June 3, 1930. O. J. ZOBEL 1,760,973

ELECTRICAL NETWORK

Filed March 27, 1928 2 Sheets-Sheet 1

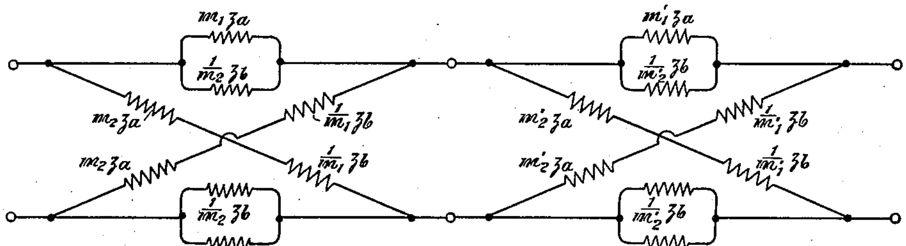


Fig. 1

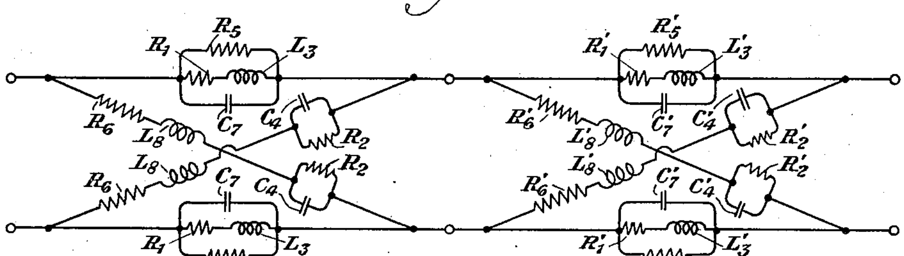


Fig. 2

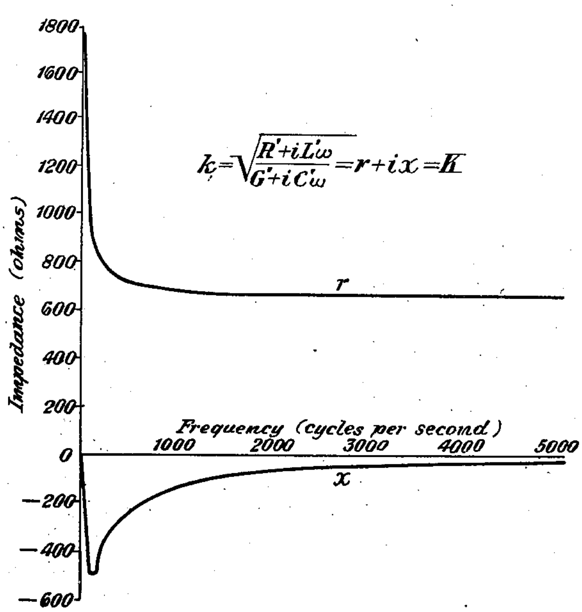


Fig. 4

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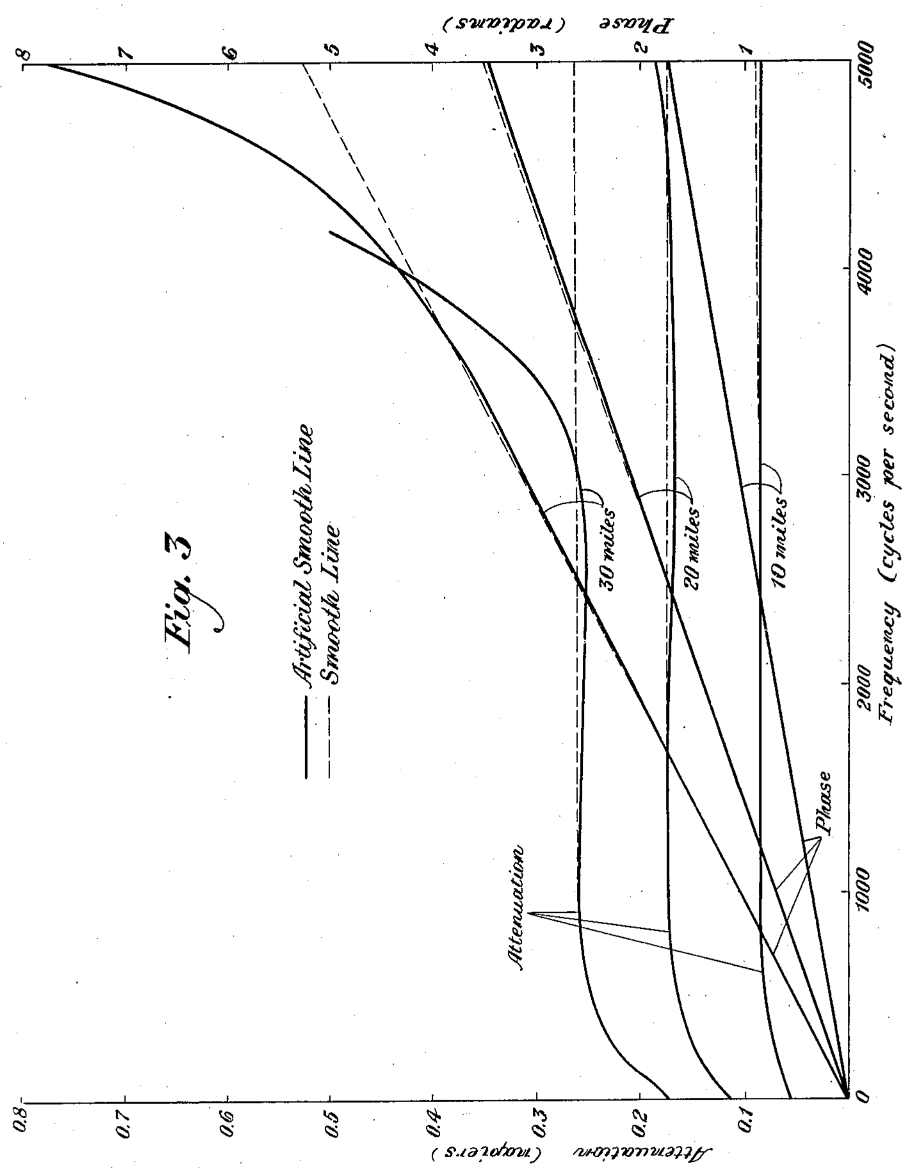


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# UNITED STATES PATENT OFFICE

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## ELECTRICAL NETWORK

Application filed March 27, 1928. Serial No. 265,200.

It is an object of my invention to provide a new and improved electrical network to simulate an extended electrical transmission circuit, or other transducer. Another object of my invention is to provide a compact network that shall accurately simulate a smooth line of a certain length. Still another object of my invention is to provide a network of fixed general pattern with certain parameters capable of determination or adjustment to make the network simulate accurately a transmission line of known properties. These objects and various other objects of my invention will become apparent on consideration of an example of practice according to the invention which I will disclose in the following specification taken with the accompanying drawings. It will be understood that this specification relates principally to this particular example of the invention, and that its scope will be indicated in the appended claims.

Referring to the drawings, Figure 1 is a diagram of my network with the impedances shown in generalized symbols; Fig. 2 is a diagram with the impedances shown more specifically; and Figs. 3 and 4 are characteristic diagrams showing the properties of this network over a certain range of frequencies.

It is desired to construct a network that shall simulate a smooth transmission line of length $l$, propagation constant $\gamma$, characteristic impedance $k$, series impedance $z_a$ and shunt impedance $z_b$, these impedance values being for the line as a whole.

