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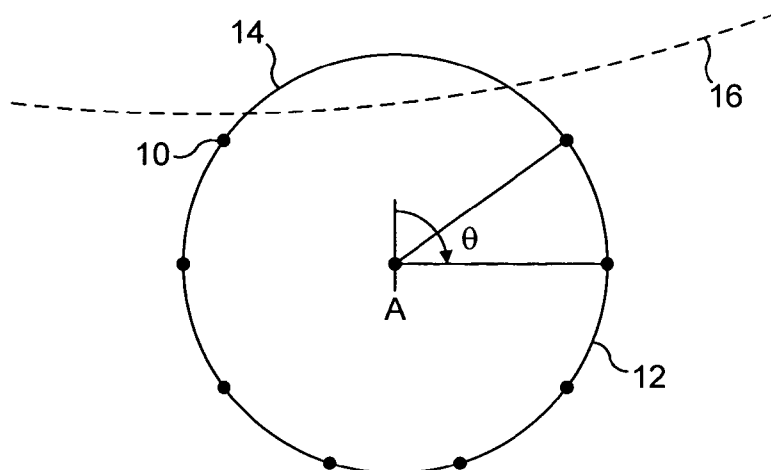
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(54) Title: SHAPE REPRESENTATION USING COSINE TRANSFORMS



**FIG. 1**

(57) Abstract: A method of approximating the inner or outer boundary of an iris comprises generating an approximate boundary representation (20) comprising a least squares approximation by a cosine transform series of a function of the angle ( $\theta$ ) about a fixed point (A) of the distance of measured points (10) on the boundary from the fixed point (A). More broadly, the method may be used to approximate the shape of any two-dimensional curve or figure.

## SHAPE REPRESENTATION USING COSINE TRANSFORMS

The present invention relates to shape representation using Cosine Transforms. The invention finds particular although not exclusive application  
5 in biometrics, for example in the generation of approximate representations of the outer and/or inner boundary of the iris in the human eye.

Ghosh and Jain ("An Algebra of Geometric Shapes", *IEEE Computer Graphics and Applications*, 1993, 50) describe the use of Fast Fourier  
10 Transforms (FFTs) to model the outline of a shape by tracking around its outer periphery.

It is of considerable importance in biometric systems that rely on iris recognition to be able to identify and map accurately both the outer edge of the  
15 iris and also the inner edge (the periphery of the pupil). Many iris recognition systems assume that the shape of the pupil is always circular, an assumption which may be inaccurate in many cases. Indeed even when pupils are circular, they tend to become elongate or oblong when viewed from an angle.

20 Some research into non-circular pupil localisation has been carried out: See B. Bonney, R. Ives, D. Etter, and D. Yingzi, "Iris pattern extraction using bit planes and standard deviations," *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers*, 2004, Y. Du, B. L. Bonney, R. W. Ives, D. M. Etter, and R. Schultz, "Analysis of Partial Iris  
25 Recognition Using a 1-D Approach", *Proceedings of the 2005 IEEE International Conference on Acoustics, Speech, and Signal Processing*, March 18-23, 2005. However, in spite of these earlier approaches, there still remains a need for a system which can in a straightforward way approximate a

## 2

boundary given a number of points (which may not be equally spaced) on that boundary.

According to a first aspect of the present invention there is provided a method  
5 of approximating an iris boundary, comprising the steps of:

- acquiring an image of an eye, including an iris boundary;
- noting a plurality of spaced boundary points on the boundary;
- selecting a fixed reference point; and
- generating an approximate boundary representation comprising a  
10 least squares approximation by a cosine transform series as a  
function of angle about said fixed point of the distance of said  
boundary points from said fixed point.

Because the points on the boundary may not be equally spaced, standard  
15 methods of calculating coefficients such as the Discrete Cosine Transform  
(DCT) cannot be used.

Preferably, this method is used to map the inner boundary of the iris (or,  
equivalently, the outer boundary of the pupil) of a human eye. Alternatively, it  
20 may be used to map the outer iris boundary.

In the method described, the use of higher harmonics provides excellent  
pupil localisation, both on general and on non-ideal eye images. The method  
provides excellent results on the vast majority of pupils which are significantly  
25 non-circular.

According to a second aspect of the present invention there is provided a  
method of approximating a two-dimensional shape, comprising the steps of:

## 3

- selecting a fixed reference point; and
- generating an approximate shape representation comprising a least squares approximation by a cosine transform series as a function of angle about said fixed point of the distance of said measured points from said fixed point.

