March 17, 1964 R. J. KOEHLER 3,125,758

INFLATED ANTENNA

Filed Oct. 17, 1960 3 Sheets-Sheet 1

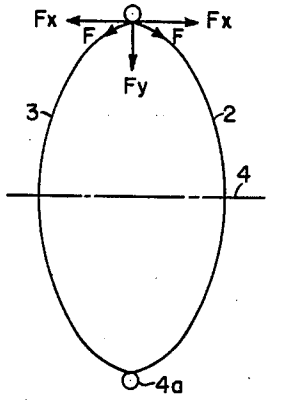


Fig. 1.

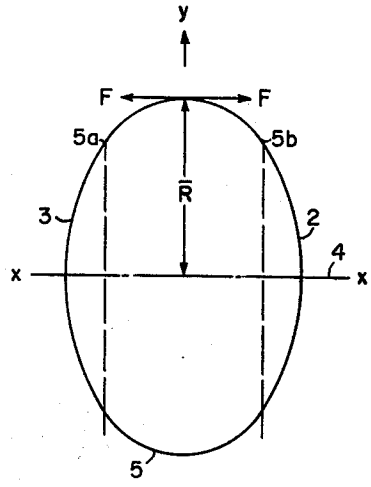


Fig. 2.

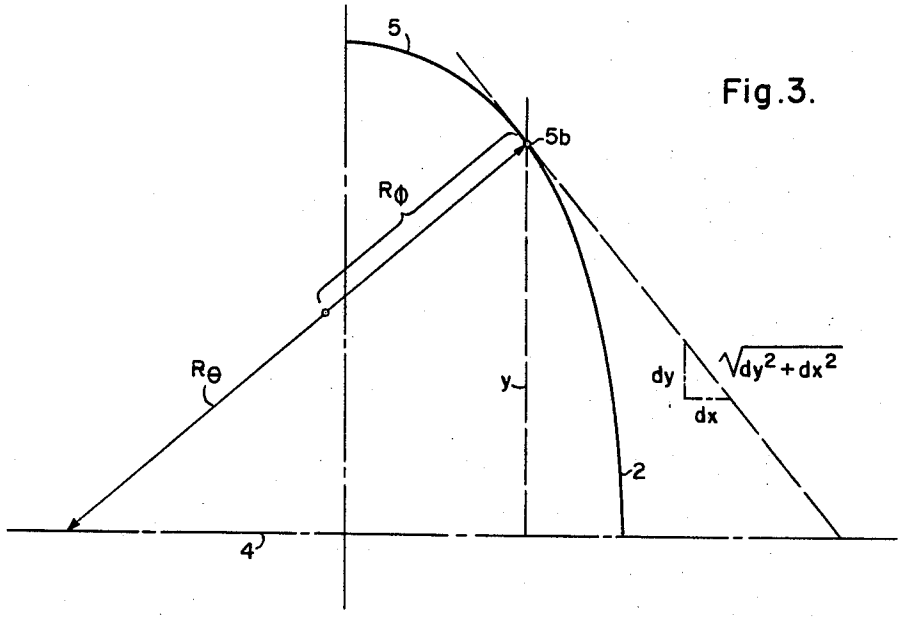


Fig. 3.

