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(71) Applicant *(for all designated States except US)*: **MONASH UNIVERSITY** [AU/AU]; Wellington Road, Clayton, Victoria 3168 (AU).

(72) Inventors; and

(75) Inventors/Applicants *(for US only)*: **LOWERY, Arthur James** [AU/AU]; 136 Derby Street, Kew, Victoria 3101 (AU). **VITERBO, Emanuele** [IT/AU]; 15 Leaf Court, Clayton, Victoria 3168 (AU). **HONG, Yi** [CN/AU]; 15 Leaf Court, Clayton, Victoria 3168 (AU).

(74) Agent: **WATERMARK PATENT & TRADE MARKS ATTORNEYS**; GPO Box 5093, Melbourne, Victoria 3001 (AU).

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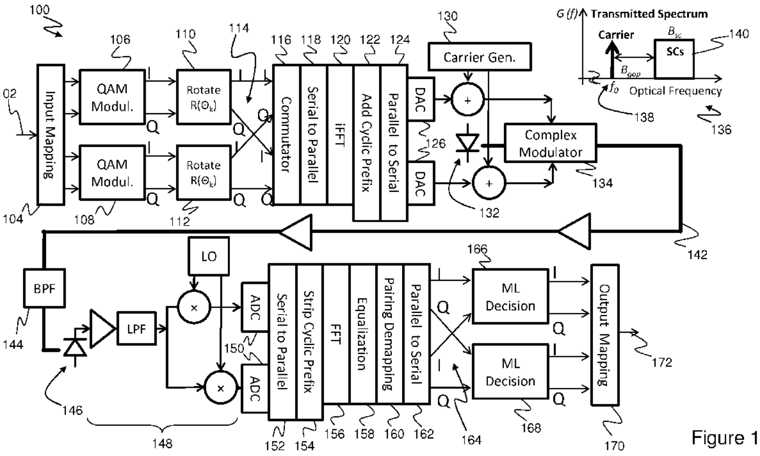

Figure 1

(57) **Abstract**: An optical orthogonal frequency division multiplexing (O-OFDM) transmitter comprises a digital processor which is configured to receive input digital information bits at a data input (102), and map the received bits to a plurality of orthogonal sub-carriers. The subcarriers are partitioned to form a plurality of subsets of subcarriers, and a joint coding algorithm (110, 112, 114) is applied to encode information for transmission via each subset of subcarriers. The subcarriers are selected (116, 118) such that an improvement in sensitivity of an associated optical receiver is obtained, when compared with a corresponding independent coding of each subcarrier. The digital processor further performs a transformation (120) of the coded subcarriers to produce a corresponding time sequence of OFDM signal samples. The transmitter also includes at least one digital-to-analog converter (126) operatively coupled to the digital processor and configured to convert the OFDM signal samples to at least one corresponding electrical OFDM signal. An optical source (132, 134) comprising at least one electrical input and an optical signal output is arranged to receive the electrical OFDM signal, and to modulate an optical carrier using the electrical OFDM signal to generate a corresponding optical OFDM signal. A corresponding receiver and O-OFDM transmission system are also described.

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JOINT CODING FOR OPTICAL OFDM

The present invention relates to optical communications, and more particularly to methods and apparatus for improving the sensitivity of receivers, and hence the overall available power budget, in optical Orthogonal Frequency Division Multiplexed (OFDM) transmission systems.

BACKGROUND OF THE INVENTION

Optical Orthogonal Frequency Division Multiplexing (O-OFDM) is a term referring to a range of proposals and technologies enabling highly spectrally-efficient and potentially robust transmission of high-bandwidth information channels over optical links and networks.

One form of O-OFDM is so-called Coherent Optical Orthogonal Frequency Division Multiplexing (CO-OFDM), in which an optical signal comprising numerous orthogonal subcarrier channels is transmitted via an optical link or network without the inclusion of a corresponding optical carrier tone. The recovery of a CO-OFDM signal at the receiver requires the use of a separate local oscillator laser, along with the necessary optical and optoelectronic components (e.g. an optical hybrid and balanced detectors) to perform coherent detection of the received signal. In addition, the independence of the local oscillator laser from the transmitter laser results in phase noise due to the randomly-evolving relative phase between the two lasers, which must be compensated using suitable techniques, such as the inclusion of a pilot tone within the CO-OFDM signal band.

Accordingly, while CO-OFDM theoretically provides the highest possible receiver sensitivity amongst all O-OFDM techniques, it also involves additional complexity in the system overall, and particularly at the receiver.

An alternative form of O-OFDM, Direct Detection Optical Orthogonal Frequency Division Multiplexing (DDO-OFDM) avoids much of the complexity inherent in the CO-OFDM receiver by transmitting an optical carrier along with the O-OFDM signal band, thereby avoiding the need to provide a local oscillator, and associated components, at the receiver. The price of this reduction in complexity is a decrease in receiver sensitivity, compared with CO-OFDM, of around 7 dB. In practical terms, this means that the DDO-OFDM receiver requires approximately 7 dB more input optical power (in a receiver-noise-limited scenario), or an equivalently higher Optical Signal-to-Noise Ratio (OSNR) (in an optical-noise-limited scenario) when compared with an otherwise equivalent CO-OFDM receiver.

There are three main contributors to the reduction in overall sensitivity of the DDO-OFDM receiver. Firstly, the transmitted carrier optimally comprises approximately half of the received optical power, but contributes nothing to the information content. Secondly, intermixing noise and interference can be substantially eliminated within a CO-OFDM receiver, using balanced detectors. Thirdly, electrical noise generated by the mixing of optical noise components can also be substantially eliminated through the use of balanced detectors. It is possible to improve the performance of the DDO-OFDM receiver in respect of the second and third contributing factors, by extracting the transmitted carrier and using it as a 'local oscillator' (so-called self-coherent optical OFDM), enabling the benefits of balanced detection to be obtained. However, this approach reintroduces much of the complexity of the CO-OFDM system.

It is a characteristic property of DDO-OFDM, however, that the post-detection noise varies with frequency, i.e. the signal quality is a function of the subcarrier number within the OFDM signal band. This is a consequence of the particular manner in which optical signal and noise components mix with one another in the course of direct detection, which is a square-law process. Notably, the error rate (i.e. overall performance) of an O-OFDM system is dominated by the performance of the 'worst' subcarriers. That is, subcarriers having the highest noise and/or interference levels relative to the signal power have the lowest quality, and contribute the largest number of errors to the overall error rate.

While a variation in signal quality with frequency is a particular characteristic of DDO-OFDM systems, similar effects may occur in any O-OFDM system, for example due to non-ideal electrical and/or optical component characteristics, nonlinear effects, and so forth. In particular, electrical components may exhibit frequency-dependent noise, gain and/or attenuation, optical amplifiers may exhibit gain slope, and both electrical components and long-haul transmission links may exhibit nonlinearities that result in frequency-dependent distortions and interference.

It is, accordingly, an object of the present invention to exploit frequency dependence of signal quality in O-OFDM systems to provide improvements in system performance.

In particular, the invention is directed to mitigating the impact of the performance of the subcarriers having lower received signal quality, in order to improve the overall performance of the transmission system. Embodiments of the invention seek to reduce overall error rate for a given received optical signal power, or equivalently enable a given error rate to be achieved with a lower received optical signal power or OSNR, i.e. to improve the receiver sensitivity. In turn, this results in an increase in the available power budget in an O-OFDM transmission system.

SUMMARY OF THE INVENTION

In one aspect, the invention provides a method for generating an Optical Orthogonal Frequency Division Multiplexed (O-OFDM) signal providing improved receiver sensitivity in an optical communications system which comprises an optical channel, wherein a received signal quality is variable with frequency, the method comprising:

mapping input digital information bits to a plurality of orthogonal subcarriers;

partitioning the subcarriers to form a plurality of subsets of subcarriers, and applying a joint coding algorithm to encode information for transmission via each subset of subcarriers, wherein the subcarriers in each subset and the joint coding algorithm are selected such that an improvement in receiver sensitivity is obtained when compared with a corresponding independent coding of each subcarrier; and

generating an O-OFDM signal comprising the plurality of jointly coded subcarriers.

Advantageously, by coding information bits jointly across selected groups of subcarriers, wherein each subcarrier in each group generally exhibits a different signal quality, an overall improvement in receiver sensitivity can be obtained. In particular, the effects of subcarriers having low signal quality on the overall error rate are mitigated by coding information on these subcarriers jointly with subcarriers having a higher signal quality. This can be shown to result in an overall net improvement, which translates directly into improved receiver sensitivity.

For example, the improvements provided by joint coding may allow a reduction in transmitted carrier power within a DDO-OFDM system, thereby assigning a larger proportion of available power to the subcarriers for a given OSNR, resulting in direct improvements to the sensitivity of the receiver, and to the power budget of the transmission system overall.

