

[54] **METHOD, APPARATUS AND SYSTEM FOR THE IDENTIFICATION OF THE RELATIONSHIP BETWEEN TWO SIGNALS**
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Related U.S. Application Data

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[52] U.S. Cl. **333/6, 333/19, 333/20, 333/70 CR**
[51] Int. Cl. **H03h 7/06, H03k 5/00**
[58] Field of Search **307/160, 161; 235/183; 333/6, 19, 20**

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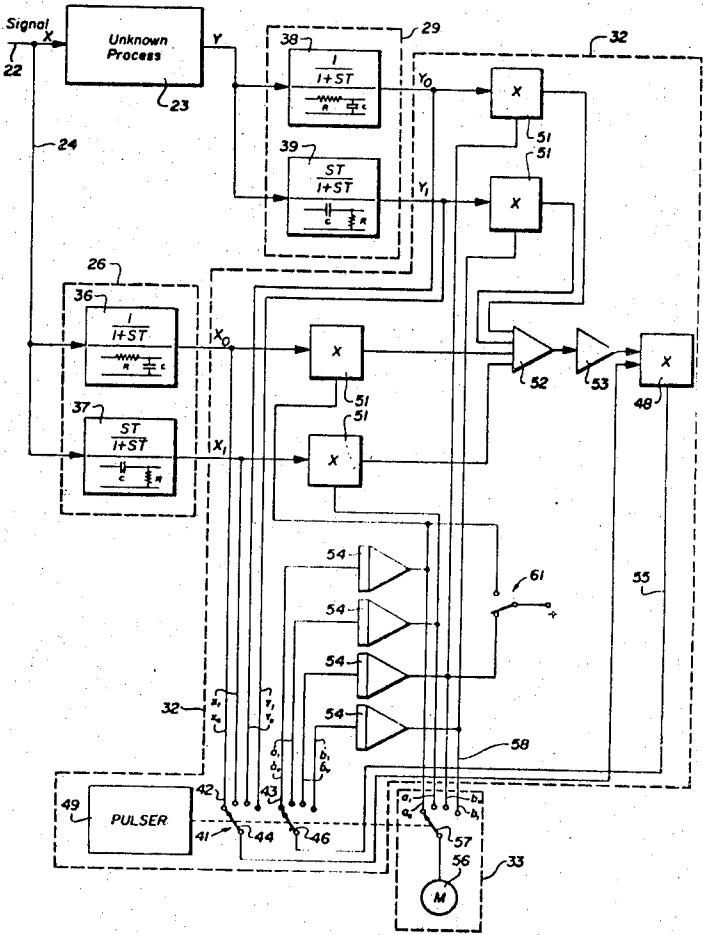
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[57] **ABSTRACT**

A set of circuits with a common input and several outputs, wherein the second output signal is the time derivative of the first output signal, and the third output signal is the second time derivative of the first output signal, and wherein the transfer functions from the input signal to each output signal have the same denominator roots.

