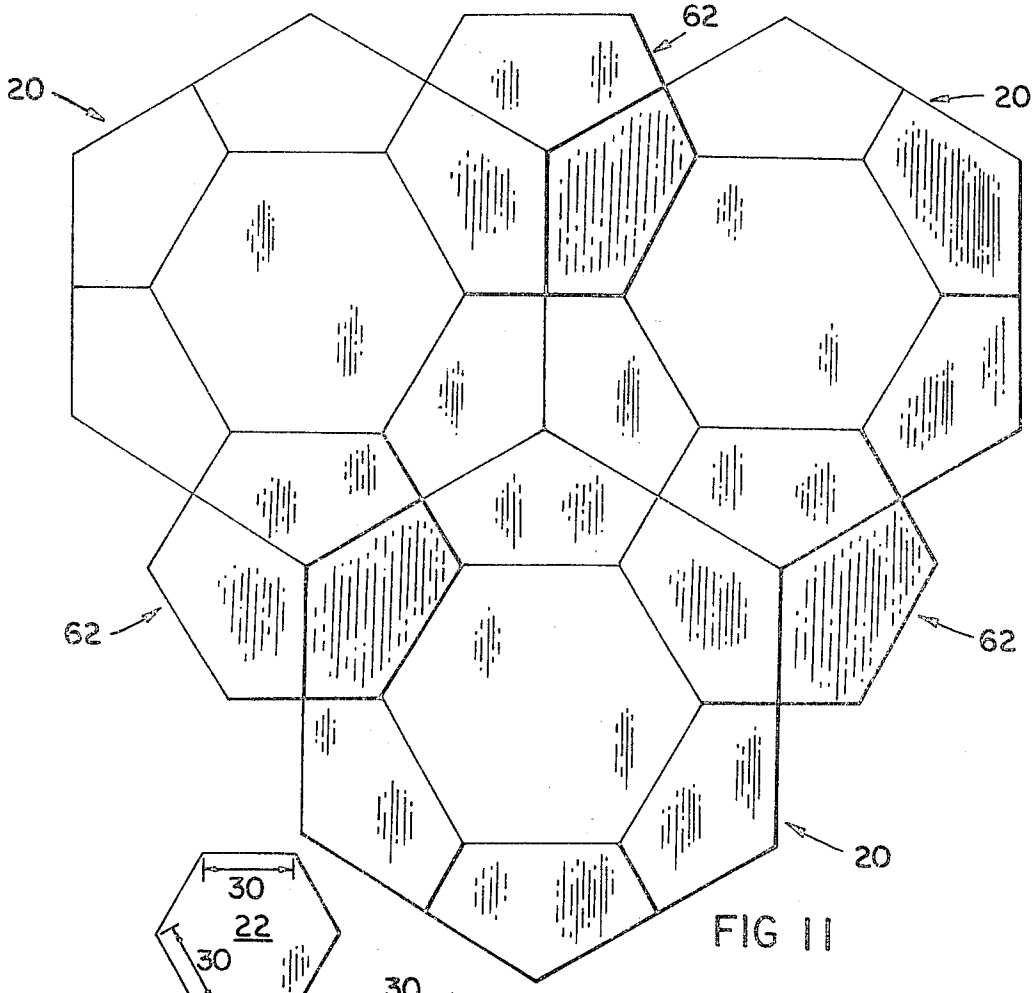


FIG 11

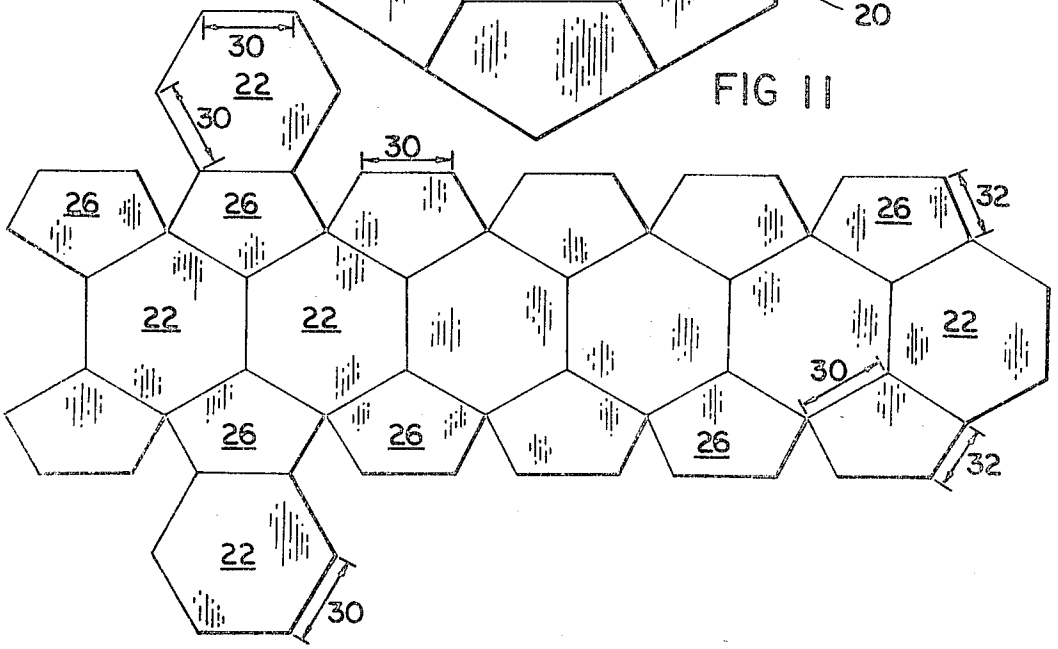


FIG 9