In general, O-OFDM systems may transmit OFDM signals on either one or two orthogonal polarisation states of the optical carrier. In a dual polarisation system, subsets of subcarriers may be selected within each polarisation state and/or across polarisation states.

Embodiments of the invention preferably employ a small number of selected subcarriers within each subset, for example at least two subcarriers, and no more than four subcarriers.

A preferred embodiment employs ‘pairwise’ coding, wherein each subset comprises a selected pair of subcarriers, wherein at least one pair is selected according to an imbalance of received signal quality.

The use of coding over subsets comprising a small number of subcarriers, such as pairwise coding, is advantageous because it enables improvements in receiver sensitivity to be achieved without requiring significant processing resources for encoding or decoding. Pairwise coding is therefore practical in O-OFDM systems, which may be required to operate at very high bit rates, e.g. tens of gigabits per second or higher.

In an embodiment, a measure of the imbalance of received signal quality is based upon a ratio of the received signal quality of subcarriers in a selected pair. This may be, for example, a ratio between the Signal-to-Interference and Noise Ratio (SINR) of the subcarriers, or of some convenient function of this ratio, such as the square root.

A preferred form of the method comprises the following steps for selection of pairs:

selecting a first pair having a maximum imbalance of received signal quality amongst all available subcarrier pairs; and selecting each subsequent pair having a decreasing imbalance of received signal quality.

Advantageously, this pair selection approach results in a pairing of the ‘worst’ subcarrier with the ‘best’ subcarrier, the ‘second best’ with the ‘second worst’, and so forth.

Preferably, the joint-coding algorithm does not introduce redundancy, i.e. a rate 1 code is employed. Advantageously, the use of a rate 1 code avoids additional complexity in the decoder of a corresponding receiver.

In an embodiment, the joint coding algorithm comprises a phase rotation applied to each selected pair of subcarriers, wherein the method includes coding steps of:

determining, for each pair, a rotation angle based upon the corresponding imbalance of received signal quality; and

applying a phase rotation corresponding with the determined rotation angle to the pair of subcarriers.

In an embodiment, the method includes a further coding step of applying an in-phase/quadrature (IQ) component interleaving over each pair of phase-rotated subcarriers.

In one embodiment, the method includes performing a theoretical calculation of predicted received signal quality of each subcarrier as a basis for partitioning of the subcarriers, wherein the theoretical calculation is based upon estimated and/or measured properties of the optical channel. In one particular such embodiment, the O-OFDM transmission system is a DDO-OFDM system, and the theoretical calculation is based upon a computation of the interference and noise components generated by square-law detection.

In another embodiment, the method includes performing a numerical simulation of predicted received signal quality of each subcarrier as a basis for partitioning of the subcarriers, wherein the simulation is based upon estimated and/or measured properties of the optical channel.

In yet another embodiment, the method includes performing a measurement of received signal quality of each subcarrier as a basis for partitioning of the subcarriers. The method may include measurement steps of transmitting a predetermined training data set over the optical channel, and determining an error rate of each subcarrier of the received signal. The error rates may be determined at the receiver, and corresponding signal quality information sent back to the transmitter via a separate (e.g. out-of-band) channel.

In another aspect, the invention provides a method for recovering digital information in an O-OFDM receiver of an optical communication system, wherein a received optical signal comprises a plurality of orthogonal subcarriers to which the digital information has been mapped and which have been partitioned to form a plurality of subcarrier pairs, each pair having been coded by applying a phase rotation and an in-phase/quadrature (IQ) component interleaving, the method comprising:

receiving the O-OFDM signal;

processing the received signal to recover the plurality of orthogonal subcarriers;

de-interleaving the IQ components of each subcarrier pair; and

performing a Maximum Likelihood (ML) detection over each subcarrier pair to recover the digital information mapped to each subcarrier.

In yet another aspect, the invention provides an O-OFDM transmitter comprising:

a digital processor configured and/or programmed at least to

receive input digital information bits, and map the received bits to a plurality of orthogonal subcarriers,

partition the subcarriers to form a plurality of subsets of subcarriers, and apply a joint coding algorithm to encode information for transmission via each subset of subcarriers, wherein the subcarriers are selected such that an improvement in sensitivity of an associated optical receiver is obtained, when compared with a corresponding independent coding of each subcarrier, and

perform a transformation of the coded subcarriers to produce a corresponding time sequence of OFDM signal samples;

at least one digital-to-analog converter (DAC) operatively coupled to the digital processor and configured to convert the OFDM signal samples to at least one corresponding electrical OFDM signal; and an optical source comprising at least one electrical input and an optical signal output, the electrical input being arranged to receive the electrical OFDM signal, the optical source being configured to modulate an optical carrier using the electrical OFDM signal to generate a corresponding optical OFDM signal.

Preferably, the digital processor is configured and/or programmed to partition the subcarriers such that each subset comprises a selected pair of subcarriers, wherein at least one pair is selected according to an imbalance of received signal quality, according to a method comprising the following steps for selection of pairs:

selecting a first pair having a maximum imbalance of received signal quality amongst all available subcarrier pairs; and

selecting each subsequent pair having a decreasing imbalance of received signal quality.

Preferably, the digital processor is further configured and/or programmed to apply a joint coding algorithm which comprises a phase rotation applied to each selected pair of subcarriers, according to a method comprising coding steps of:

determining, for each pair, a rotation angle based upon the corresponding imbalance of received signal quality; and

applying a phase rotation corresponding with the determined rotation angle to the pair of subcarriers.

Preferably, the digital processor is further configured and/or programmed to execute a further coding step of applying an IQ component interleaving over each pair of phase-rotated subcarriers.

In another aspect, the invention provides an O-OFDM receiver comprising:

an optical-to-electrical (O/E) conversion unit having an optical input and at least one electrical output, which is configured to detect an O-OFDM signal received at the input and to generate a corresponding electrical OFDM signal at the electrical output;

at least one analog-to-digital converter (ADC) operatively coupled to the electrical output of the O/E conversion unit, and arranged to produce a corresponding sequence of signal samples; and

a digital processor configured and/or programmed at least to receive the signal samples from the ADC,

perform a transformation of the signal samples to produce a corresponding plurality of coded subcarriers, to which transmitted digital information bits have been mapped in an associated optical transmitter, and which have been partitioned to form a plurality of subcarrier pairs, each pair having been coded by applying a phase rotation and an IQ component interleaving,

de-interleave the IQ components of each subcarrier pair, and

perform a Maximum Likelihood (ML) detection over each subcarrier pair to recover the digital information mapped to each subcarrier.

Further features and advantages of embodiments of the present invention will be apparent from the following description of preferred embodiments, which is provided by way of example only, and without limitation to the scope of the invention as defined in the preceding statements, and in the claims appended hereto.

BRIEF DESCRIPTION OF THE DRAWINGS

Embodiments of the invention will now be described with reference to the accompanying drawings, in which:

Figure 1 is a schematic diagram of a DDO-OFDM transmission system embodying the invention;

Figure 2 shows schematic spectra illustrating frequency dependence of received noise and interference in the DDO-OFDM system of Figure 1;

Figure 3 is a graph of simulated and calculated received signal, interference and noise levels as a function of frequency within the DDO-OFDM system of Figure 1;

Figure 4 is a graph illustrating variation of signal-to-interference noise ratio across the subcarrier band according to embodiments of the invention;

Figure 5 is a graph illustrating optimal angle of pairing of subcarriers according to embodiments of the invention; and

Figure 6 is graph showing required optical signal-to-noise ratio for a bit error ratio of 10^{-3} as a function of the ratio of carrier power to sideband power, with and without pairing in accordance with an embodiment of the invention.

DETAILED DESCRIPTION OF PREFERRED EMBODIMENTS

Embodiments of the present invention are applicable to a range of Optical Orthogonal Frequency Division Multiplexing (O-OFDM) transmission systems and networks, in which the signal quality varies across the OFDM subcarriers. A central principle exploited by the invention is the recognition that by performing joint coding over subcarriers having different signal quality, it is possible to 'share' the benefits of higher-quality subcarriers amongst those of lower quality or, conversely, to 'distribute' the adverse impact of poor quality subcarriers across those of higher quality.

For specificity, embodiments are described herein, without limitation of the above general principle, in which Direct Detection Optical Orthogonal Frequency Division Multiplexing (DDO-OFDM) is employed. Such a system 100 is shown schematically in Figure 1.

The DDO-OFDM system 100 has a data input 102 at which digital information bits are received. The input digital data is then processed in the transmitter by a digital processor which implements, conceptually, a number of distinct processing functions represented as blocks within the schematic diagram of the system 100. In a practical implementation, the digital processor may be implemented in software executing on a suitable central processing unit (e.g. a DSP device), or as custom, or semi-custom, hardware, such as an Application-Specific Integrated Circuit (ASIC), or programmable hardware, such as a Field-Programmable Gate Array (FPGA). Such implementations, and others relying on a combination of applicable digital processing components, will be readily apparent to persons skilled in the art of digital signal processing.

