Aug. 19, 1958 H. J. WOLL 2,848,161

ANALOGUE MULTIPLICATION DEVICE

Filed Oct. 31, 1952

Fig. 1.

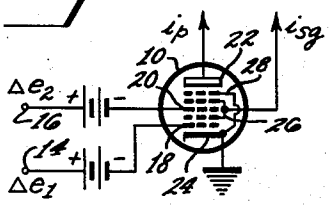


Fig. 2.

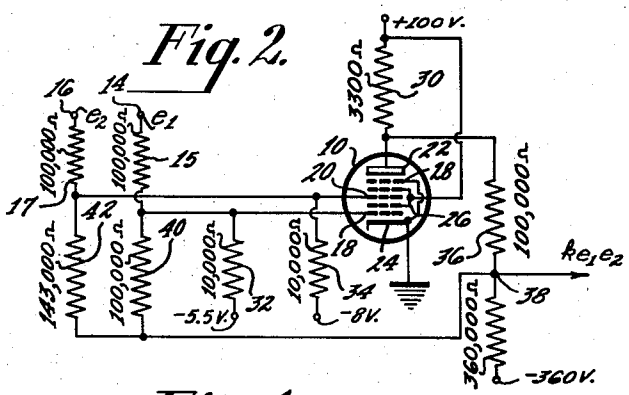


Fig. 3.

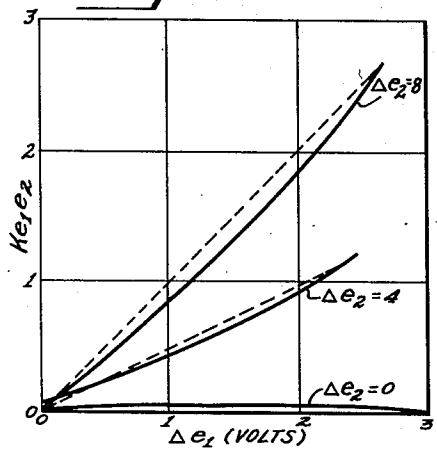


Fig. 4.

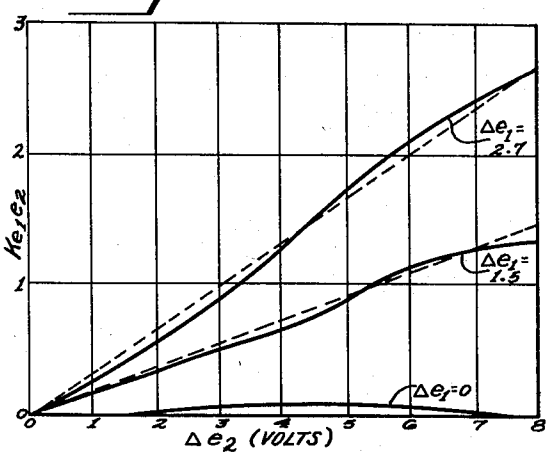


Fig. 5.

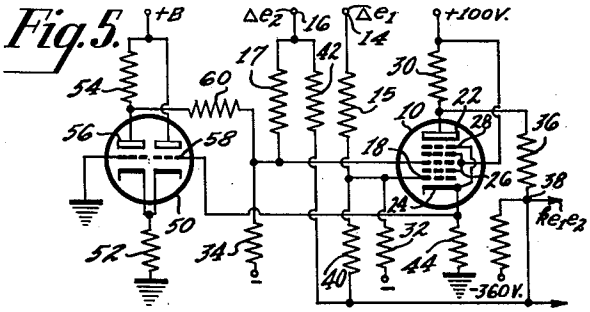


Fig. 6.