13 Claims, 16 Drawing Figures



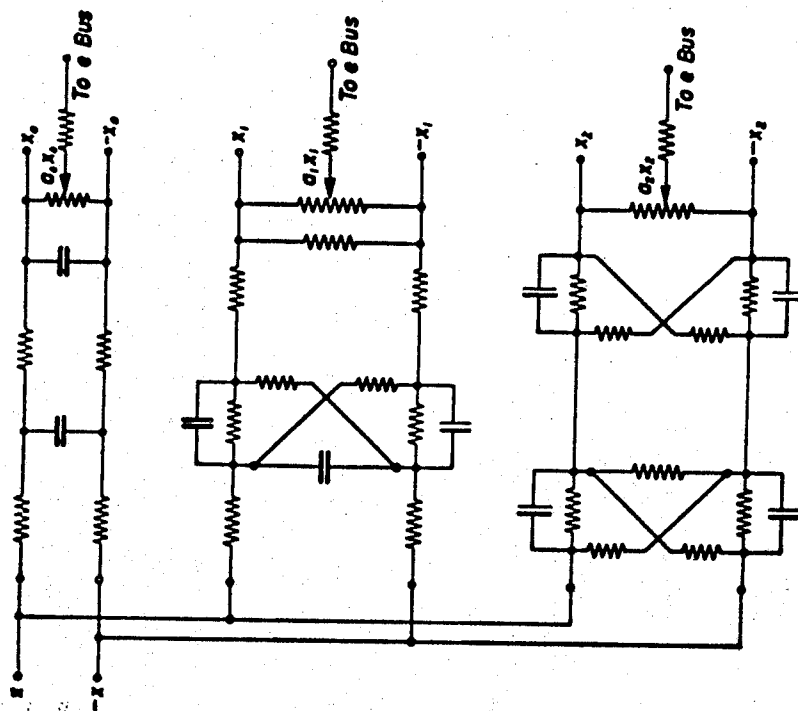


Fig. 12

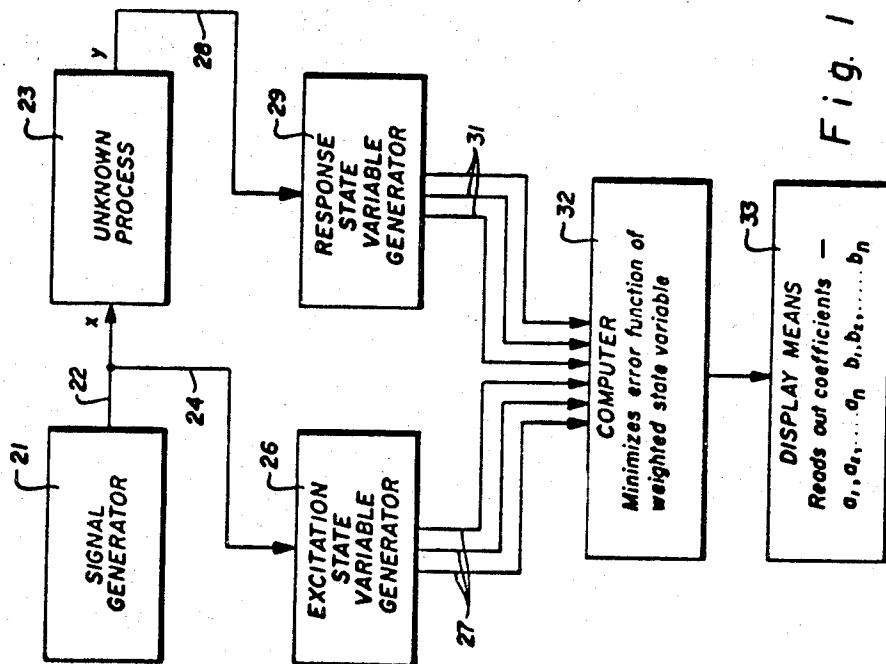


Fig. 1

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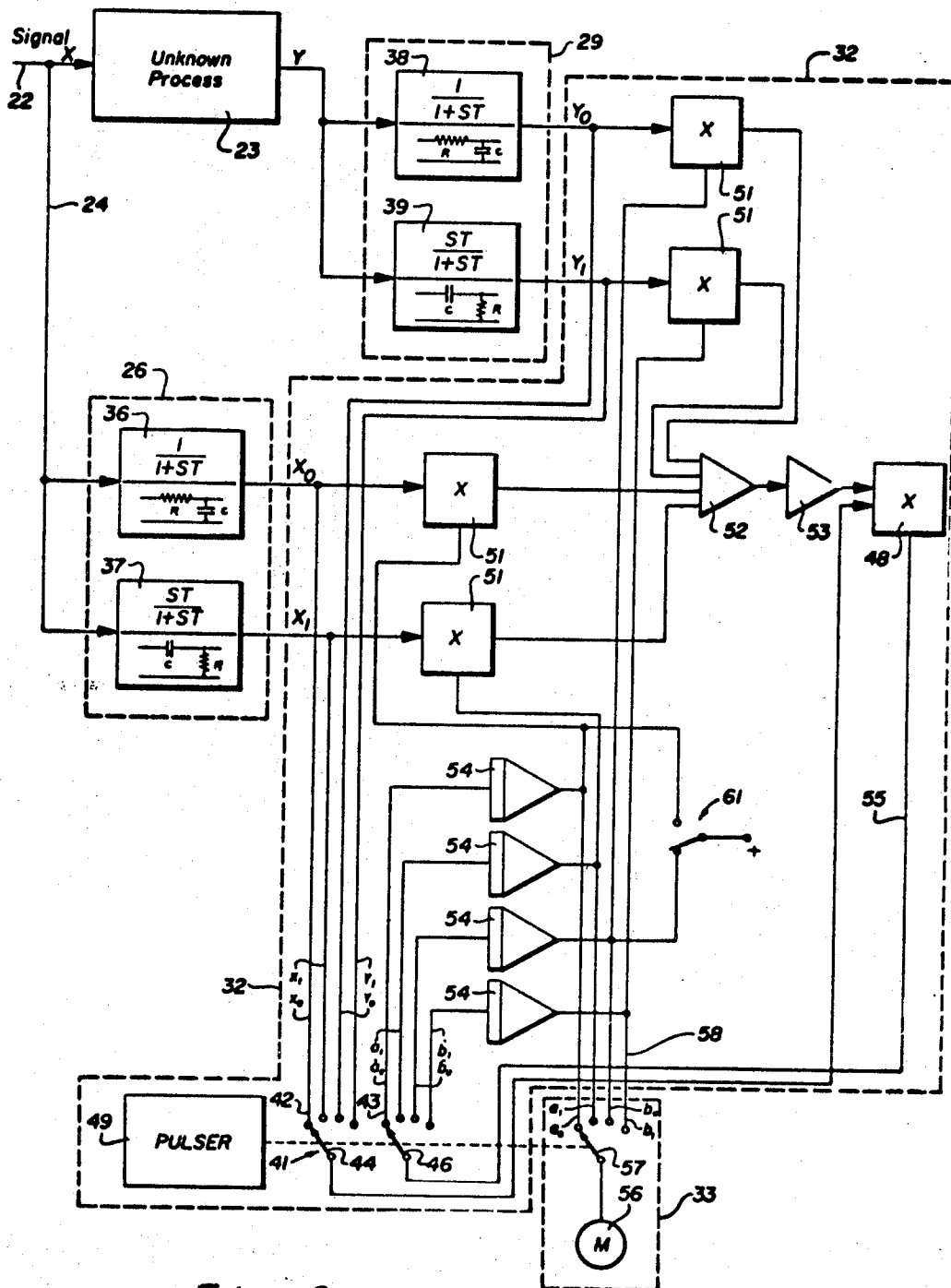
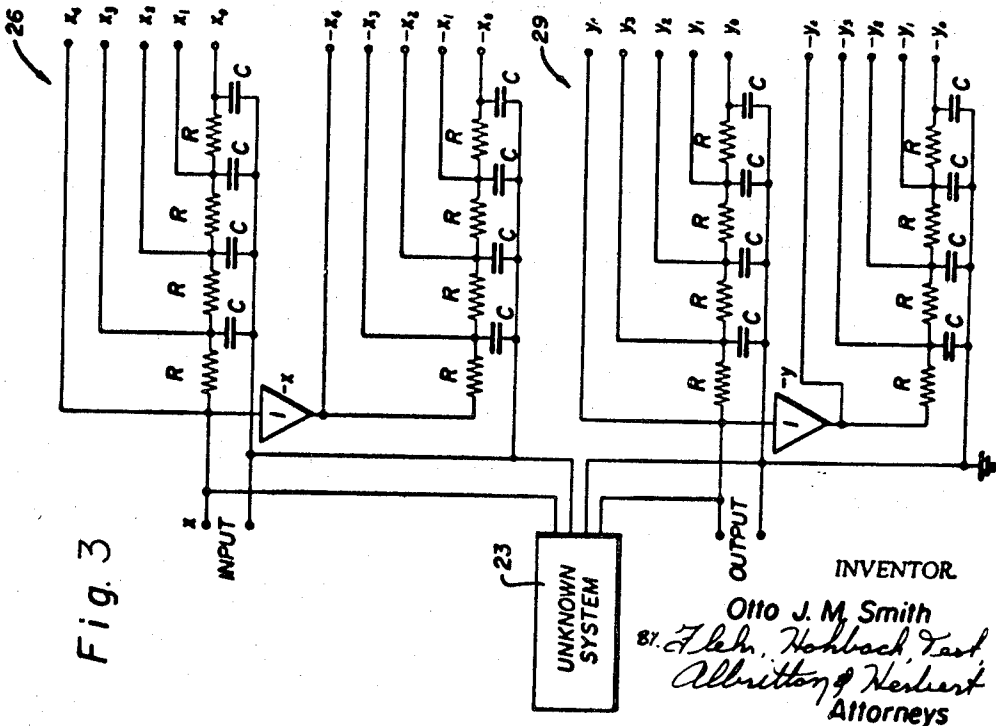
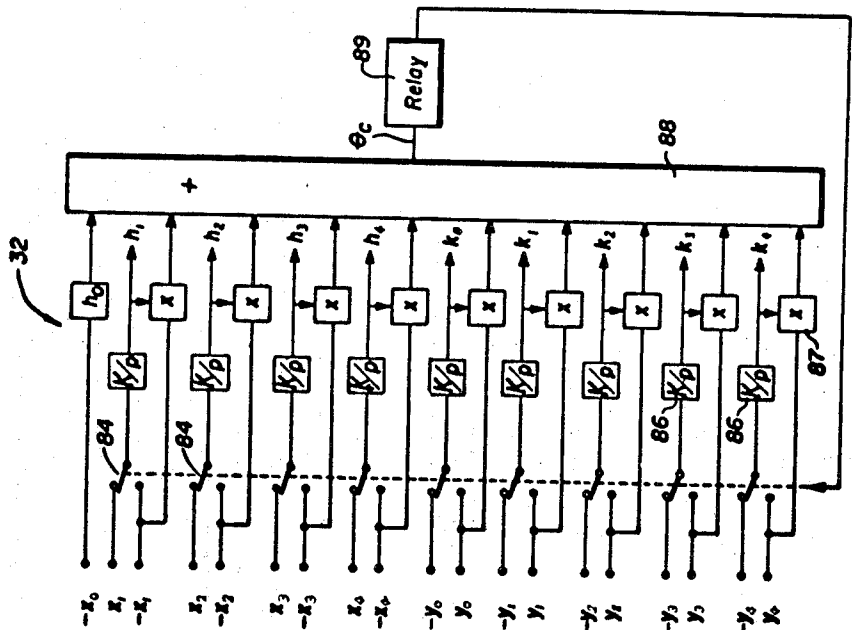


