Feb. 18, 1969

M. MOONEY

3,427,915

ACOUSTIC PANELS

Filed Aug. 19, 1964

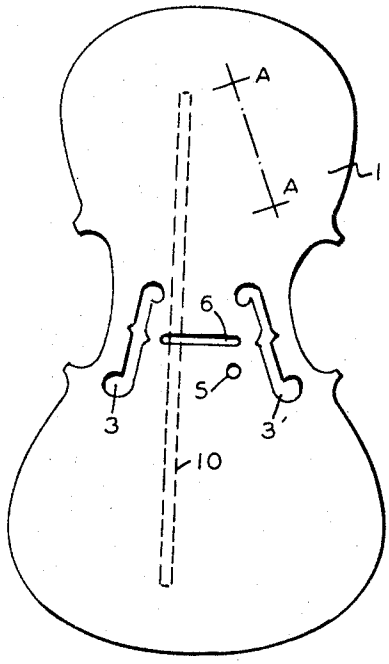


FIG.1

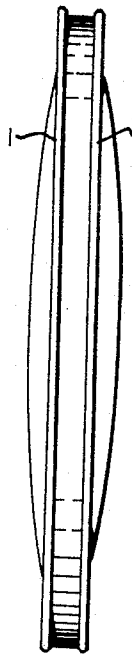


FIG.2

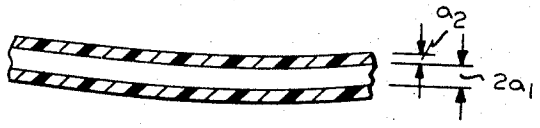


FIG. 3



FIG.4

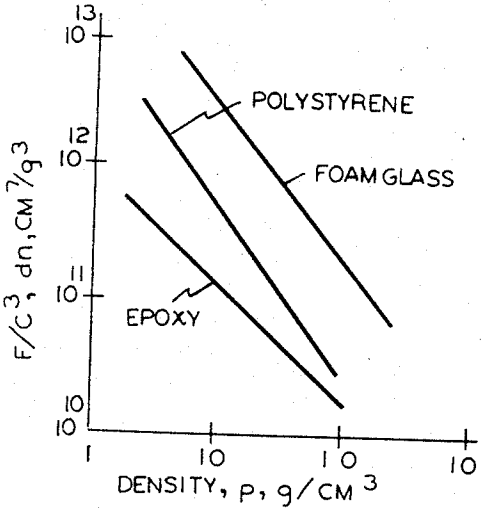


FIG. 6

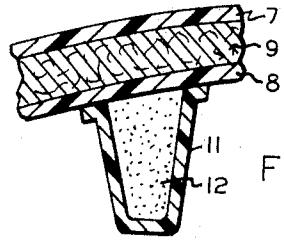


FIG. 5

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HIS AGENT

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ACOUSTIC PANELS

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13 Claims

ABSTRACT OF THE DISCLOSURE

Improved acoustic panels, especially front and back plates for violins, are made from non-wood materials in such a way as to match the acoustic properties of a high-grade wood violin. This is accomplished by application of mathematical rules for selecting materials having usable combinations of density, flexural modulus, and damping factor. The optimal panel thickness for the particular materials selected is then determined mathematically by formulas relating thickness to density and flexural modulus, so as to achieve the desired areal density and pitch characteristic.

By this method violins can be made in quantity whose tone quality and responsiveness match those of a high-grade hand-made violin of wood.

The present invention applies to acoustic vibrating panels, normally made of wood, such as the front and back plates of a violin or guitar, the sounding board of a piano, and in general, any vibrating panel of a stringed musical instrument normally made of wood.

Stringed musical instruments, particularly those of the violin family, may be divided into two groups, those that are hand-made and are good in tone but are necessarily expensive, and those that are factory-made and are inexpensive and with but few exceptions are poor in tone. An object of my invention is to make acoustic panels that are as good in tone as hand-made panels but at the same time no more expensive than factory-made panels.

I have discovered that violins approximately of the same tone quality and responsiveness as high-grade, hand-made violins of wood can be made by constructing the vibrating panels essentially of man-made materials, the panels having approximately the same mechanical and other acoustical properties as the panels in the best wooden instruments.

My invention applies equally well to panels of either single-ply or multi-ply constructions. Of the possible multi-ply constructions the three-ply sandwich is preferred, having a core, or middle ply, of low density, usually of low modulus, and two similar faces, or outer plies, of high elastic modulus. The core material, because it has a low density, will normally have also a low modulus, relatively speaking; but the low modulus is not essential for my invention. I could equally use for my core a material of low density and high modulus, if any such material exists. As a possible example, a foamed beryllium of density 0.20 g./cm.^{3} would have a modulus of 1.6×10^{11} d./cm.^{2}, according to an estimate based partially on the data in FIG. 6 for foamed glass. (I use the abbreviation *d* for dynes.)

In the application of my invention to any musical instrument conventionally employing a wooden vibrating panel, the panel shape and size are copied without change, but the choice of material and the thickness thereof are

determined in accordance with the methods of my invention.

In the drawing:

FIGURE 1 is a plan view of the body of a violin having front and back panels constructed in accordance with my invention;

FIGURE 2 is a side elevational view of the violin body of FIG. 1;

FIGURE 3 is a cross-sectional view taken through the front panel of the violin along the line A—A of FIG. 1, and showing the details of the sandwich construction;

FIGURE 4 is a cross-sectional view, taken along the line A—A of FIG. 1 wherein the material of the panel is homogeneous.

FIGURE 5 is a cross-section of a bass-bar as used with a sandwich front-plate.