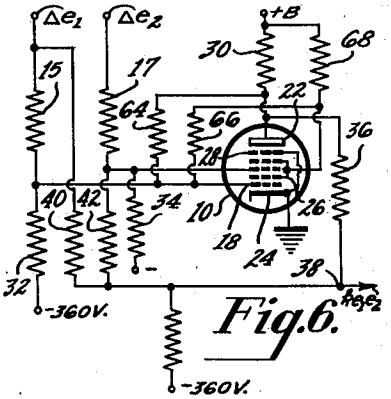
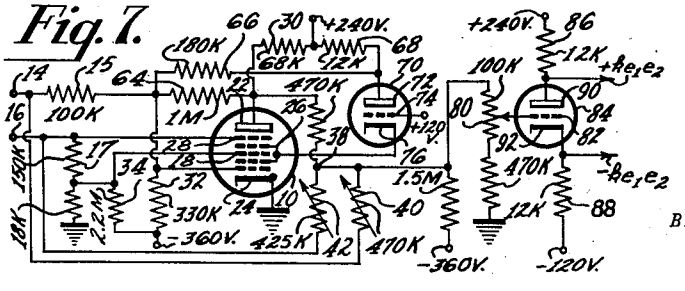


Fig. 7.



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ANALOGUE MULTIPLICATION DEVICE

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Application October 31, 1952, Serial No. 317,991

16 Claims. (Cl. 235—61)

This invention relates to electronic analogue computing devices and more particularly to an improvement in an electronic analogue multiplication circuit.

The problem of obtaining the product of two voltages is one which occurs in analogue electrical systems which perform computations aimed at solving mathematical problems. The prior art is replete with a plurality of devices all intended to provide a product of two or more voltages. These devices are more or less accurate, depending upon what definition real accuracy is given. It will be seen, however, that the higher the degree of accuracy required, and the greater the speed required for the multiplication operation, the more complex the circuitry to provide such multiplication.

A single multigrid tube of the type wherein one grid controls the current in the electron stream while a second grid controls the portion of the total electron stream that flows into the plate circuit, is admirably suited as a simple and rapid multiplication device and has long been so recognized.

A brief description of a circuit of this type is found on page 152 of a book by Seely, entitled "Electron Tube Circuits," published by McGraw-Hill Book Company. The accuracy of such a multigrid tube, when used as a multiplier is not too good, but it has been used despite this defect, in view of the simplicity of the circuit.

It is an object of the present invention to provide an improved multiplication circuit using a single tube for the multiplying process.

It is a further object of the present invention to provide a novel and simple multiplication circuit.

Still a further object of the present invention is to provide a multiplication circuit using a single tube which provides more accurate results than heretofore obtainable with such circuits.

These and other objects of the invention are achieved by providing circuitry wherein at least two voltages to be multiplied are applied to two grids of a single multigrid electron tube. The first of these two grids controls the cathode current of the tube and the second of these two grids controls the ratio of plate current to cathode current. Output from the tube is applied across a voltage divider which has an output tap thereon. Negative potential is applied to one end of the voltage divider and its other end is connected to the anode of the tube. Undesirable current components which flow in the voltage divider from the plate of the tube are cancelled by means of resistors which are connected between the two concerned control grids and the voltage tap. Thereby, when signals are applied, the cancellation current flows through these resistors. The tube is biased to operate on the most linear portion of its characteristic curve. In a further embodiment of the invention, still better linearization is provided by supplementing the above structure with feedback from the plate and screen of the tube to the grid which controls the gain of the tube.

The novel features of the invention as well as the invention itself, both as to its organization and method of

operation, will best be understood from the following description when read in connection with the drawings, wherein

Figure 1 is a drawing of the basic element used in presently known single tube multiplication circuits,

Figure 2 is a drawing of the circuit of an embodiment of the invention,

Figures 3 and 4 are curves, respectively, of output voltage vs. the voltage applied to the first control grid and the voltage applied to the second control grid.

Figures 5, 6 and 7 are circuit diagrams of further embodiments of the invention.