5

The invention may be carried into practice in a number of ways and one specific embodiment will now be described, by way of example, with reference to the accompanying drawings, in which:

10

Figure 1, shows a non-circular pupil shape, as imaged; and  
Figure 2, shows an approximation to that shape.

15

The way in which an embodiment of the invention may be applied to a particular problem, that of characterising a non-circular pupil shape, will now be described.

20

25

First, an eye may be imaged, and the image analysed to identify a plurality of points 10, which occur on the imaged pupil/iris boundary 12. To do this, an approximate pupil location may be first determined by searching for a dark area of significant size close to the image centre. A histogram analysis may then be carried out to find a more exact centre as well as the average pupil radius. This approximate circular pupil boundary may then be examined in detail to obtain the required number of edge points 10. In the preferred embodiment, 16 such points are identified. It will be understood by those skilled in the art that other methods may be employed to locate points on the pupil/iris boundary and the scope of the claimed subject matter is not limited in this respect.

## 4

It will also be understood that the points 10 may not necessarily be equally spaced around the edge of the pupil. Indeed, in some images part of the boundary 14 may be obscured by an eyelid and/or eyelashes 16.

5           Once the boundary points 10 have been identified, those points can be used to generate a mathematical approximation 20 of the actual curve 12, as is shown in Figure 2. In the present invention, the fitted curve 20 is a Cosine Transform least squares approximation, as a function of angle  $\theta$ , of the distance of the points 10 from a notional fixed point A (see Figure 1).

10

We wish to describe the distance  $d(\theta)$  of a curve in the plane from a known or assumed centre as a harmonic function of the angle  $\theta$  in the form of a variation of the 1D Discrete Cosine Transform (DCT). For simplicity, it will be assumed that the function  $d(\theta)$  is single valued in  $\theta$ . The inverse transform from N

15 coefficients  $\{C_n; n = 0 \dots N-1\}$  is

$$d(\theta) = \frac{C_0}{\sqrt{2}} + \sum_{n=1}^{N-1} C_n \cos n \left( \frac{\theta}{2} + \frac{\pi}{2N} \right)$$

A standard Discrete Cosine Transform is a fit of regularly spaced data related to the Chebyshev polynomial, which has equi-ripple properties between the  
20 sample points.

However, in the present application, the points to be fitted  $\{r_i, \theta_i; i = 1 \dots M\}$  may be irregular in  $\theta_i$ .

25 If the error in the fit is

$$E(\theta_i) = d(\theta_i) - r_i$$

Then we wish to find  $\{C_n; n = 0 \dots N\}$  which minimizes the sum of squares of the

## 5

$$E^2 = \sum_{i=1}^M E(\theta_i)^2 = \sum_{i=1}^M (d(\theta_i) - r_i)^2$$

$$= \sum_{i=1}^M \left[ \left\{ \frac{C_0}{\sqrt{2}} + \sum_{n=1}^{N-1} C_n \cos n \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right) \right\} - r_i \right]^2$$

To do this we differentiate with respect to  $C_k$  and equate to zero:

5

$$\frac{\partial E^2}{\partial C_0} = \sum_{i=1}^M \sqrt{2} \left[ \frac{C_0}{\sqrt{2}} + \sum_{n=1}^{N-1} C_n \cos n \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right) - r_i \right] \cos k \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right)_i = 0$$

$$\frac{\partial E^2}{\partial C_k} = \sum_{i=1}^M 2 \left[ \frac{C_0}{\sqrt{2}} + \sum_{n=1}^{N-1} C_n \cos n \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right) - r_i \right] \cos k \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right)_i = 0, k = 1 \dots N-1$$

10

From which

$$\sum_{i=1}^M \frac{C_0}{\sqrt{2}} + \sum_{n=1}^{N-1} C_n \cos n \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right) = \sum_{i=1}^M r_i$$

$$\sum_{i=1}^M \left[ \frac{C_0}{\sqrt{2}} + \sum_{n=1}^{N-1} C_n \cos n \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right) \right] \cos k \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right) = \sum_{i=1}^M r_i \cos k \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right), k = 1 \dots N-1$$

15

This gives  $N$  linear equations in the unknowns  $\{C_n; n = 0 \dots N-1\}$  which can be expressed in matrix form as

$$PC=S$$

20

Where the unknowns are given by

## 6

$$C = \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_N \end{bmatrix}$$

and the right hand side is given by

$$S = \begin{bmatrix} S_0 \\ S_1 \\ \vdots \\ S_N \end{bmatrix}$$

5

where

$$S_k = \sum_{i=0}^M r_i \cos k \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right)$$