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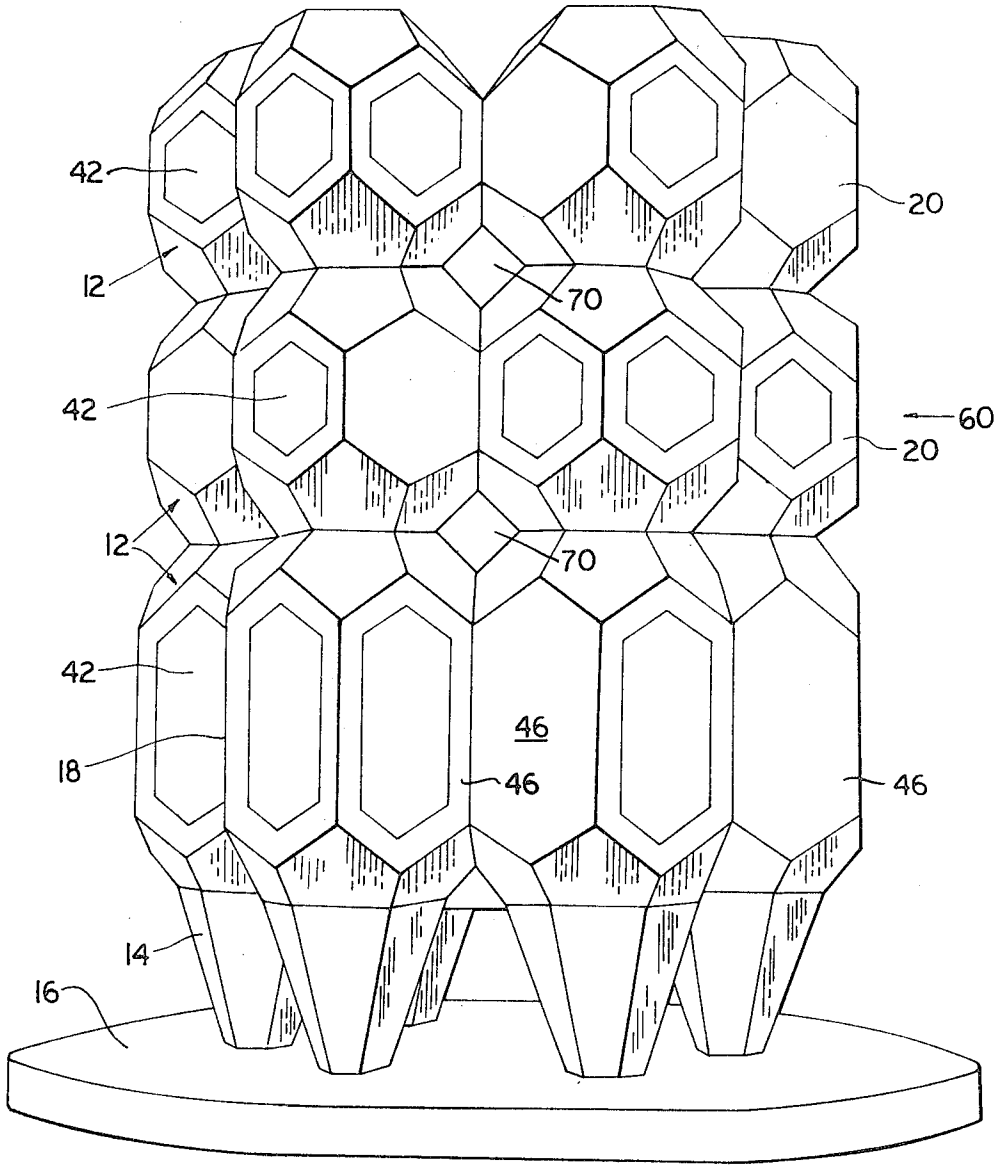