WITNESSES

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INFLATED ANTENNA

Filed Oct. 17, 1960 3 Sheets-Sheet 2

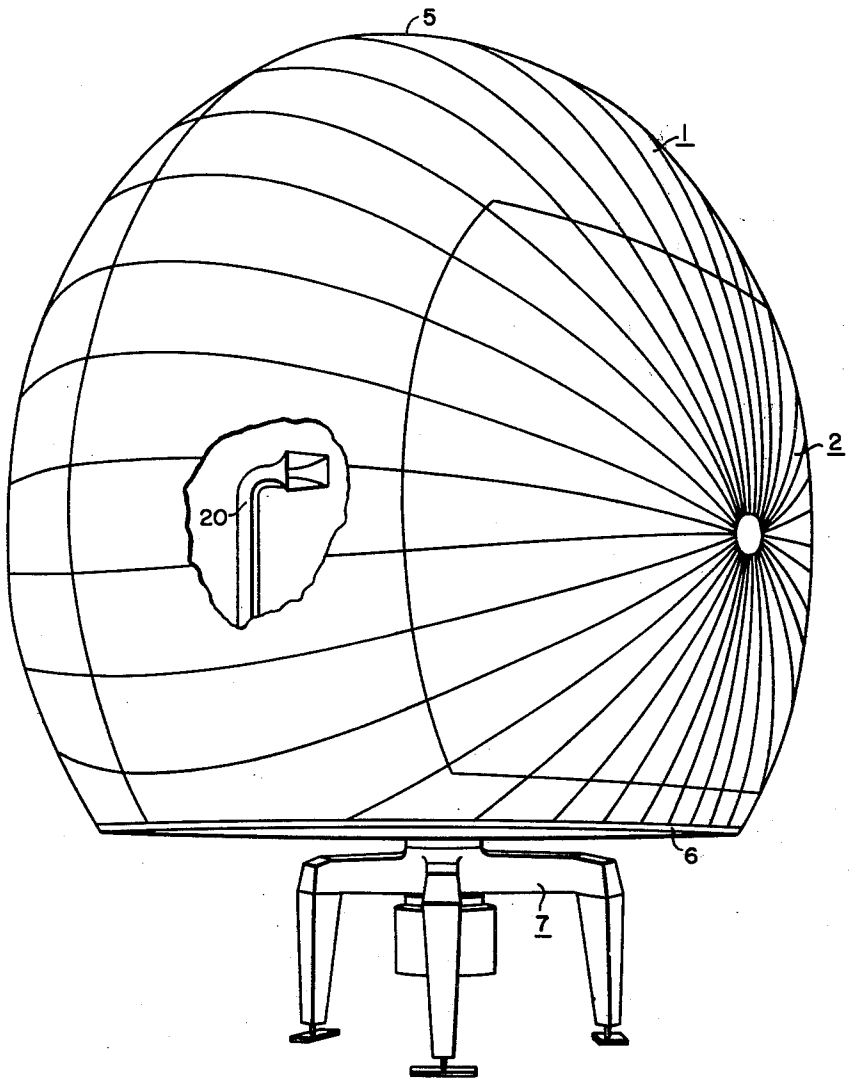


Fig. 4.

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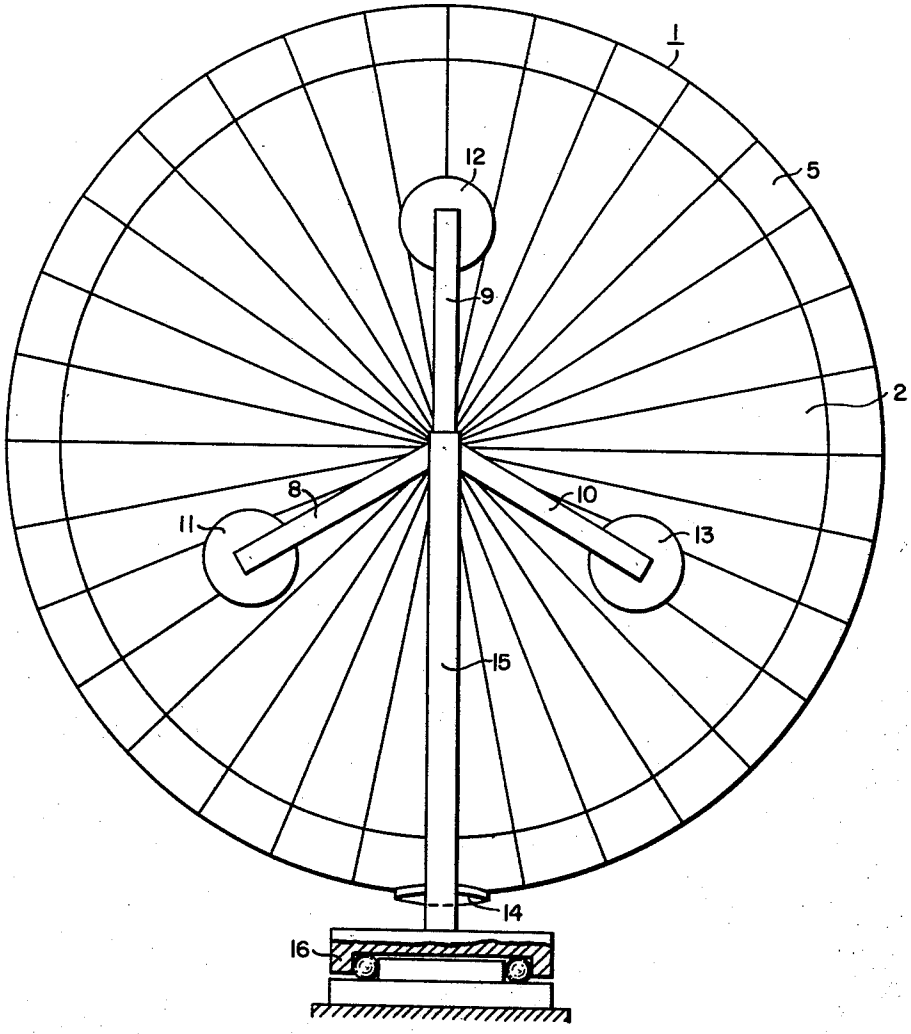


