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Auray

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[54] **DEVICE FOR CALCULATING THE SUCCESSIVE DERIVATIVES OF A FUNCTION OF A VARIABLE**

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Oct. 8, 1973	France	73.35895
Nov. 21, 1973	France	73.41916

[52] **U.S. Cl.** **235/183;** 235/150.2; 235/197

[51] **Int. Cl.²** **G06G 7/18**

[58] **Field of Search** ... 235/150.2, 193, 197, 150.51, 235/183; 328/127; 307/229

[56] **References Cited**

UNITED STATES PATENTS

3,022,009	2/1962	Fogarty	235/197 X
3,142,822	7/1964	Martin	235/183 X
3,458,695	7/1969	Tait	235/193
3,590,229	6/1971	Humphreys	235/183 X
3,786,241	1/1974	Pukhov et al.	235/183

OTHER PUBLICATIONS

G. A. Korn and T. M. Korn, *Electronic Analog and Hybrid Computers*, McGraw-Hill Book Company, 1964, pp. 541–546.

Primary Examiner—Malcolm A. Morrison
Assistant Examiner—Jerry Smith

[57] **ABSTRACT**

This invention relates to a device for calculating the successive derivatives of a function of a variable. This device comprises, for calculating the *n* first derivatives, means for sensing in succession the function values for $(n + p)$ successive values of the variable. The sensed $(n + p)$ function values are memorized in a storage device, then simultaneously transferred into computing circuits, which determine the values of the *n* first derivatives according to known mathematical formulas.

5 Claims, 13 Drawing Figures

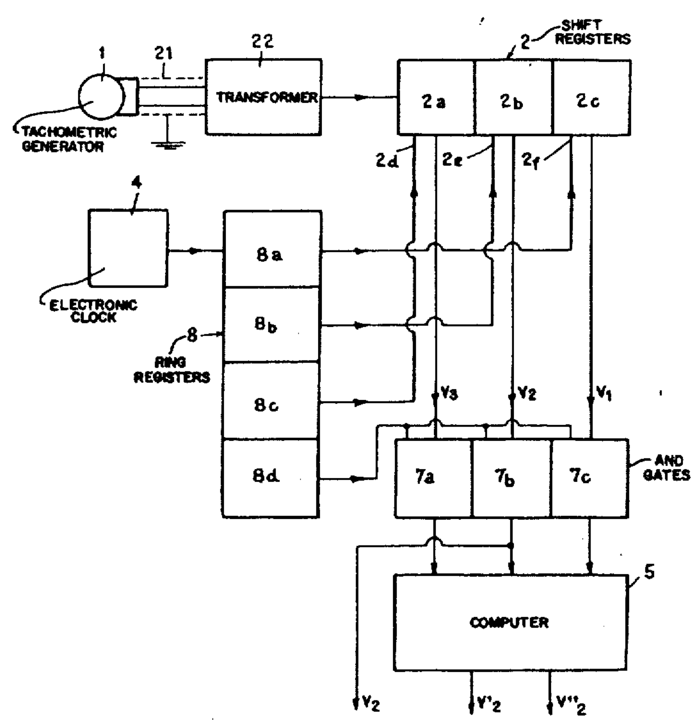
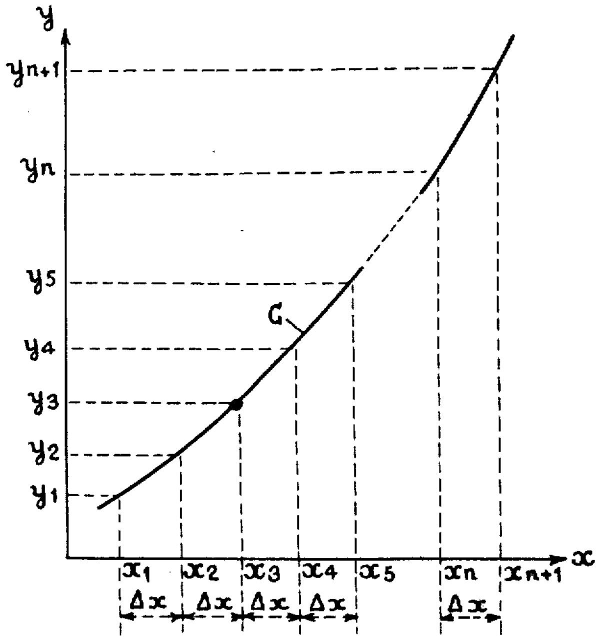


Fig.1.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yn+1
yn
y5
y4
y3
y2
y1
G
x
x1
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x3
x4
x5
xn
xn+1
Δx
Δx
Δx
Δx
Δx

Fig. 2.

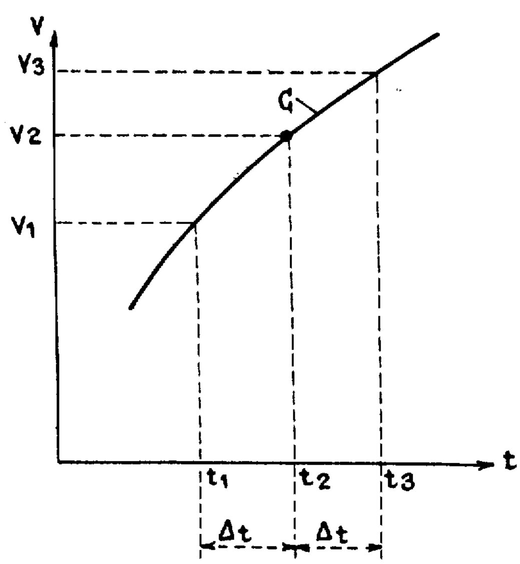

V
V3
V2
V1
G
t
t1
t2
t3
Δt
Δt

Fig.3.

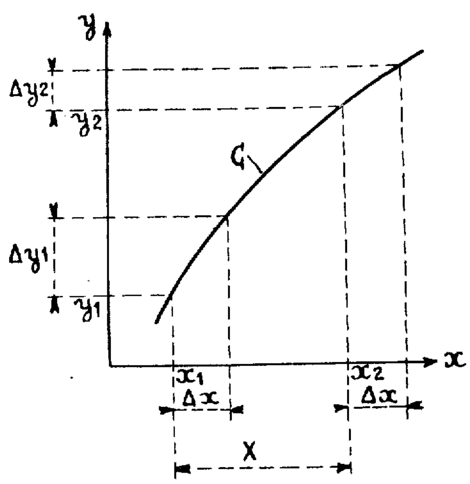


Fig.4.

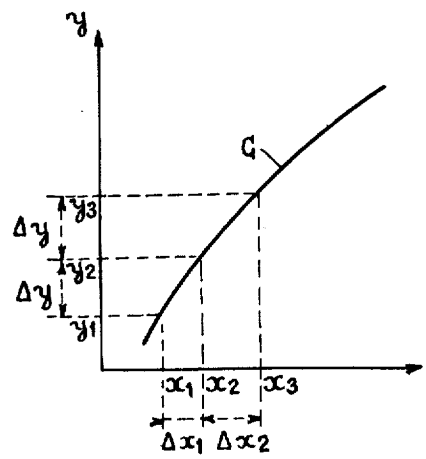


Fig.5.