An input mapping 104 is applied, in which groups of input data bits are buffered and transferred to modulation blocks 106, 108. In the embodiment 100 shown in Figure 1, 4-QAM modulation is employed, whereby two input data bits are provided to each modulation block 106, 108, each of which outputs a corresponding complex number, which takes on one of four values. These complex values are conveniently represented in Cartesian form, as a real, or in-phase (I) component, and an imaginary, or quadrature (Q) component.

In the embodiment 100, the joint coding algorithm employed is a 'pairwise' algorithm, i.e. one which performs coding across subsets comprising two subcarriers each. As such, the two complex-valued outputs of the modulation blocks 106, 108 represent the signal information that is to be coded across a corresponding pair of subcarriers.

In accordance with the exemplary pairwise coding algorithm, the pairs of 4-QAM values output from the modulators 106, 108 are jointly coded by firstly applying a phase rotation, in the rotation blocks 110, 112, which is dependent upon an imbalance in received signal quality of a corresponding pair of subcarriers of the DDO-OFDM signal. The pair of rotated complex values are then in-phase/quadrature (IQ) interleaved, as indicated by the crossed signal paths 114. That is, the in-phase, or real, components of both input values are mapped to one output value, while the quadrature, or imaginary, components are mapped to the other output.

The resulting pair of jointly coded values are then mapped to a corresponding selected pair of subcarriers of the transmitted signal via the operation of the commutator 116 and serial-to-parallel conversion 118 processing blocks.

Further details of the exemplary pairwise coding algorithm, and the subcarrier partitioning process by which the subcarrier pairs are selected, are described in the Appendix.

A frequency/time transformation, implemented by the inverse Fast Fourier Transform (FFT) block 120 is then applied to produce a corresponding time sequence of OFDM signal samples from the coded subcarriers. In accordance with the embodiment 100, a cyclic prefix is added 122, and then parallel-to-serial conversion performed 124, in order to produce two serial sequences of real-valued digital samples, corresponding with in-phase and quadrature components of a transmitted signal. These sequences of digital samples are converted to corresponding electrical signal waveforms via a pair of digital-to-analog converters (DACs) 126. A carrier tone 130 is then added to the electrical waveforms, which will act as the optical carrier during direct detection of the transmitted optical signal. The RF frequency of the carrier tone 130 is selected so as to provide a frequency spacing between the resulting carrier and the edge of the subcarrier signal bandwidth of B_{gap}. The addition of a carrier tone within the electrical domain enables the transmitted electrical signal waveforms to remain at baseband, so as to take maximum advantage of the bandwidth of the relevant electrical components.

An optical source, such as a semiconductor laser 132, is then modulated with the in-phase and quadrature components of the signal using a complex optical modulator 134. The resulting optical spectrum 136 comprises the carrier tone 138, and an OFDM subcarrier band 140, having a bandwidth of B_{sc}.

The O-OFDM signal 136 is then transmitted via amplified fibre link 142, which comprises a number of spans of single-mode optical fibre, along with optical amplifiers, such as Erbium-Doped Fibre Amplifiers (EDFAs), arranged to compensate for attenuation within the fibre spans.

At the receiving end, an optical bandpass filter 144 is used to reject Amplified Spontaneous Emission (ASE) noise generated by the EDFAs outside the desired signal bandwidth. The resulting signal is directly detected, for example by the photodiode 146.

Front-end analog components 148 amplify, filter and down-convert the received signal, resulting in detected in-phase and quadrature signal waveforms, which are converted to corresponding sequences of digital signal samples using a pair of analog-to-digital converters (ADCs) 150. These real-valued digital samples represent the real and imaginary components of complex-valued received digital signals corresponding with in-phase and quadrature components of the received RF signals.

The receiver includes a digital processor configured to demodulate and recover the transmitted digital information. As in the transmitter, the receiver digital processor may be implemented via software, custom ICs, ASICs, FPGAs, other suitable digital processing components, and/or any combination of these. As in the transmitter, the required processing is described with reference to specific conceptual functional blocks.

A serial-to-parallel converter 152 receives fixed-length blocks (i.e. received OFDM symbols) and converts them into a corresponding parallel array of complex values. The cyclic prefix is then stripped in block 154. A time/frequency transformation, in the form of FFT 156, is applied to recover a corresponding block of frequency domain, i.e. subcarrier, values. Equalisation 158 is applied, to compensate for transmission impairments such as chromatic dispersion, polarisation-mode dispersion, and linear distortions due to non-ideal characteristics of optical and electrical components. The jointly coded pairs of subcarriers are then selected by the demapping block 160. A parallel-to-serial converter 162 then outputs the values corresponding with these selected pairs in sequence.

At the output of the parallel-to-serial converter 162, IQ de-interleaving is performed, as indicated by the crossed signal paths 164, which reverses the interleaving process performed in the transmitter 114. The resulting pair of complex received subcarrier values, which have been degraded by processes such as additive noise and nonlinear distortion, are then processed in order to recover (i.e. estimate) the corresponding values of transmitted information bits.

The process of information recovery/estimation is performed by a pair of Maximum Likelihood (ML) decision blocks 166, 168. The relevant ML estimators are described in greater detail in the Appendix.

Finally, an output mapping 170 is applied, to produce a digital data output 172.

In general, a transmission system such as the exemplary system 100 will not be designed to achieve a zero raw Bit Error Ratio (BER), which is neither possible nor optimal in practice. More typically, suitable Forward Error Correction (FEC) algorithms are applied to enable correction of bit errors up to a BER of approximately 10^{-3}. The improvement in overall system power budget achieved by utilising FEC is more than sufficient to compensate for the additional overhead required for the error detection/correction coding. For present purposes, it is assumed that any error coding is applied prior to the data input 102, and subsequent to the data output 172, and will therefore not be discussed in any greater detail.

As has previously been mentioned, embodiments of the invention are applicable to O-OFDM systems in which the signal quality varies across the subcarrier band. In general, subcarrier signal quality is affected by additive noise, distortion, and interference, such as crosstalk from other subcarriers and other signals within the system, which may be generated, for example, through nonlinear processes, including the square-law detection process occurring at the photodiode 146 of the system 100. For present purposes, it is convenient to represent the signal quality of a particular subcarrier by a single Signal-to-Interference and Noise Ratio (SINR), encompassing all of the sources of signal degradation.

If all of the received subcarriers posses the same SINR, then there will be no gain from the use of joint coding. However, this will not generally be the case in real systems. In particular, in a DDO-OFDM system, such as the exemplary system 100, the SINR is frequency-dependent, even for flat ASE spectrum, and even if all of the characteristics of the various electrical and optical components are ideal. This arises because of the mixing that occurs at the photodetector 146, which outputs a photo current proportional to the square of the input optical field. Figure 2 shows schematically a series of spectra illustrating the frequency dependence of received noise and interference in the DDO-OFDM system 100.

The optical spectrum 200 of the received signal comprises the carrier tone 202, the band of information-bearing subcarriers 204, and an approximately flat ‘floor’ of ASE noise 206. (Note that all of the schematic spectre in Figure 2 are shown in two-sided form.)

Following square-law detection the spectrum 208 of the desired OFDM signal, which results from mixing between the subcarrier band 204 and the carrier tone 202, comprises the received subcarriers 210. The spectrum 212 shows the interference 214 which results from mixing of the subcarriers 204 with themselves. The spectrum 216 comprises the spontaneous-spontaneous beat noise 218, which results from mixing of ASE noise 206 with itself. The spectrum 220 represents the carrier-spontaneous beat noise, resulting from mixing of the ASE noise 206 with the carrier tone 202. This noise component has a ‘step’ form, having a higher noise power level 222 around DC, and a lower noise power level 224 across the subcarrier signal band. Finally, the spectrum 226 resulting from mixing of the ASE noise 206 with the subcarriers 204 has the relatively complex form 228.

The parameters shown in Figure 2, which are used to define the various spectral features and power levels, are defined as follows:

- f_0 is the carrier frequency;
- A is the amplitude of the optical carrier field;
- S_0 is the mean power spectral density of the optical OFDM signal;

- N_{ASE} is the ASE noise power spectral density;
- B_m is the bandwidth of the optical bandpass filter 144;
- B_{gap} is the width of the frequency gap between the carrier 202 and the subcarrier band 204;
- B_{sc} is the total bandwidth of the OFDM subcarrier band 204;
- B_L is the bandwidth of unrejected ASE noise at frequencies lower than the carrier frequency; and
- B_H is the bandwidth of unrejected ASE noise above the optical OFDM subcarrier band.