Fig.5.

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3,125,758

INFLATED ANTENNA

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Filed Oct. 17, 1960, Ser. No. 63,110
3 Claims. (Cl. 343—872)

My invention relates to inflated antennas and more particularly to such antennas suitable for use in the transmission and reception of high-frequency electromagnetic wave energy.

Previous inflated antennas of coated fabric having a wave-energy-reflective parabolic surface have employed a peripheral rim which acts to maintain the proper shape and disposition of the wave-energy-reflective surface. While such rim is usually also inflatable, in compatability with the portability of such antennas, it does add somewhat to the complexity of the assemblage and to the effort involved in its erection.

A prime object of my invention is to provide an inflatable high frequency antenna which includes a parabolic reflector of flexible material as an integral wall, without the need for an independent reflector-wall-shape-maintaining member or rim.

Another object of my invention is to provide an inflatable high frequency antenna assemblage including an integral parabolic reflector wall without the usual rim, which occupies a minimal space for an assemblage of the type.

Other objects and advantages of the invention will become apparent from the following detailed description, taken in connection with the accompanying drawing in which:

FIGURE 1 is a diagrammatic representation showing vector forces along a meridian curve on a surface of revolution corresponding to a pressurized enclosure formed by the juncture of two oppositely-arranged adjoining walls in the shape of paraboloids of revolution;

FIG. 2 is a diagrammatic representation showing a meridian curve of a surface or revolution corresponding to an inflatable antenna shaped in accord with the present invention and employing flexible material having a high modulus of elasticity with oppositely-arranged walls shaped as paraboloids of revolution joined by an intermediate wall which affords shape-stability to such opposite walls;

FIG. 3 is an expanded view of a segmental portion of the meridian curve of FIG. 2, showing geometric and mathematical relationships relative to the shapes of different wall surfaces of such antenna;

FIG. 4 is a perspective view of the novel antenna as truncated for support on a base; and

FIG. 5 is a rear elevation view of the novel antenna as cantilever-supported from its reflective surface.

Description

In accord with a general feature of the present invention, the inflatable antenna, FIG. 2, when inflated is oblate to afford minimal size and includes oppositely-arranged walls **2** and **3** shaped as paraboloids of revolution about a horizontal axis of revolution **4**. Mathematical solutions for the adaptation of paraboloids of revolution to form air-supported antennas for radar and communications applications have been developed in behalf of fabricating the present invention. A brief summary of these developments will prove beneficial in understanding the oblate antenna of this invention and its distinctions over other types of air-inflated antennas.