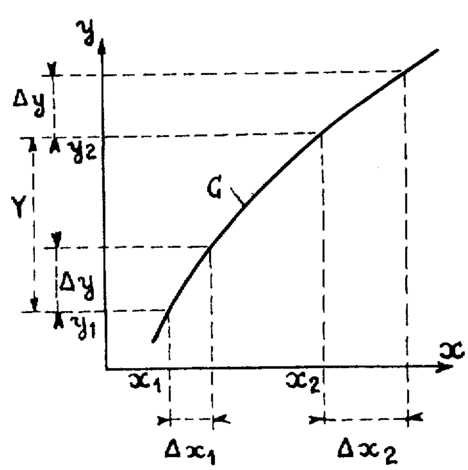
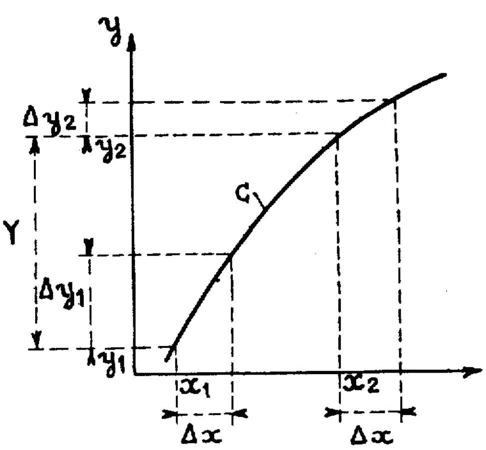


Fig.6.



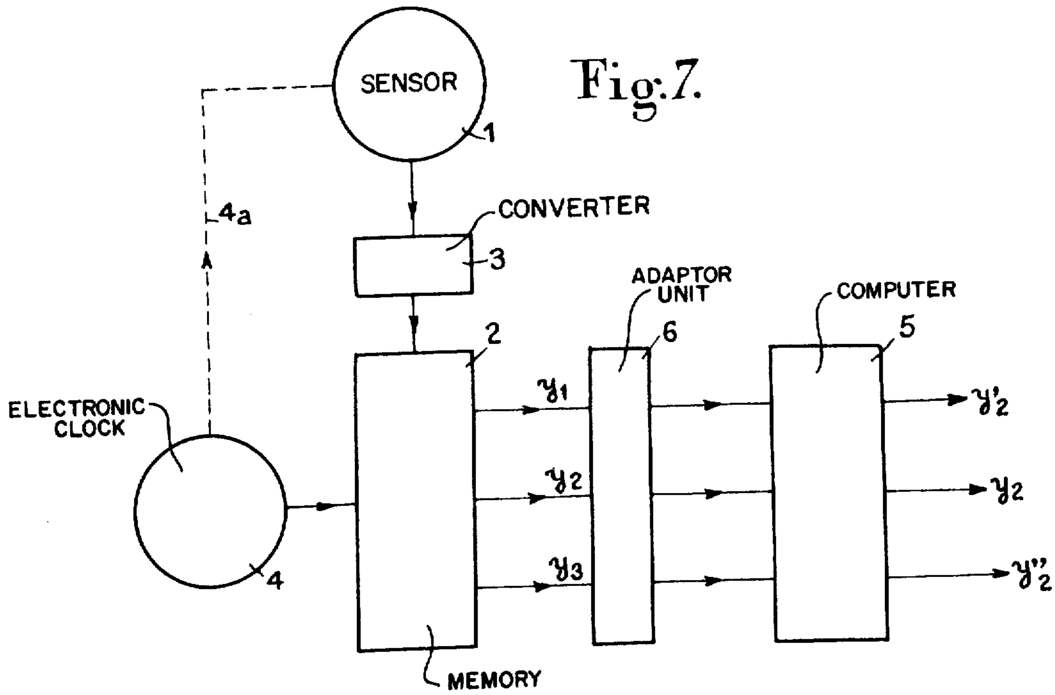

SENSOR
1
4a
CONVERTER
3
ADAPTOR UNIT
6
COMPUTER
5
2
ELECTRONIC CLOCK
4
MEMORY

Fig. 7.

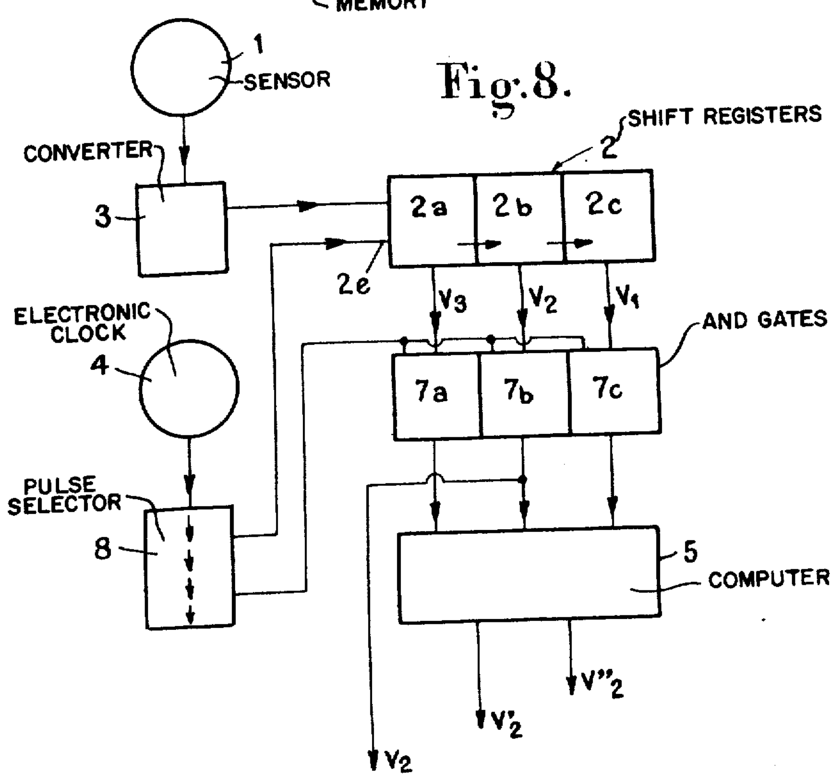

1
SENSOR
CONVERTER
3
SHIFT REGISTERS
2
2a
2b
2c
2e
V3
V2
V1
AND GATES
ELECTRONIC CLOCK
4
7a
7b
7c
PULSE SELECTOR
8
5
COMPUTER
V''2
V'2
V2

Fig. 8.

Fig. 9.

