June 24, 1924. 1,498,915

R. S. HOYT

ARTIFICIAL LINE

Filed Nov. 11, 1919 3 Sheets-Sheet 1

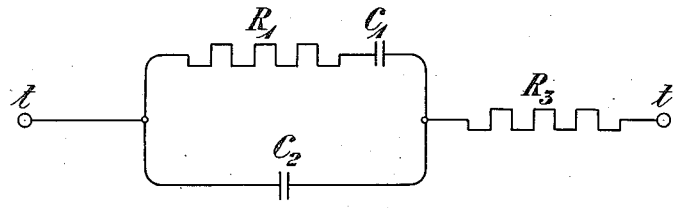


Fig. 1

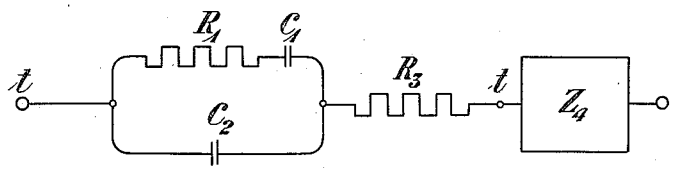


Fig. 1a

Fig 2

INVENTOR.
R. S. Hoyt
BY
ATTORNEY

June 24, 1924. 1,498,915

R. S. HOYT

ARTIFICIAL LINE

Filed Nov. 11, 1919 3 Sheets-Sheet 2

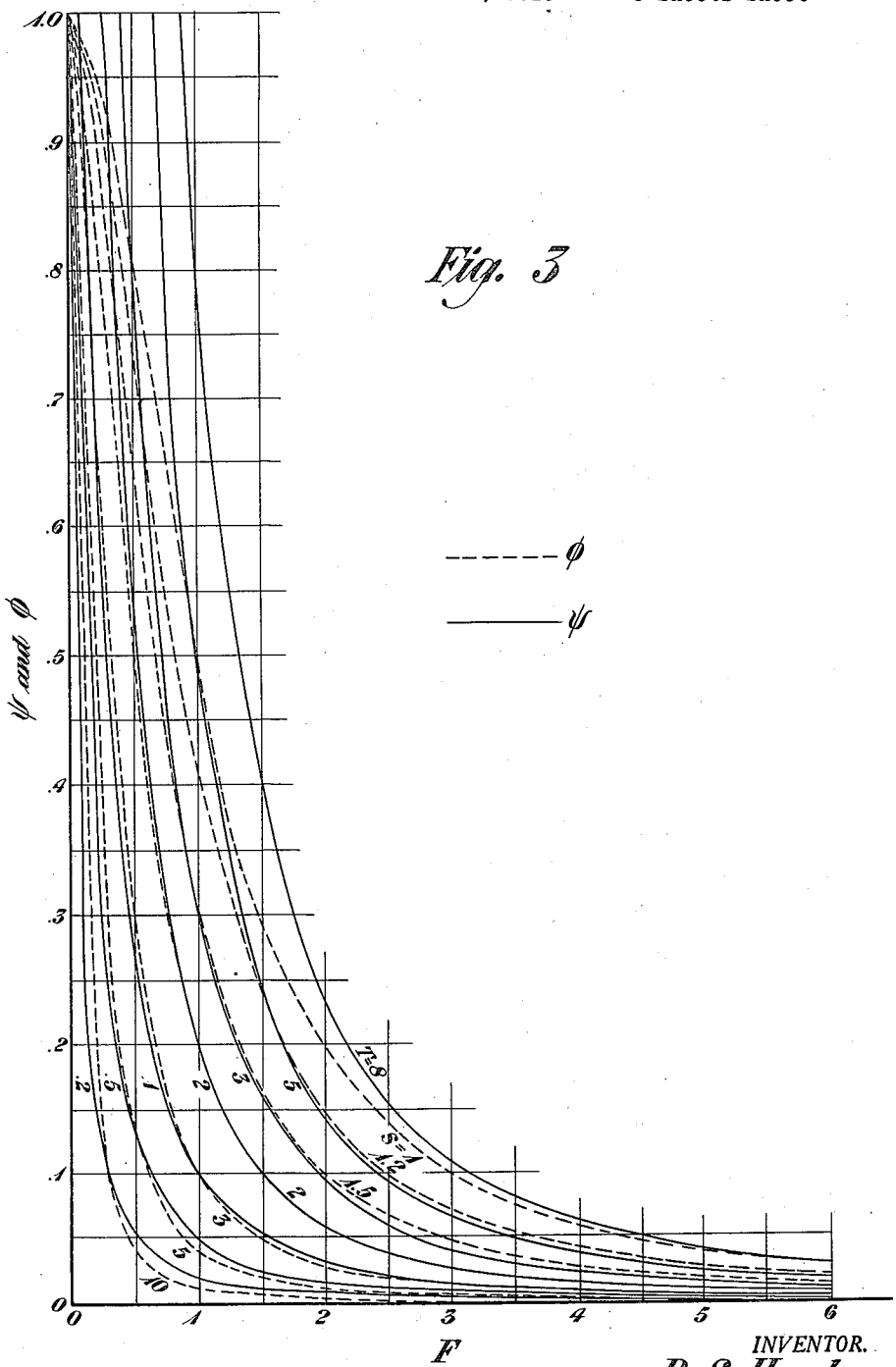


Fig. 3

------ φ

——— ψ

INVENTOR.
R. S. Hoyt
BY
ATTORNEY

June 24, 1924. 1,498,915

R. S. HOYT

ARTIFICIAL LINE

Filed Nov. 11, 1919 3 Sheets-Sheet 3

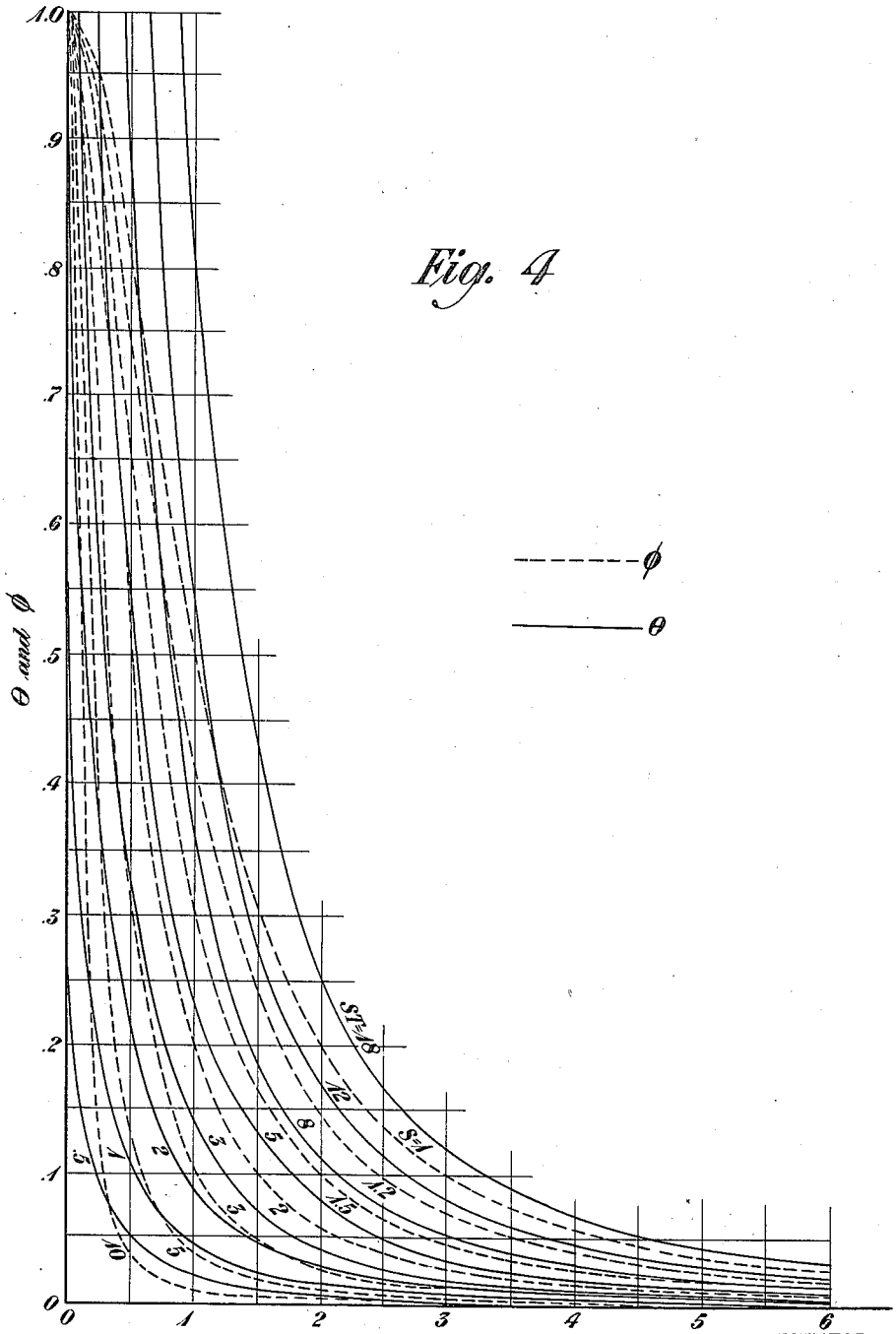


Fig. 4

INVENTOR.
R. S. Hoyt
BY
ATTORNEY

Patented June 24, 1924. 1,498,915

UNITED STATES PATENT OFFICE.

RAY S. HOYT, OF RIDGEWOOD, NEW JERSEY, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

ARTIFICIAL LINE.

Application filed November 11, 1919. Serial No. 337,347.

To all whom it may concern:

Be it known that I, RAY S. HOYT, residing at Ridgewood, in the county of Bergen and State of New Jersey, have invented certain Improvements in Artificial Lines, of which the following is a specification.

This invention relates to an artificial line for simulating the impedance of a transmission line. More particularly, it relates to an artificial line comprising two main portions: a "fundamental artificial line" for simulating the impedance that an actual line would have if devoid of wire-resistance; and a "supplementary artificial line" for simulating the increase produced in the impedance of said actual line by its wire-resistance.