One of the problems heretofore unsolved has been how to completely form and support an inflated antenna using

only coated fabric as a construction element without resorting to bulky high pressure support tubes or rigid hoops which are difficult to erect and transport. Considering the employment of oppositely-arranged walls **2** and **3** shaped as two paraboloids of revolution, if these walls are immediately joined along their edge as shown in FIG. 1, the internal stresses in the fabric which result from the pressure within cannot be balanced along the junction between the two surfaces without the use of a rigid rim member **4***a*. This member takes the unbalanced radial component, Fy, which exists at this junction as is shown in FIG. 1. The wrinkling of inflated beach toys at their edges is a commonplace example of the effect of unbalanced radial loads. It was realized that in order to eliminate the rim member **4***a*, the radial component Fy had to be eliminated, and, that in order to eliminate Fy, the primary force F had to be reoriented parallel to the axis of revolution of the paraboloidal walls **2** and **3**. In accord with the concept of the invention, this realization led to the introduction of an intermediate curved surface **5**, FIG. 2, which joins the two paraboloidal surfaces **2**, **3** at junctions **5***a* and **5***b* to permit the transition from the paraboloidal shapes and their corresponding slopes to a shape of larger diameter, the general slope of which is parallel to the axis as shown in FIG. 2. The final solution to the problem thus rested in determining the curve that permitted this transition.

In accord with the object of affording minimal weight, size, volume, etc., it is essential also that the joining or intermediate curved surface **5** be minimized or at least made optimum commensurate with other considerations.

Generalized formulas for load distribution in surfaces of revolution subjected to internal pressure may be found in R. J. Roark, Formulas for Stress and Strain, McGraw-Hill, 3rd edit., 1954, and S. Timoshenko, Theory of Plates and Shells, McGraw-Hill, 2nd edit., 1959. The formulas $N_\theta = N_\phi(2 - R_\theta/R_\phi)$ and $N_\phi = PR_\theta/2$ were applied; N designating the fabric unit load in pounds per inch, R being the radius of curvature of the surface in inches, and P being the inflation pressure in pounds per square inch. In these equations the subscript θ pertains to parallel circles formed by passing a plane through the surface of revolution perpendicular to the axis of revolution. The subscript ϕ pertains to meridians formed by passing a plane which includes the axis of revolution, through the surface. It follows that N_θ is the unit load in pounds per inch along a given parallel circle; R_θ is the radius of curvature of a given parallel circle element measured as the distance extending normally from such element to the axis of revolution; N_ϕ is the unit load in pounds per inch along a given meridian; and R_ϕ is the radius of curvature of a meridian.

The above formulas show that the value of the ratio of the radii of curvature, R_θ/R_ϕ, in the two principal axes determines whether or not the joining surface **5** of revolution is in tension or compression at a given point or elemental segment. If this ratio does not exceed the numerical value of 2, the resulting surface will always be in tension. As stated heretofore, minimum size for a given diameter of the parabolic reflector surface **2** is desirable. Again examining the formula for N_θ, it is evident, that to achieve minimum diameter of the antenna **1**, R_ϕ must be the minimum possible at each point on the joining surface **5**. Since compressive loads in the fabric antenna cannot be tolerated, the value of R_ϕ is limited by the ratio of R_θ/R_ϕ being not greater than 2. Therefore, if this ratio is held at its maximum numerical value of 2 throughout the joining surface **5**, the resulting surface **5** is the minimum size possible for a given diameter of parabolic reflector surface **2**. This minimum size is accompanied by a minimal degree of mechanical rigidity of the joining surface **5** caused by the reduction of the N_θ

loads to zero leaving only the tensile meridian loads on the surface. In other words, at the R_θ/R_ϕ ratio of 2 the transition surface **5** has no hoop stress except the small amount transferred by the parabolic portions **2** and **3** near their junction with such surface **5**.

It is to be noted, however, that such lack of hoop stress is insignificant where the antenna is stationary or being rotated at a constant rate and within a radome which protects such antenna from wind loads. In certain arrangements and modes of operation, such as in tracking where acceleration and deceleration forces must be sustained by the antenna, stability of the joining surface **5** becomes significant and a compromise made between minimal size and stability. For this reason, the continuing description of the antenna of this invention is generalized to embrace these compromise ratios of radii of curvature, and afford an objective comparison between the ratio of loading N_θ/N_ϕ and the required overall diameter of the antenna **1**.