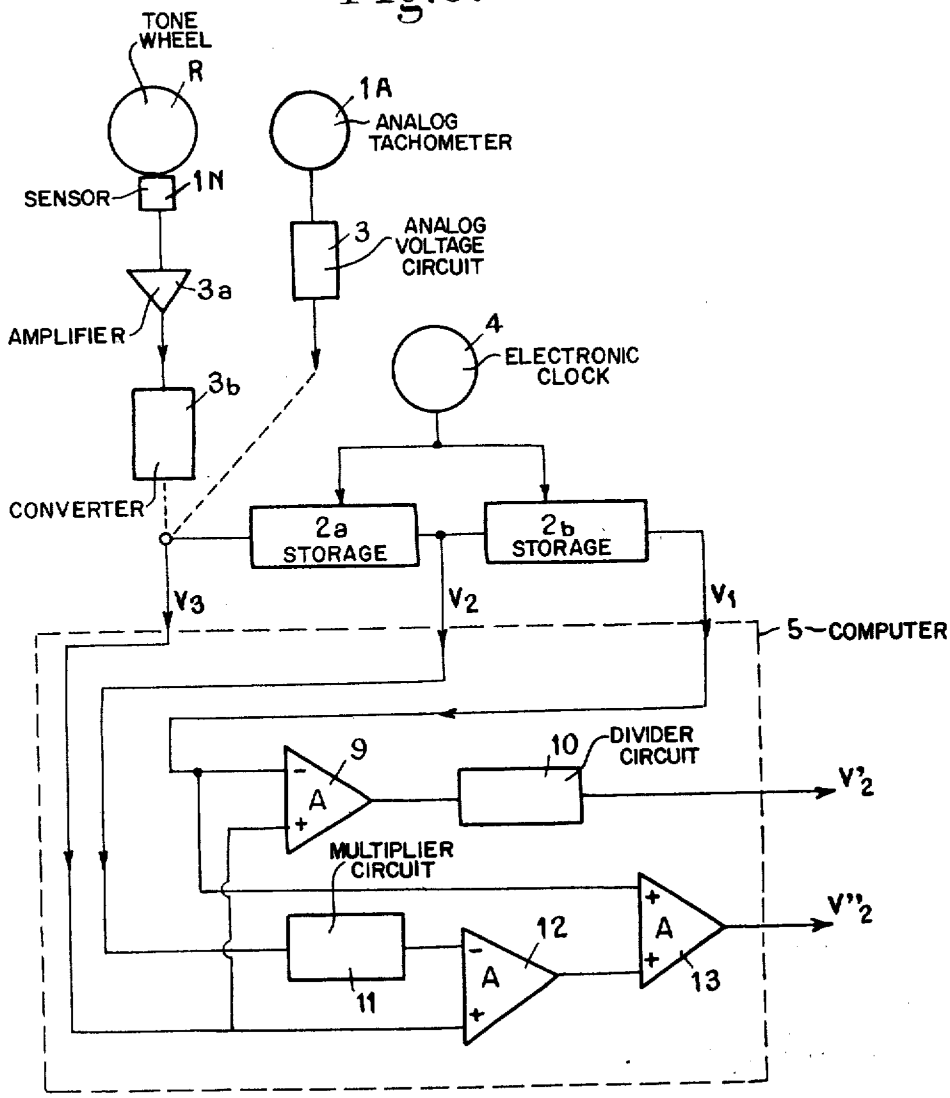

TONE WHEEL
R
1A
ANALOG TACHOMETER
SENSOR
1N
3
ANALOG VOLTAGE CIRCUIT
3a
AMPLIFIER
4
ELECTRONIC CLOCK
3b
CONVERTER
2a STORAGE
2b STORAGE
V3
V2
V1
5~COMPUTER
9
A
10
DIVIDER CIRCUIT
V'2
MULTIPLIER CIRCUIT
A
12
A
13
V''2
11
A

Fig.10.

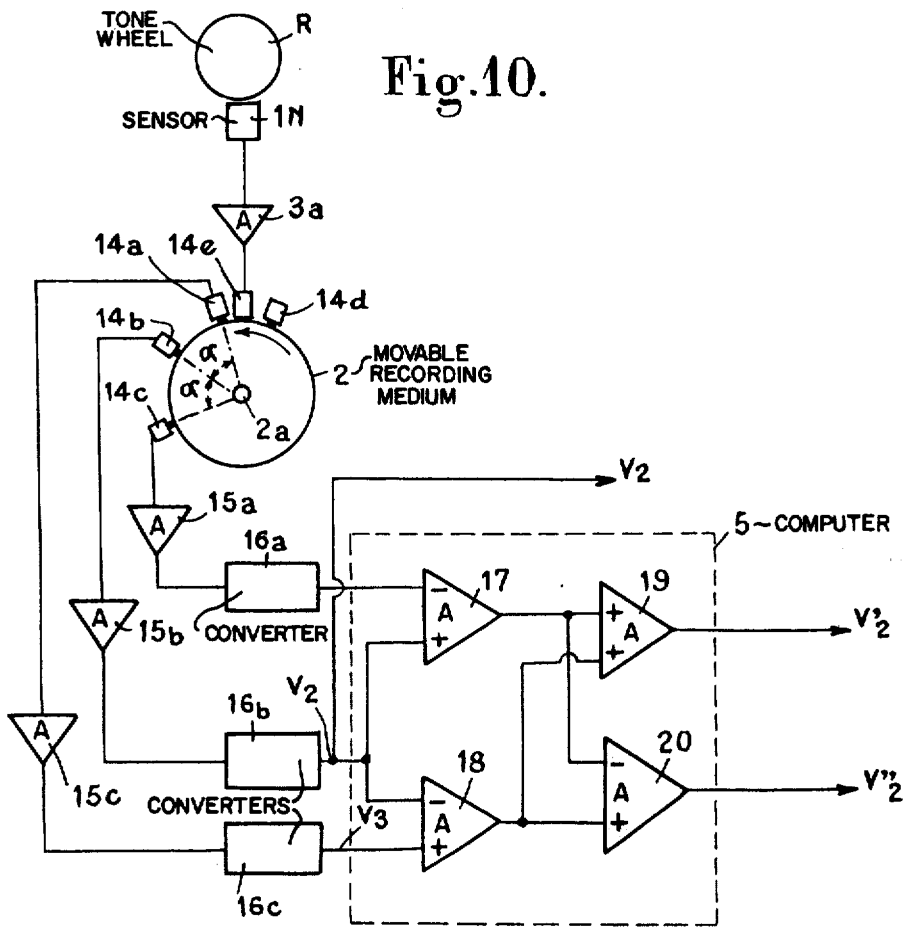

TONE WHEEL
R
SENSOR
1N
A
3a
14a
14e
14d
14b
α
α
2
MOVABLE RECORDING MEDIUM
14c
2a
V2
A
15a
5~COMPUTER
16a
17
19
A
15b
CONVERTER
V'2
16b
V2
20
A
18
15c
CONVERTERS
V3
V''2
16c

Fig. 11.