For brevity in what follows, this increase in the impedance of a line caused by its wire-resistance will be termed the "excess impedance"; or, more precisely, the "excess characteristic impedance", since this invention is concerned primarily with lines that are long or at least effectively long.

The form of the fundamental artificial line and also the method of proportioning it, depend on the type of the actual line involved (smooth line or periodically loaded line). For instance, in the case of a smooth line, the requisite fundamental artificial line is merely a constant resistance; while in the case of a loaded line the requisite fundamental artificial line consists in general of a combination of several different elements (resistance, inductance, capacity) as fully set forth, for instance, in my previous patents on artificial lines Nos. 1,124,904 and 1,167,693 and in my pending application Serial No. 309,633, filed July 9, 1919, which disclose various forms of such fundamental artificial lines and the corresponding methods of proportioning.

On the other hand my supplementary artificial line herein described has the same form and the same method of proportioning for all existing types of actual lines, and is therefore universally adaptable for supplementing any of the above-cited or any other suitable forms of fundamental artificial lines. The need for a supplementary artificial line of this character has arisen particularly in connection with recent repeater development where, to admit of large gains without singing, the artificial lines must simulate with high precision over an extremely wide frequency-range the impedance characteristics of the transmission lines with which they are associated, the types of artificial lines heretofore used having been found inadequate for the lower frequencies of the range.

My invention may readily be understood from the following description, reference being had to the accompanying drawings in which Fig. 1 is a diagram of one form of my supplementary artificial line; Fig. 1A is a diagram illustrating the employment of this form of supplementary artificial line to supplement any type of fundamental artificial line, the combination thus constituting a complete artificial line; Fig. 2 is a plot of the impedance characteristics of smooth lines; and Figs. 3 and 4 are charts for use in evaluating the requisite constants of my supplementary artificial line for any specific case.

As prerequisite to an adequate understanding of the proportioning and use of my supplementary artificial line the exact nature and magnitude of the above-mentioned "excess characteristic impedance" will now be elucidated, after which the supplementary artificial line itself will be fully treated.

It is fairly well-known that the characteristic impedance of an efficient telephone transmission line, over most of the telephonic frequency-range, depends mainly on its inductance and capacity and only slightly on its wire resistance. Hence the "excess characteristic impedance" of such a line is relatively small; in fact, for most of the lines and frequency-ranges employed in practice until recently, the excess characteristic impedance has been either entirely negligible, or else small enough to be sufficiently simulated by means of a mere condenser (or, at most, by a condenser in series with a very small inductance).