In deriving the required formulas, general formulas were established for the two radii of curvature involved in the ratio. Referring to FIG. 3, the solution for the radius of curvature R_θ is a simple right triangle in which

$$R_\theta = y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

The radius of curvature of a meridian, R_ϕ, is given by the general formulas for curvature as

$$R_\phi = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$

Letting n equal $-R_\theta/R_\phi$, the ratio of the respective radii of curvature (the introduction of the minus sign is necessary for the solution of the problem and is mathematically correct since curvature for such a surface is negative) and substituting the values given above for the radii of curvature, we have

$$n = \frac{YY''}{(Y')^2+1}$$

This can be reduced to a first order equation

$$\frac{dy}{dx} = \sqrt{\left(\frac{R}{y}\right)^{2n}-1}$$

which is an expression for the slope of the joining curve at a point. The solution of this equation to produce an equation for the required curve can only be achieved by approximate methods between limits where the expression is real. This results in

$$X = \frac{R}{an}\left[\theta-\left(\frac{b}{n}\right)\theta^3-\left(\frac{c}{n}\right)\theta^5-\left(\frac{d}{n}\right)\theta^7-\left(\frac{e}{n}\right)\theta^9\cdots\right]$$

The values a, b, c, d, e, etc. are constants determined from the binomial theorem and are too lengthy for inclusion here. The term θ in this equation is a parameter of x and y and is defined as

$$\theta \text{ equals } \cos^{-1}\left(\frac{y}{R}\right)^n$$

so that with these two formulas a joining surface **5** can be plotted to achieve any desired ratio of load. An IBM 704 was programmed to solve the equation of the joining curve and to convert the curve into fabric gore patterns.

The above formulas were derived as follows:

$$n = \frac{-R_\theta}{R\phi}$$

$$n = \frac{-y\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^2+1}$$

$$y\frac{d^2y}{dx_2} = -n\left[\left(\frac{dy}{dx}\right)^2+1\right]$$

$$\frac{R_\theta}{y} = \frac{\sqrt{(dy)^2+(dx)^2}}{dy} \quad \text{See Fig. 2}$$

$$R_\theta = y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$R_\phi = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \quad \text{general form radius of curvature}$$

if $P = \frac{dy}{dx}$

$$yP\frac{dp}{dy} = -n(P^2+1)$$

$$\frac{PdP}{P^2+1} = \frac{-ny}{dy}$$

Then

$$\int\frac{PdP}{P^2+1} = \int\frac{-ny}{dy}$$

$$\frac{1}{2}\log(P^2+1) = -n\log y+\log c$$

$$P^2+1 = \frac{C'}{y^{2n}} \text{ or } \frac{dy}{dx} = \sqrt{\frac{C'}{y^{2n}}-1}$$

Then $\frac{dy}{dx}=0$ where $y=\overline{R}$ See Fig. 2

$$0 = \frac{C'}{R^{2n}}-1 \qquad C' = R^{2n}$$

$$\therefore \frac{dy}{dx} = \sqrt{\left(\frac{R}{y}\right)^{2n}-1}$$

Then:

$$dx = \frac{y^n dy}{R^{2n}-y^{2n}}$$

substitution

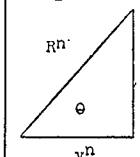
$$dx = \frac{dy}{\tan\theta}$$

$$= \frac{-R^n\sin\theta d\theta}{n\tan\theta[R^n\cos\theta]^{\frac{n-1}{n}}}$$

$$= \frac{-R}{n}\frac{\cos\theta d\theta}{[\cos\theta]^{\frac{n-1}{n}}}$$

$$= \frac{-R}{n}\cos^{1/n}\theta d\theta$$

Trig. substitution



$R^{2n} - y^{2n}$

$$\cos\theta = \left(\frac{y}{R}\right)^n$$

$$y^n = R^n\cos\theta$$

$$ny^{n-1}dy = -R^n\sin\theta d\theta$$

Then $y^{n-1} = [R^n\cos\theta]^{\frac{n-1}{n}}$

$$\therefore dy = \frac{-R^n\sin\theta d\theta}{n[R^n\cos\theta]^{\frac{n-1}{n}}}$$