FIG 12

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POLYHEDRAL STRUCTURAL SYSTEM AND SPATIAL ENCLOSURE

This invention relates to a new and improved structural system which, at the same time, provides a framework for enclosing space.

It is well known that traditional rectilinear architecture incorporates basically unstable and inefficient structural units. In the construction of rectilinear cubical building systems, initial structural members are required to support a load of dead weight resulting from the inefficient use of materials. Thus, certain materials are utilized for providing spatial enclosure without contributing to the structural support.

In order to increase the structural efficiency of materials and buildings, diagonal and triangulated systems have been adopted as, for example, in geodesic structure in which tetrahedral units are in effect projected onto generally spherical surfaces. While geodesic systems approach optimum efficiency in the volume of space enclosed with respect to the amount of material required and incorporate the structural efficiency of self-triangulated elements, the geodesic dome has limited adaptability and application.

It is therefore an object of the present invention to provide a new and improved structural system and framework which defines a space intermediate the generally spherical geodesic structure and the traditional cubical or rectilinear building structure and which combines the structural and volume efficiency of the former with the simplicity and iterative modularity of the latter.

Another object of the invention is to provide a basic architectural unit which by means of self-triangulated elements distributes the stress and load about the material comprising the structure and at the same time utilizes the material for providing a spatial enclosure.

In order to provide these results, the present invention contemplates providing a structural system and framework for enclosing space defined by a new but basic polyhedron, designated by the present inventor as the isoicosahedron. The isoicosahedron is a polyhedron generally comprising 20 faces consisting of eight hexagons and 12 pentagons bounded by 54 straight line elements and 36 vertices. In a preferred form of the basic polyhedron, the 36 vertices lie on the surface of an imaginary sphere circumscribing the polyhedron. The eight hexagons forming faces of the structure are regular and congruent while the 12 pentagons though congruent are irregular having three longer edges equal in length to the edge length of the hexagons and two shorter edges equal in length to the longer edges divided by $\sqrt{2}$. The shorter edges are separated by the longer. In this preferred form, the ratio of the radius of the enclosing sphere to one-half an edge length of the hexagon is equal to the $\sqrt{13}$.

The invention contemplates that the primary unitary structure, the isoicosahedron, can be reiterated in a modular fashion to provide simple and inexpensive building and housing construction with rigid and stable geometric configuration and high volume and structural efficiency. When the basic polyhedra of the present invention are arranged in a closely packed array, adjacent isoicosahedra define and enclose interspaces or intermediate spaces in the configuration of polyhedra with nine faces, six faces consisting of pentagons congruent with the pentagons comprising the isoicosahedra and three square faces having a side length equal in length to the shorter sides of the pentagons. A nine-faced polyhedron formed in the interspace between closely packed isoicosahedra is named by the present inventor, the endoenneahedron. According to other aspects of the invention, the endoenneahedral interspaces between the isoicosahedra of an iterated modular structure can themselves be sealed off to form part of the building system thereby providing optimum use of the volume of space enclosed by the iterated structure further increasing also the stability.

In other forms contemplated by the invention, 36 vertices of the isoicosahedron lie on the surface of an imaginary enclosing spheroid, rather than a sphere. Thus, the imaginary enclosing sphere can by transformation be converted to an oblate spheroid or an elongate spheroid such as, for example, ellipsoids of various shape formed by ellipses of rotation rotated about either the minor or major axis thereby providing either flattened or elongate structural units respectively. Furthermore, the vertices can lie on a surface of an irregular spheroid such as an egg-shaped spheroid or a skewed spheroid. In each case, the shapes and sizes of the hexagons and pentagons comprising the isoicosahedra are altered in a mathematically determinable and definable manner according to the transformation selected.