Reference is now made to Figure 1 wherein there is shown a multigrid tube **10** of the type employed for multiplying two voltages. A preferred tube is commercially sold as an RCA 6SA7. Two voltages to be multiplied, designated as Δe_1 and Δe_2 are respectively applied to first and second input terminals **14**, **16** respectively connected to the first control grid **18** and the second control grid **20** of the tube **10**. The anode **22** of the tube is connected to B+. The cathode **24** of the tube is connected to ground. The screen grid **26** is connected to a screen grid voltage supply. The suppressor grid **28** of the tube is connected internally to the cathode. Over a limited range in the linear portion of the tube's operation characteristic the plate current, i_p, of a pentagrid tube is essentially

$$(a_0+a_1\Delta e_1)(b_0+b_1\Delta e_2)$$

where a_0, a_1, b_0, b_1 are constants and Δe_1 and Δe_2 are grid voltage deviations from given reference voltages. The formula may be expanded thus:

$$i_p=a_0b_0+b_0a_1\Delta e_1+a_0b_1\Delta e_2+a_1b_1\Delta e_1\Delta e_2$$

where

a_0b_0 is the plate current

$\frac{\partial i_p}{\partial \Delta e_2}=b_0a_1$ is the transconductance from grid #1 to the plate

$\frac{\partial i_p}{\partial \Delta e_2}=a_0b_1$ is the transconductance from grid #3 to the plate

with both grid voltages at reference, i. e., $\Delta e_1=\Delta e_2=0$.

It is easy to obtain a current,

$$i_c=a_0b_0+b_0a_1\Delta e_1+a_0b_1\Delta e_2$$

If this is subtracted from i_p, the remainder

$$i_p-i_c=a_1b_1\Delta e_1\Delta e_2$$

is proportional to the product of the instantaneous grid voltages.

One type of pentagrid converter tube that may be used for analogue multiplication, as previously indicated, is the commercially available 6SA7. From the plate characteristics of the tube it may be seen that the linearity of the tube is improved as the amplitude of grid voltage deviations is reduced. Accordingly, if e_1 is allowed to vary from −2.8 to −5.5 volts and e_2 is allowed to vary from 0 to −8 volts, the resultant plate current i_p, is essentially

$$i_p=.65+.82\frac{5.5+e_1}{5.5-2.8}+1.70\frac{8+e_2}{8-0}+2.68\frac{5.5+e_1}{5.5-2.8}\times\frac{8+e_2}{8-0}\text{ ma}$$

If Δe_1 is referenced with respect to −5.5 volts and Δe_2 with respect to −8 volts, then

$$\Delta e_1=5.5+e_1$$

$$\Delta e_2=8+e_2$$

$$i_p=65+\frac{.82}{2.7}\Delta e_1+\frac{1.70}{8}\Delta e_2+\frac{2.68}{2.7\times 8}\Delta e_1\Delta e_2\text{ ma}$$

One way of referencing the grids and thus placing the tube on the most linear portion of its characteristic curve is to bias the tube grids in the manner shown in Fig. 1 by inserting a biasing battery in the grid lead. Another way is the well known one of applying a bias to one end of a grid leak resistor (not shown here).

Referring now to Figure 2, there is shown an embodiment of the invention wherein there is shown a means to provide a current to cancel the undesired plate current terms

$$\frac{.82}{2.7}\Delta e_1+\frac{1.70}{8}\Delta e_2$$

Two terminals, **14**, **16** to which the Δe_1 and Δe_2 voltages are respectively applied, are shown. The amplitude ranges of Δe_1 and Δe_2 fall within the limitations previously indicated. This is not to be construed as any limitation upon the invention, since scale factors may be easily worked out by anyone skilled in this art. Voltages to be multiplied can be attenuated by a factor to fall within the operating range of the device shown. The product output of the circuit may then be multiplied by the attenuation factor applied to the input voltages. The first and second terminals are respectively connected through isolating resistors **15**, **17**, to the first and second control grids **18**, **20** of the tube. An anode load resistor **30** is connected to the anode **22** of the tube. Operating potential is applied to the tube through this resistor and also, directly to the screen **26** of the tube. The two control grids are biased through their respective grid return resistors **32**, **34** so that the tube operates over the most linear portion of its operating characteristics. The tube cathode **24** is grounded.