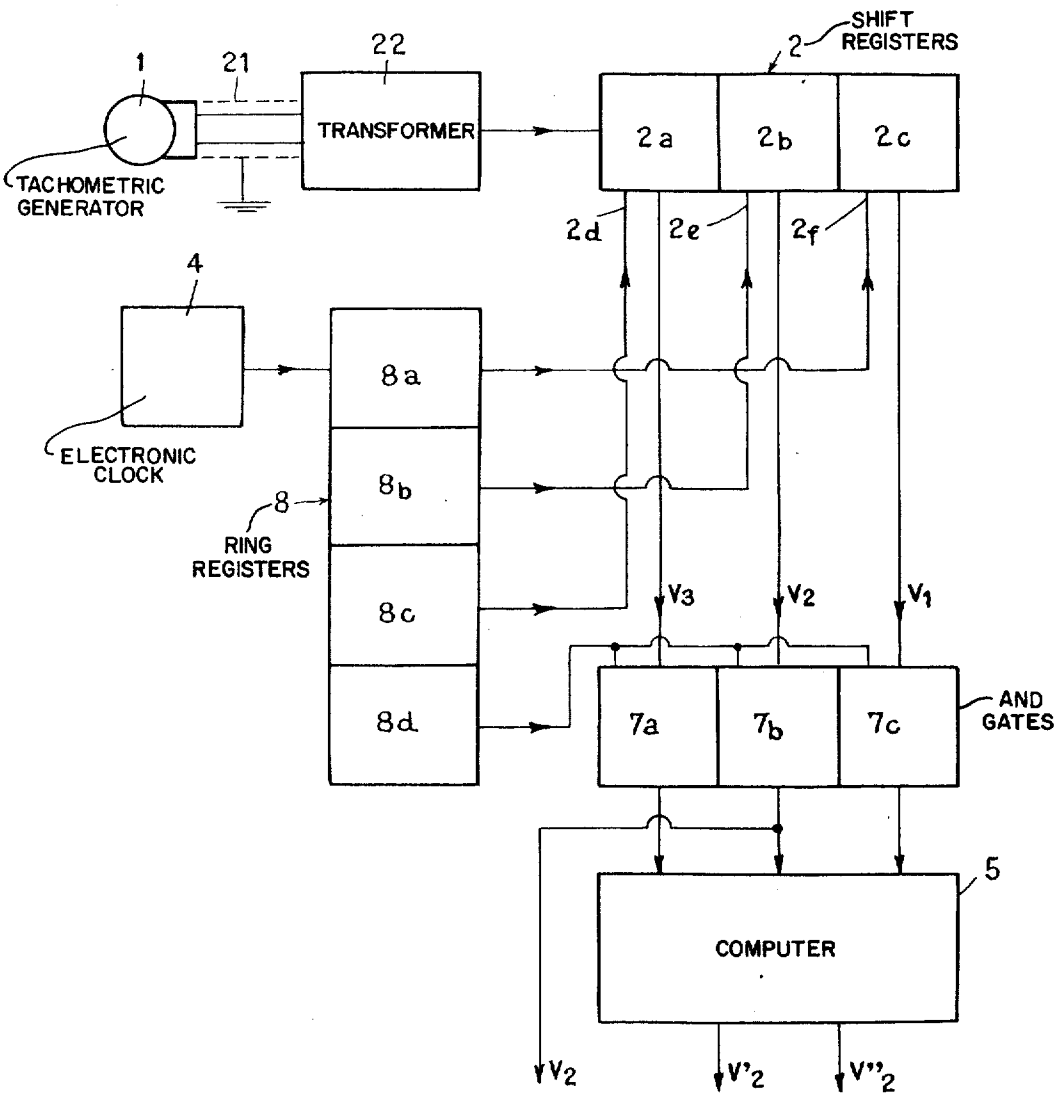
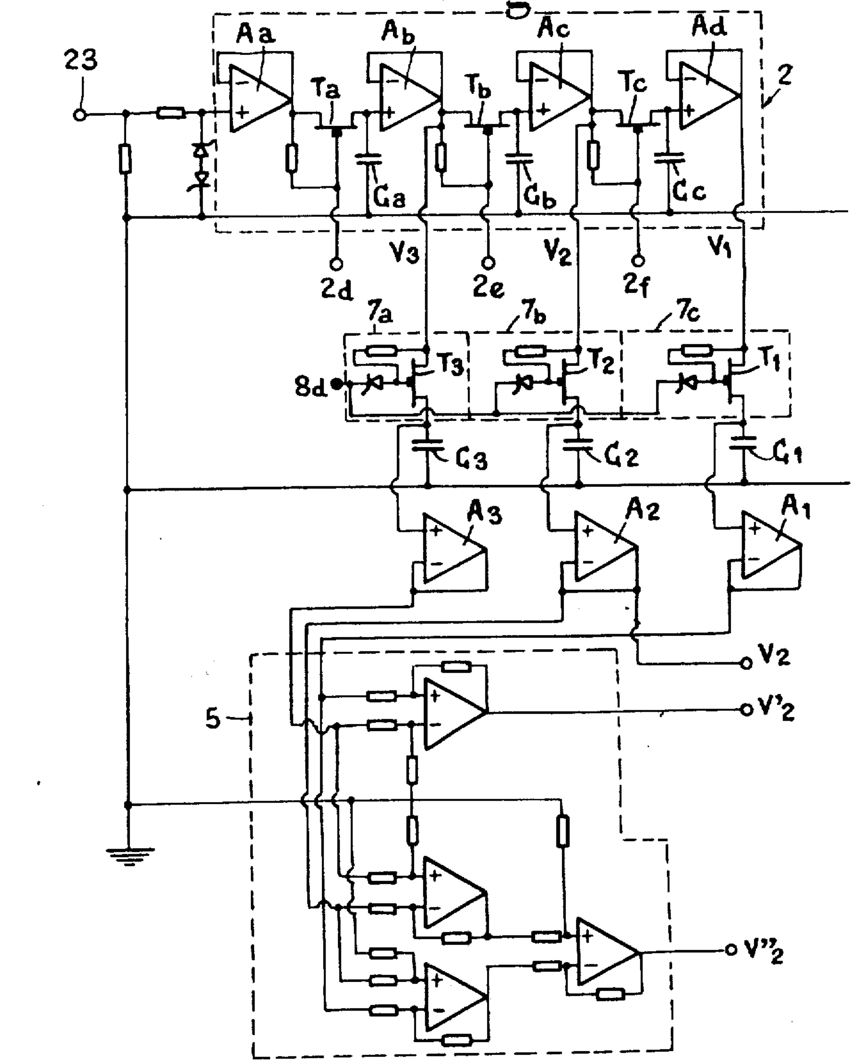

1
21
22
TRANSFORMER
TACHOMETRIC GENERATOR
2
SHIFT REGISTERS
2a
2b
2c
2d
2e
2f
4
ELECTRONIC CLOCK
8a
8b
8c
8d
8
RING REGISTERS
V3
V2
V1
7a
7b
7c
AND GATES
5
COMPUTER
V2
V'2
V''2

Fig. 12.


Aa
Ab
Ac
Ad
23
Ta
Tb
Tc
2
Ca
Cb
Cc
V3
V2
V1
2d
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2f
7a
7b
7c
T3
T2
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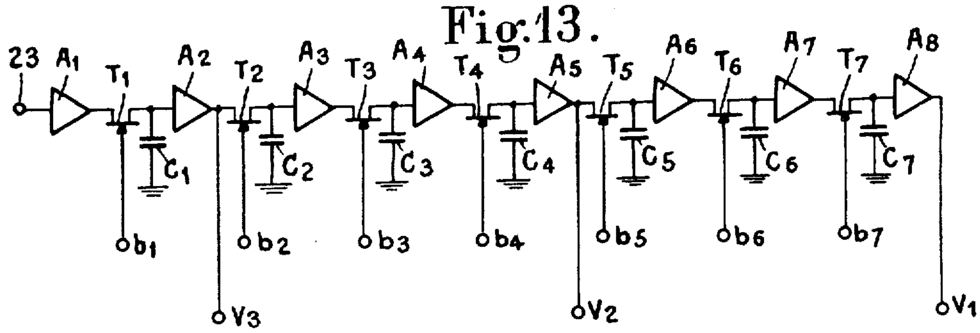
8d
C3
C2
C1
A3
A2
A1
V2
V'2
5
V''2

Fig. 13.


23
A1
T1
A2
T2
A3
T3
A4
T4
A5
T5
A6
T6
A7
T7
A8
C1
C2
C3
C4
C5
C6
C7
b1
b2
b3
b4
b5
b6
b7
V3
V2
V1

DEVICE FOR CALCULATING THE SUCCESSIVE DERIVATIVES OF A FUNCTION OF A VARIABLE

FIELD OF THE INVENTION

This invention relates to a device for calculating the successive derivatives of a function of a variable, specially for regulating or controlling said function, for example the velocity of a moving body.

It is known that, for regulating a function of a variable, or for controlling the same according to a predetermined law, for example for controlling the velocity of a moving body according to a predetermined programme, it is necessary to calculate at least one, and generally a plurality, of the successive derivatives of said function, for example the velocity of the moving body. Thus, it is essential to know the first derivative of velocity for taking due account of the vis inertia; if the moving body is a transport vehicle, the knowledge of the second derivative of its velocity, which is bound up with the jolts to which the vehicle is subjected, is necessary when it is desired to improve the comfort of the vehicle passengers, and the knowledge of the third derivative provides information as to the shocks applied to the vehicle.