The  $N$  by  $N$  matrix  $P$  is then

10

$$P_{k,0} = \frac{1}{\sqrt{2}}$$

$$P_{k,n} = \sum_{i=1}^M C_n \cos n \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right) \cos k \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right) \quad \text{for } k = 0 \cdots N-1 \text{ and } n = 1 \cdots N-1$$

The matrix  $P$  is symmetric. This can be solved using standard numerical methods for any  $M$  and  $N$ , giving an approximation by  $N$  coefficients to  $M$  given points:

15

$$C = P^{-1}S$$

20

Where  $M \geq N$ , the equations above minimise the Root Mean Square (RMS) error between each measured point 100 on the actual boundary, and the closest corresponding point 101 of the fitted boundary 20. When the number of coefficients is 1, the fitted curve is a circle, and as the number of coefficients increases the RMS error generally decreases. It has been found in practice that  
5 good results in iris approximation can be obtained by using 5 coefficients.

In addition to modelling the boundary of the pupil (or, equivalently, the inner boundary of the iris), the present embodiment may also be used to model  
10 the shape of the outer boundary of the iris. Once the inner and outer boundaries have been determined, biometric identification can proceed in the normal way based on the characteristics of the iris image between the inner and outer boundaries.

The position of the fixed point A (Figure 1) is not of great importance, and although the approximate centre of the pupil is a convenient point to take other points are not excluded – even points which lie outside the boundary being fitted. Of course, if the fixed point lies outside the boundary, the resulting function will no longer be single valued in  $\theta$ , and a corresponding  
15 allowance for that will need to be made.  
20

If there is a large variability in distances, an improved fit may sometimes be achieved using a multi pass approach: carry out a first fit, exclude any outliers which are greater than a cut-off value, and repeat the calculation. The cut-off  
25 value may be fixed, or may be data dependent, for example a given number of standard deviations.

It will be understood that the method described above may find application in the fitting of a variety of other curves and/or boundaries, in



addition to fitting of the inner and outer iris boundaries. With a suitable choice of fixed reference point, the method could even be used to approximate open shapes such as simple curve fragments.

## CLAIMS

1. A method of approximating an iris boundary, comprising the steps of:
- (a) acquiring an image of an eye, including an iris boundary;
- 5 (b) noting a plurality of spaced boundary points on the boundary;
- (c) selecting a fixed reference point; and
- (d) generating an approximate boundary representation comprising a least squares approximation by a cosine transform series as a function of angle about said fixed point of the distance of said
- 10 boundary points from said fixed point.

2. A method as claimed in claim 1 in which the approximate boundary representation is generated by numerically solving the system of linear equations

15 
$$PC=S$$

or a mathematical equivalent thereof, for the unknown matrix C, where:

$$C = \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_N \end{bmatrix}$$

and  $S = \begin{bmatrix} S_0 \\ S_1 \\ \vdots \\ S_N \end{bmatrix}$  where  $S_k = \sum_{i=0}^M r_i \cos k \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right)$

20

and where P is an N by N matrix given by

$$P_{k,0} = \frac{1}{\sqrt{2}}$$

and 
$$P_{k,n} = \sum_{i=1}^M C_n \cos n \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right) \cos k \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right)$$

## 10

for  $k = 0 \cdots N - 1$  and  $n = 1 \cdots N - 1$

3. A method as claimed in claim 1 in which the boundary is the pupil/iris boundary.
4. A method as claimed in claim 1 in which the boundary is the outer iris boundary.
5. A method as claimed in claim 1 in which the boundary points are not all equally spaced.
6. A method as claimed in claim 1 in which the fixed reference point is in the approximate centre of the iris boundary.
- 10 7. A method as claimed in claim 1 in which, following step (d), any boundary points which lie more than a selected distance from the boundary representation are excluded, and step (d) is then repeated.
8. A method of approximating a two-dimensional shape, comprising the steps of:
  - (a) Noting a plurality of spaced measured points on the shape;
  - (b) selecting a fixed reference point; and
  - (c) generating an approximate shape representation comprising a least squares approximation by a cosine transform series as a function of angle about said fixed point of the distance of said measured points from said fixed point.
9. A method as claimed in claim 9 in which the approximate shape representation is generated by numerically solving the system of linear equations

$$PC=S$$

## 11

or a mathematical equivalent thereof, for the unknown matrix  $C$ , where:

$$C = \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_N \end{bmatrix}$$

$$\text{and } S = \begin{bmatrix} S_0 \\ S_1 \\ \vdots \\ S_N \end{bmatrix} \quad \text{where } S_k = \sum_{i=0}^M r_i \cos k \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right)$$