With these definitions, an analytical expression for the electrical power spectral density resulting from summing the various components shown in Figure 2 may be written as follows:

$$G_y(f) = \underbrace{2G_s(f) * \frac{A^2}{4}\left[\delta(f-f_0)+\delta(f+f_0)\right]}_{(1)} + \underbrace{G_s(f)*G_s(f)}_{(2)} + \underbrace{G_n(f)*G_n(f)}_{(3)}$$

$$+\underbrace{2G_n(f)*\frac{A^2}{4}\left[\delta(f-f_0)+\delta(f+f_0)\right]}_{(4)} + \underbrace{2G_s(f)*G_n(f)}_{(5)} + [DC,\ 2f_0\ \text{component}]$$

where * denotes the convolution operation and $\delta(f)$ the Dirac function. The terms $G_s(f)$ (equal to S_0 over the subcarrier band) and $G_n(f)$ (equal to N_{ASE} over the optical filter bandwidth) are the PSD of the OFDM signal and of the ASE noise in both polarizations, respectively. The above expression is valid under the assumption that both noise and signal are zero-mean Gaussian random processes. This assumption is valid when the number of subcarriers is sufficiently large.

In the above equation term (1) represents the power spectrum of the useful OFDM signal which has been down-converted by the subcarriers mixing with the carrier upon photodetection; (*2*) is the autocorrelation of $G_s(f)$ accounting for the unwanted mixing tones; (3) is the autocorrelation of $G_n(f)$; (4) is noise that has been down converted by mixing with the carrier; and (5) arises from mixing of noise with the OFDM subcarriers. The sum of the terms (2), (3), (4) and (5) represents the noise and interference impairments. The DC component arises from the optical carrier and the unwanted tones, while the ($2f_0$) arises from the optical carrier.

As can be seen from the spectra shown in Figure 2, or from a consideration of the terms appearing in the above equation, there is an inherent variation in the noise level across the subcarrier band in a DDO-OFDM receiver. This results from the frequency dependence of the spontaneous-spontaneous beat noise 218 and the subcarrier-spontaneous beat noise 228, across the subcarrier signal band. As a result, the SINR of the subcarriers also varies inherently across this band in the DDO-OFDM receiver.

Figure 3 provides a further illustration of the inherent frequency dependence of SINR in a DDO-OFDM receiver. The figure comprises a graph 300 of simulated and calculated received signal, interference and noise levels in the receiver as a function of frequency. This exemplary case represents a 48 Gb/s optical OFDM signal using 4-QAM modulation with 1,024 bits per OFDM symbol, using $N_{sc} = 512$ subcarriers. The OFDM signal occupies $B_{sc} = 30$ GHz bandwidth. The bandgap between carrier 202 and subcarriers 204 is $B_{gap} = 30$ GHz. The total ASE noise bandwidth is $B_m = 60$ GHz. The resolution bandwidth used in generating the spectrum is 14.648 MHz. This bandwidth is one quarter of the subcarrier spacing, and accordingly the SINR that may be read from the graph 300 (i.e. the difference between the signal power density and the associated noise power density at a particular frequency) is 6 dB more than the actual SINR experienced by an individual subcarrier after demodulation.

The graph 300 shows RF power in dBm (within the resolution bandwidth) on the vertical axis 302, as a function of RF frequency, in gigahertz, on the horizontal axis 304. Three sets of points are shown in the graph 300, being the RF signal level across the subcarrier band 306, the inter-modulation components 308 (i.e. resulting from subcarrier mixing), and RF noise 310, resulting from the combined ASE mixing terms. In each case, the scattered points result from a system simulation performed using VPItransmissionMaker™ Version 8.5, from VPIphotonics™, while the smooth curves are computed using the above analytical expression of the electrical power spectral density. As can be seen, excellent agreement is obtained between the full simulation and the analytical expression, indicating the effectiveness of performing a theoretical computation of the SINR in a DDO-OFDM transmission system.

From the data shown in the graph 300 (which corresponds with equal power in the carrier 202 and subcarrier band 204, and an OSNR of 13 dB), the SINR of the subcarriers spans the range from 6 dB (at 30 GHz) to about 9dB (at 60 GHz).

Having confirmed the accuracy of the analytical expression for the electrical power spectral density, it is possible to compute the SINR across the subcarrier band for different values of the OSNR, without the use of simulation. Example calculations are shown in the graph 400 of Figure 4. The graph 400 shows SINR, in dB, on the vertical axis 402, as a function of the RF frequency, in gigahertz, on the horizontal axis 404. The curves 406, 408, 410, 412, respectively, represent the SINR for an OSNR of 19 dB, 16 dB, 13 dB and 10 dB.

For all cases, a slope is observed in the SINR across the subcarrier band, due to the contributions of spontaneous-spontaneous beat noise, and spontaneous-subcarrier mixing, which are both frequency dependent. For a 10 dB OSNR (curve 412) the increase is 4 dB, while for a 19 dB OSNR (curve 406) this reduces to 3.2 dB. This difference is due to the relative contribution of spontaneous-spontaneous beat noise, which becomes negligible at high OSNR.

A detailed theoretical description of the subcarrier pairing and joint coding algorithms, along with the ML estimation procedure, is set out in the Appendix, with reference to the graph 500 shown in Figure 5. These algorithms have been implemented in MATLAB™ software, from MathWorks, using Monte Carlo simulation of the receiver with the variance of the random electrical noise for each subcarrier being set using the calculated SINR.

In particular, MATLAB™ simulations have been conducted comparing the OSNRs at which the DDO-OFDM system can achieve a raw BER of 10^{-3} as a function of the carrier-to-signal power ratio. In the simulations, the OSNR was swept to obtain values of the BER as a function of OSNR, from which the OSNR corresponding with a BER of 10^{-3} was derived. The results are summarised in the graph 600 of Figure 6, which shows results with and without 'pairing' (i.e. joint coding).

The graph 600 shows the required OSNR for a BER of 10^{-3}, in dB, on the vertical axis 602, as a function of the carrier-to-sideband power ratio on the horizontal axis 604. The upper set of points 606 represents the results without pairwise coding, i.e. in accordance with the prior art. Three additional points, e.g. 608, have been obtained using a full system simulation in VPItransmissionMaker™ software. Excellent agreement is obtained between the MATLAB™ and VPItransmissionMaker™ simulations, demonstrating the suitability of the Gaussian approximation for the interference terms used within the MATLAB™ model.

The lower set of points 610 corresponds to MATLAB™ simulations including optimum pairwise coding (see appendix), in combination with ML estimation in the receiver. In all cases simulated, a reduction in the required OSNR to achieve a BER of 10^{-3} is achieved. This reduction in required OSNR corresponds with an improvement in receiver sensitivity or, equivalently, an increase in overall system power budget.

In accordance with the prior art, it is generally considered that a ratio between the carrier and sideband power of around 1.0 (i.e. equal power in each) produces a near-optimum result, and the graph 600 confirms this, although it is apparent that there is a reasonably broad range of carrier-to-sideband power ratio over which the required OSNR does not vary significantly.

By comparison, there is a definite advantage in reducing the carrier-to-sideband power ratio when pairwise coding is employed. This is because the improvement in receiver sensitivity allows for the use of a lower carrier power level, which in turn reduces the proportion of transmitted power that is devoted to the non-information-bearing carrier tone.

Overall, the exemplary simulations demonstrate an improvement (i.e. reduction in required OSNR) of 0.7 dB using a carrier-to-sideband power ratio of 60%.

As the above examples demonstrate, in a basic DDO-OFDM system it is possible to perform subcarrier partitioning and pairwise coding for improved system performance based purely upon theoretical calculations. For more complex systems, for example those having substantially non-ideal optical or electrical components, or in which nonlinear effects are significant, more-detailed system simulations may be performed in order to obtain predictions of received SINR as a function of frequency, for the purposes of subcarrier partitioning and optimum pairwise coding.

In the most general case, the received SINR of each subcarrier may be performed online in a deployed system. In particular, by transmitting a training sequence of predetermined OFDM symbols the BER of each channel may be directly determined at the receiver. From the measured BER, the corresponding SINR may be calculated, for example based upon a Gaussian approximation, or more-detailed model of the system and receiver. The measured SINR may then be communicated back to the transmitter, for example using an out-of-band channel, such as a signalling channel. The quantity of information involved in this feedback step (i.e. a single SINR value representing each subcarrier) is relatively small and accordingly a high-bandwidth return channel is not required. The measured data is then used in the transmitter in order to compute optimised pairwise partitions, and corresponding coding parameters.

For optical channels with relatively slowly time-varying characteristics that impact upon the SINR, predetermined OFDM training symbols may be periodically re-transmitted, in order to update the measured SINR values, and track the time variations.