A feature and advantage of the present invention is that whereas the volume-to-surface-area ratio of a circumscribing sphere is approximately 1.20, the volume-to-surface-area ratio for the basic isoicosahedron circumscribed by the sphere is about 1.065 while the volume-to-surface-area ratio for a cube circumscribed in the sphere is only approximately 0.682. Thus, the volume efficiency of the basic polyhedral structural unit of the present invention is approximately 89 percent that of a sphere whereas the volume efficiency of a cube is only about 57.8 percent that of a sphere. Thus, while the volume of enclosure for the structural unit of the present invention approaches that of a sphere, it preserves the linearity of traditional cubical and rectilinear systems and lends itself to iterative modularity. Furthermore, the present invention provides a structural system of rigid and stable geometric configuration incorporating diagonality and self-triangulation achieving high structural efficiency by distribution of stress and load over the entire material comprising the structural unit and enclosure.

Other objects, features and advantages of the present invention will become apparent in the following specification and accompanying drawings.

In the drawing:

FIG. **1** is a perspective view of an isoicosahedron and an adjacent endoenneahedron.

FIG. **2** is a plan view of the isoicosahedron.

FIG. **3** is a side view of the isoicosahedron.

FIG. **4** is a side view from another direction of the isoicosahedron.

FIG. **5** is a plan view of the endoenneahedron.

FIG. **6** is a side view of the endoenneahedron.

FIG. **7** is a side view from another direction of the endoenneahedron.

FIGS. **8***a*, **8***b* and **8***c* are plan views of the hexagon, pentagon and square which form the surface faces of the isoicosahedron and the endoenneahedron.

FIG. **9** is a cross-sectional plan view of an isoicosahedral building structure.

FIG. **10** is a diagrammatic plan view of a structure consisting of three adjacent isoicosahedral units with three endoenneahedral units positioned in appropriate spaces above the isoicosahedran units.

FIG. **11** is a side perspective view of modular building structure in which the modular units consist of regular isoicosahedra and elongate isoicosahedra mounted on supported bases.

FIG. **12** is an expanded cutout showing the faces of the isoicosahedron.

The basic structural unit, the isoicosahedron contemplated by the present invention is shown in perspective in FIG. **1**. This polyhedron has 20 faces consisting of eight regular and congruent hexagons **22** and twelve irregular and congruent pentagons **26**. The faces are arranged to form 54 straight line elements and 36 vertices **28** which in a preferred form of the invention lie on the surface of an imaginary sphere circumscribing or enclosing the polyhedron. As shown in the plan and side views of FIGS. **2**, **3** and **4**, the faces of the polyhedron are arranged so that six of the regular hexagons form a ring around the center of the polyhedron, while the other two hexagons form top and bottom bases, respectively. The 12 pentagons are arranged in two rings of six pentagons each which join the ring of hexagons with the top and bottom base hexagons, respectively. A cutout suitable for forming an isoicosahedron and showing all of the faces of such a polyhedron is shown diagrammatically in FIG. **9**.

As shown in FIGS. 8*a* and 8*b*, each of the hexagons **22** is a regular hexagon, each of the sides **30** having a length *d*. Each of the pentagons **26**, however, is irregular, having three sides **30** of length *d* and two sides **32** of length $d/\sqrt{2}$.

In the regular isoicosahedron, the 36 vertices **28** lie on the surface of an imaginary sphere circumscribing and enclosing the polyhedron. The volumetric efficiency of the isoicosahedron is apparent in that it occupies approximately 89 percent of the volume of the circumscribing sphere, whereas a cube would occupy only approximately 57.8 percent of the volume of a circumscribing sphere. Moreover, the structural stability is significantly improved by the elimination of rectilinear elements and the incorporation of self-triangulation and diagonality in the structural system.

In a regular isoicosahedron whose vertices lie on the surface of an imaginary enclosing sphere, the ratio of the radius of the imaginary sphere to one-half the side length, $d/\sqrt{2}$ of a side of the hexagons forming faces of the polyhedron is equal to the $\sqrt{13}$. The basic isoicosahedron can be flattened or elongated, however, by transforming the enclosing sphere into an ellipsoid or an oblate spheroid. The mathematically determinable transformation results in a translation of the vertices relative to each other and a consequent elongation or flattening of elements of the polyhedron.