Fig. 2

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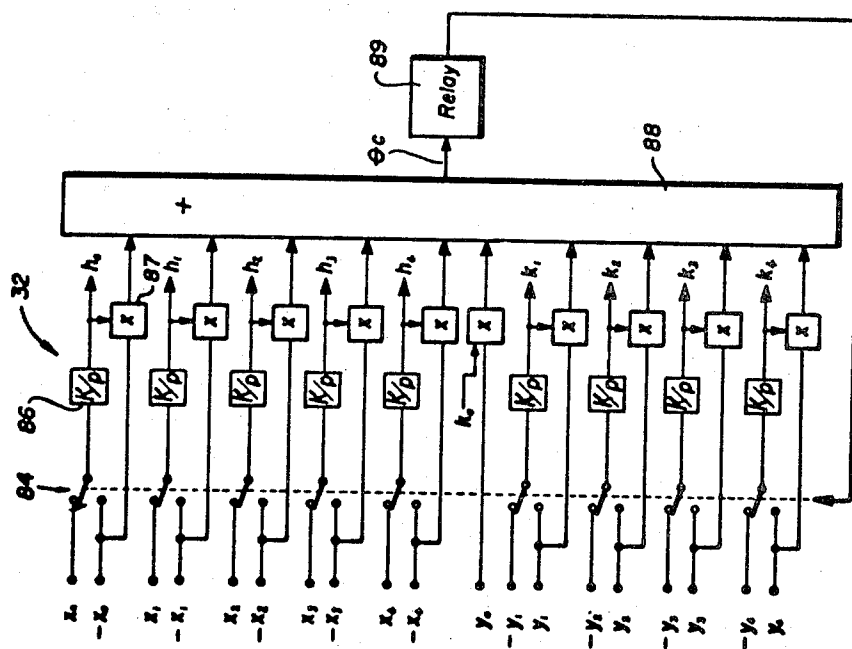