Recently, however, a more precise simulation of the excess characteristic impedance has become necessary; partly because of the employment of small gauge and therefore less efficient lines (rendered possible by repeater developments), and partly because of a need for extending the frequency-range to lower frequencies—the excess characteristic impedance being greater for lines of low efficiency than for lines of high efficiency, and for any given line increasing

with decrease of frequency (as brought out below). This new need is met by my supplementary artificial line herein disclosed.

To arrive at a formula for the excess characteristic impedance of a transmission line let $K=M+iN$ denote the characteristic impedance of the line, M and N thus denoting its characteristic resistance and charteristic reactance respectively; let $K'=M'+iN'$ denote the characteristic impedance if the line were devoid of wire-resistance; and let $k=m+in$ denote the excess characteristic impedance of the actual line, m thus denoting its excess characteristic resistance and n its excess characteristic reactance. Then, by the definition of excess characteristic impedance, we have

$$k=K-K' \quad (1)$$

so that

$$m=M-M' \quad (2)$$

and

$$n=N-N' \quad (3)$$

In the following analysis of the excess characteristic impedance it suffices to treat only smooth lines explicitly (a smooth line being one whose constants are all uniformly distributed), because the excess characteristic impedance of a periodically loaded line is approximately the same, over most of the frequency-range below the "critical frequency" of the loaded line, as the excess characteristic impedance of a smooth line having the same total constants as the given loaded line. It will thus be seen that my supplementary artificial line herein disclosed is applicable to periodically loaded lines (including periodically loaded cables), as well as to smooth lines (including non-loaded cables, non-loaded aerial lines, smoothly loaded cables, and smoothly loaded aerial lines).

For a smooth line possessing inductance L, capacity C, and resistance R per unit length, the value of K is given by the well-known formula

$$K=M+iN=\sqrt{(R+ipL)\,/\,ipC} \quad (4)$$

wherein p denotes 2π times the frequency f, and i denotes the imaginary operator $\sqrt{-1}$.

Since for any given line we are concerned with the impedance as a function of the frequency f, the study of the above formula (4) for K will be simplified and at the same time rendered more comprehensive by introducing the two symbols g and F defined by the equations

$$g=\sqrt{L/C} \quad (5)$$

$$F=pL/R=(2\pi L/R)f \quad (6)$$

F thus being proportional to the frequency f; for then formula (4) becomes

$$K=M+iN=g\sqrt{1-i/F} \quad (7)$$

which involves only the one parameter g, F being regarded as the independent variable since it is proportional to f. In particular, for a smooth line devoid of wire-resistance (R=O) the values of K, M, N—then denoted by K', M', N'—are merely

$$K'=M'=g \quad (8)$$

$$N'=O \quad (9)$$

(Thus the parameter g is interpretable as being the characteristic impedance that the line would have if devoid of wire-resistance.) Hence the excess characteristic impedance $k=K-K'$ is given by the formula

$$k=g(\sqrt{1-i/F}-1) \quad (10)$$

To secure formulas for the two components m and n of k and also formulas for the two components M and N of K, it is necessary to carry out the indicated root-extraction

$$\sqrt{1-i/F}$$

to obtain the real and imaginary components of this complex quantity

$$\overline{1-i/F}$$

which occurs in the formula (10) for k and in the formula (7) for K. Performing this operation with reference to (7), the resistance and reactance components M and N of the characteristic impedance K will be found to have the values given by the formulæ

$$\frac{M}{g}=\sqrt{\frac{F+\sqrt{1+F^2}}{2F}} \quad (11)$$

$$\frac{-N}{g}=\frac{1}{2F}\sqrt{\frac{2F}{F+\sqrt{1+F^2}}} \quad (12)$$

whence the excess characteristic resistance $m=M-M'$ and the excess characteristic reactance $n=N-N'$ are given by the formulas

$$\frac{m}{g}=-1+\sqrt{\frac{F+\sqrt{1+F^2}}{2F}}=1/\left(F+\sqrt{1+F^2}\right)\left[\sqrt{2F}+\sqrt{F+\sqrt{1+F^2}}\right]\sqrt{2F} \quad (13)$$

$$\frac{-n}{g}=\frac{1}{2F}\sqrt{\frac{2F}{F+\sqrt{1+F^2}}} \quad (14)$$

For use below, it may be here noted that

$$\frac{-n}{m}=\left[\sqrt{2F}+\sqrt{F+\sqrt{1+F^2}}\right]\sqrt{F+\sqrt{1+F^2}} \quad (15)$$

which is obviously greater than unity for all positive values of F.