When a plurality of regular isoicosahedra are packed in a close array with adjacent hexagonal faces touching, interspaces or intermediate spaces are defined, forming a nine-faced polyhedron, the endoenneahedron, **62**, also shown in perspective in FIG. **1**. The nine faces of the endoenneahedron consist of six pentagons **26** and three squares **66** arranged in the manner further set forth in the plan and side views of FIGS. **5**, **6**, and **7**. The pentagons **26** are, of course, congruent to the pentagons forming faces of the isoicosahedron and as shown in FIG. 8*b*. The square face **66** of each endoenneahedron has a side length **32** equal to the shorter side length $d/\sqrt{2}$, of the pentagons. The endoenneahedron is itself a structurally efficient unit which can be used to advantage in architectural and building structures as hereinafter described. Thus, a plurality of isoicosahedra and endoenneahedra can be packed in a close array occupying generally 100 percent of an enclosed space. Three adjacent isoicosahedra **20** are shown diagrammatically in FIG. **11** with three endoenneahedra **62** nested in appropriate interspaces formed between the isoicosahedra. The volume occupied by each endoenneahedron is approximately 7/24 of the volume of the corresponding isoicosahedron.

A modular building element **30** embodying the present invention is shown in FIG. **10**. In applying the basic polyhedron to architectural and building construction, a load-carrying skeletal framework, structural wall member, or both can be used. Thus, a load-carrying framework can first be constructed following the 54 straight-line elements of the basic polyhedron. Alternatively, and as illustrated in FIG. **9**, each modular unit can be constructed of wall members **34** which enclose the space defined by the isoicosahedron and simultaneously constitute the load carrying and transmitting structural material. The wall members can be constructed of any suitable material such as precast concrete slabs, wood, plastic, etc. Edges of each panel are joined to the edges of adjoining panels in any conventional manner, as by welding or bonding or by brackets **36** and bolts **38**. The modular units can be prefabricated and assembled at the manufacturing site or at the construction site, thereby lending themselves to inexpensive housing units. Because only two building blocks, namely, the regular hexagons and irregular pentagons, are required for each polyhedral building element **30**, low-cost, high-volume, mass-production building units are possible. Further shown in FIG. **10** are openings **40** defining windows or doorways to the outside or doorways between adjoining modular elements **30**. A flat duodecagon brace or 12-sided floor is particularly suitable for placement near the bottom of the modular unit with every other side of the floor piece resting against one of six pentagons forming a ring around the bottom of the unit.

Referring to FIG. **12**, a building structure **60** is constructed of a plurality of vertically and horizontally aligned stacked building modules or elements **12** which are supported by base pillars **14** resting on ground **16**. The first or lowermost horizontal layer of building modules comprises vertically elongate isoicosahedral elements **18** to enhance the architectural appearance of the structure and available space while the remainder of the building modules consists of elements **20** having a regular isoicosahedral shape. Windows **42** are provided. The vertically elongate modular elements **18** are identical to the above-described elements **20** except that they include vertically oriented hexagonal walls **46** which are elongate and, therefore, irregular. In addition to the architectural effects obtained from the use of such vertical elongate modular elements, additional interior space is provided. In all other respects the elements **18** are similar to the regular modular elements **20** and they are constructed of the same building blocks or slabs except for the irregularly shaped vertical hexagons. The vertices of the elongate modular elements **18** lie on the surface of a circumscribing ellipsoid rather than a sphere. By varying the relative positions and arrangement of the modular building elements further shapes and configurations of building structures can be obtained.

Turning now to the construction of building **60**, modular building elements **18** and **20** are assembled, either on site or at the factory, and then hoisted onto the supporting pillars **14**. Alternatively, if weight or size considerations require, the elements can be individually assembled on the pillars. Thereafter the next horizontal layer of building elements **20** are placed on top of elongate elements **18** to define the next building floor. This is repeated until the desired height of the building is obtained.

During the assembly it will be noted that the individual building elements **18** and **20** can each be provided with all twenty members to define the 20-faced polyhedron or, alternatively, wall members which overlie the wall member of an adjoining modular building element can be omitted on one of the two adjoining elements to reduce construction costs and lower the overall weight of the elements and the building.