Fig. 6

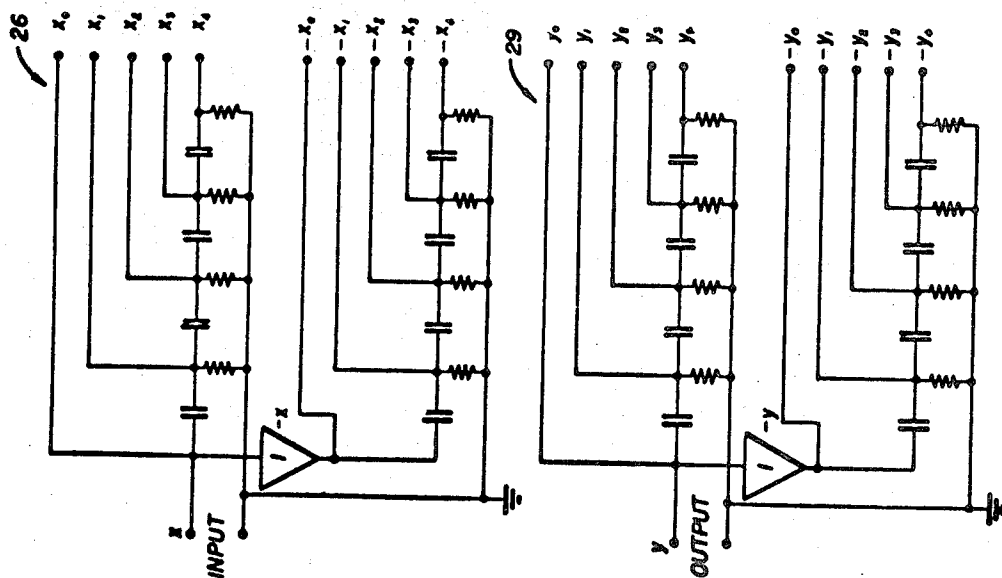


Fig. 5

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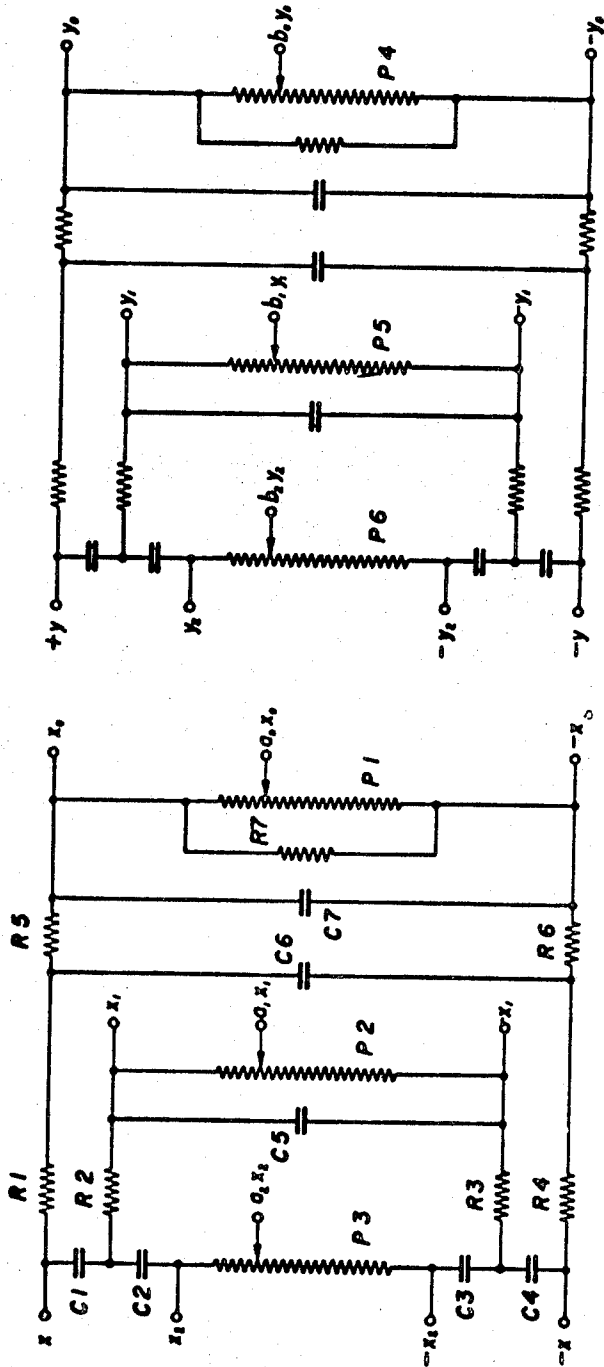