To furnish a clear and exact idea of how the quantities M, N, m, n depend on the fre-

quency f, Fig. 2 gives graphs of M/g, $-N/g$, m/g, $-n/g$ as functions of F—which, for any fixed line, is proportional to f, it will be remembered. These graphs bring out the fact—indicated by formula (15)—that the excess characteristic resistance m is, at all values of F, smaller than the negative excess characteristic reactance $-n$; in fact that m is much smaller than $-n$ except at very small values of F, m there approaching $-n$. The graphs show also that m and $-n$ are both small compared with g at the larger values of F but that they both increase with decreasing F until at small values of F they become comparable with g, and at still smaller values of F they even exceed g.

Since $F=(2\pi L/R)f$, the curves of Fig. 2 show also that, at any fixed frequency f the excess characteristic impedance of any given line (L and C fixed) increases as R increases—the above formula for F showing that increasing R has the same effect as decreasing f.

Thus it is seen that ordinarily the increase produced in the characteristic impedance of a transmission line by its wire-resistance is mainly an excess characteristic reactance, which may be very large at low frequencies but rapidly decreases with increase of frequency, ordinarily becoming negligible at high and at medium frequencies and often at low frequencies even. In fact, for the frequency-ranges and the lines employed in practice until recently, the excess characteristic resistance has been entirely negligible; while the excess characteristic reactance, though not always negligible—and even rather large in some cases—has been simulated sufficiently closely by means of a mere condenser.

Having thus furnished an exact and comprehensive idea of the nature and magnitude of the excess characteristic impedance, I may now proceed to describe the construction and proportioning of my artificial line, whose impedance is to simulate the excess characteristic impedance above described.

I have found that an artificial line (supplementary artificial line) of the type shown in Fig. 1 can, by proper proportioning of the elements thereof, be caused to simulate the above-defined excess impedance very closely over a wide range of frequencies; whence the total impedance of the actual line can be simulated by the complete artificial line, represented by Fig. 1^A, consisting, of said supplementary artificial line connected in series with the requisite fundamental artificial line, whose impedance is there denoted by Z_4. For the purpose of evaluating the elements of said supplementary artificial line, I may regard the same as consisting of two parts serially connected between the two terminals t, t. Part 1, which is the major part, comprises a resistance element R_1 and a capacity element C_1 in series with each other, and a capacity element C_2 in shunt therewith; while part 2 comprises merely a resistance element R_3. The artificial line is thus made up of four impedance-elements only. The resultant impedance of the combination, when the elements are proportioned as hereinafter set forth, with reference to the constants of the contemplated actual line, closely simulates the excess characteristic impedance of said actual line over a wide range of frequencies.

Let $Z=X+iY$ denote the impedance of the entire supplementary artificial line, between the terminals t, t, represented by Fig. 1; X and Y thus denoting respectively the resistance component and the reactance component of said impedance Z. Then

$$Z=X+iY=R_3+\frac{R_1C_1^2-i(C_1+C_2+R_1^2C_1^2C_2p^2)/p}{(C_1+C_2)^2+(R_1C_1C_2p)^2} \quad (16)$$

whence

$$X=R_3+\frac{R_1C_1^2}{(C_1+C_2)^2+(R_1C_1C_2p)^2} \quad (17)$$

and

$$-Y=\frac{(C_1+C_2+R_1^2C_1^2C_2p^2)/p}{(C_1+C_2)^2+(R_1C_1C_2p)^2} \quad (18)$$