Up to now, for the above-mentioned applications, and specially for controlling the running conditions of a vehicle, the velocity was measured by using a tachometer of any suitable type, such as an alternator, a tachometric dynamo or generator, a phonic or tone wheel, or the like, and the acceleration of the vehicle was measured by means of a suitable sensor such as an inertia accelerometer; however, as accelerometers of this type cannot operate satisfactorily under transient operating conditions, and they are also responsive to gravity, attempts were made to derive the vehicle acceleration from its measured velocity by using a differentiating electronic circuit of the analog type; unfortunately, this last-mentioned device is not capable of taking due account of the velocity variations of the vehicle or movable body during a time long enough to permit of forecasting the future running conditions of the vehicle or moving body with a sufficient degree of precision.

SUMMARY OF THE INVENTION

A first object of this invention is to provide a device for calculating successive derivatives of a function of a variable, for example the velocity of a moving body, while taking due account of the variations of said function, for example the velocity of the moving body, during a preceding time period long enough to permit of forecasting the future variations of the function, for example the velocity of the moving body, with a very high degree of precision; this precision may be increased very easily inasmuch as the device according to this invention permits of calculating without any undue complication a great number of the successive derivatives of said function.

A second object of this invention is to provide a device for calculating the successive derivatives of a function of a variable, specially for regulating or controlling said function, for example the velocity of a moving body, said device comprising, for calculating the n first derivatives, means for sensing in succession the values of the function for $(n + p)$ uccessive values of the variable, n and p being integers, storage means for memorizing the sensed $(n + p)$ values of said function, and computing circuits fed from said storage means and adapted to determine the values of the n first derivatives according to known mathematical formulas.

In a preferred embodiment of the device according to this invention, the computing circuits are adapted to determine the value of the nth derivative of the function y of the variable x according to the formula:

$$\frac{d^n y}{dx^n} = \frac{1}{k \Delta x^n} \sum_{i=0}^{i=n} a_i \, y(x_1 + i \Delta x),$$

wherein k is a constant factor, Δx is a predetermined algebraic increment of said variable x, and a_i designates a plurality of mathematically calculable coefficients, each of which depends on the respective value of the integer index i.

This preferred embodiment of the device according to this invention is particularly advantageous in that it permits of calculating the n first derivatives of the function y through easy mathematic operations on the values of said function y which are measured for stepped values of the variable x, for example at regularly spaced time intervals if y is a function of time; in fact, as k and Δx, in the above mathematical formula, are constant, the only mathematical operations required are weighting by coefficients a_i the measured values of function y, as well as adding and subtracting the weighted values, which can be performed both in the analog technique and in the digital technique, by using plain, rugged and reliable electronic circuits, which have a high degree of precision and are free of the inconveniences characterising the differentiating electronic circuits previously used to this end.

A typical embodiment of the device according to this invention is intended more particularly for use in a system for controlling the motion of a movable body, specially a vehicle; this special embodiment of the invention comprises, for calculating the acceleration dV/dt of the moving body at time t and its variation d^2V/dt^2, computing circuits which are adapted for determining the values of

$$\frac{dV}{dt} = \frac{V(t + \Delta t) - V(t - \Delta t)}{2 \Delta t} \text{; and}$$

$$\frac{d^2V}{dt^2} = \frac{V(t + \Delta t) - 2V(t) + V(t - \Delta t)}{\Delta t^2}$$

Δt being a predetermined time period.

A particularly advantageous embodiment of this invention, permitting of calculating the successive derivatives of a time function at regularly spaced times, for example to control the running conditions of a moving body or of a vehicle, comprises a memory of the shift or displacement type, having at least one input connected to the output of the sensor, and a plurality of outputs stepped in the direction of shift or displacement of said memory, said outputs being connected to the corresponding inputs of the computing circuits. In this specific embodiment the memory may consist of a shift register comprising at least one shift control input connected to an electronic clock; said memory may also consist of a delay line or network, or a movable recording medium such as a magnetic drum, disc or tape, a plurality of output sensors being disposed stepwise along said delay line or said recording medium.

BRIEF DESCRIPTION OF THE DRAWINGS

FIGS. 1 to 6 are diagrams for explaining the mode of operation of various embodiments of the device according to this invention.

FIG. 7 is the general block-diagram of a typical embodiment of the device according to this invention, intended for calculating the first two derivatives of a time function.

FIG. 8 is the block diagram of a specific embodiment of the device illustrated in FIG. 7, wherein the memory consists essentially of a shift register.

FIG. 9 is the block diagram of another specific embodiment of the device illustrated in FIG. 7, provided with static storages of the analbg type;

FIG. 10 is the block diagram of another specific embodiment of the device illustrated in FIG. 7, wherein the memory consists essentially of a movable recording medium.

FIG. 11 is another block-diagram showing a modified embodiment of the device illustrated in FIG. 8;

FIG. 12 is the detailed wiring diagram of the device illustrated in FIG. 11, and

FIG. 13 is a part of the wiring diagram of a modified embodiment of the device shown in FIG. 12.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

In FIG. 1, the reference letter C designates the curve of variation of a function y of the variable x, and y_1, y_2 ... y_n, y_{n+1} designate the values of said function y for different values x_1, x_2 ....x_n, x_{n+1} of said variable, which are regularly stepped, that is, separated by constant intervals Δx. With this notation, the derivative of order n of function y may be calculated by using the following algebraic formula:

$$\frac{d^n y}{dx^n} = \frac{1}{k\,\Delta x^n}[a_0 y_1 + a_1 y_2 + ... + a_n y_{n+1}] \quad (1)$$

wherein k is a constant factor, a_0, a_1 .....a_n are mathematically calculable coefficients, depending each on the value of its integer index. The above formula (1) may be expressed in a more condensed form:

$$\frac{d^n y}{dx^n} = \frac{1}{k\,\Delta x^n} \sum_{i=0}^{i=n} a_i\, y\,(x_1 + i\,\Delta x) \quad (2)$$

wherein i is a zero or integer index at the most equal to n.

Both formulas (1) and (2) hereinabove give the value of the derivative of order n of the function y for the middle value of the variable x, if n is even, that is, for example for $x = x_3$ if $n = 4$, or for the value of the variable x nearest of this middle value, if n is odd, that is, for example for $x = x_2$ if $n = 3$. The following Table gives the values of coefficients a_i in formula (2) hereinabove for the first four integer values of n:

TABLE 1

n \ a_i	a_0	a_1	a_2	a_3	a_4
1	−1	+1	0	0	0
2	+1	−2	+1	0	0
3	−1	+3	−3	+1	0
4	+1	−4	+6	−4	+1

However, this invention should not be construed as being limited by the application of the above mathematical formula (2); it can be applied by using other equivalent mathematical formulas, for example the following one, giving the value of the derivative of order n of function y of the variable x, for the value $x = x_1$ of this variable, with the same notation as in formula (2):

$$\frac{d^n y}{dx^n} = \frac{1}{k\,\Delta x^{n+1}} \sum_{i=0}^{i=n+l} a_i y(x_1 + i\,\Delta x) \quad (3)$$

With this formula, the value of the derivative of order n is derived from the values of the function y for (n = 2) values of the variable x equally spaced from each other by a distance or value Δx. The following Table 2 illustrates the values of the coefficients a_i of the above formula (3) for the first three derivatives:

TABLE 2

n \ a_i	a_0	a_1	a_2	a_3	a_4
1	−3	+4	−1	0	0
2	+2	−5	+4	−1	0
3	−5	+18	−24	+14	−3

In the diagram of FIG. 2, C designates the curve of variation of the velocity V of a movable body as a function of time t. The first derivative of V in relation to t, i.e. the acceleration of the vehicle, as well as its second derivative may be calculated according to the present invention from values V_1, V_2 and V_3 of the velocity V at three successive instants, t_1, t_2 and t_3 separated by equal intervals Δt, through the following relationships:

$$\frac{dV}{dt} = \frac{V_3 - V_1}{2\Delta t} = \frac{V(t_2 + \Delta t) - V(t_2 - \Delta t)}{2\Delta t}$$

$$\frac{d^2V}{dt^2} = \frac{V_3 - 2V_2 + V_1}{\Delta t^2} = \frac{V(t_2 + \Delta t) - 2V(t_2) + V(t_2 - \Delta t)}{\Delta t^2} \quad (4)$$

giving the values of these two derivatives at the middle or intermediate instant t_2.