FIGURE 6 is a graph showing the relation between the flexural modulus and the volume density (g./cm.^{3}) for certain rigid foams suitable for core materials.

The basic features of my invention as applied to a violin are shown in the drawing. The vibrating panels—the front **1** and back **2** of the instrument—have a three-ply sandwich construction. In shape, size, and curvatures the panels are copies of the panels of a Stradivarius violin. This is true also of the size and shape of the accessory parts—the *f*-holes **3** and **3'**, the bass-bar **10**, the sound post **5**, and the bridge **6**.

FIGURE 3 shows a section through the front panel of a violin, wherein **7** and **8** are the two outer plies, made of a hard or high-modulus material, such as a metal or a glass fiber-reinforced plastic; and **9** is the inner ply, made of a material of low density, and usually but not necessarily of low modulus, such as balsa wood or a rigid plastic foam.

The bridge **6** and the sound post **5** are made of wood, as in conventional wooden violins. The bass-bar **10** may also be of wood, but I prefer a bass-bar constructed as shown with an outer layer **11** of hard material and a core **12** of low-density material (as shown in FIG. 5), the two materials being the same as are used in the faces and core, respectively, of the panels.

The back **2** has the same sandwich construction as does the front, but there are differences in thickness.

In matching the acoustical properties of a good wooden panel, matching the damping factor involves only the selection of materials, not the panel design. Materials are selected which, singly or in combination, have the same or a lower damping factor than the best wood. If the damping factor is lower, it can be raised, if desired, by adding suitable damping materials, such as rubber or a soft varnish, to a panel or to the bridge.

The areal density, D, and the pitch characteristic, P, are simultaneously matched by properly selecting the materials as to volume density, ρ, and flexural modulus, F, and by properly designing the panel as to thicknesses $2a_1$ and a_2. The necessary density relation is

$$D_{12}=D_0$$

or

$$2(\rho_1 a_1+\rho_2 a_2+b)=2\rho_0 a_0 \qquad (1)$$

Equality of pitch characteristics requires

$$P_{12}=R_{12}/D_{12}=P_0=R_0/D_0$$

or

$$\frac{2\{F_1 a_1^3+F_2[(a_1+a_2)^3-a_1^3]\}/3}{2(\rho_1 a_1+\rho_2 a_2+b)}=\frac{2F_0 a_0^3/3}{2\rho_0 a_0} \qquad (2)$$

In these equations the subscript $_0$ refers to a selected wooden reference panel; subscript $_{12}$ refers to a sandwich panel, subscript $_1$ refers to the core, and subscript $_2$ to the face; b is the weight per unit area of the adhesive employed to bond a face to the core; F_0 is the effective mean value of the two flexural moduli of the wood in the two directions, with and across the grain; R is the flexural rigidity of a panel. The definitions of a_0, a_1, and a_2 are shown graphically in FIGS. 3 and 4.

The above Equations 1 and 2 are valid for either a sandwich panel or a homogeneous panel. In the latter case I merely set $b=0$ and either a_1 or $a_2=0$.

Equations 1 and 2 are my thickness design equations. Thus, if ρ_0, a_0 and F_0 are assigned the values which are observed in a selected wooden panel, and if ρ_1, ρ_2, F_1 and F_2 are assigned the correct values characteristic of particular chosen materials in my sandwich panel, and if b is assigned the lowest value which results in an adequate bond between face and core, then these two equations can be solved for a_1 and a_2. A similar statement holds for a homogeneous panel. These thickness values thus obtained are physically significant only if they are mathematically real and positive. The same Equations 1 and 2 are also the basis of my rules for the selection of face and core materials such that the solutions for a_1 and a_2 are physically significant. This requires that both a_1 and a_2 be mathematically real and positive. The selection rules eliminate the need for constructing experimental panels to test the suitability of unsuitable materials. Details of the selection rules and their use will be disclosed below.

In the construction of a sandwich panel, according to my invention, while it is important that the damping factor of the face plies be approximately equal to or less than the damping factor of the single-ply wood panel being duplicated, the damping factor of the core and middle layer of the sandwich panel is relatively less important. It can usually be ignored, provided only that it is not of a higher order of magnitude than the damping factor of the face layers.

Although in the prior art the properties of damping factor, areal density, and pitch characteristic have been recognized as being qualitatively of importance in the design and construction of stringed instruments, my contribution to this part of the art is the development of quantitative methods by which all these properties can be simultaneously and quantitatively controlled to achieve the desired acoustical properties in a non-wooden panel.

It is well recognized, in the violin industry, for example, that the objective of my invention, good instruments at a low cost, cannot be achieved so long as wood is retained as the principal material of construction of the panels, or plates, of the instrument. This statement applies specifically to the surface layers of a plate, which almost alone support the bending stress on the plate; but it does not necessarily apply to the central layer, which is merely a bonding and spacing element, and supports very little of the bending stress. For example, no harm results from the normal variations in balsa wood when used in this layer.

The tone quality of a violin is very sensitive to small variations in the texture of the wooden plates and to small departures from the optimum thickness, density and rigidity. Yet all species of wood are both variable and non-uniform in their texture and physical properties; and hence it is impossible to make uniformly good violins of wood by a factory method with single, fixed designs of the front and back plates and fixed thickness and thickness gradations. Uniformly good violins of wood can be made only by hand by trial and error, by the most skilled artisans, who select the wood with the utmost care, and cut and graduate each plate individually to give it the proper weight and proper pitch characteristic as revealed by the tap tone in free vibration. It is customary in the thinning process to glue the violin parts together temporarily several times; for it is only by testing the complete instrument that it is possible to determine with certainty whether further thinning is necessary. With so many required man-hours of highly skilled labor, hand-made violins are necessarily expensive. Another item of appreciable expense is the wood itself, which is quite costly if it is of fine, uniform grain, of proper density and damping characteristics and has been properly seasoned for 25 to 50 years.