To secure the desired impedance-simulation the impedance elements R_1, C_1, C_2, R_3 of the supplementary artificial line must be proportioned with reference to the given values of the constants L, C, R of the actual line. There are various procedures that might be followed in arriving at suitable values of the impedance-elements R_1, C_1, C_2, R_3. In that systematic procedure which is now to be set forth the impedance-elements R_1, C_1, C_2, R_3 are not determined directly, but are determined indirectly in terms of certain parameters whose direct determination is more readily accomplished, namely

the parameters S, T, r, C_{21} defined by the following equations:

$$S=\left(\frac{R_1C_1C_2}{C_1+C_2}\right)\frac{R}{L} \quad (19)$$

$$T=\frac{(C_1+C_2)^2}{R_1C_1^2}\sqrt{\frac{L}{C}} \quad (20)$$

$$r=R_3(C_1+C_2)^2/R_1C_1^2 \quad (21)$$

$$C_{21}=C_2/C_1 \quad (22)$$

whence the constants R_1, C_1, C_2, R_3 of the supplementary artificial line in terms of the parameters S, T, r, C_{21}, and in terms of the constants L, C, R of the actual line, are

$$R_1=\frac{(1+C_{21})^2}{T}\sqrt{\frac{L}{C}} \quad (23)$$

$$C_1=\frac{ST}{C_{21}(1+C_{21})}\sqrt{\frac{LC}{R}} \quad (24)$$

$$C_2=\frac{ST}{1+C_{21}}\sqrt{\frac{LC}{R}} \quad (25)$$

$$R_3=\frac{r}{T}\sqrt{\frac{L}{C}} \quad (26)$$

By means of the parameters S, T, r, C_{21} and the variable F, the values of X and Y as given originally by (17) and (18) can be expressed in the following simple and convenient forms:

$$\frac{X}{g}=\frac{r}{T}+\frac{1}{T(1+S^2F^2)} \quad (27)$$

$$\frac{-Y}{g}=\frac{SF}{T(1+S^2F^2)}+\frac{C_{21}}{TSF} \quad (28)$$

Since the purpose of the supplementary artificial line is to simulate by its impedance $Z=X+iY$ the excess characteristic impedance $k=m+in$ of the actual line, the most fundamental design-formula is evidently

$$Z=k \quad (29)$$

and the proportioning of the supplementary artificial line is to be such as to fulfill this equation as closely as may be. Since the impedances are complex, this equation implicity contains the two fundamental design-formulæ

$$X=m \quad (30)$$

$$Y=n \quad (31)$$

or, what is equivalent,

$$X/g=m/g \quad (32)$$

$$Y/g=n/g \quad (33)$$

Since the main part of the design-work consists in evaluating the parameters S, T, r, C_{21} characterizing the supplementary artificial line, in terms of the known constants of the actual line, the quantities occurring in the fundamental design-formulæ (30) and (31) must next be expressed in terms of the above-mentioned parameters. This can be done by means of (13), (14), (27), (28); and, for design-purposes, the two resulting relations can be written most conveniently in the forms

$$r=\psi(F,T)-\phi(F,S) \quad (34)$$

$$-1-C_{21}+ST/2=\theta(F,ST)-\phi(F,S) \quad (35)$$

where the functions on the right-hand sides have the following explicit meanings

$$\phi(F,S)=1/(1+S^2F^2) \quad (36)$$

$$\psi(F,T)=T/\left(\sqrt{2F}+\sqrt{F+\sqrt{1+F^2}}\right)\left(F+\sqrt{1+F^2}\right)\sqrt{2F} \quad (37)$$

$$\theta(F,ST)=ST/2\left(\sqrt{2F}+\sqrt{F+\sqrt{1+F^2}}\right)\left(F+\sqrt{1+F^2}\right)^{3/2} \quad (38)$$

In these formulæ the quantity F, which for any fixed line is by (6) proportional to the frequency f, is to be regarded as the independent variable, since we are concerned with impedance-simulation over a wide frequency-range; and the remaining quantities are to be treated as parameters.

Referring now to the two design formulæ (34) and (35) it will be recalled that they were established from the assumption that the impedance of the supplementary artificial line is equal to the excess characteristic impedance of the actual line; in fact they are nothing more nor less than indirect statements of such assumed equality. Now, r and C_{21} are constants of the supplementary artificial line (r being proportional to the resistance-element R_3, and C_{21} being the ratio of two capacity-elements). Hence, mathematically stated, the problem is reduced to determining what values of the parameters S and T will render r and $-1-C_{21}+ST/2$, expressed by (34) and (35), as nearly as possible independent of F over the contemplated F-range, for positive values of r and C_{21}. (Negative values of r are discussed below).