5 and where  $P$  is an  $N$  by  $N$  matrix given by

$$P_{k,0} = \frac{1}{\sqrt{2}}$$

$$\text{and } P_{k,n} = \sum_{i=1}^M C_n \cos n \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right) \cos k \left( \frac{\theta_i}{2} + \frac{\pi}{2N} \right)$$

$$\text{for } k = 0 \dots N-1 \text{ and } n = 1 \dots N-1$$

- 10 10. A method as claimed in claim 9 in which the measured points are not all equally spaced.
11. A method as claimed in claim 9 in which the fixed reference point is in the approximate centre of the shape.
12. A method as claimed in claim 9 in which, following step (c), any boundary points which lie more than a selected distance from the shape representation are excluded, and step (c) is then repeated.

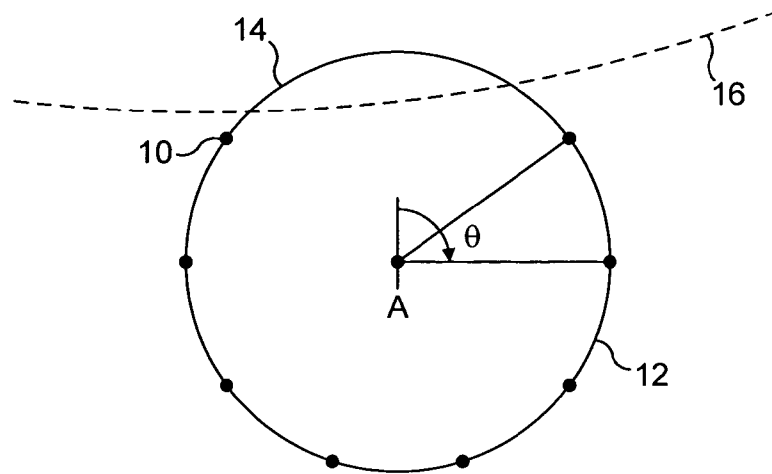


FIG. 1

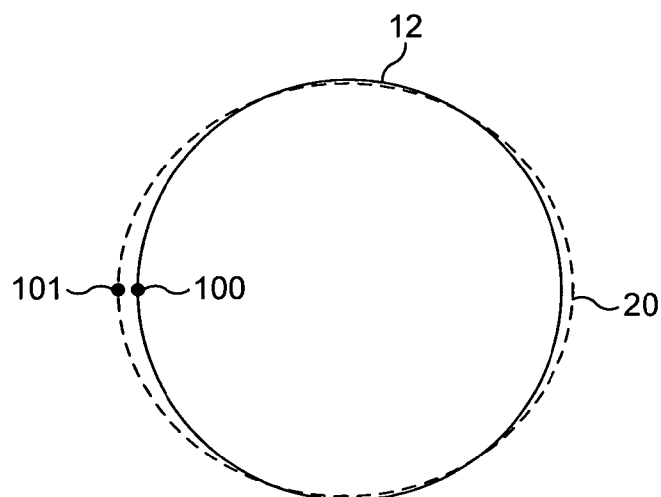


FIG. 2

# INTERNATIONAL SEARCH REPORT

International application No  
PCT/EP2008/050372

A. CLASSIFICATION OF SUBJECT MATTER  
INV. G06K9/48 G06K9/00

According to International Patent Classification (IPC) or to both national classification and IPC

## B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)  
G06K

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal

## C. DOCUMENTS CONSIDERED TO BE RELEVANT

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☒ Further documents are listed in the continuation of Box C.

☒ See patent family annex.

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Date of the actual completion of the international search

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## INTERNATIONAL SEARCH REPORT

International application No

PCT/EP2008/050372

## C(Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
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X	COSTA L DA F: "Estimating derivatives and curvature of open curves" 1 November 2002 (2002-11-01), PATTERN RECOGNITION, ELSEVIER, GB, PAGE(S) 2445 - 2451, XP004819285 ISSN: 0031-3203 Section 2. "Methodology", Section 3. "Experimental results"	8,10-12
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# INTERNATIONAL SEARCH REPORT

International application No

PCT/EP2008/050372

C(Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	<p>US 2006/147094 A1 (YOO WOONG-TUK [KR])  6 July 2006 (2006-07-06)  the whole document</p> <p>-----</p>	1-12



# INTERNATIONAL SEARCH REPORT

Information on patent family members

International application No

PCT/EP2008/050372

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US 2006147094	A1	06-07-2006	WO 2005024708 A1 17-03-2005
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