The superiority of man-made materials over wood for instrument construction lies in the fact that they are, or can be, made much more uniform in physical properties than the natural woods. Hence, after one good instrument of man-made materials has been made, any number of similar instruments can be made to precise specifications by factory methods employing molds and other suitable tools for quantity production. Thus, as compared with hand-made wooden instruments, both material costs and labor costs are greatly reduced by my invention.

If for any reason it should be necessary or desirable to relax somewhat the requirements of Equations 1 and 2, I would consider that, for the reproduction of tone quality, Equation 2, referring to pitch characteristic, is the more important, and should be retained; while slight departure—as much as 10 to 20%—from Equation 1, referring to areal density, can be tolerated with less damaging effects on tone. But so long as permitted departures from either equation are slight, the construction will still be within the scope of my invention.

Heretofore, non-wooden stringed musical instruments have been constructed of various non-wooden materials, for example, violins made of aluminum or of a transparent acrylic ester plastic, and ukeleles made of polystyrene. However, the aluminum violins were notably weak in tone in the first octave of the violin range. The violins of acrylic plastic, a material of a high damping factor, are weak in tone over the entire range. Polystyrene, the material used in plastic ukeleles, is somewhat higher than wood in its damping factor, and it is too low in the ratio of its modulus of rigidity to the cube of its density. The importance of this ratio is brought out later in my discussion of the selection rules.

Cellos and base viols also have been made of aluminum, but, like the aluminum violins, they are no longer on the market. According to acoustical principles, such instruments will be poor in quality, compared with good wooden instruments, for they must necessarily be either too low in the pitch characteristic, and mechanically weak, or too high in areal density, or both.

While a wooden panel, as in a violin for example, normally is made to vary in thickness over its area, it is not always necessary in a duplicate panel to match these local variations. Good results have been obtained with a duplicate panel in which the areal density was uniform and equal to the mean areal density of the wooden panel. A similar statement applies to local variations in the flexural rigidity and the pitch characteristic of a panel. Nevertheless, obtaining the very best possible results in a duplicate panel will require, as in a wooden panel, suitable thickness gradations.

In considering pitch characteristic and panel rigidity it is desirable, for clarity, to introduce here the definitions of the flexural rigidity of an elastic plate and the flexural modulus of an elastic material. The flexural rigidity is defined by reference to a thin, straight, uniform strip which is wide in comparison with its thickness and long in comparison with its width. If the strip, when subjected to the bending couple C, is bent to the curvature γ, then its flexural rigidity, R, is

$$R=C/w\gamma \qquad (3)$$

where w is the width.

If the uniform strip under consideration is also homogeneous, the flexural rigidity varies as the cube of the thickness. Hence, a material property, F, called the flexural modulus, can be defined by the equation

$$R=2Fa_0^3/3 \quad (4)$$

where a_0 is the half-thickness of the strip. With the value 2/3 for the numerical factor in this definition, F is approximately equal to the Young's modulus of the material; and for the purposes of this invention the value of Young's modulus can be used for F if F itself is not known.

For a sandwich structure with two faces of the same material and thickness, the flexural rigidity, R_{12}, has the form of the numerator in the first term of Equation 2.

The above Equations 3 and 4 are valid for materials that are isotropic. But woods are highly anisotropic. In spruce and maple, the two woods usually employed for violin plates, the ratio of the elastic moduli with and across the grain is 10 to 1 in order of magnitude; and the same is true of the corresponding flexural moduli. The effective mean flexural modulus in a wooden plate is a weighted average of the values, with and across the grain. The correct weighting in this average is variable and unknown; but as an adequate approximation for the purpose of this invention, I use the arithmetic mean value. Then, instead of Equation 4, the equation to be used here for determining the reference standard, the rigidity of a wood structure of thickness=a_0, is

$$R_0=2F_0a_0^3/3 \quad (5)$$

where R_0 is the effective mean rigidity; and, approximately,

$$F_0=0.55F_g \quad (6)$$

F is the effective mean flexural modulus of the wood, and F_g is the flexural modulus parallel to the grain—that is, the modulus for a bending which stretches or shortens the surface fibers lying parallel to the grain but does not alter the length of fibers lying across the grain. Equation 6 gives a value of F_0 which can be used in Equation 2 when only the flexural modulus with the grain is known for the wood concerned.

The same averaging method is required also for the fabricated material if it also is anisotropic.

For a panel made of isotropic fabricated materials and satisfying the density Equation 1, the equality of flexural rigidities requires

$$F_1a_1^3+F_2[(a_2+a_2)^3-a_1^3]=F_0a_0^3 \quad (7)$$

This equation is valid for either a sandwich or a homogeneous structure in my panels. In the latter case, either a_1 or a_2 would be set equal to zero in Equations 1 and 2.

With an arbitrary, unguided choice of fabricated materials in my panel, it would often be impossible to satisfy, simultaneously with real values, both of the Equations 1 and 7. To ensure that both equations can be simultaneously satisfied, it is necessary to impose conditions which constitute selection rules for materials of suitable properties for the purpose. These rules, derived from Equations 1 and 7, are

$$F_1/\rho_1^3 \gtrless F_0/\rho_0^3 \quad (8)$$

and

$$F_2/\rho_2 \gtrless F_0/2\rho_0 \quad (9)$$

The rule of Equation 8 applies when my panel is a single, homogeneous ply, with $a_2=0$. Then, if the equality sign in Equation 8 holds for the material considered, the correct value of a_1 is

$$a_1=a_0(F_0/F_1)^{1/3}=a_0\rho_0/\rho_1 \quad (10)$$

If the inequality sign holds in 8, the fabricated material has a higher modulus for its density than is necessary. But Equations 1 and 7 could still be satisfied by increasing D through the addition of a coat of heavily loaded paint, or of small dead weights attached to the panel surface.