This cannot be integrated except within its real limits. Therefore writing the expansion for cos θ:

$$dx = \frac{-R}{n}\left[1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-\frac{\theta^6}{6!}+\frac{\theta^8}{8!}\right]^{1/n}d\theta$$

using binomial theorem and grouping as follows, let $m=1/n$

$$x = -Rm\int_0^\theta\left[\left(1-\left[\frac{\theta^2}{2!}-\frac{\theta^4}{4!}+\frac{\theta^6}{6!}\right]\cdots\right)^m\right]d\theta$$

$$x = -Rm\left[\theta-\frac{m}{6}\cdot\theta^3+\left(\frac{m}{120}+\frac{m(m-1)}{40}\right)\cdot\theta^5 - \left(\frac{m}{5040}+\frac{m(m-1)}{336}+\frac{m(m-1)(m-2)}{336}\right)\theta^7\right.$$

$$+\left(\frac{m}{36288}+\frac{m(m-1)}{5760}+\frac{m(m-1)(m-2)}{1728}+\frac{m(m-1)(m-2)(m-3)}{3456}\right)\theta^9$$

$$-\left(\frac{m}{3,991,680}+\frac{m(m-1)}{60480}+\frac{m(m-1)(m-2)}{7920}+\frac{m(m-1)(m-2)(m-3)}{13152}+\frac{m(m-1)(m-2)(m-3)(m-4)}{42240}\right)\theta^{11}$$

$$+\cdots\theta=\cos^{-1}\cdot\left(\frac{y}{R}\right)^n$$

This series converges sufficiently for values of $\theta>1.0$ $\therefore$ as m varies from ½ to 1 the coefficient of θ^{11} varies from −.00004974 to −.00000025. The constant

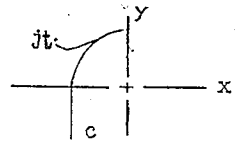
$$\frac{-R}{n}$$

is a function of antenna size, but generally 5 place accuracy is required for values of the coefficients.

Since for a given design it may be desirable to compromise between minimum size (n equals 2) and increased size with more rigidity (n less than 2), it is advantageous to have relationships between overall size and usable parabolic diameter for a given f/d ratio. At the point where the parabolic surfaces **2**, **3** and joining surface **5** meet, the slopes and their respective values of x and y coordinates are equal by definition, so at this point it is possible to determine the value of dy/dx for either curve by the use of the formulas of the parabola. Simple differentiation yields an expression for the slope of the parabolic portion as $dy/dx=2f/y$, where f is the focal length of the parabola and y is the common radius to the point at the junction. Solving this equation simultaneously with the equation for slope of the joining curve given previously yields $K=[16(f/d)^2+1]^{½n}$. Then K is equal to the ratio of overall diameter of the antenna to useful parabolic diameter of the reflective surface **2**. Given an f/d ratio one can evaluate the effects of the choice of the constant n in terms of load and overall size. In the design of the ablate antenna of this invention, the actual choice involves factors related to weight, tolerance requirements, mechanical stability and mounting configuration.

Derivation of the formula for K is as follows:

Determination of constant K $y^2=4f(x-c)$ equation of parabola



$$\frac{dy}{dx}=\frac{2f}{y}$$

at, point of junction

$$(jt.)\left.\frac{dy}{dx}\right]_{\text{parabola}}=\left.\frac{dy}{dx}\right]_{\text{oblate}}$$

also

$$y_{\text{parabola}}=y_{\text{oblate}}$$

$$\therefore\frac{dy}{dx}=\frac{\sqrt{R^{2n}-y^{2n}}}{y^n}=\frac{2f}{y}$$

$$\frac{R^{2n}-y^{2n}}{y^{2n}}=\frac{4f^2}{y^2}$$

$$\frac{R^{2n}}{y^{2n}}=\frac{4f^2}{y^2}+1\qquad ,y\Big]_{\text{max. parabola}}=\frac{D}{2}$$

$$K=\frac{R}{y}=\sqrt[2n]{16\left(\frac{F}{D}\right)^2+1}$$

This expression is valid only at the point of junction between parabolic and oblate surfaces and is used to determine the constant K when F/D ratio, reflector diameter and the loading constant n are given.