In summary, embodiments of the invention employ joint coding, exemplified by pairwise coding in the present examples, which is shown to be beneficial in systems for which the received SINR varies as a function of frequency, i.e. subcarrier number. In particular, this is an inherent characteristic of DDO-OFDM systems, for which a performance improvement of 0.7 dB has been demonstrated for particular exemplary embodiments. A further advantage of the joint-coding approach in the exemplary embodiments is that an improvement may be achieved in conjunction with a reduction in the relative power of an optical carrier tone. This reduction in power is potentially useful for reducing the effects of fibre nonlinearity.

While particular embodiments have been described, it will be understood that these are not intended to be limiting of the scope of the invention, as defined in the claims appended hereto. In particular, the principles of the invention are applicable to any O-OFDM system in which subcarriers with lower SINR can be paired with subcarriers of higher SINR, or more-generally in which joint coding can be performed across groups of subcarriers having different SINR.

APPENDIX

Consider a set of pairs $\Psi = \{(p_k,q_k), k=1,\ldots,N_{sc}/2\}$ forming a *partition* of N_{sc} subcarriers, where k is the index of pairs. Sub-channels with high SINR are paired with sub-channels with low SINR. In a DDO-OFDM system the SINR is monotonically increasing with subcarrier number (see Figure 3), thus the pairing of the corresponding subcarriers is:

$$\Psi = \left\{ (p_k, q_k) = (k, N_{sc} - k + 1); k = 1, \cdots, \frac{N_{sc}}{2} \right\}$$

By way of example, for a DDO-OFDM system with six subcarriers ($N_{sc} = 6$), there are three pairs of sub-channels: the first pair ($k = 1$) includes the 1st and the 6th sub-channels, i.e., $(p_1,q_1) = (1, 6)$; the second pair ($k = 2$) includes the 2nd and the 5th sub-channels, i.e., $(p_2,q_2) = (2, 5)$; the third pair ($k = 3$) is $(p_3,q_3) = (3, 4)$.

In a more general case, the sub-channels may first be reordered into a list with monotonically increasing (or decreasing) SINR, before applying the same pairing method, and mapping back to the original subcarrier numbers.

The actual coding is performed across a pair (indexed by k) of M-QAM information symbols a_k and b_k by multiplying by rotation factor $e^{j\theta k}$, yielding two rotated complex symbols $a_k\, e^{j\theta k}$ and $b_k e^{j\theta k}$, where θ_k is the *rotation angle* for the k-th pair. The optimal rotation angle, denoted by θ_k^{opt}, is derived analytically in S. K. Mohammed, E. Viterbo, Y. Hong, and A. Chockalingam, "MIMO precoding with X- and Y-codes," *IEEE Trans. on Information Theory* (2010) for 4-QAM, to minimize the total error probability and is given by:

$$\theta_k^{opt} = \begin{cases} \pi/4 & \beta_k \le \sqrt{3} \\ \tan^{-1}\left[(\beta_k^2 - 1) - \sqrt{(\beta_k^2 - 1)^2 - \beta_k^2} \right] & \beta_k > \sqrt{3} \end{cases}$$

where $\beta_k = \lambda_{q_k} / \lambda_{p_k}$ is called *condition number* of the pair of subcarriers (p_k, q_k), and

$$\lambda_{p_k} = \sqrt{SINR_{p_k}} \quad ; \quad \lambda_{q_k} = \sqrt{SINR_{q_k}}$$

The condition number describes the SINR unbalance between the two subcarriers.

Figure 5 is a graph 500 illustrating the optimum rotation angle for a carrier-to-sideband power ratio $\eta = 0.5$. The graph 500 shows the optimum rotation angle on the vertical axis 502 as a function of the subcarrier pairing index k on the horizontal axis 504. The curves 506, 508, 510, 512, 514 correspond with OSNR of 7 dB, 10 dB, 13 dB and 19 dB respectively. For the higher OSNR values there are fewer subcarrier pairs with optimal rotation angles θ_k^{opt} that are not 45°. Specifically, when OSNR = 7 dB and 19 dB, there are 54 and 6 subcarrier pairs whose condition number is greater than $\sqrt{3}$, respectively.

After rotation, IQ Component interleaving is used over the two precoded symbols $a_k e^{j\theta k}$ and $b_k e^{j\theta k}$. The real part of $b_k e^{j\theta k}$ and the imaginary part of $a_k e^{j\theta k}$, are interchanged (IQ interleaving) to obtain two coded complex symbols for transmission:

$$X_{p_k} = \mathrm{Re}(a_k e^{j\theta_k}) + j\,\mathrm{Re}(b_k e^{j\theta_k}) \quad X_{q_k} = \mathrm{Im}(a_k e^{j\theta_k}) + j\,\mathrm{Im}(b_k e^{j\theta_k})$$

where Re(.) and Im(.) denote the real and imaginary parts of a complex symbol. These complex symbols are transmitted on separate subcarriers with different SINRs, so that the received complex symbols are given by:

$$\begin{cases} Y_{p_k} = \lambda_{p_k} X_{p_k} + n_{p_k} \\ Y_{q_k} = \lambda_{q_k} Y_{q_k} + n_{q_k} \end{cases}$$

where n_{p_k}, n_{q_k} are unit variance complex Gaussian noise samples. Then IQ component deinterleaving results in two complex symbols $\mathrm{Re}(Y_{p_k}) + j\,\mathrm{Re}(Y_{q_k})$ and $\mathrm{Im}(Y_{p_k}) + j\,\mathrm{Im}(Y_{q_k})$. The ML estimates are given by:

$$\begin{cases} \hat{a}_k = \arg\min_{a_k} \left|\mathrm{Re}(Y_{p_k}) - \lambda_{p_k}\,\mathrm{Re}(X_{p_k})\right|^2 + \left|\mathrm{Re}(Y_{q_k}) - \lambda_{p_k}\,\mathrm{Re}(X_{q_k})\right|^2 \\ \hat{b}_k = \arg\min_{b_k} \left|\mathrm{Im}(Y_{p_k}) - \lambda_{p_k}\,\mathrm{Im}(X_{p_k})\right|^2 + \left|\mathrm{Im}(Y_{q_k}) - \lambda_{p_k}\,\mathrm{Im}(X_{q_k})\right|^2 \end{cases}$$

In the encoding and the decoding processes, constellation rotation and component interleaving both play important roles. A benefit is achieved because the real and imaginary components of the deinterleaved symbols are affected by the two independent channel coefficients. For example, when decoding a_k, if one channel loses one component, say Re(X_{pk}), due to a small coefficient λ_{pk}, the other component Re(X_{qk}) is still valid and available to be decoded. This provides modulation diversity gain or, equivalently, additional protection on decoding against the effect of noise and/or fading.

CLAIMS:

1. A method for generating an Optical Orthogonal Frequency Division Multiplexed (O-OFDM) signal providing improved receiver sensitivity in an optical communications system which comprises an optical channel, wherein a received signal quality is variable with frequency, the method comprising:

mapping input digital information bits to a plurality of orthogonal subcarriers;

partitioning the subcarriers to form a plurality of subsets of subcarriers, and applying a joint coding algorithm to encode information for transmission via each subset of subcarriers, wherein the subcarriers in each subset and the joint coding algorithm are selected such that an improvement in receiver sensitivity is obtained when compared with a corresponding independent coding of each subcarrier; and

generating an O-OFDM signal comprising the plurality of jointly coded subcarriers.

2. The method of claim 1 wherein each subset comprises a selected number of at least two and no more than four subcarriers.

3. The method of claim 2 wherein each subset comprises a selected pair of subcarriers, wherein at least one pair is selected according to an imbalance of received signal quality.

4. The method of claim 3 wherein a measure of the imbalance of received signal quality is based upon a ratio of the received signal quality of subcarriers in a selected pair.

5. The method of claim 3 comprising the following steps for selection of pairs:

selecting a first pair having a maximum imbalance of received signal quality amongst all available subcarrier pairs; and

selecting each subsequent pair having a decreasing imbalance of received signal quality.

6. The method of claim 1 wherein the joint coding algorithm encodes the information for transmission without redundancy (rate 1 code).

7. The method of claim 3 wherein the joint coding algorithm comprises a phase rotation applied to each selected pair of subcarriers, wherein the method includes coding steps of:

determining, for each pair, a rotation angle based upon the corresponding imbalance of received signal quality; and

applying a phase rotation corresponding with the determined rotation angle to the pair of subcarriers.

8. The method of claim 7 comprising a further coding step of applying an in-phase/quadrature (IQ) component interleaving over each pair of phase-rotated subcarriers.

9. The method of claim 1 which includes performing a theoretical calculation of predicted received signal quality of each subcarrier as a basis for partitioning of the subcarriers, wherein the theoretical calculation is based upon estimated and/or measured properties of the optical channel.