The rule of Equation 9 applies to the face material of a sandwich panel. It is only a preliminary or tentative selection rule, because its derivation from Equations 1 and 7 involves several approximations, some of which are valid, in general, only as to order of magnitude. Specifically, the approximations are: b, F_1/F_2 and $(a_2/a_0)^2$ are negligible; and

$$\left.\begin{array}{l}\rho_1a_1/\rho_2a_2=1/2\\ a_1(a_1+a_2)\gtrless a_0^2\end{array}\right\} \quad (11)$$

The exact selection rule for sandwich materials is developed below.

The following equation is derived from Equations 1 and 2 without any approximations:

$$\phi(\beta)=\frac{(1-\mu\beta)^3}{1-\sigma\beta^3}=\frac{(1-2b/D_o)^3F_2/\rho_2^3}{F_o/\rho_o} \quad (12)$$

where

$$\left.\begin{array}{l}a_1=\beta T\\ a_2=(1-\beta)T\\ a_1+a_2=T\\ \sigma=1-K_f=1-F_1/F_2\\ \mu=1-K_\rho=1-\rho_1/\rho_2\end{array}\right\} \quad (13)$$

also

$$T=\frac{D_o-2b}{2[\beta\rho_1+(1-\beta)\rho_2]} \quad (14)$$

Equations 12, 13, and 14 are the sandwich design equations in their final, preferred form. With given F_1, F_2, ρ_1, ρ_2, and b, they determine the values of a_1 and a_2 required to satisfy Equations 1 and 7 simultaneously.

In the computational procedure the parameter β is first determined by Equation 12. Since the equation is of the third degree in β, a graphical or numerical method of solution seems best. Under certain conditions to be discussed later, $\phi(\beta)$ passes through a minimum at a point within the range $0<\beta<1$. When this is the case, there may be two solutions of Equation 12. That solution with the smaller β is to be preferred, because the smaller β results in a sandwich with the thicker and stronger face.

If with a given set of property values there is no real solution of Equation 12 with the range $0<\beta<1$, then the chosen core and skin materials cannot be used in combination to produce a plate which matches the reference plate. Thus Equation 12 is the final, exact selection rule; but when it is used purely as a selection rule, certain changes are helpful.

The basic question is whether the right member of Equation 12 is greater than or less than the lowest possible value of the left member. By standard mathematical operations it can be shown that under the conditions of my problem, the slope, $d\phi/d\beta$, is negative at $\beta=0$; and that it remains negative up to $\beta=1$, if $\mu/\sigma>1$. In this case, $\phi(1)$ is the lowest value of ϕ in the physically significant range of β. This lowest value is $\phi_1=K\rho^3/K_f$. On the other hand, $\phi(\beta)$ has a minimum at

$$\beta=\sqrt{\frac{\mu}{\sigma}}=\beta_m$$

and if $\mu/\sigma<1$, this minimum lies at a point within the range of physically real β. The value of ϕ at the minimum is

$$\phi(\beta_m)=\phi_m=\left(1-\mu\sqrt{\frac{\mu}{\sigma}}\right)^2$$

The exact and final selection rule for sandwich materials is therefore

$$\frac{(1-2b/D_o)^3F_2/\rho_2^3}{F_o/\rho_o^3}\gtrless\frac{K_\rho^3}{K_f}, \text{ if } \mu/\sigma>1 \quad (15a)$$

or

$$\frac{(1-2b/D_o)^3F_2/\rho_2^3}{F_o/\rho_o^3}\gtrless(1-\mu\sqrt{\mu/\sigma})^2, \text{ if } \mu/\sigma<1 \quad (15b)$$

To summarize the new teaching regarding sandwich-type acoustical panels based on mathematical analysis:

If $F_2/\rho_2^3 \gtrless F_0/\rho_0^3$, a sandwich panel with the particular skin material is neither necessary nor desirable. If $F_2/\rho_2^3 < F_0/\rho_0^3$, and if condition (15*a*) or (15*b*) is satisfied, whichever is appropriate, then a matching sandwich panel with the particular core and skin materials is possible. In this case the design is obtained by solving Equation 12 for β, as was previously indicated.

In the teaching here presented I have ignored the highly improbable case $\sigma < 0$; but this would cause no difficulty. There would be no minimum in ϕ and Equation 15*a* would apply.

The use of Equations 8 and 9 as selection rules can be further exemplified in application to the data in Table I. I consider here first Equation 8, valid for a homogeneous panel. The approximate values of F_0/ρ_0^3 for spruce and mple are, respectively, 9 and 4×10^{11} d. cm.^{7}/g.^{3}. Among the few materials listed in Table I that have such high values of F/ρ^3 are beryllium and foamed glass. Either of these two materials can be used in a homogeneous matching panel. The design details are given in Examples 1 and 2 below. Rigid foams of the plastics polystyrene or epoxy can be made with acceptable values of F/ρ^3, but their strengths at the required densities are too low. The same inadequacy is found also in balsa, aside from the fact that balsa is a highly variable product of nature. Of the metals in Table I, aluminum is nearest to beryllium in the value of F/ρ^3; but it is an order of magnitude too low. Silver, which serves so well in flutes, is useless for a violin. A silver violin plate thick enough to have the required pitch characteristic would be much too heavy to respond adequately under the vibratory force of the violin strings. Of the ceramics the beryllium oxide ceramic would be better than aluminum, but still not good enough.