Fig. 8

Fig. 7

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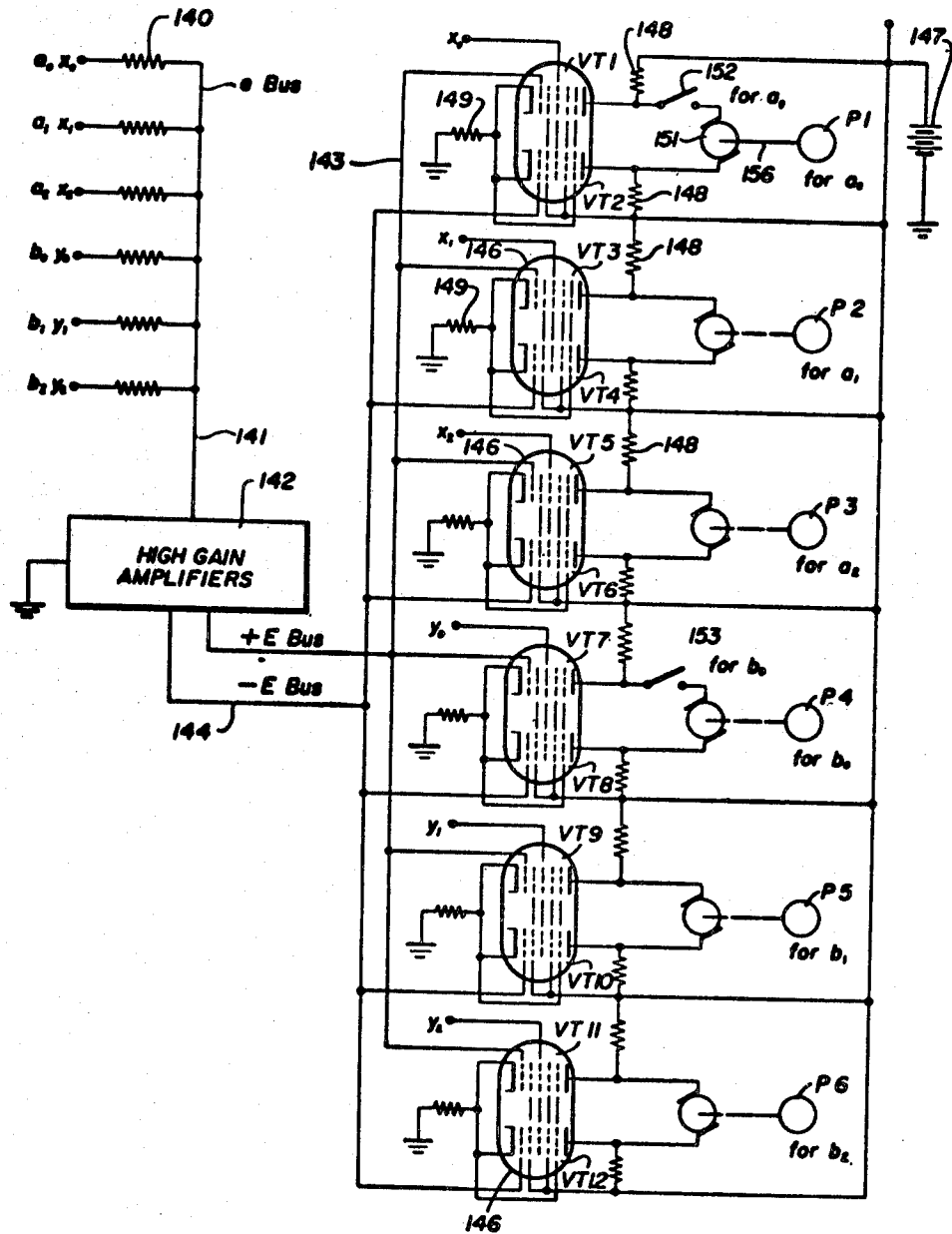


Fig. 9

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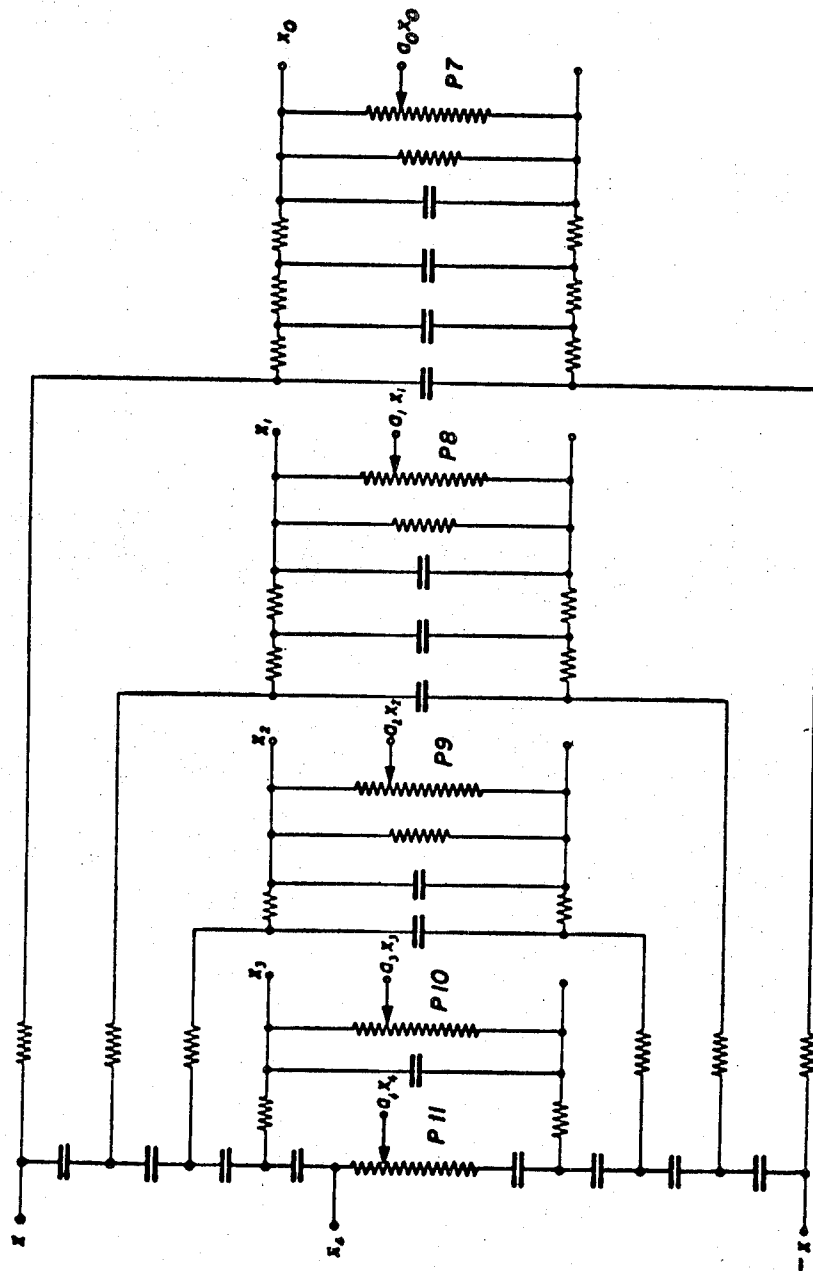