It is known that for a smooth line the square of the propagation constant is equal to the product of the series impedance by the shunt admittance, and that the square of the characteristic impedance is equal to the quotient of the series impedance divided by the shunt admittance, such impedance and such admittance being taken per unit length of the line. See, for example, J. A. Fleming's book "The Propagation of Electric Currents in Telephone and Telegraph Conductors", third edition, 1919, page 84. From these properties, it can readily be deduced that for this smooth line

$$\gamma l = \sqrt{z_a/z_b} \quad (1)$$

and

$$k = \sqrt{z_a . z_b} \quad (2)$$

Let X= the open circuit impedance of this smooth line section, and let Y=its short-circuit impedance. Then it can readily be proved that

$$\gamma l = \sqrt{z_a/z_b} = \tanh^{-1}\sqrt{Y/X} \quad (3)$$

and

$$k = \sqrt{z_a . z_b} = \sqrt{XY} \quad (4)$$

These results can be obtained by the aid of formula 61 on page 99 of Fleming's book, above referred to.

From Equations (3) and (4), it follows readily that

$$z_a = k\gamma l = \sqrt{XY}\tanh^{-1}\sqrt{Y/X} \quad (5)$$

and

$$z_b = k/\gamma l = \sqrt{XY}/\tanh^{-1}\sqrt{Y/X} \quad (6)$$

Thus, $z_a$ and $z_b$ are inverse networks of impedance product $k^2$. In a physical smooth line, $z_a$ is simulated by series resistance and inductance, and $z_b$ by parallel resistance and capacity (assuming the line constants to be independent of frequency), both represented by simple physical networks. In other cases, they may be realized in desired frequency ranges, more or less approximately, by physical networks. It will be assumed in what follows that $z_a$ $z_b$ are given by the foregoing Equations (5) and (6).

The structure which is to simulate the smooth line consists of a finite number of lattice network sections, as shown in Fig. 1. In this particular illustrative example, the number of these sections is taken at two. It will be seen that in each section certain multiples and sub-multiples of the two impedances $z_a$ and $z_b$ are present, and that the multiples and sub-multiples involve the factors $m_1$ and $m_2$ in the first section, and $m'_1$ and $m'_2$ in the second section. We shall proceed to obtain more definite values for the $m$'s based on the assumption that the simulation is to be accurate only for moderate propagation lengths $\gamma l$.

Let $z_{11}$ equal twice the impedance of the parallel combination in Fig. 1, $m_1 z_a$ and $\frac{1}{m_2} z_b$; then

$$z_{11}=\frac{1}{1/2m_1 z_a+m_2/2z_b} \quad (7)$$

In a sequence of like lattice networks, the characteristic impedance is given by

$$K=\sqrt{z_{11}z_{21}} \quad (8)$$

wherein for each section $z_{11}/2$ represents the series impedance in each of two non-adjacent sides, and $2z_{21}$ represents the lattice impedance in each of the remaining two non-ad-

The foregoing result is for the first section of Fig. 1. By a similar procedure, the following equation is obtained for the second section of that figure,

$$e^{\Gamma'}=\frac{1+m'_1y+m'_1m'_2y^2}{1-m'_1y+m'_1m'_2y^2} \quad (15)$$

For the composite structure made up of these two sections in tandem, as shown in Fig. 1, the characteristic impedance condition is satisfied whatever the values of the $m$'s in Equations (14) and (15) since Equation (8) holds for each section. Therefore, the propagation constant for the two-section network of Fig. 1 will be given by multiplying together Equations (14) and (15), and, accordingly,

$$e^{\Gamma_c}=e^{\Gamma+\Gamma'}$$
$$=\frac{1+(m_1+m'_1)y+(m_1m_2+m_1m'_1+m'_1m'_2)y^2+(m_1m_2m'_1+m_1m'_1m'_2)y^3+m_1m_2m'_1m'_2y^4}{1-(m_1+m'_1)y+(m_1m_2+m_1m'_1+m'_1m'_2)y^2-(m_1m_2m'_1+m_1m'_1m'_2)y^3+m_1m_2m'_1m'_2y^4} \quad (16)$$

jacent sides. To satisfy the condition for the desired characteristic impedance at all frequences, we should have