In considering the selection rule of Equation 9, applicable to the face of a sandwich panel, it is seen that many materials in Table I are possible, having values of F/ρ that are equal to or greater than 0.7 or 0.5 *d.* cm./g. corresponding to half the values for spruce or maple, respectively. Hence in the final choice of core materials it is often possible to consider such minor characteristics as appearance, toughness, or brittleness, and cost. With regard to the glass reinforced plastics, it is clear that the continuous fibers, in the form of a woven fabric, are better than chopped fibers. In fact, the chopped glass reinforcement is not quite adequate.

The exact selection rule corresponding to the approximate rule of Equation 9 is the exact Equation 15. The latter equation must ultimately decide in cases which are border-line according to Equation 9; and the final decision will depend partly on the density of the core and the weight of the bonding agent used.

These selection rules are an important part of the teaching of my invention, and they constitute a novel step in advance over the prior art.

In addition to the acoustical suitability conditions, there are certain other requirements that must be observed for satisfactory results. If a panel is too thick, it will not vibrate readily as a sheet, particularly in the high frequencies where the nodal lines in the vibration pattern are closer together than at the lower frequencies. A good working rule is that the panel should not be more than twice as thick as the reference wooden panel. Thus

$$a_1+a_2 \not> 2a_0 \tag{16}$$

An alternative working rule is that the thickness should not be greater than 1/20 of the panel width.

Any serviceable panel must have sufficient strength to withstand accidental knocks and stresses encountered in normal handling. No precise strength requirement can be stated; but, again as a good working rule, it can be required that, for a one-ply panel or for the face of a sandwich panel, the crushing strength of the material shall not be less than 500 lb./in.^{2}, or 3.5×10^6 d./cm.^{2}.

In a sandwich panel, the core material must resist shear and compressional stresses. As a working rule, it can be required that the compression modulus shall not be less than 2×10^3 lb./in.^{2}, or 0.15×10^9 d./cm^2.

It is not necessary to make the surface hardness over the entire panel adequate to withstand high local stresses such as exist at the feet of the bridge of a violin and at the ends of the sound-post. Such concentrated stresses can be distributed over a larger area by incorporating in the panels reinforcing patches at the stressed points.

The above designing method has not given any consideration to the curvature of certain panels, those in the instruments of the violin family, for example. These curvatures are slight and have only minor effects on the acoustical properties, effects which are roughly the same in either wooden or fabricated panels. Hence, for practical purposes, I have ignored the curvature in setting up the rigidity equations.

Instead of choosing a particular wooden violin plate as a standard to be duplicated in its acoustical behavior, it is possible to determine the ranges of the physical properties exhibited by the panels of good violins as a class and then to choose for the property values of a new panel any set of values within these ranges.

In good violins the properties of areal density, D, and pitch characteristic, P, expressed by the right members of Equations 1 and 2 respectively, have the following approximate range limits:

For the front plate

$$\left.\begin{aligned} 0.09 \lesssim D_f \lesssim 0.16 \text{ g./cm.}^2 \\ 8\times10^8 \lesssim P_f \lesssim 11\times10^8 \text{ cm.}^2\text{/sec.}^2 \end{aligned}\right\} \tag{17}$$

For the back plate

$$\left.\begin{aligned} 0.15 \lesssim D_b \lesssim 0.24 \text{ g./cm.}^2 \\ 1.0\ P_f \lesssim P_b \lesssim 1.4\ P_f \text{ cm.}^2\text{/sec.}^2 \end{aligned}\right\} \tag{18}$$

The statement of the range of P_b, for the back plate, in terms of P_f, of the front plate, is in accordance with an established rule in the art of violin making. This rule is that, whatever the tap tone of the front plate may be, the tap tone of the back plate must not be greatly different. It is usually higher than that of the front plate by one to two half-tones.

Any violin with sufficiently low damping, and with other plate properties lying within the above indicated ranges, will be a good one. Minor differences in tone quality and loudness will be associated with particular values of the plate properties that are chosen in the violin design, and with gradations in plate thickness. Thinning around the periphery results in a slight increase in the volume of tone and in the depth of tone quality, that is, a tone with relatively higher energy in the lower frequencies of the tone spectrum.

The equation for the damping factor, δ_{12}, of a sandwich panel is

$$\delta_{12}=\frac{F_1\delta_1 a_1^3+F_2\delta_2[(a_1+a_2)^3-a_1^3]}{F_1 a_1^3+F_2[(a_1+a_2)^3-a_1^3]} \tag{19}$$

With a pre-assigned desired value for δ_{12}, this equation, when solved for $F_1\delta_1$, constitutes a selection rule for the core material with respect to the damping factor. The damping selection rule is

$$\delta_1 F_1 \lesssim (\delta_{12}-\delta_2)F_2\left[\left(\frac{a_1+a_2}{a_1}\right)^3-1\right]+\delta_{12}F_1 \tag{20}$$

If the inequality in Equation 20 holds for a particular core material, that material can be used; but an additional damping element might be desired, such as one of those previously suggested.

In the following four examples which demonstrate the use of my teaching in designing violin plates, I use in all cases the same reference front and back plate, made of spruce and maple, respectively. The properties of the two woods and plates are given in Table II. In all four examples, as in any other application of this invention, all parts of the violin except the plates and the bass-bar are of conventional design and may be of wood, the conventional material.