The diagrams of FIGS. 3 to 6 inclusive illustrate other methods of calculating the successive derivatives of a function y of a variable x, which may be the time t; although the embodiments of the device of this invention which will be described presently by way of examples, utilize essentially the simplest calculating method as exemplified in FIGS. 1 and 2, the present invention also covers all the embodiments utilizing the calculating methods to be disclosed hereinafter.

In the case illustrated in FIG. 3, the values y_1, y_2 ... etc. of the function y, corresponding to values x_1, x_2 ... etc., respectively, of the variable, which are regularly spaced by a distance X, are considered; in this diagram the symbols Δy_1, Δy_2 ...designate the algebraic increments of the function y which correspond to equal algebraic increments, Δx, of the variable x, respectively, from these values x_1, x_2 ... etc.; the second derivative of function y, for instance, is given by the formula:

$$\frac{d^2y}{dx^2} = \frac{\Delta y_2 - \Delta y_1}{x \cdot \Delta x} \qquad (5)$$

If for instance y designates the velocity of a rotating member, this calculation method is tantamount to considering the velocity variations at regularly spaced instants, for example spaced by the reference time period T of the rotational movement, during equal time periods Δt considerably shorter than T, which may be obtained by means of tachometric sensor operating at regular intervals T and each time during a predetermined time period Δt, considerably shorter than T.

According to the diagram of FIG. **4**, the second derivative for example of said function y may be deducted from the values x_1, x_2 and x_3 of the variable x for which said function assumes the respective values, y_1, y_2 and y_3 regularly spaced by the increment Δy, according to the formula:

$$\frac{d^2y}{dx^2} = \frac{\Delta y}{2\Delta x_1 \cdot \Delta x_2} \qquad (6)$$

wherein $\Delta x_1 = x_2 - x_1$ and $\Delta x_2 = x_3 - x_2$. In the case already contemplated hereinabove of the velocity of a rotating body, this mode of calculation is tantamount to considering the successive time instants t_1, t_2 and t_3 between which the velocity of the moving body undergoes equal variations.

The method of calculation illustrated in the diagram of FIG. **5** corresponds to that illustrated in FIG. **3**, with the permutation of y and x, which, in the example of the rotational movement of a body or member, is like starting a stop-watch each time the tachometer has detected a same variation Y of the velocity of said movable body, and then stopping said stop-watch when said tachometer has detected a same variation ΔY of the velocity of said movable body, which is considerably lower than Y. With this calculation method:

$$\frac{d^2y}{dx^2} = \frac{\overline{\Delta y}^2}{2Y} \left(\frac{1}{\overline{\Delta x_2}^2} - \frac{1}{\overline{\Delta x_1}^2} \right) \qquad (7)$$

The calculation method exemplified in FIG. **6** consists in considering values y_1, y_2...etc. of the function y, equally spaced from each other by a distance Y, and the variations Δy_1, Δy_2 ...etc. of y, from its values y_1, y_2 ... etc., corresponding to equal variations Δx of the variable x; under these conditions, the second derivative, for example, may be calculated by applying the formula:

$$\frac{d^2y}{dx^2} = \frac{1}{2Y \cdot \overline{\Delta x}^2} (\overline{\Delta y_2}^2 - \overline{\Delta y_1}^2); \qquad (8)$$

in the case contemplated hereinabove, which relates to a rotating body, this calculation method consists in recording the velocity variation Δy detected by the tachometric sensor during equal time periods Δt, each time it has previously detected a same variation Y of said velocity, which is considerably greater than Δy.

The general block diagram of the device according to this invention, which is illustrated in FIG. **7**, comprises a sensor **1** of a type consistent with the function of the time y (t) to be measured; if the function to be measured is the velocity of a moving body, this sensor will consist of a tachometric sensor pertaining to one of the types mentioned hereinabove, or of another known type; according to the specific calculation method selected among these mentioned hereinabove and utilized for all practical purpose, the sensor **1** must operate either permanently (as in the case illustrated in FIGS. **1**, **2** and **4**), or intermittently (as illustrated in FIG. **3**), each time during a predetermined time period. In all cases, the information produced by this sensor, whether in the analog or digital form, is transmitted to the input of a storage or memory **2**, which may also be of the analog or digital type; if the specific nature (analog or digital) of the information produced in said sensor **1** is not consistent with that of said storage or memory **2**, a suitable convertor **3** must be inserted. The transfer of information to or from the storage **2** is controlled by means of a suitable electronic clock **4** adapted to deliver equally spaced pulses to said storage **2**, according to a cycle Δt. The dash line **4***a* designates these clock pulses which may also be transmitted to the sensor **1** in case the latter operates by periodic sampling. The storage **2** comprises output lines equal in number to the successive values of the variable x, which are to be used simultaneously for calculating the n first derivatives of the function y (t), with the assistance of one of the above-given formulas or equivalent mathematical formulas; these output lines of storage **2** will thus transmit simultaneously for instance the values y_1, y_2 and y_3 (FIGS. **1** and **2**) to the computer unit **5**, of which the output lines then deliver simultaneously the values y_2, y'_2 and y''_2 respectively of the function y of time t, and of its first and second derivatives at the intermediate time t_2. The construction of said computer unit **5** depends of course on the mathematical formulas utilised for these calculation; a unit **6** may further be inserted between the outputs of storage **2** and the computer unit **5** for adapting the output characteristics of one unit to the input characteristics of the other unit (shaping, analog-to-digital conversion or vice-versa, etc..).

The mode of operation of the device illustrated in FIG. **7** is obvious, insofar as the successive values y_1, y_2 ....y_n (FIG. **1**) of function y at regular time intervals Δt are fed to the storage **2** under the control of clock **4** and then returned by groups of three (such as y_1, y_2, y_3) to the computer unit **5**, also under the control of clock **4**. In other words, if at an arbitrary instant T the group of values y_1, y_2, y_3 is transferred to the computer unit **5**, the group of values y_2, y_3, y_4 is transferred thereto at a later instant T + Δt, the group of values y_3, y_4, y_5 is transferred at a still later instant T + $2\Delta t$, and so forth.

In the diagram of FIG. **8** the same reference numeral as those of FIG. **7** are used for designating similar functional blocks. In this embodiment the storage **2** comprises essentially a shift register having three stages **2***a*, **2***b* and **2***c*, of which the shift control input **2***e* receives the pulses generated by the electronic clock **4**. Logic AND gates **7***a*, **7***b* and **7***c* have first inputs connected directly to the respective outputs of register stages **2***a*, **2***b* and **2***c*, and outputs connected to three inputs of the computer unit **5**: furthermore, a pulse selector **8**, such as a shift register or a delay line, is inserted between the output of electronic clock **4** and the second inputs of

AND gates 7a to 7c, so that said second inputs of AND gates 7a to 7c cannot receive a clock pulse before the shift register 2 has been filled completely. The pulse selector 8 may consist for instance of a multiple-tap delay line, of which the first tap, nearest to the electronic clock is connected to the shift control input 2e of shift register 2, the last tap being connected to the second inputs of AND gates 7a to 7c.