10. The method of claim 1 which includes performing a numerical simulation of predicted received signal quality of each subcarrier as a basis for partitioning of the subcarriers, wherein the simulation is based upon estimated and/or measured properties of the optical channel.

11. The method of claim 1 which includes performing a measurement of received signal quality of each subcarrier as a basis for partitioning of the subcarriers.

12. The method of claim 11 including measurement steps of transmitting a predetermined training data set over the optical channel, and determining an error rate of each subcarrier of the received signal.

13. A method for recovering digital information in an O-OFDM receiver of an optical communication system, wherein a received optical signal comprises a plurality of orthogonal subcarriers to which the digital information has been mapped and which have been partitioned to form a plurality of subcarrier pairs, each pair having been coded by applying a phase rotation and an in-phase/quadrature (IQ) component interleaving, the method comprising:

receiving the O-OFDM signal;

processing the received signal to recover the plurality of orthogonal subcarriers;

de-interleaving the IQ components of each subcarrier pair; and

performing a Maximum Likelihood (ML) detection over each subcarrier pair to recover the digital information mapped to each subcarrier.

14. An O-OFDM transmitter comprising:

a digital processor configured and/or programmed at least to

receive input digital information bits, and map the received bits to a plurality of orthogonal subcarriers,

partition the subcarriers to form a plurality of subsets of subcarriers, and apply a joint coding algorithm to encode information for transmission via each subset of subcarriers, wherein the subcarriers are selected such that an improvement in sensitivity of an associated optical receiver is obtained, when compared with a corresponding independent coding of each subcarrier, and

perform a transformation of the coded subcarriers to produce a corresponding time sequence of OFDM signal samples;

at least one digital-to-analog converter (DAC) operatively coupled to the digital processor and configured to convert the OFDM signal samples to at least one corresponding electrical OFDM signal; and

an optical source comprising at least one electrical input and an optical signal output, the electrical input being arranged to receive the electrical OFDM signal, the optical source being configured to modulate an optical carrier using the electrical OFDM signal to generate a corresponding optical OFDM signal.

15. The transmitter of claim 14 wherein the digital processor is configured and/or programmed to partition the subcarriers such that each subset comprises a selected number of at least two and no more than four subcarriers.

16. The transmitter of claim 15 wherein the digital processor is configured and/or programmed to partition the subcarriers such that each subset comprises a selected pair of subcarriers, wherein at least one pair is selected according to an imbalance of received signal quality, according to a method comprising the following steps for selection of pairs:

selecting a first pair having a maximum imbalance of received signal quality amongst all available subcarrier pairs; and

selecting each subsequent pair having a decreasing imbalance of received signal quality.

17. The transmitter of claim 14 wherein the digital processor is configured and/or programmed apply the joint coding algorithm to encode the information for transmission without redundancy (rate 1 code).

18. The transmitter of claim 16 wherein the digital processor is further configured and/or programmed to apply a joint coding algorithm which comprises a phase rotation applied to each selected pair of subcarriers, according to a method comprising coding steps of:

determining, for each pair, a rotation angle based upon the corresponding imbalance of received signal quality; and

applying a phase rotation corresponding with the determined rotation angle to the pair of subcarriers.

19. The transmitter of claim 18 wherein the digital processor is further configured and/or programmed to execute a further coding step of applying an IQ component interleaving over each pair of phase-rotated subcarriers.

20. An O-OFDM receiver comprising:

an optical-to-electrical (O/E) conversion unit having an optical input and at least one electrical output, which is configured to detect an O-OFDM signal received at the input and to generate a corresponding electrical OFDM signal at the electrical output;

at least one analog-to-digital converter (ADC) operatively coupled to the electrical output of the O/E conversion unit, and arranged to produce a corresponding sequence of signal samples; and

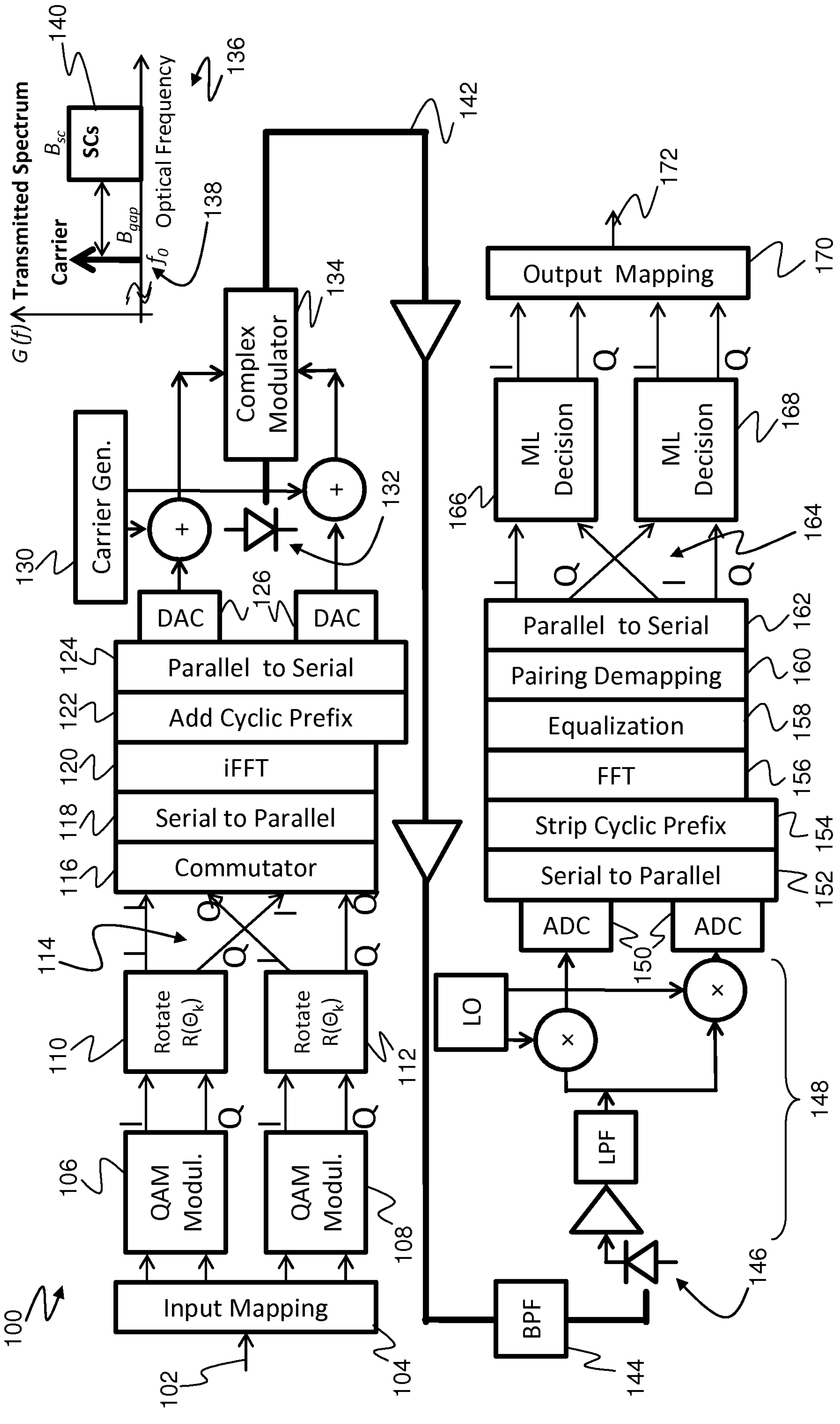
a digital processor configured and/or programmed at least to

receive the signal samples from the ADC,

perform a transformation of the signal samples to produce a corresponding plurality of coded subcarriers, to which transmitted digital information bits have been mapped in an associated optical transmitter, and which have been partitioned to form a plurality of subcarrier pairs, each pair having been coded by applying a phase rotation and an IQ component interleaving,