Fig. 10

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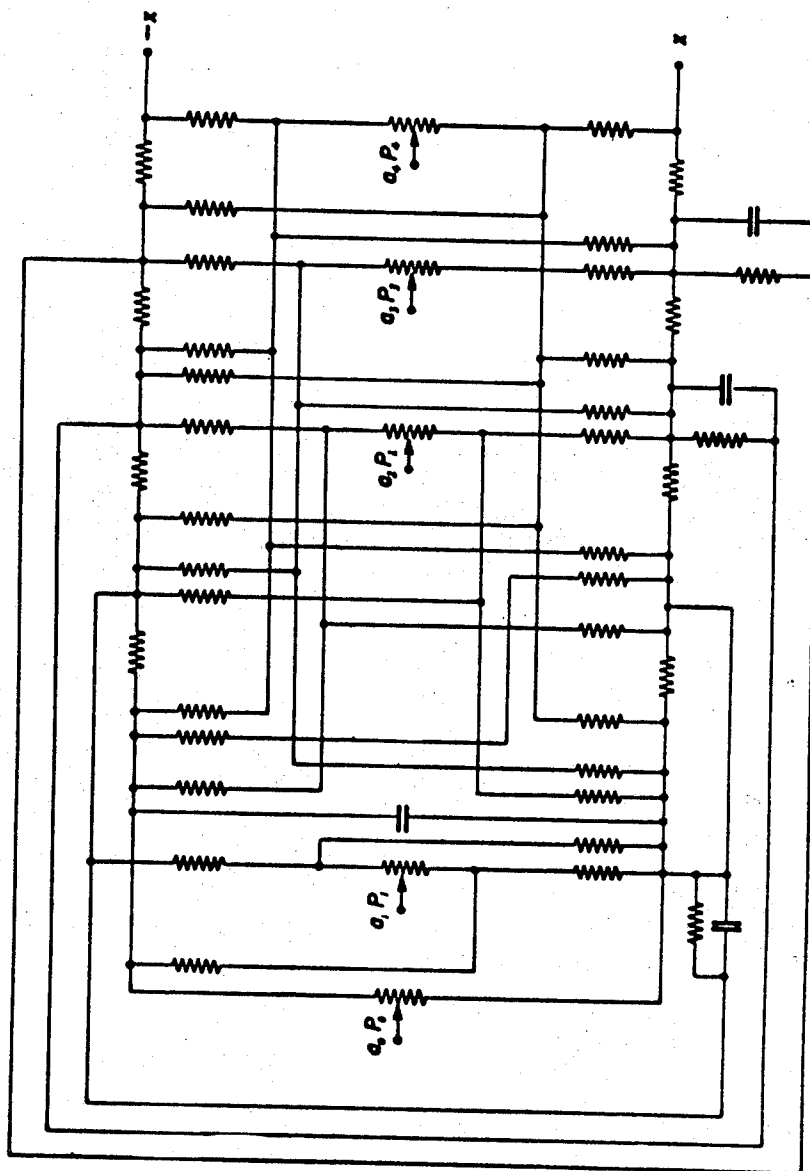


Fig. 11

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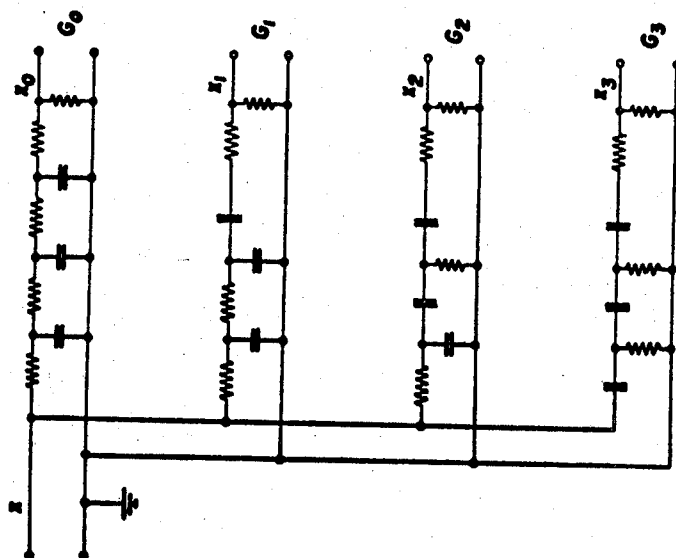