The device illustrated in FIG. 8 operates as follows: assuming that the shift register 2 is initially empty, the value V_1 of the velocity at the instant t_1 is recorded in its first stage 2a when its shift control input 2e receives the first clock pulse; the second clock pulse will cause V_1 to be transferred from stage 2a to stage 2b of said shift register 2 and, at the same time, the velocity value V_2 at time t_2 of the second clock pulse to be recorded in said stage 2a; when the third clock pulse takes place, the successive velocity values V_3, V_2 and V_1 are recorded in stages 2a, 2b and 2c of shift register 2, respectively, so that said third pulse, or one of the following pulses, transmitted to the second input of each AND gate 7a to 7c will open these gates and cause them to transmit at the same time the values V_3, V_2 and V_1 to the computer unit 5; the outputs of this unit 5 display the values V'_2 and V''_2 (at instant t_2 of the second clock pulse) of the first and second derivatives of velocity V_2 at the same instant, whereas V_2 may be taken directly from the output of AND gate 7b.

The embodiment illustrated in FIG. 9 is specially adapted for determining the first and second derivatives of the velocity of a rotary member, for instance a shaft; it may comprise either a digital tachometer such as a phonic or tone wheel R, and a sensor 1N generating an electric voltage having a frequency proportional to the velocity of rotation, which is amplified by an amplifier 3a and then converted into an analog voltage in a converter 3b, or an analog tachometer 1A, generating directly an analog voltage, shaped by a suitable circuit 3; the analog voltage generated by 3 or 3b is fed permanently to the input of a first static analog storage 2a having its output connected to the input of another static analog storage 2b; the inputs and outputs of these storages 2a and 2b are controlled simultaneously by means of clock pulses 4 at a cycle Δt, so that at each clock pulse the analog voltage available at the input of storage 2a be transferred thereto, and the information contained in said storage 2a is transferred to storage 2b having its content transmitted to its output. With the notation of FIG. 2, the three successive values V_1, V_2 and V_3 of the rotational velocity of the shaft are thus transmitted simultaneously at the instant t_3 of the third clock pulse to the corresponding inputs of computer unit 5. This unit 5 comprises a first operational amplifier 9 receiving at its positive and negative inputs the values V_3 and V_1, respectively, the output of said amplifier 9 being connected to a suitable divider circuit 10, so that the output of this divider circuit 10 will display a magnitude:

$$V'_2 = \frac{V_3 - V_1}{2\Delta t}, \qquad (9)$$

which is therefore equal to the first derivative of velocity V according to formula (4) hereinabove. The computer unit 5 further comprises a circuit 11 adapted to multiply by the factor two, which receives the value V_2 at its input and has its output connected to the negative input of a second operational amplifier 12 receiving the value V_3 at its positive input; the output of this operational amplifier 12 is connected to the first input of a third operational amplifier 13 receiving at its other input the value V_1, so that the following magnitude:

$$V''_2 = \frac{V_3 - 2\,V_2 + V_1}{\Delta t^2} \qquad (10)$$

appears at the output of operational amplifier 13, this magnitude corresponding to the second derivative of the velocity of the rotating shaft, according to formula (4) hereinabove.

The modified embodiment illustrated in FIG. 10 is also intended for controlling the movement of rotation of a shaft and comprises a phonic or tone wheel R wedged to this shaft and a sensor 1N adapted to transmit, via an amplifier 3a, a variable voltage having a frequency proportional to the rotational velocity of said shaft to a recording head 14e, disposed in close vicinity of a movable recording medium 2, such as a magnetic drum rotating at a uniform speed about a shaft 2a; this recording medium may also consist of a magnetic cylinder or disc or a magnetic tape, without entailing substantial modifications in the structure of the device. Three reading heads 14a, 14b and 14c are disposed close to the magnetic drum 2 downstream of the recording head 14e in the direction of rotation of said drum 2, so as to provide equal angular spacing α; an erasing head 14d is disposed close to the magnetic drum 2 between said heads 14e and 14c, preferably nearer to the head 14e than to the head 14c. The reading heads 14a to 14c are connected to the corresponding inputs of amplifiers 15a, 15b and 15c, respectively, having their corresponding outputs connected in turn to the relevant inputs of computer unit 5 via analog-to-digital converters 16a, 16b and 16c, respectively.

Considering the uniform velocity of rotation of the magnetic drum 2, the frequencies of the electric voltage induced in the reading heads, 14a, 14b and 14c correspond to those of the electric voltages transmitted to the recording head 14e, respectively, at three successive instants separated by a same interval Δt corresponding to the angular distance α between the reading heads; consequently, with the notation of FIG. 2, the analog voltages transmitted via converters 16a, 16b and 16c to the inputs of computer unit 5 correspond to the three values V_3, V_2 and V_1, respectively, of the rotational velocity of said shaft.

In the embodiment illustrated in FIG. 10, the computer unit 5 comprises a first and a second operational amplifiers 17 and 18, receiving the values V_1, V_2 and V_2, V_3, respectively, and having their outputs connected to the first and second inputs of third and fourth operational amplifiers 19 and 20, respectively. The outputs of these operational amplifiers 19 and 20 will thus produce two magnitudes:

$$V'_2 = (V_2 - V_1) + (V_3 - V_2) = V_3 - V_1 \qquad (11)$$

$$V''_2 = (V_3 - V_2) - (V_2 - V_1) = V_3 - 2\,V_2 + V_1 \qquad (12)$$

proportional to the first and second derivatives respectively, of the rotational velocity V of said shaft, as shown by the above formulas (9) and (10).

Of course, the embodiments of the computer unit 5 which are illustrated in FIGS. 9 and 10 may be permuted with each other; however, if it is contemplated to feed the computer unit illustrated in FIG. 10 by means of two static analog storages such as those shown at 2a and 2b in FIG. 9, it would also be possible, without departing from the basic scope of this invention, to cause the difference $V_2 - V_1$ to be formed by the differential amplifier **17** (FIG. **10**) from the very second clock pulse, and to connect the output of this operative amplifier **17** to the input of the second static analog storage *2b*; with this assembly, the magnitudes $V_2 - V_1$ and $V_3 - V_2$ are obtained, at the third clock pulse, from the output of the second storage *2b* and from the output of the operational amplifier **17**, respectively, and these amagnitudes may be combined by another pair of operative amplifiers such as **19** and **20** (FIG. **10**) to deliver the magnitudes V'_2 and V''_2 according to formulas (11) and (12) hereinabove; it will be seen that with this particular embodiment the fourth operative amplifier **18** (FIG. **10**) may be dispensed with.

The above-described embodiments of the device according to the present invention are liable to many changes and modifications without departing from the scope of the invention; thus, the movable recording medium **2** (FIG. **10**) may be replaced by a delay line along which a plurality of output sensors, substituted for the reading heads **14***a*, **14***b* and **14***c* are disposed at regularly spaced intervals.

The device shown in block diagram in FIG. **11** differs from the device of FIG. **8** only by the following points:

The sensor **1** is a tachometric generator or dynamo generating an electric voltage directly proportional to the velocity of the rotating member; this electric voltage is fed via a bifilar shielded cable **21** to a differential transformer **22** for eliminating possible strays, the output of said transformer **22** being connected to the input of the first stage *2a* of the shift register **2**. Each stage *2a*, *2b*, *2c* of this register comprise a control input for transferring to said stage the information contained either in the preceding stage or at the output of the differential transformer **22**, for the first stage *2a*; these control inputs *2d*, *2e* and *2f* are connected to the outputs of the first three stages *8a*, *8b* and *8c*, respectively, of a ring register **8**, comprising a fourth stage *8d*; the input of the first stage *8a* of said ring register **8** is connected to the outputs of an electronic clock **4**. The first inputs of the three AND gates *7a*, *7b*, and *7 c* are connected to the outputs of stages *2a*, *2b* and *2c*, respectively, of register **2**, and their second inputs are connected jointly to the output of the fourth stage *8d* of said ring register **8**; the outputs of the three AND gates *7a*, *7b* and *7c* are connected to corresponding inputs of the computer unit **5** determining the values V'_2 and V''_2, as already explained hereinabove with reference to FIG. **8**.