de-interleave the IQ components of each subcarrier pair, and

perform a Maximum Likelihood (ML) detection over each subcarrier pair to recover the digital information mapped to each subcarrier.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B_{gap}
f_0
Optical Frequency
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136
138
142
Complex Modulator
134
Carrier Gen.
130
+
+
132
DAC
DAC
126
Parallel to Serial
124
Add Cyclic Prefix
122
iFFT
120
Serial to Parallel
118
Commutator
116
I
Q
114
Rotate R(Θ_k)
110
Rotate R(Θ_k)
112
QAM Modul.
106
QAM Modul.
108
Input Mapping
104
102
100
Output Mapping
170
172
ML Decision
166
ML Decision
168
164
Parallel to Serial
162
Pairing Demapping
160
Equalization
158
FFT
156
Strip Cyclic Prefix
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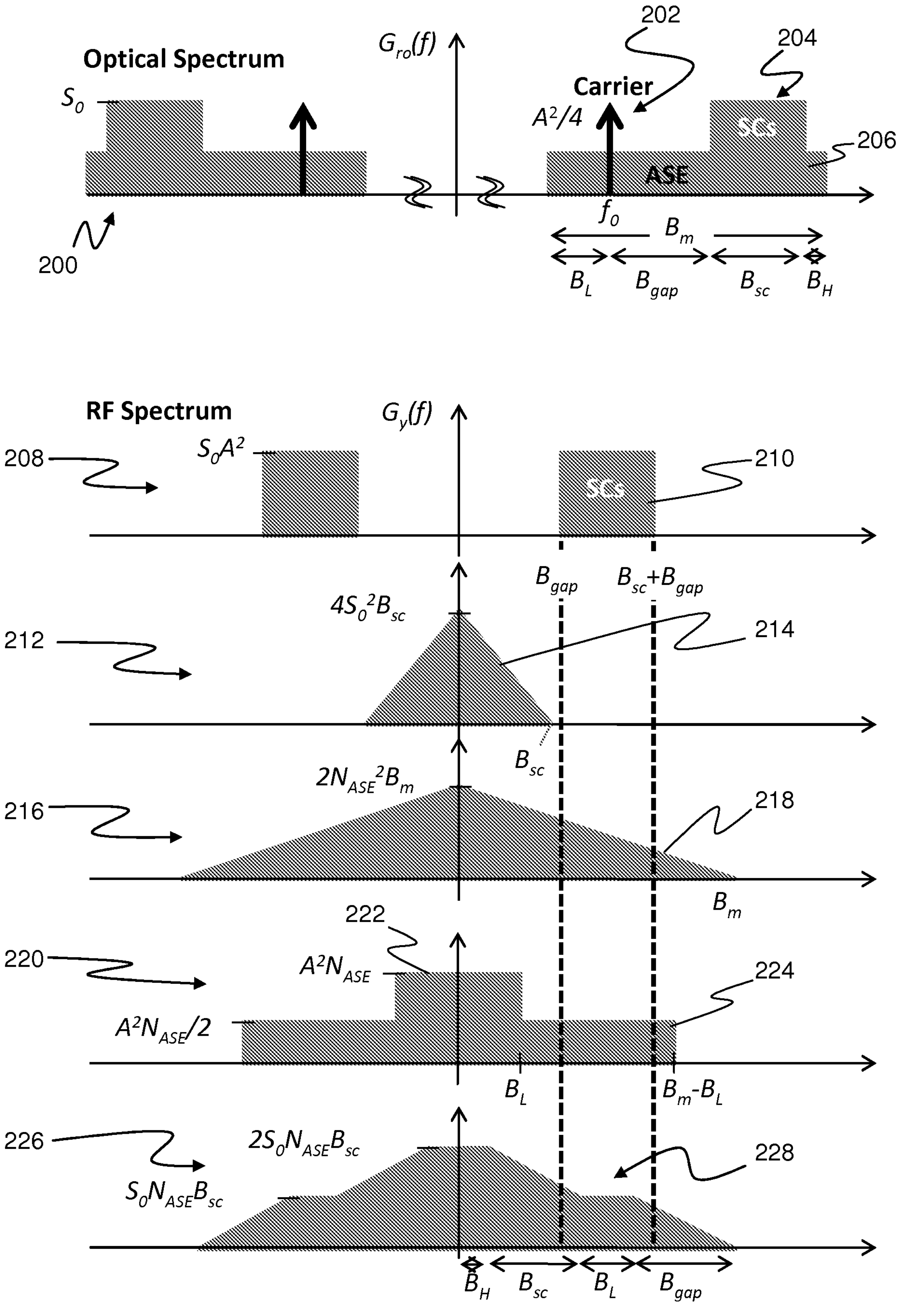
Serial to Parallel
152
ADC
ADC
150
LO
×
×
LPF
148
BPF
144
146

Figure 1


Optical Spectrum
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202
204
Carrier
$A^2/4$
S_0
SCs
206
ASE
f_0
B_m
B_L
B_{gap}
B_{sc}
B_H
200
RF Spectrum
$G_y(f)$
208
S_0A^2
SCs
210
B_{gap}
$B_{sc}+B_{gap}$
$4S_0^2B_{sc}$
212
214
B_{sc}
$2N_{ASE}^2B_m$
216
218
B_m
222
220
A^2N_{ASE}
224
$A^2N_{ASE}/2$
B_L
B_m-B_L
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$S_0N_{ASE}B_{sc}$
B_H
B_{sc}
B_L
B_{gap}