Fig. 13

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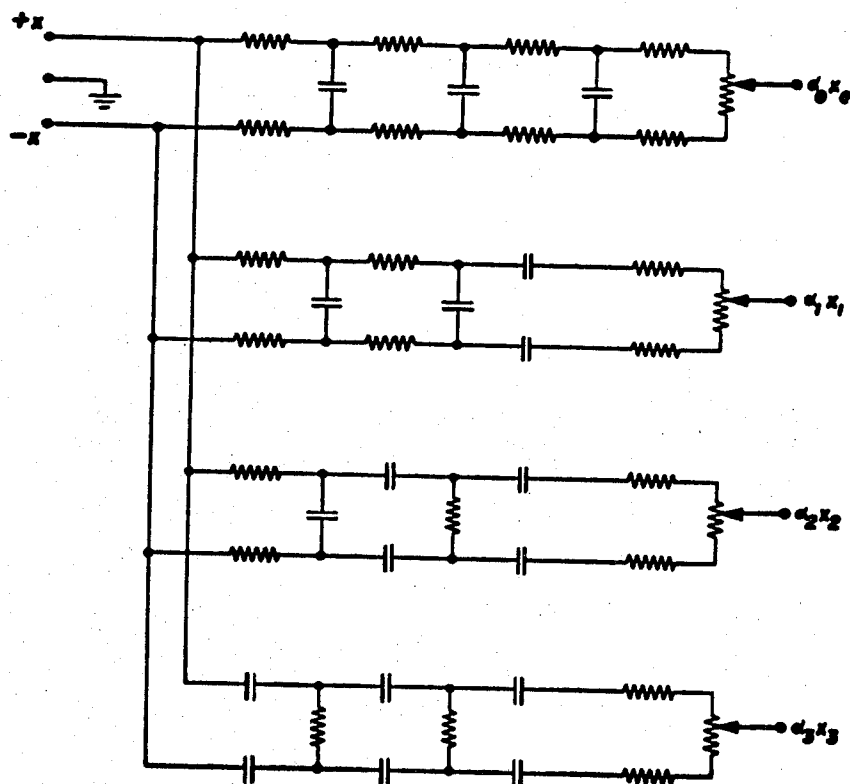
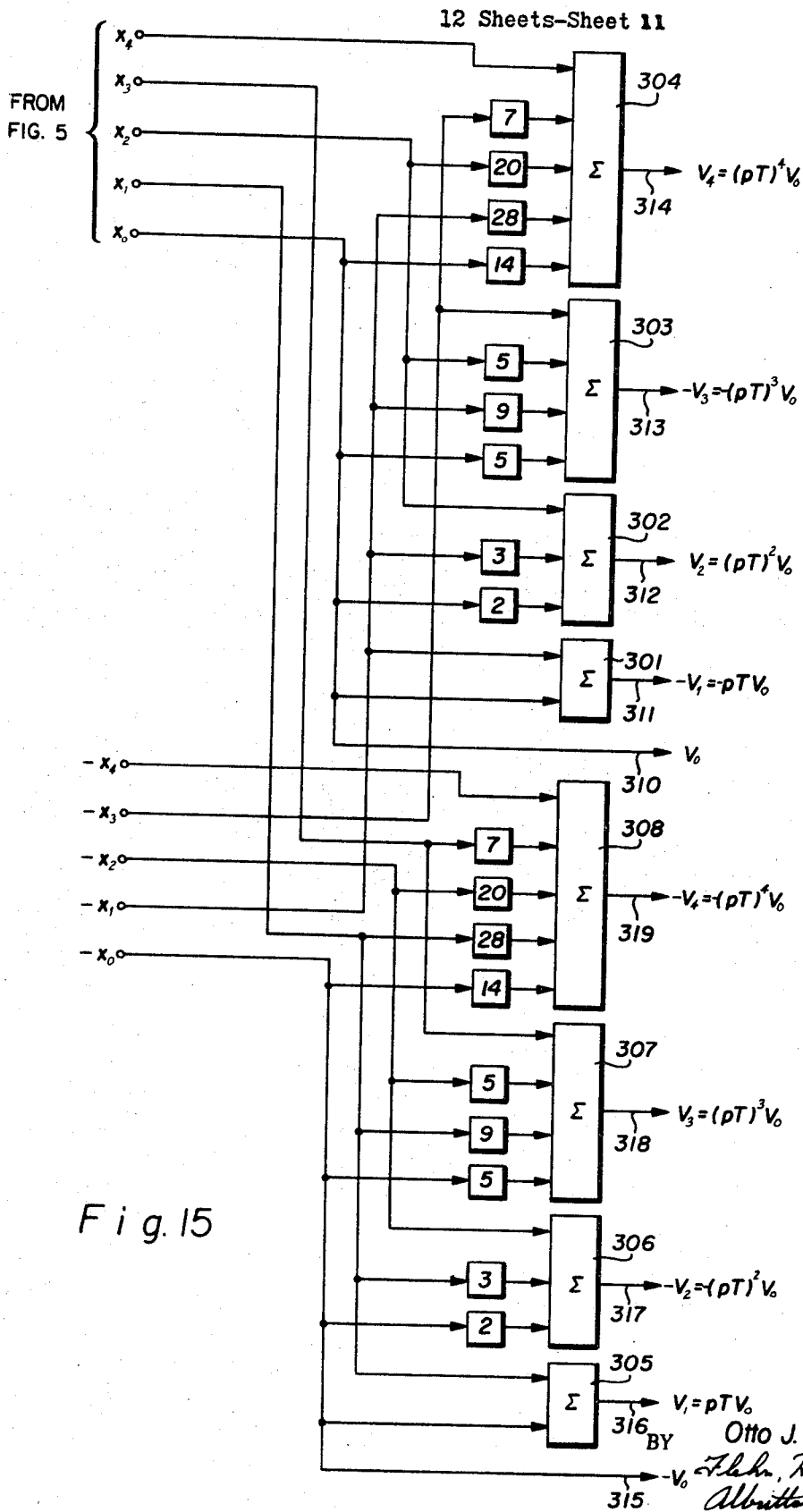