The evaluation of the parameters for specific cases can be accomplished by means of the sets of curves furnished herewith in Figs. 3 and 4. Fig. 3 gives two superposed sets of curves; solid curves, which are graphs of ψ (F,T) as function of F with T as parameter; and dashed curves, which are graphs of ϕ (F,S) as function of F with S as parameter. Similarly Fig. 4 gives two superposed sets of curves; solid curves, which are graphs of θ (F,ST) as function of F with the product ST as parameter; and dashed curves, which are graphs of ϕ (F,S)

as function of F with S as parameter. (Except for complexity and resulting confusion, the ϕ, ψ and θ curves might evidently be plotted in a single diagram as one three-fold set of curves).

To evaluate the parameters for any specific case, the sets of curves in Figs. 3 and 4 are to be inspected, over the particular F-range contemplated in said specific case, with the object of selecting a certain ϕ-curve common to the two sets of curves; and in Figs. 3 and 4, respectively, selecting a ψ-curve and a θ-curve that are approximately parallel to the common ϕ-curve, and moreover are such that the value of ST for the θ-curve is equal to the product of the values of S and T for the ϕ-curve and the ψ-curve; the selection, moreover, to be such as to yield a positive value for C_{21} when computed from (35).

When my supplementary artificial line is to be used independently, i. e., as an addition to an already existing artificial line and without change in the elements of either, it is necessary that the chosen ψ-curve be not below the chosen ϕ-curve in order that r, and hence R_3, shall not be negative. This is not always necessary, however, when the supplementary artificial line is not to be used independently, for then the remaining artificial line in series with the supplementary artificial line can sometimes be so proportioned as to absorb a negative R_3-element; this being accomplished in the design by reducing by an amount equal to $-R_3$ the resistance component of the impedance of the remaining artificial line. (In the case of non-loaded cables, the design can often be so carried out that the negative R_3-element will be exactly absorbed, whence then the resulting total artificial line will consist merely of the principal part (R_1,C_1,C_2) of the supplementary artificial line represented in Fig. 1).

Although the sets of curves given in Figs. 3 and 4 cover a fairly wide range of practical cases, no attempt has been made to render them sufficiently extensive to cover all cases that might arise; their purposes here being mainly illustrative. For cases falling outside of their range they can be readily extended by means of formulæ (36), (37), (38). Moreover, for illustrative purposes, it has sufficed to plot them rather sparsely, whence they can serve only for approximate evaluation of the parameters, more refined evaluations requiring more densely plotted sets for values of the parameters in the neighborhoods of the values first determined approximately.

From a little study of the design-charts of Figs. 3 and 4, it will be seen that in many cases there will be a considerable range in the choice of the parameters and hence a considerable range in the choice of values for the constants of the artificial line to simulate approximately the impedance of the specified actual line over the preassigned frequency-range. By taking advantage of this fact, it is often possible to arrive at constants having values more suitable for commercial applications than would otherwise be obtained.

As a simple example illustrating the application of my invention, let it be required to construct an artificial line for closely simulating, over the frequency-range extending from f=200 to f=2500 cycles per second, the impedance of a long smooth transmission line having the following specifications: An aerial line consisting of two parallel #12 N.B.S. gauge copper wires having an inductance L=0.00367 henrys per mile, a capacity C=0.00835×10^{-6} farads per mile, and a resistance R=10.4 ohms per mile.

If this line were devoid of wire-resistance (that is, if R=O) the line impedance at all frequencies would be equal merely to

$$\sqrt{L/C}=663 \text{ ohms},$$

and hence would be exactly simulatable by a mere constant resistance of 663 ohms, which therefore is in this example the particular value of Z_4 in Fig. 1A. As explained above, the impedance of the actual line (R not zero) exceeds this limiting value

$$\sqrt{L/C}$$

by an amount, there termed the "excess impedance," which it is the province of the supplementary artificial line of my present invention to simulate; such supplementary artificial line therefore to be connected in series with Z_4, the combination thus constituting the required complete artificial line for simulating the total impedance of the specified actual line.

As the first step in the proportioning of the supplementary artificial line, the values of L and R are to be substituted in equation (6). It will be thus found that F=0.00222 f, and hence that the preassigned f-range of 200 to 2500 corresponds to an F-range of 0.444 to 5.55—which is therefore the F-range over which impedance simulation is required. Next, inspecting over this particular F-range the sets of curves in Figs. 3 and 4 in the manner above described for evaluating the parameters S and T it will be found by interpolation that suitable values of S and T are S=2.3, T=1.7, and hence ST=3.9. For these values of S and T, the ψ-curve and ϕ-curve are substantially coincident, so that by (34) we may regard r as equal to zero. Moreover, the θ-curve and ϕ-curve also are substantially coincident, so that by (35) we find that C_{21}=0.95.