The open spaces or interspaces between modular elements have a nine-face configuration, six faces having the shape of pentagons **26** while three faces have a square shape and communicate with the exterior of the building through holes **70**. Since the interspaces are of considerable size (their volume is 7/24 of the volume of the polyhedral modular building elements **20**) and all but one of their faces are already enclosed by the walls of the adjoining building elements, a wall member covering the hole **70** can be provided to thereby close the spaces and make them usable for living quarters, storage spaces or the like. The spaces are then interconnected with one or more of the adjoining modular elements through suitable door openings and stairways (not shown). The interior of the modular building elements **18** and **20** can be constructed and finished as desired.

In addition to architectural and building applications the structural system of the present invention is applicable for three-dimensional corrugate or "honeycomb" material consisting of closely packed arrays of the isoicosahedral and/or endoenneahedral elements.

It is apparent that the basic polyhedral unit contemplated by the present invention, the isoicosahedron, can be modified in a variety of ways by truncating the polyhedron at some or all of its vertices. Thus, a variety of geometrical configurations can be introduced while maintaining the structural integrity of the isoicosahedral structural system. As used herein and in the following claims, the phrase "truncated isoicosahedron" refers to the basic polyhedron truncated at one or more of its vertices.

What is claimed:

1. A building structure formed of a plurality of modular elements comprising:

a plurality of first polyhedral elements, each having 20 faces comprising eight hexagons and 12 pentagons, 54 straight-

line edge elements defining the faces, and 36 vertices, said faces arranged in a first ring of six side-by-side hexagons, second and third rings of six side-by-side pentagons adjacent either side of the first ring, said second and third rings each surrounding a hexagon on the side away from the first ring, said eight hexagons bring regular and congruent and said 12 pentagons being irregular and congruent, each said pentagon formed with three edges of a first length equal to an edge length of said hexagons and wherein the other two edges are formed with a second length equal to said first length divided by $\sqrt{2}$, said two edges of the second length separated by edges of the first length, and wherein the 36 vertices lie on the surface of an imaginary sphere circumscribing the polyhedral element, the ratio of the radius of said sphere to one-half the edge length of said hexagon being substantially equal to $\sqrt{13}$;

said plurality of first polyhedral elements arranged side by side in a three-dimensional array with hexagonal faces in adjacent relationship;

and a plurality of second polyhedral elements defined by the interspaces between said plurality of first polyhedral elements, each said second polyhedral element having nine faces comprising six pentagons and three squares, said faces arranged so that each square is surrounded by four pentagons, said six pentagons being irregular and congruent, each said pentagon also being congruent with the pentagons comprising said first polyhedral elements and wherein the vertices of each second polyhedral element lie in the surface of an imaginery sphere circumscribing the second polyhedral element, the ratio of the radius of the sphere circumscribing the first polyhedral element to the radius of the sphere circumscribing the second polyhedral element being substantially equal to $\sqrt{13}/2$.

2. A building structure formed of a plurality of modular elements comprising:

a plurality of first polyhedral elements, each having 20 faces comprising 8 hexagons and 12 pentagons, 54 straight-line edge elements defining the faces, and 36 vertices, said faces arranged in a first ring of six side-by-side hexagons, second and third rings of six side-by-side pentagons adjacent either side of the ring, said second and third rings each surrounding a hexagon on the side away from the first ring, said eight hexagons being regular and congruent and said 12 pentagons being irregular and congruent, each said pentagon formed with three edges of a first length equal to an edge length of said hexagons and wherein the other two edges are formed with a second length equal to said first length divided by $\sqrt{2}$, said two edges of the second length separated by edges of the first length, and wherein the 36 vertices lie on the surface of an imaginary sphere circumscribing the polyhedral element, the ratio of the radius of said sphere to one-half the edge length of said hexagon being substantially equal to $\sqrt{13}$;

said plurality of first polyhedral elements arranged side-by-side in a three-dimensional array with hexagonal faces in adjacent relationship whereby the modular polyhedral elements define interspaces substantially enclosed by the pentagonal faces of the polyhedral elements with square holes opening between the interspaces and to the exterior of the building structure and wherein the building structure also includes flat, rectangular faces closing the square holes between the interspaces and between the interspaces and the exterior of the building structure.

* * * * *