A voltage divider **36** is provided with an output tap **38** thereon. This voltage divider has one end connected to the anode of the tube and the other end to a source of negative potential. Resistors **40**, **42** are connected between the input terminals **14**, **16** and the output tap **38**.

A voltage, ke_1e_2, representative of the product of the two voltages is obtainable at the output tap **38**. The values of the resistors **40**, **42** connected between the two input terminals and the output tap as well as the position of the output tap are selected so that a current flows to the output tap from the input terminals which has an amplitude sufficient to cancel the undesirable terms in the current which flows in the voltage divider from the plate of the tube. Thus the output obtainable from the output tap is a term 0.136 times the product of the two voltages applied to the input terminals. Values of the resistors used to illustrate an embodiment of the invention which was built are shown on the drawing. Considering scale factors again, the maximum voltage applied to each of the input terminals should be considered as units. Accordingly, the product obtainable at the output tap should be considered as a number less than one. In other words, if the voltages applied to be multiplied are on the order of 0.6 and 0.7 referenced to unity, the output product is $.136\times0.6\times0.7=.05712$.

The solid curve of Fig. 3 is a plot of the voltage applied to the first control grid against the output at tap **38** which is obtained with the circuit shown in Fig. 2. Three separate curves for three conditions for the second control grid voltages are shown (1) wherein the applied voltage is zero, (2) wherein the applied voltage equals 4 volts, and (3) wherein the applied voltage is equal to 8 volts. Ideal multiplication is shown as a dotted line.

Figure 4 shows the results obtained with the circuit shown in Fig. 2 by varying the voltage applied to the second grid from zero to 8 volts where the voltage applied to the first grid is fixed at "0" volts, 1.5 volts and 2.7 volts. The variation from the ideal multiplication may be seen by comparing the dotted lines representing such ideal condition with the solid curve. The linearity of the circuit can be considerably improved and a closer realization to the ideal condition obtained if inverse feedback is applied to the first control grid.

Figures 5, 6 and 7 show circuit diagrams of embodiments of the invention wherein inverse feedback is applied to the first control grid. This can be applied in a manner to improve linearity without reducing the range of operation. Since the voltage applied to the second control grid has little effect on the cathode current, either an unbiased cathode resistor as shown in Fig. 5, or feedback from the sum of the plate and screen current to grid #1 as shown in Figs. 6 and 7, can be used to linearize these curves.

Referring to Fig. 5, the basic multiplication circuit is shown with similar functioning parts having the same reference numerals as are used in Fig. 2. The circuit linearity is improved by using a cathode resistor **44**, connected between cathode **24** and ground of the pentagrid converter tube **10**. Care must be taken, however, that the voltage applied to the second control grid is applied from that grid to cathode and not from that grid to ground. This is done by feeding back the cathode voltage to be added to the voltage applied to the second control grid.

Accordingly, a buffer and a phase inverter circuit are used consisting of a double triode **50** having a common cathode resistor **52** and a plate load resistor **54** connected to one of the anodes **56** in the tube. The voltage from the cathode **24** is applied to the grid **58** of the buffer portion of the double triode which, acting as a cathode follower, applies a signal to the phase inverter portion of the double triode. Voltage equal to the cathode voltage is applied through a summing resistor **60** to the second input terminal. Operational level setting bias is applied through the resistor **34** connected between the junction of the two resistors **60**, **17** coupled to the second control grid. Similarly, bias is applied to the first control grid through another resistor **32**.

Referring to Figure 6, there may be seen a second method of using feedback to correct for linearity irregularities. The tube **10** is biased and connected as previously shown. As in Figure 5, similarly functioning components have the same reference numerals applied. Feedback between the plate **22** and screen grid **26** and the first control grid **18** is obtained by means of a first and a second feedback resistor **64**, **66** respectively connected between the plate and screen and the first control grid.