Thus, summarized, the parameters of the supplementary artificial line have the values

S=2.3, T=1.7, ST=3.9, r=0, C_{21}=0.95; and the constants of the actual line have the values L=0.00367, C=0.00835×10^{-6}, R=10.4. Hence, by formulæ (23) (24), (25), (26), the requisite constants of the supplementary artificial line are R_1=1483 ohms, C_1=1.12×10^{-6} farads, C_2=1.06×10^{-6} farads, R_3=0; and, to form the complete artificial line required, this supplementary artificial line is to be connected in series with a resistance of 663 ohms, as represented by Fig. 1[A] when Z_4=663.

Computations made for the purpose of determining the simulative precision of the complete artificial line, as represented in per cent by the function d=100|Z-K|/|K| gave the following values of d (from which additional values can be obtained by plotting a curve, if desired):

f, Cycles per second.	d, Per cent.
200	1.78
300	.54
500	.55
800	.37
1200	.20
1600	.13
2000	.09
2300	.06
2500	.06

Thus, over most of the contemplated frequency-range of 200 to 2500 cycles per second, the complete artificial line of this example does not differ from the actual line in impedance by more than about one-half of one per cent; while its greatest departure (occurring at f=200) is less than two per cent. As above remarked, in the design-work of this illustrative example the parameters S and T—which determine (indirectly) the constants of the supplementary artificial line—were evaluated by a mere inspectional process of interpolation in the rather sparse sets of curves furnished in Figs. 3 and 4; that is, S and T were evaluated without recourse to any curves more densely plotted, and without recourse to any tentative computations of the precision. By such recourse, higher and more uniform simulative precision could be secured if desired, but the precision attained in this example without such recourse is sufficiently good for most applications.

Although only one form of apparatus embodying this invention is shown and described herein it is readily understood that various changes and modifications may be made therein within the scope of the following claims without departing from the spirit and scope of the invention.

What I claim is:

1. An artificial line comprising resistance and reactance elements designed and combined so that the impedance over a wide range of frequencies is substantially the same as the precomputed excess impedance of an actual line.

2. An artificial line comprising a plurality of impedance elements whose values are computed as functions dependent upon the resistance component of the impedance effect caused by the resistance of the conductors of an actual line.

3. An artificial line for simulating that part of the impedance of an actual line which is caused by the resistance of its conductors, said artificial line comprising a plurality of impedance elements whose values are determined by a computation based on the resistance and reactance components of that part of the impedance due to the resistance of the line conductors.

4. An artificial line comprising a plurality of portions connected in series with each other, one of said portions being comprised of a plurality of parallel branches, and impedance elements in said branches and in said other portions, some of said impedance elements being resistances and some of them being reactances, the values of said elements being determined by a computation based on the variation over a wide range of frequencies of the impedance effect due to the resistance of the conductors of an actual line.

5. An artificial line comprising a plurality of portions connected in series with each other, one of said portions comprising a resistance element; another of said portions comprising a plurality of parallel branches, one branch comprising a capacity element, another branch a resistance element in series with a capacity element, the values of said elements being determined by a computation based on the variation over a wide range of frequencies of the impedance effect due to the resistance of the conductors of an actual line.

6. An artificial line which consists of a plurality of portions, one having its impedance elements computed to simulate over a wide range of frequencies the impedance of a line assuming the line conductors to have no resistance, and another having its impedance elements computed to simulate over a wide range of frequencies both the resistance and reactance components of the impedance effect due to the actual resistance of the said line conductors.

7. An artificial line which consists of two portions, one simulating a given line on the assumption that its ohmic resistance is zero, and the other computed as a function of the constants of the given line to supply the impedance defect involved in the assumption on which the first mentioned portion of the artificial line is based, said other portion

comprising resistance and reactance elements in combination.

8. The method of balancing the impedance of a line which consists in balancing its impedance on the assumption that its ohmic resistance is zero, and then balancing out the impedance defect involved in that assumption by means of a network computed as a function of the constants of the given line.

In testimony whereof, I have signed my name to this specification this 10th day of November, 1919.

RAY S. HOYT.