$$K=k \quad (9)$$

and, accordingly, it follows that

$$z_{21}=\frac{m_2 z_a}{2}+\frac{z_b}{2m_1} \quad (10)$$

It remains to choose the $m$'s so that for moderate propagation lengths, $y=\gamma l$, the composite network will give

$$\Gamma_c \text{ approximately}=y=\gamma l \quad (17)$$

At this point, I shall introduce an important simplification by utilizing the following identity:

$$e^y=\frac{e^{y/2}}{e^{-y/2}}=\frac{1+y/2+y^2/8+y^3/48+y^4/384+y^5/3840+\ldots}{1-y/2+y^2/8-y^3/48+y^4/384-y^5/3840+\ldots} \quad (18)$$

It will be seen that not only are $z_a$ and $z_b$ inverse networks of impedance product $k^2$, as mentioned heretofore, but $z_{11}$ and $z_{21}$ are also inverse networks of impedance product $k^2$.

Let $\Gamma$ stand for the propagation constant per section of a sequence of like network sections such as shown in Fig. 1. It is known that

$$\cosh \Gamma=\frac{1}{2}(e^{\Gamma}+e^{-\Gamma})=1+\frac{2z_{11}}{4z_{21}-z_{11}} \quad (11)$$

Regarding the second and third members of the foregoing equation as a quadratic with $e^{\Gamma}$ as the unknown quantity, and solving accordingly, and substituting from Equation (8), the result is obtained that

$$e^{\Gamma}=\frac{1+z_{11}/2K}{1-z_{11}/2K} \quad (12)$$

Substituting in Equation (12) from Equations (7) to (10), and simplifying, and letting

$$y=\sqrt{z_a/z_b}=\gamma l \quad (13)$$

the result is obtained that

$$e^{\Gamma}=\frac{1+m_1y+m_1m_2y^2}{1-m_1y+m_1m_2y^2} \quad (14)$$

Comparing Equations (16) and (18), it will at once be seen that for small values of $y$, we can satisfy Equation (17) if we identify the coefficients of powers of $y$ in equation (16) as follows:

$$m_1+m'_1=\frac{1}{2},$$
$$m_1m_2+m_1m'_1+m'_1m'_2=\frac{1}{8}, \quad (19)$$
$$m_1m_2m'_1+m_1m'_1m'_2=\frac{1}{48},$$
$$m_1m_2m'_1m'_2=\frac{1}{384}$$

These four Equations (19) involve the four $m$'s as unknown quantities. By elimination, the following sixth degree equation is obtained for $m_1$:

$$m_1^6-3m_1^5/2+m_1^4-3m_1^3/8+5m_1^2/64-m_1/128+1/4608=0 \quad (20)$$

and from the first of equations (19),

$$m'_1=\frac{1}{2}-m_1 \quad (20^a)$$

In terms of $m_1$ and $m'_1$, the remaining $m$'s are expressed as follows:

$$m_2=\frac{6m_1-48m_1^2m'_1-1}{48m_1(m_1-m'_1)},$$
$$m'_2=\frac{1+48m_1(m'_1)^2-6m'_1}{48m'_1(m_1-m'_1)} \quad (21)$$

Of the real positive roots of Equation (20), the following is found as suitable for our procedure:

$$m_1=0.45737 \quad (22)$$

From this, by the aid of Equations ($20^a$) and (21), the values for the remaining $m$'s are obtained as follows:

$$\begin{aligned} m_2 &= 0.14456 \\ m'_1 &= 0.04263 \quad (23) \\ m'_2 &= 0.92403 \end{aligned}$$

These values for the $m$'s are fixed for all cases, no matter what values $z_a$ and $z_b$ may have for the given smooth line. With these fixed values for the $m$'s, and with Equations (5), (6), (7), (10) and corresponding equations for the second section of Fig. 1, the network can be constructed which is to simulate any smooth line having physically realizable impedance values $z_a$ and $z_b$. This simulation is very accurate for small values of $y$. As $y$ increases, the departure of the network propagation characteristic from the smooth line value also increases, but it amounts to less than 1.4% even at $|y|=3.0$; this may be inferred from a comparison of Equations (16) and (18).