Figure 7 shows a preferred embodiment of the invention having a refinement which corrects for some of the difficulties occasioned by use of the feedback resistors from the plate and screen. In order to obtain sufficient feedback voltage to correct for the non-linearities in the transfer characteristic between the first control grid and the cathode of the tube, the resistors **30**, **68** in the plate and screen circuits in some cases have to be made rather large. This may result in a voltage appearing, from the screen **26** to cathode **24**, which is large enough to have undesirable effects on the accuracy of multiplication. This occurs because an A.-C. voltage appearing on the screen generates higher order products of Δe_1 and Δe_2 in the plate current. This difficulty is overcome by inserting a means to isolate these alternating currents from the screen. This consists of a vacuum tube **70** having a plate **72**, cathode **76**, and control grid electrode **74**. This vacuum tube is connected with its plate **72** in series with the screen load resistor **68** and with its cathode in series with the screen grid **26**. The control grid **74** of the tube **70** has applied thereto a fixed potential. The feedback resistor **66** to control grid No. 1 is connected from the anode of this isolation tube **70**. Output from the output tap **38** is applied to a variable potentiometer **80** which is connected to the control grid **82** of a following amplifier tube **84**. This following tube **84** has resistors **86**, **88** respectively connected to anode **90** and cathode **92** to provide a push-pull output. The potentiometer **80** in the output sets the operating level for the subsequent tube **84** in the event that the characteristics of this tube change or the tube is changed altogether.

It is desirable to apply the voltages to be multiplied to the first and second input terminals from low impedance sources so that these voltages are not affected by any currents flowing through these sources. The values for the various resistors used, as well as operating potentials applied, are shown in the circuit diagram in order to show an operative embodiment. This, however, is not to be construed as a limitation upon the invention, since other values may be used, as well as other tubes, and the inventive concept described herein will still apply. The suppressor grid may be coupled to receive a voltage equal to e_2 instead of being returned to the cathode. This may improve the linearity of operation of some tube types.

There has been described and shown above a simple, efficient and novel analogue multiplication circuit employing a single tube as a multiplier.

What is claimed is:

1. An analogue multiplication circuit for obtaining the product of a plurality of voltages comprising an electron discharge tube having an anode, a cathode, and a plurality of control electrodes, means applied to the electrodes of said tube to bias it on the linear portion of its operating characteristic, means to apply each of said plurality of voltages to a different one of said control grids, an output terminal, means coupling said output terminal to said tube anode, means coupled from said output terminal to each of the control electrodes to which said plurality of voltages are applied to supply currents to said output terminal to be added in opposition to undesirable linear anode current components to provide an output at said output terminal representative of the product of said plurality of voltages.

2. An analogue multiplication circuit for obtaining the product of a first and a second voltage comprising an electron discharge tube having a cathode, an anode, and a plurality of control electrodes, an anode load connected to said anode, an output voltage divider having a tap and being connected at one end to said anode, means to apply a bias to said tube to cause it to operate on the linear portion of its characteristics, a first and a second input terminal to which said first and second voltages are applied, means to couple said first and second input terminals to a first and second one of said control electrodes, and means coupling said first and second input terminals to said tap to supply currents thereto in linear proportion to said first and second voltages to be added in opposition to undesirable portions of anode signal present at said tap whereby to obtain an output from said voltage divider tap representative of the product of said two voltages.

3. A circuit for obtaining the analogue product of first and second voltages comprising an electron discharge tube having a cathode, an anode and at least a first and second control grid electrode, an anode load resistor connected to said anode, a voltage divider resistor having an output tap, said voltage divider resistor having one end connected to said anode, means to apply a negative bias to the other end of said voltage divider resistor, first and second input terminals to which said first and second voltages are applied, means respectively coupling said first and second input terminals to a first and second one of said plurality of control electrodes, means to bias said tube electrodes to cause said tube to operate over a linear portion of its characteristics, and first and second resistors respectively coupling said first and second input terminals to said output tap to supply to said output tap currents proportional to said first and second voltages, said currents to be added in opposition to undesirable linear anode current components present at said output whereby the output at said output tap is representative of the product of said first and second voltages.