Each time the clock **4** delivers a pulse, the first stage *8a* of ring register **8** transmits this pulse to the control input *2f* of the last stage of shift register **2**, thus causing the information contained in the preceding stage *2b* to be transferred to this last stage *2c*. When the next clock pulse is delivered, the output of stage *8b* of ring register **8** controls the transfer of the information contained in the first stage *2a* to the second stage *2b* of register **2**. At the next clock pulse the value of the voltage delivered by the differential transformer **22** is memorized in the first stage *2a* of register **2** due to the pulse transmitted to the control input *2d*. Finally, when the fourth puls s from the clock is transmitted to the second inputs of AND gates *7a* to *7c*, the information contained in the three stages *2a*, *2b* and *2c* of register **2**, which correspond to the values of the rotational velocities V_3, V_2 and V_1 at three successive instants, respectively (FIG. 2), are transmitted to the computer unit **5**, possibly through the medium of corresponding storages.

More generally, to calculate the first n derivatives, the shift register **2** should comprise $(n + 1)$ stages disposed in series, and the ring register **8** should comprise $(n + 2)$ stages.

In the actual embodiment illustrated in FIG. **12**, the analog voltage produced by the sensor (not shown) is fed to an input terminal **23** connected in turn to the positive input of an operational amplifier *Aa*. Each stage of the shift register (**2**, FIG. **11**) comprises essentially a capacitor, notably *Ca*, *Cb* or *Cc*, of which the first armature or plate is grounded while the other armature or plate is connected either to that of the capacitor of the preceding stage or to the input terminal **23** via an AND gate connected in series with an operational amplifier *Aa*, *Ab* or *Ac*. In the embodiment illustrated in FIG. **12**, each AND gate is illustrated in the form of a field-effect transistor *Ta*, *Tb* or *Tc* or which the gate electrode constitutes the second input of the corresponding AND gate, *2d*, *2e* or *2f*, which is connected to one of the three first stages of the ring register **8** (not shown in FIG. **12**). As already explained in the foregoing, after the third clock pulse the capacitors *Ca*, *Cb* and *Cc* are charged with voltages proportionals to V_3, V_2 and V_1, respectively (with the notation of FIG. **2**); at the fourth pulse, the transfer pulse transmitted via terminal *8d* from the last stage of the ring register **8** (FIG. **11**) releases second AND gates, also shown in the form of field-effect transistors **T3**, **T2** and **T1**, inserted between the second plates of capacitors *Ca*, *Cb* and *Cc* and the first plates of capacitors **C3**, **C2** and **C1**, respectively, through operational amplifiers *Ab*, *Ac* and *Ad*. Capacitors C_3, C_2 and C_1, have their second plates grounded. The charge transfers resulting therefrom are attended by the shift register **2**, consisting in this example of capacitors *Ca*, *Cb* and *Cc*, being emptied preliminary to the next measuring cycle. After said fourth clock pulse the voltages proportional to V_3, V_2 and V_1 respectively, which charge the capacitors C_3, C_2 and C_1, respectively, are fed through operational amplifiers **A3**, **A2** and **A1**, respectively, to the corresponding inputs of computer unit **5**, which, in the structure shown in FIG. **12**, is the functional equivalent of the embodiment illustrated in FIG. **9**.

FIG. **13** illustrates diagrammatically a modified embodiment of the shift register of the device of FIG. **12**; this modified embodiment comprises seven capacitors C_1 to C_7 having inserted therebetween AND gates shown in the form of seven field-effect transistors T_1 to T_7 in series with operational amplifiers A_1 to A_7; the gate electrodes of transistors T_1 to T_7 must be connected through terminals b_1 to b_7 to the first seven stages, respectively, of an eight-stage ring register, according to a diagram similar to that shown in FIG. **11**.

Although preferred embodiments of this invention have been shown and described herein, it will be readily understood by those conversant with the art that various modifications and changes may be brought thereto without departing from the gist of the invention, as claimed in the appended claims.

What is claimed as new is:

1. A device for calculating the successive derivatives of a function of a variable, especially for regulating or controlling said function, comprising: for calculating the n first derivatives, means for sensing in succession the function values for $(n + p)$ successive values of the variable, n and p designating two integers, storage means for memorizing the sensed $(n+p)$ values of said function, and computing circuits fed from said storage means and adapted to determine the values of the n first derivatives according to known mathematical formulas; said computing circuits being adapted to determine the value of the nth derivative of the function y of variable x according to the following formula:

$$\frac{d^n y}{dx^n} = \frac{1}{k \overline{\Delta x^n}} \sum_{i=O}^{i=n} a_i y\,(x_1 + i\Delta x),$$

k being a constant factor, Δx a predetermined algebric increment of the variable x, and a_i mathematically calculable coefficients, depending each on the respective value of integer index i; for calculating the acceleration dV/dt of a moving body at a time t, and its variation d^2V/dt^2, said computing circuits being adapted to determine the values of:

$$\frac{dV}{dt} = \frac{V(t+\Delta t) - V(t-\Delta t)}{2\,\Delta t}, \text{ and}$$

$$\frac{d^2V}{dt^2} = \frac{V(t+\Delta t) - 2\,V(t) + V(t-\Delta t)}{\Delta t^2},$$

Δt being a predetermined time interval; for calculating the derivatives of a function of time at regularly spaced time intervals, said storage means essentially comprising a shift or displacement memory, having at least one input connected to the output of said sensing means, and a plurality of outputs stepped in the direction of shift or displacement of said memory, said memory outputs being connected to the corresponding inputs of said computing circuits; said memory being a shift register having at least one shift control input, connected to an electronic clock; first AND gates having each a first and a second input, said first AND gates being respectively inserted through their respective first inputs between the outputs of said shift register and the corresponding inputs of said computing circuits, and means for transmitting a transfer pulse to the second inputs of said first AND gates at a time not earlier than the moment when said shift register is filled; for calculating the n first derivatives of a function of time, said device comprising a shift register having $(n+1)$ stages connected in series, and a ring register having $(n+2)$ stages, the input of the first stage of said ring register being connected to said electronic clock, whereas the outputs of the $(n+1)$ first stages of said ring register are connected to the respective shift control inputs of the corresponding stages of said shift register, and the output of the $(n+2)$th stage of said ring register is connected to the second inputs of said first AND gates.

2. A device as set forth in claim 1, wherein the $(n+1)$ stages of said shift register consist essentially of $(n+1)$ first capacitors, each of which is coupled to the preceding first capacitor through a second AND gate, having a first and a second input, said second input being connected to the output of the corresponding stage of the ring register, and $(n+1)$ second capacitors are inserted between said first capacitors and the corresponding inputs of said computing circuits, respectively through the first inputs of said second AND gates, the second inputs of said first AND gates being connected together to the output of the $(n+2)$th stage of said ring register.

3. A device as set forth in claim 2, wherein each first and second AND gate comprises a field-effect transistor, of which the gate electrode constitutes the second input of the corresponding AND gate.

4. A device as set forth in claim 3, wherein an operational amplifier is mounted before each of said first and second AND gates.

5. A device as set forth in claim 3, wherein operational amplifiers are inserted between said second capacitors and the corresponding inputs of said computing circuits.

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