Further to illustrate my invention, the foregoing results were applied in the case of a 104-mil open wire smooth line having the constants per loop mile (for wet weather, and assumed independent of frequency):

$$\begin{aligned} R &= 10.12 \text{ ohms} \\ L &= 3.66 \text{ mh.} \\ G &= 3.20 \text{ micromhos} \quad (24) \\ C &= 0.00837 \text{ m}^f. \end{aligned}$$

where R, L, G and C stand, respectively, for series resistance, series inductance, shunt conductance and shunt capacity per loop mile. The corresponding network for a length $l$ is shown in Fig. 2. The values of its elements are determined by comparison with Fig. 1, taking the values of the $m$'s found above, and observing that

$$\begin{aligned} z_a &= (R+iL\omega)l, \\ z_b &= 1/(G+iC\omega)l. \quad (25) \end{aligned}$$

More particularly, the values of the resistances, inductances and capacities in Fig. 2 are as follows, expressed in terms of Equations (22), (23) and (24);

$$\begin{array}{llll} R_1=m_1Rl, & R_2=1/m_1Gl & R'_1=m'_1Rl, & R'_2=1/m'_1Gl \\ L_3=m_1Ll, & C_4=m_1Cl & L'_3=m'_1Ll, & C'_4=m'_1Cl \\ R_5=1/m_2Gl, & R_6=m_2Rl & R'_5=1/m'_2Gl, & R'_6=m'_2Rl \\ C_7=m_2Cl, & L_8=m_2Ll & C'_7=m'_2Cl, & L'_8=m'_2Ll \end{array}$$

A comparison of the propagation characteristic of the given smooth line and its simulating network is shown in Fig. 3 for three different line lengths, namely, 10, 20 and 30 miles, respectively. Even for the longest length, the simulation is close, up to frequencies of 3,000 cycles per second. The characteristic impedances are, of course, the same for all lengths and are as shown in Fig. 4. For greater lengths, one may use more sections. For example, if the 2-section network here considered is equivalent to 20 miles of smooth line then two such 2-section networks in tandem would be equivalent to 40 miles of such smooth line.

While I have exemplified my invention for simulating a smooth line of moderate length, it will be apparent that the same procedure will applying in the case of any symmetrical general section of any passive transducer where $z_a$ and $z_b$ are determined from the open circuit and short circuit impedances, X and Y, of the section and can be simulated in the desired frequency range by simple physical elements.

In the following claims I use the term "multiple" to include sub-multiple, unless expressly stated otherwise, i. e., I use multiple to mean a factor less than unity as well as greater than unity.

I claim:

1. A smooth line having a certain series impedance and a certain shunt impedance and a network associated therewith and simulating it, said network comprising a plurality of lattice sections each with two impedances in each of its arms, these last mentioned impedances being respectively real number multiples of said series impedance and shunt impedance.

2. A smooth line having a certain series impedance and a certain shunt impedance and a network associated therewith and simulating it, said network comprising a plurality of lattice sections each with impedances in its arms, each such impedance being a real number multiple of one of said first mentioned series and shunt impedances.

3. A smooth line of series impedance $z_a$ and shunt impedance $z_b$ and a network of several lattice sections associated therewith and simulating it, one pair of non-adjacent arms of one section each having the admittance value $1/m_1z_a+m_2/z_b$ and the other pair each having the impedance value $m_2z_a+z_b/m_1$, where the $m$'s are determinate real number constants.

4. A network of several lattice sections to simulate a smooth line of series impedance $z_a$ and shunt impedance $z_b$, one pair of non-adjacent arms of one section each having the admittance value $1/m_1z_a+m_2/z_b$ and the other pair each having the impedance value $m_2z_a+z_b/m_1$, where, for one section, $m_1=0.45737$ and $m_2=0.14456$, and for another section, $m'_1=0.04263$ and $m'_2=0.92403$.

In testimony whereof, I have signed my name to this specification this 24th day of March, 1928.

OTTO J. ZOBEL.