4. An analogue multiplication circuit for obtaining the product of two voltages comprising an electron discharge tube having an anode, a cathode and at least two control grid electrodes, an anode load resistor connected to said anode, a cathode resistor connected to said cathode, a voltage divider having an output tap and having one end connected to said anode, means to apply a negative potential to the other end of said voltage divider, first and second input terminals to which a different one of said two voltages to be multiplied are applied, means to couple said first and second input terminals to a different one of said two control grids, means to apply an operation level setting bias to said first terminal, means to add said cathode voltage to said voltage applied to said second terminal, and resistance means coupled from each of said input terminals to said output tap to supply currents in opposition to undesirable plate current components to provide an output at said tap representative of the product of said two voltages.

5. An analogue multiplication circuit as recited in claim 4 wherein said means to add said cathode voltage to said voltage applied to said second terminal includes a buffer connected to receive said cathode voltage, a phase inverter connected to receive output from said buffer, and means coupling output from said phase inverter to said second terminal.

6. An analogue multiplication circuit for obtaining the product of a first and a second voltage comprising an electron discharge tube having an anode, a cathode, first, second and screen grid electrodes, an anode load resistor connected to said anode, a screen resistor, means coupling said screen resistor to said screen grid, first and second input terminals to which said first and second voltages are respectively applied, connections between said first and second input terminals and said first and second grid electrodes, means to bias said electrodes to position said tube in its linear operating region, a voltage divider having an output tap, one end of said divider being connected to said anode, means to apply a negative potential to the other end of said divider, first and second resistors respectively connected between said first and second input terminals and said output tap, said resistors having their values selected to permit current flow to said output tap in proportion to said input voltages and to oppose undesirable plate current components flowing in said voltage divider, and separate feedback resistors respectively connected between said screen resistor and said anode resistor and said first grid electrode, the values of said feedback resistors being selected to reduce errors in multiplication due to any non-linearity in the transfer characteristic between said first grid electrode and the cathode current of said tube.

7. An analogue multiplication circuit for obtaining the product of a first and a second voltage comprising an electron discharge tube having an anode, a cathode, first, second and screen grid electrodes, an anode load resistor connected to said anode, a screen resistor, means coupling said screen resistor to said screen grid, first and second input terminals to which said first and second voltages are respectively applied, connections between said first and second input terminals and said first and second grid electrodes, means to bias said electrodes to position said tube in its linear operating region, a voltage divider having an output tap, one end of said divider being connected to said anode, means to apply a negative potential to the other end of said divider, first and second resistors respectively connected between said first and second input terminals and said output tap, said resistors having their values selected to permit current flow to said output tap to oppose undesirable plate current components flowing in said voltage divider, and separate feedback resistors respectively connected between said screen and said anode and said first grid electrode, the values of said feedback resistors being selected to reduce errors in multiplication due to any non-linearity in the transfer characteristic between said first grid electrode and the cathode current of said tube, said means coupling said screen resistor to said screen includes means to prevent undesirable voltages de-

veloped across said screen resistor from being applied to said screen.

8. An analogue multiplication circuit for obtaining the product of a first and a second voltage comprising an electron discharge tube having an anode, a cathode, first, second and screen grid electrodes, an anode load resistor connected to said anode, a screen resistor, means coupling said screen resistor to said screen grid, first and second input terminals to which said first and second voltages are respectively applied, connections between said first and second input terminals and said first and second grid electrodes, means to bias said electrodes to position said tube in its linear operating region, a voltage divider having an output tap, one end of said divider being connected to said anode, means to apply a negative potential to the other end of said divider, first and second resistors respectively connected between said first and second input terminals and said output tap, said resistors having their values selected to permit current flow to said output tap to oppose undesirable plate current components flowing in said voltage divider, and separate feedback resistors respectively connected between said screen and said anode and said first grid electrode, the values of said feedback resistors being selected to reduce errors in multiplication due to any non-linearity in the transfer characteristic between said first grid electrode and the cathode current of said tube, said means coupling said screen resistor to said screen includes a second electron discharge tube having anode, control grid and cathode electrodes, said second tube cathode being connected to said screen grid, said second tube anode being connected to said screen resistor, said first feedback resistor having one end connected to said second tube anode, and means to apply a fixed potential to said control grid.