Example 1.—Foamed glass plates

Front plate.—The first question to be decided is what density of foamed glass must be used in order that its value of F/ρ^3 shall match the value of F_0/ρ_0^3 for spruce, given in Table II as 9.60×10^{11} d. cm.^{7}/g.^{3}. The answer is given by FIG. 6, which shows that the required density is 0.33 g./cm^3. Then, by Equation 10, the thickness of the foamed glass plate must be $2a_1=0.324$ cm. This completes the design of the front plate, because the equality of pitch characteristics, stated by Equation 2, is necessarily satisfied if Equation 10 is satisfied, and if also Equation 8 is satisfied with the = sign, not the > sign.

Back plate.—The required value, 3.87×10^{11} d. cm.^{7}/g.^{3}, for the foamed glass is found in FIG. 6 at $\rho=0.60$ g.^{7}/cm^3. Then, again by Equation 10, the thickness is $2a_1=0.313$ cm.

The bass-bar, attached to the underside of the front plate, is made of the same foamed glass as is the front plate itself. A coat of varnish, used to keep dirt out of the glass, can be so chosen that it will increase the damping factor, if that is desired.

Example 2.—Beryllium plates

Table I shows that the value of F/ρ^3 for beryllium is only about one-half that for the reference spruce. Consequently exact and complete acoustical matching of the spruce plate is not possible with beryllium. An imperfect but acceptable design is one which matches the pitch characteristic and lets the areal density be somewhat larger than that of the spruce plate.

In order to match the pitch characteristic, P_0, for the spruce plate given in Table II, it is necessary that

$$\frac{F_1a_1^2}{3\rho_1}=9.18\times10^8$$

With the beryllium values $F_1=30\times10^{11}$ and $\rho_1=1.85$ given in Table I, the above equation is solved for a_1. It is found that the thickness is $2a_1=0.0824$ cm. Also $D_1=2a_1\rho_1=0.1526$ g./cm.^{2}. This value of D_1, the areal density, is roughly 40 percent higher than the value for the spruce plate. Such a value is undesirable, but it lies within the range set by Equation 17.1 and is not ruinous for the tone. While exact matching of the spruce plate is thus not complete, beryllium in this use is still far superior to aluminum, which was used for a period in commercial violins. A computation similar to the above, when applied to aluminum, gives the value $D_1=0.557$ g./cm.^{2}, which is over five times the value for the spruce plate.

A complete and exact match of a spruce plate can be accomplished with beryllium as the principal material if a sandwich construction is used.

In the problem of matching the maple back plate with beryllium, it is seen from Table I that the value of F/ρ^3 for beryllium is higher than is required. The preferred procedure in this case is to alloy the beryllium with another metal, copper, for example, in order to obtain the desired value of F/ρ^3 in the alloy.

By Table II, the desired value is 3.87×10^{11} d. cm.^{7}/g^3. On the assumption that both the modulus and the specific volume of the alloy vary linearly with the volume-percent composition, the required alloy has the properties: composition, by weight, Be, 92.13%; Cu, 7.87%;

$F_a=29.7\times10^{11}$ d. cm.^{7}/g.^{3}, $\rho_a=1.97$ g./cm.^{3}

The required thickness of the back plate made of the alloy is 0.095 cm., computed in the same manner as in the case of the foamed glass plates.

An alternative structure for a beryllium back plate is possible, in which, instead of alloying with a baser metal, pure beryllium is used; and a heavy coating of varnish is added to increase the weight. For such a plate a_2 is set equal to zero in Equations 1 and 2, and b is interpreted as the areal density of the varnish coat. With assigned values in the right members of these two equations, the equations can be solved for a_1 and b; and the panel design is thereby obtained.

Example 3.—Sandwich core, balsa; face, glass fabric in epoxy resin

The material properties are:

For the core, $\rho_1$0.165 g./cm.^{3};

$$F_1 \text{ (tabulated } F_0)=0.27\times10^{11} \text{ d./cm.}^2$$

For the face, $\rho_1=1.70$ g./cm.^{3}; $F_2=1.75\times10^{11}$ d./cm.^{2}.

In examining the suitability of these sandwich materials it is to be noticed, first, that the proposed face material satisfies the tentative selection rule stated by the inequality 9. For rigorous testing of the two proposed sandwich materials in combination, inequality (15a) is the relevant selection rule. With b assigned the reasonable value 0.005 g./cm.^{2}, it is found that the inequality is obeyed for both the front and the back plate. For the front, the left and right members of the inequality are respectively 0.0286 and 0.0059. For the back the figures are 0.0787 and 0.0069. Thus a solution of the design problem is assured.

The value of β is found by solving Equation 12. For the front plate the right member of the equation is 0.0286. The parameters K_f and K_w are, $K_f=0.1537$, $K_w=0.0972$. By numerical solution of Equation 12, $\beta=0.838$.

Equation 14 now gives for T, the half-thickness of the sandwich, 0.118 cm. Then, from the first two of Equations 13, $a_1=0.0984$ cm. and $a_2=0.0390$ cm. The total plate thickness is 0.235 cm. These figures complete the design specifications of the sandwich front plate.

For the back plate the right member of Equation 12 is 0.0787. The parameters K_f and K_ρ are the same as for the front plate. The value of β given by Equation 12 is $\beta=0.680$. The additional design calculations proceed as for the front plate, the results being, for the back plate, $T=0.135$ cm.; $a_1=0.0918$ cm.; $a_2=0.0432$ cm. The plate thickness is $2T=0.270$ cm.

Example 4.—Sandwich, core, aluminum honey comb; face, aluminum

The material properties are:

For the core, $\rho_1=0.083$ g./cm.^{3}, $F_1=0.108\times10^{11}$ d./cm^2.

For the face, $\rho_2=2.7$ g./cm.^{3}, $F_2=7.0\times10^{11}$ d./cm.^{2}.