The determination of actual construction patterns for the inflatable oblate antenna, once the choice of F/D ratio and the value of the constant n have been made, requires expressions for the developed arc in the two principal axes of curvature. Formulas for the parabolic surfaces **2**, **3** exist and will not be covered here. Formulas for the curve of the joining surface **5** must be derived. The length of arc of the curve of surface **5** may be determined by integration in a conventional manner as follows:

$$S=\int_a^b\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx$$

Substituting the values for slope obtained above we have

$$S=\int_{y/R}^1\frac{dz}{\sqrt{1-z^{2n}}}$$

where z equals y/R. This is an expression for the developed length from a point where the slope equals zero (y equals R) to the particular point in question. The corresponding length of arc in a parallel circle is found in the same fashion as in the parabolic case. From the information thus derived, the mathematical gore pattern for the joining surface may be determined and, after correction for fabric properties, the antenna constructed from the gores will have the desired initial shape to permit precision contouring of the parabolic surface.

In FIG. 4 is shown the novel antenna of the invention as truncated for support on a mounting base **6** rotatably mounted on a support structure **7**. An antenna of this type has been constructed using vinyl-coated fiberglass fabric and having an overall diameter of 41 feet, a reflector surface **2** diameter of 30 feet and a depth of 28 feet, with an interior pressure measured as about one inch of water column. The usual feed horn **20** is disposed within the inflated antenna **1**, directed toward the parabolic reflector surface **2**.

Another specific oblate antenna designed in accord with the invention is shown in FIG. 5. By eliminating the truncated base **6** of FIG. 4, increasing the inflation pressure from 1 inch of water column to 8 inches of water column, and designing for a joining curve surface **5** having loading characteristics of 40% (n equals 1.6), the resulting oblate antenna can easily be cantilevered from its reflective surface **2** and can be rotated and elevated with negligible change of contour. The antenna of FIG. 5 might, for example, have a reflector surface **2** twelve feet in diameter, a major diameter of 15.85 feet, and a depth of 12.84 feet. For this configuration the focal point naturally falls inside the antenna **1**. The antenna of FIG. 5 may be supported on three arms **8**, **9** and **10** attached to plates **11**, **12** and **13**, respectively incorporated in the antenna fabric on the back surface of the reflector **2**. One additional attachment point **14** may be located at the bottom of the antenna joined to arms **8**, **9** and **10** via a vertical support member **15**. A base **16**, shown as being rotatable, also may be provided. This method of attachment provides stability in rotation and elevation. Antenna weight, exclusive of the support structure will be approximately eighty pounds when constructed of the usual vinyl-coated fiberglass cloth including a reflective coating on wall **2**.

While the invention has been described with a certain particularity, it will be understood to embrace those changes and modifications which will be obvious to those skilled in the art.

I claim as my invention:

1. In an inflated antenna assemblage, a pair of parabolic walls of flexible stretch-resistant material, one of which is wave-energy-reflective and each of which has inwardly-facing concave surfaces; and an intermediate minimal-sized continuous joining portion of flexible stretch-resistant material for maintaining the shape of said parabolic walls, there being substantially no hoop stress in the flexible material at the junctions of said joining portion with said parabolic walls along a line in a plane intersecting at right angles the axis of symmetry of such antenna assemblage.

2. In an inflated oblate antenna assemblage, a pair of paraboloidal walls of flexible stretch-resistant material curving outwardly, at least a portion of one of which walls being wave-energy-reflective; and a joining portion of flexible stretch-resistant material interposed between and in continuous surface juncture with said paraboloidal walls, said joining portion being shaped as a surface generated by revolution of a curve about an axis concentric with said paraboloidal walls, and each elemental segment of said joining portion having a ratio of R_θ/R_ϕ less than the numerical value of 2, where R_θ is the radius of curvature of a given segment in a parallel circle direction, measured as the distance extending normally from said given segment to said axis, and R_ϕ is the radius of curvature of said given segment in the meridian direction.

3. In an inflated oblate antenna assemblage, a pair of paraboloidal walls of flexible stretch-resistant material curving outwardly, at least a portion of one of which walls is wave-energy-reflective; and a joining portion of flexible stretch-resistant material interposed between in continuous surface juncture with said paraboloidal walls, said joining portion being shaped as a surface generated by revolution of a curve about an axis concentric with said paraboloidal walls, and each elemental segment of said joining portion having a ratio of R_θ/R_ϕ less than the numerical value 2 and greater than the numerical value of 1.6, where R_θ is the radius of curvature of a given segment in a parallel circle direction, measured as the distance extending normally from said given segment to said axis, and R_ϕ is the radius of curvature of said given segment in the meridian direction.

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