Figure 2

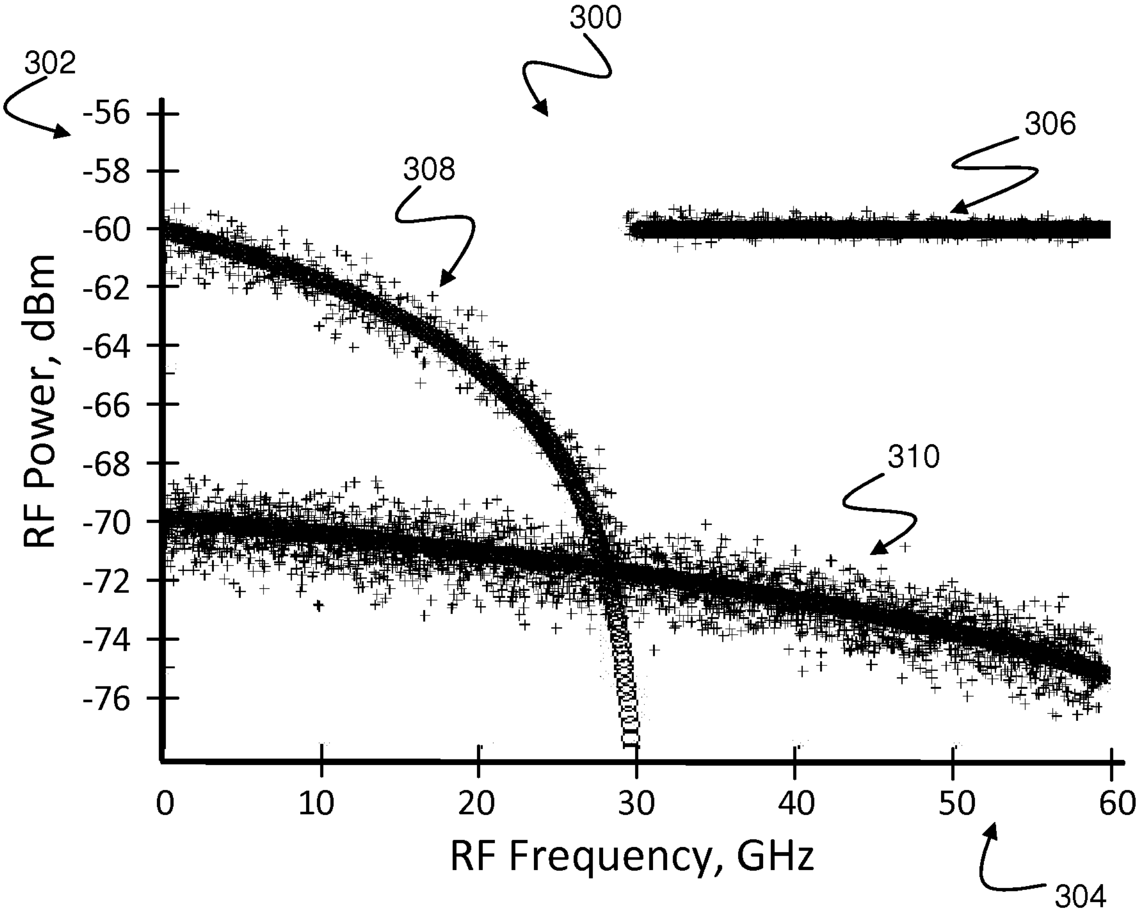

Figure 3

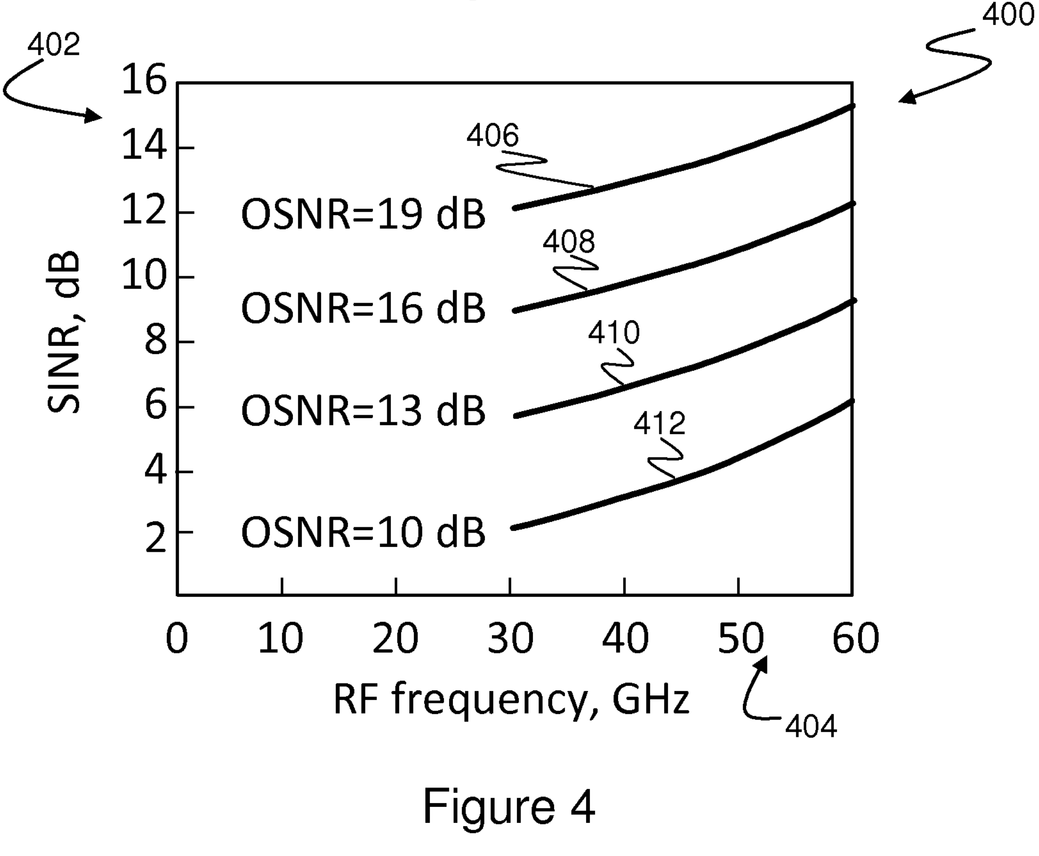

Figure 4

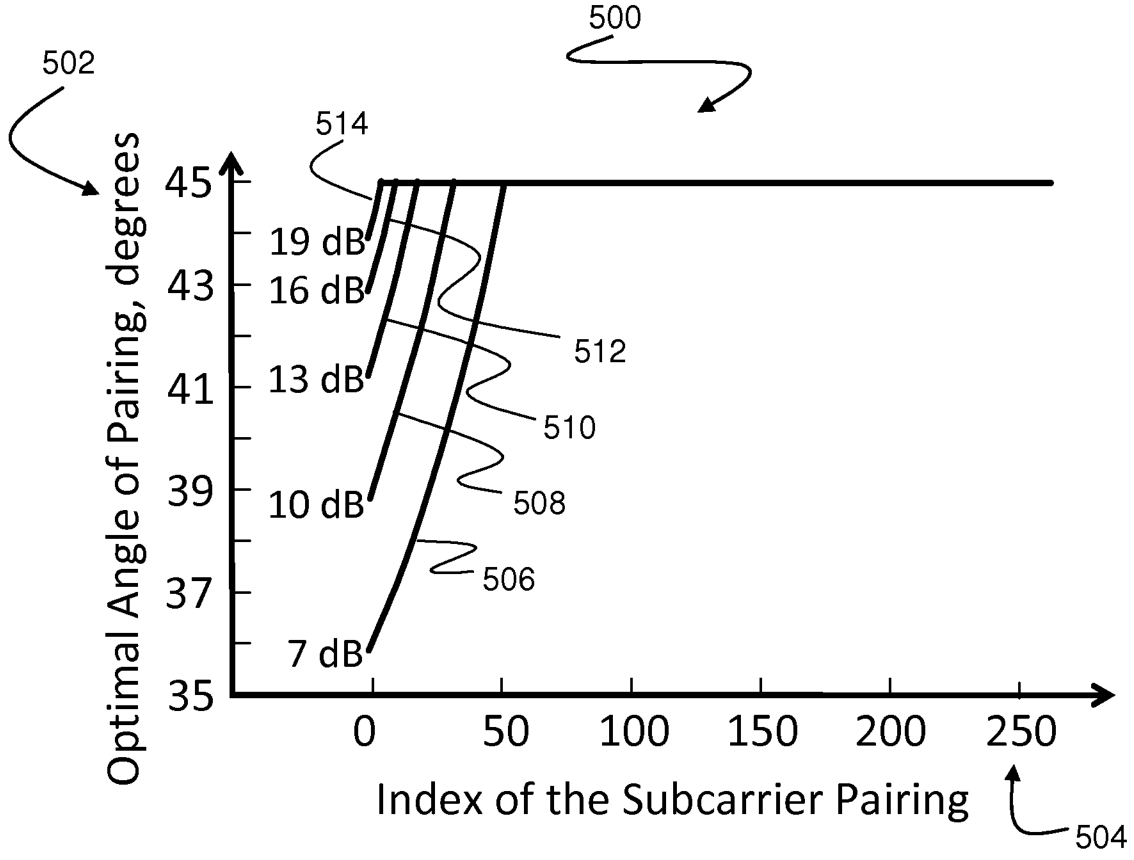

Figure 5

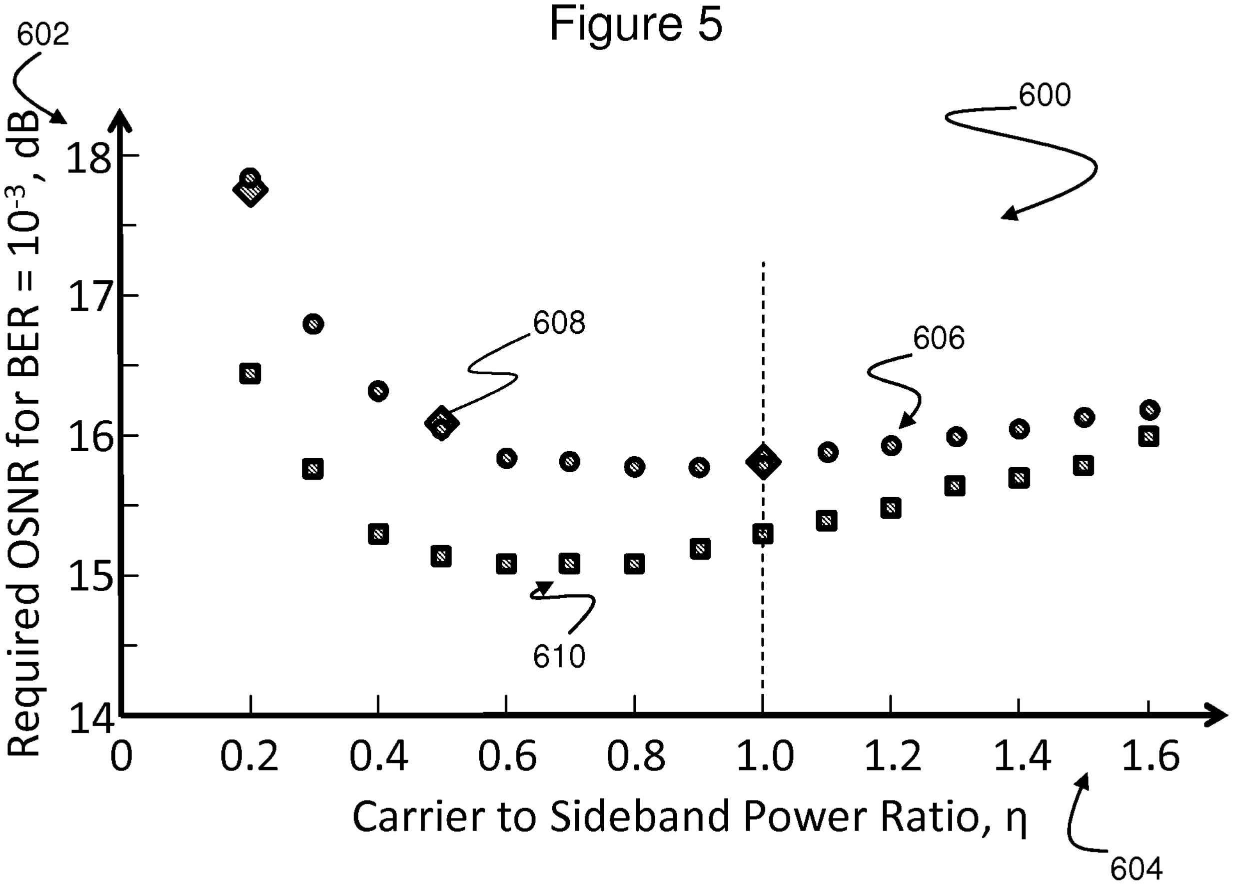

Figure 6