Parameter values are $\mu=0.969$ and $\delta=0.985$. The ratio $\mu/\delta=0.984$. Equation 9 gives $2.59>1.41$. Thus the preliminary selection rule is satisfied by a safe margin. Checking by means of the exact selection rule is really not necessary, but if the exact rule were employed, Equation 15b would be used, since $\mu/\delta<1$.

With the appropriate values of D_0 for the front and back plate, and again with $b=0.005$ g./cm.^{2}, the solution of Equation 12 yields, for the front plate, $\beta=0.781$; and for the back plate, $\beta=0.766$. By continuing the solution as before, the final results are found; for the front plate, $a_1=0.0578$ cm., $a_2=0.0162$ cm.,

$2T$=plate thickness=0.148 cm.

for the back plate, $a_1=0.0973$ cm., $a_2=0.0297$ cm.; $2T$=plate thickness=0.254 cm.

TABLE I.—RELEVANT PROPERTIES OF SELECTED MATERIALS

Units	ρ, g./cm.^{3}	F_g, d./cm.^{2} $\times 10^{11}$	F or F_0, d./cm.^{2} $\times 10^{11}$	F/ρ or F_0/ρ_0, d.-cm./g. $\times 10^{11}$	F/ρ^3 or F_0/ρ_0^3, d.-cm.^{7}/g.^{3} $\times 10^{11}$	δ, $\times 10^{-3}$	Strength, compress or tensile, lb./in.^{2}
Woods:							
Balsa	0.165	0.490	0.270	1.630	60.0		130
Spruce	0.383	0.979	0.539	1.405	9.60	20	
Maple	0.537	1.091	0.600	1.117	3.87	24	2.750
Metals:							
Beryllium	1.85		30.0	16.2	4.74		
Aluminum	2.70		7.0	2.6	0.36	3.4	
Steel	7.8		20.0	2.6	0.04	0.6	
347 Stainless Steel	7.9		19.6	2.5	0.04	1.7	
Tungsten	19.3		34.0	1.76	0.05	16.0	
Brass	8.6		10.4	1.21	0.016		
Copper	8.9		10.0	1.12	0.013	3.2	
Silver	10.5		7.5	0.72	0.007		
Ceramics:							
Beryllium Oxide	3.0		30.3	10.1	1.12		
Porcelain	2.5		9.0	3.6	0.60		
Silica	2.2		7.3	3.3	0.69		
E-Glass	2.55		7.3	2.8	0.44	6-12	
Pyrex 702	2.32		6.2	2.7	0.50		
Plastics:							
Polystyrene	1.06		0.34	0.32	0.29	30-40	
Polyester	1.10		0.21	0.19	0.16		
Epoxy	1.11		0.24	0.22	0.18	50	
Phenol-formaldehyde	1.7		0.69	0.40	0.14		
Poly (tetrafluoroethylene)	2.15		0.034	0.016	0.0034		
Poly (vinyl chloride)	1.45		0.41	0.28	0.135		
Poly (methyl methacrylate)						200	
Foams:							100
Foamed glass	0.144		0.124	0.86	41.6		1,400
	0.50		0.625	1.25	5.0		360
Polystyrene	0.16		0.0145	0.091	3.55		170
	0.10		0.0086	0.086	8.6		90
	0.08		0.0018	0.023	3.58		600
Epoxy	0.083		0.108	1.30	189		
Aluminum honey-comb							
Composites:							
Chopped glass, polyester	2.30		1.10	0.48	0.09		
Glass fabric, polyester	2.40		3.10	1.48	0.330		
Glass fabric, epoxy	1.70		1.75	1.03	0.336		
Glass roving, epoxy	2.0	4.0	2.5	1.25	0.31		
Chopped glass, epoxy	1.9		2.1	1.10	0.36		
Macerated fabric in phenol-form	1.43		0.90	0.63	0.31		
Asbestos in phenol-form	2.0		2.6	1.3	0.32		
Mica in phenolformaldehyde	1.92		3.4	1.8	0.49		
Mica in glass	3.2		8.6	2.69	0.262		

ρ=Volume density.
F_g=Flexural modulus with the grain.
F=Flexural modulus.
F_0=Mean flexural modulus for anisotropic materials.
δ=Damping factor.
d. is my abbreviation for dynes.

TABLE II.—PROPERTIES IN PLATE DESIGNS

Units	ρ, g./cm.^{3}	F_0, d./cm.^{2} $\times 10^{11}$	F_0/ρ_0^3, d.-cm.^{7}/g.^{3} $\times 10^{11}$	Front or Back	$2a_0$, cm.	D_0, g./cm.^{2}	P_0, d.-cm.^{2}/g. $\times 10^8$
Material:							
Spruce	0.383	0.539	9.60	F	0.280	0.1073	9.18
Maple	0.537	0.600	3.87	B	0.340	0.1870	10.76

Example Number	Material	ρ	F $\times 10^{11}$	F/ρ^3 $\times 10^{11}$	Front or Back	$2a_1$	a_2	2T	D	P $\times 10^8$
1	Glass Foam	0.33	0.0345	9.60	F	0.324		0.324	0.1073	9.18
	Glass Foam	0.60	0.0836	3.87	B	0.313		0.313	0.1870	10.76
2	Beryllium	1.85	30.0	4.74	F	0.0824		0.0824	0.1526	9.18
	Beryllium Copper Alloy	1.973	28.7	3.86	B	0.095		0.095	0.1870	10.76
3	Glass Epoxy (Face)	1.70	1.75	0.315	F	0.1968	0.039	0.236	0.1073	9.18
	Balsa (Core)	0.165	0.270	60.0	B	0.1836	0.0432	0.270	0.1870	10.76
4	Aluminum (Face)	2.7	7.0	0.36	F	0.1156	0.0162	0.148	0.1073	9.18
	Aluminum Honeycomb (Core)	0.083	0.108	189	B	0.1946	0.0297	0.254	0.1870	10.76

ρ is the volume density.
F is the flexural rigidity.
$2a_0$ is the one-ply plate thickness.
D is the areal density.
P is the pitch characteristic.
$2a_1$ is the core thickness.
a_2 is the face thickness.
d. is my abbreviation for dynes.