9. A multiplication circuit comprising an electron discharge tube having anode, cathode, and a plurality of control electrodes, separate input means to apply each of a plurality of input signals to a different one of said control electrodes, output means coupled to said anode, and separate impedance means coupled from said separate input means to said output means to supply linear portions of said input signals to said output means in opposition to undesirable anode signal components, said multiplication circuit being coupled throughout for direct current paths.

10. A circuit for obtaining the product of a plurality of input signals comprising an electron discharge tube having anode, cathode and a plurality of control electrodes, separate input means to apply each of said plurality of input signals to a different one of said control electrodes, an output terminal, means coupling said output terminal to said anode electrode, and means for eliminating at said output terminal certain anode signal components linearly proportional to said input signals, said eliminating means including separate means coupling said separate input means to said output terminal.

11. A circuit for obtaining the product of a plurality of input signals comprising an electron discharge tube having anode, cathode and a plurality of control electrodes, separate input means to apply each of said plurality of input signals to a different one of said control electrodes, an output terminal, means coupling said output terminal to said anode electrode, means for eliminating at said output terminal certain anode signal components linearly proportional to said input signals, said eliminating means including separate means coupling said separate input means to said output terminal, and inverse feedback means coupled to one of said tube electrodes for improving the linearity of product signals produced at said output terminal.

12. A circuit for obtaining the product of a plurality of input signals comprising an electron discharge tube having anode, cathode and a plurality of control electrodes, separate input means to apply each of said plurality of input signals to a different one of said control electrodes, an output terminal, means coupling said output terminal to said anode electrode, means for eliminating at said output terminal certain anode signal components linearly proportional to said input signals, said eliminating means including separate means coupling said separate input means to said output terminal, and inverse feedback means coupled to one of said tube electrodes for improving the linearity of product signals produced at said output terminal, wherein said electron discharge tube has two anode electrodes, and said inverse feedback means includes separate resistors connecting said anode electrodes to one of said control electrodes.

13. An electronic circuit comprising a first electron discharge tube having anode, cathode, screen and a plurality of control electrodes, a resistor connected to said anode electrode, a screen resistor, a second electron discharge tube having anode, cathode and control electrodes, said second tube anode electrode being connected to said screen resistor, said second tube cathode electrode being connected to said screen electrode, means for applying a fixed bias potential to said second tube control electrode, and first and second feedback resistors respectively connected between one of said first tube control electrodes and said first tube and said second tube anode electrodes.

14. A multiplication circuit comprising an electron control device having anode, cathode, and a plurality of control electrodes, separate input means for supplying a plurality of input signals to different ones of said control electrodes, output means coupled to said anode electrode, and separate impedance means, one for each said input means, respectively coupled from said input means to said output means to add linear portions of said input signals to signals at said output means in a sense opposite to undesirable anode signal components to obtain an output signal proportional to the product of said input signals.

15. A multiplication circuit comprising an electron control device having anode, cathode, and a plurality of control electrodes, separate input means for supplying a plurality of input signals to different ones of said control electrodes, different paths corresponding in number to said plurality, and output means coupled to said anode electrode and respectively along said paths to said input means for adding said input signals in a certain proportion to and in opposition to signals produced at said anode electrode to obtain an output signal proportional to the product of said input signals.

16. A multiplication circuit as recited in claim 15 wherein said paths each includes a separate resistor coupled to said anode electrode and to said separate input means and a common resistor connected to said separate resistors for receiving currents therein.

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