I claim:

1. In a stringed musical instrument employing one or more sounding boards or panels, a panel constructed primarily of fabricated rigid materials, the materials being so selected as to flexural modulus F, defined by the equation

$$R=2Fa_0^3/3 \quad (4)$$

and density, where a_0 is one-half the thickness of a selected solid wooden reference panel, and the new panel being so designed as to thickness, that the flexural rigidity R, defined by the equation

$$R=C/w\gamma \quad (3)$$

where γ is the curvature resulting when the bending couple C is imposed on a uniform strip of material of width w, and the areal density D of the panel are essentially equal to the flexural rigidity and areal density, respectively, of the selected solid wooden panel of which the tone quality is to be duplicated; and the fabricated materials also being so selected as to give the panel a damping factor δ, defined by the equation

$$\delta_{12}=\frac{F_1\delta_1 a_1^3+F_2\delta_2[(a_1+a_2)^3-a_1^3]}{F_1 a_1^3+F_2[(a_1+a_2)^3-a_1^3]} \quad (19)$$

not significantly greater than that of said wooden panel, where δ_1 and δ_2 are the damping factors of the core and face materials, respectively, a_1 is one-half of the core thickness, a_2 is the thickness of a face layer, and F_1 and F_2 are the flexural moduli of the core and face materials, respectively, and subscript 12 refers to a sandwich panel.

2. A panel constructed primarily of fabricated material according to claim 1, wherein the value of areal density D is in the range from 0.10 to 0.18 g./cm.^{2} and the value of pitch characteristic R/D is in the range from 5×10^8 to 11×10^8 cm.^{2}/sec.^{2}.

3. A three-ply panel according to claim 1 wherein the inequality of the damping selection rule

$$\delta_1 F_1 \gtrless (\delta_{12}-\delta_2) F_2 \left[\left(\frac{a_1+a_2}{a_1} \right)^3 - 1 \right] + \delta_{12} F_1 \quad (20)$$

holds and wherein an additional body of damping material is applied such that the condition of equality in said rule (20) is obtained.

4. A panel according to claim 2, wherein the inequality of the damping selection rule (20) defined in claim 3 holds and wherein an additional body of damping material is applied such that the condition of equality in said rule (20) is obtained.

5. In a stringed musical instrument employing one or more sounding boards or panels, a panel constructed primarily of fabricated rigid materials, the material being so selected as to flexural modulus, as defined in claim 1, and density, and being so designed as to thickness, that the flexural rigidity R, defined as in claim 1, and the areal density D of the panel are essentially equal to the flexural rigidity and the areal density, respectively, of a selected wooden panel which is to be duplicated.

6. A panel according to claim 5, wherein the value of areal density D is in the range from 0.10 to 0.15 g./cm.^{2} and the value of pitch characteristic R/D is in the range from 5×10^8 to 11×10^8 cm.^{2}/sec.^{2}.

7. A panel according to claim 1, constructed of a single homogeneous fabricated material.

8. A panel according to claim 1, constructed of a core of one material and face plies of a different material.

9. A musical instrument comprising a correlated pair of front and back panels each of which is constructed according to claim 1, the back panel having an areal density equal to from 1.2 to 2.0 times the areal density of the front panel.

10. A sandwich panel according to claim 8, wherein the materials are so selected as to density and flexural modulus and the plies are so designed as to thickness that the following equations

$$2(\rho_1 a_1+\rho_2 a_2+b)=2\rho_0 a_0 \quad (1)$$

and

$$\frac{2\{F_1 a_1^3+F_2[(a_1+a_2)^3-a_1^3]\}/3}{2(\rho_1 a_1+\rho_2 a_2+b)}=\frac{2F_0 a_0^3/3}{2\rho_0 a_0} \quad (2)$$

are satisfied, the thickness values selected being real and positive, where ρ is density, subscripts 0, 1 and 2 refer respectively to the reference material, the core material,

and the face material, and b is the weight per unit area of bonding adhesive employed.

11. A panel according to claim 7, wherein the material is so selected and the panel is so designed that the relation

$$F_1/\rho_1^3 \gtrless F_0/\rho_0^3 \quad (8)$$

is satisfied, symbols as in claims 1, 10.

12. A panel according to claim 8, wherein the materials are so selected and the panel is so designed that the relation

$$F_2/\rho_2 \gtrless F_0/2\rho_0 \quad (9)$$

is satisfied, symbols as in claims 1, 10.

13. A panel according to claim 8 wherein the selection of materials and the panel design are such that equations

$$\phi(\beta)=\frac{(1-\mu\beta)^3}{1-\sigma\beta^3}=\frac{(1-2b/D_0)^3 F_2/\rho_2^3}{F_0/\rho_0} \quad (12)$$

$$\left.\begin{aligned} a_1&=\beta T \\ a_2&=(1-\beta)T \\ a_1+a_2&=T \\ \sigma&=1-K_f=1-F_1/F_2 \\ \mu&=1-K_\rho=1-\rho_1/\rho_2 \end{aligned}\right\} \quad (13)$$

and

$$T=\frac{D_0-2b}{2[\beta\rho_1+(1-\beta)\rho_2]} \quad (14)$$

are satisfied.

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