Fig. 14

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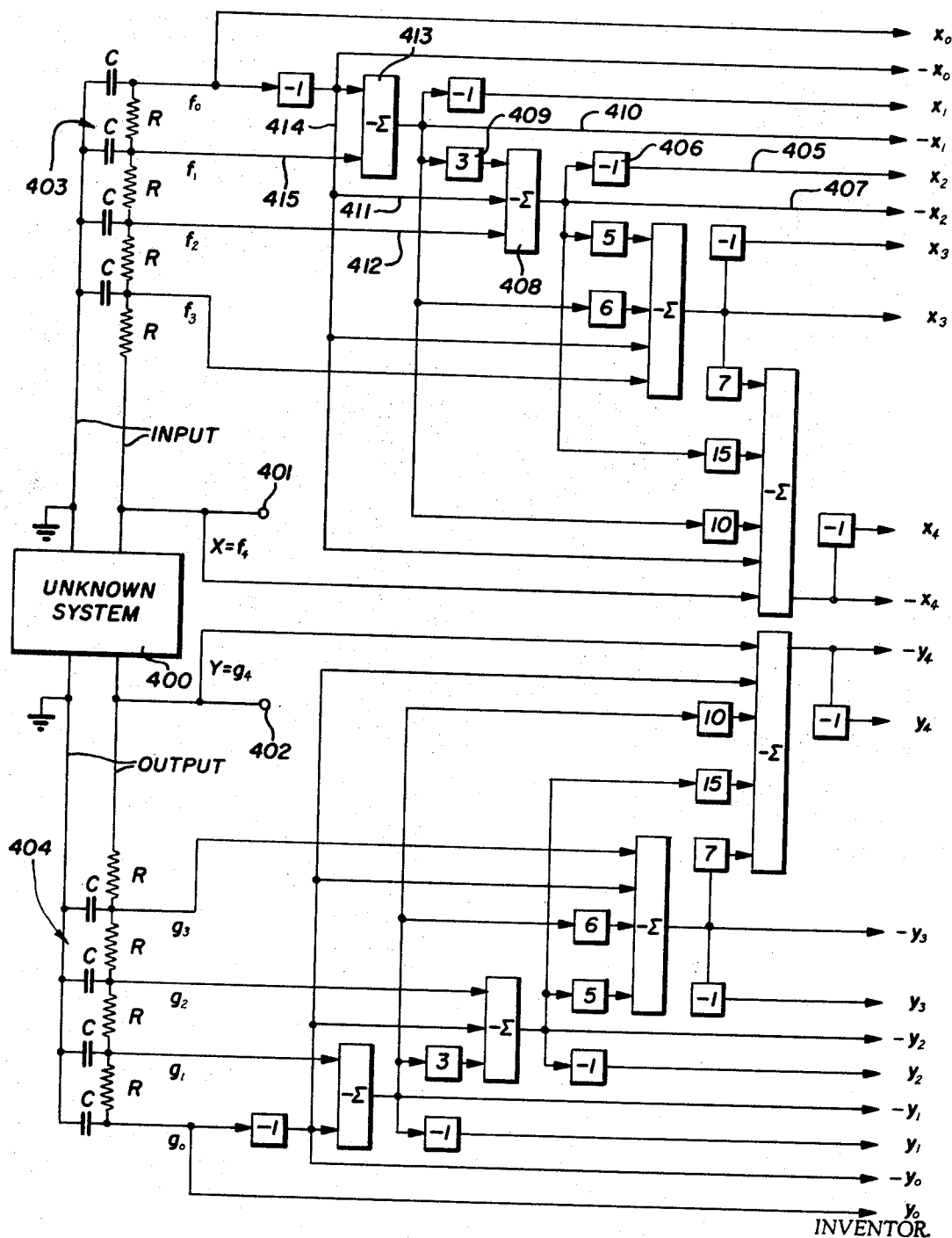


Fig. 16

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METHOD, APPARATUS AND SYSTEM FOR THE IDENTIFICATION OF THE RELATIONSHIP BETWEEN TWO SIGNALS

CROSS-REFERENCE TO RELATED APPLICATION

This application is a continuation-in-part and division of application Ser. No. 826,085, filed May 15, 1969 now U.S. Pat. No. 3,526,761, issued Sept. 1, 1970.

BACKGROUND OF THE INVENTION

This invention relates to a method, apparatus and system for the identification of the relationship between two signals, and more particularly to a method, apparatus and system for the measurement of impedance and admittance functions, gain and phase, transfer functions. Fourier transforms of the time domain functions, and power spectra.

Considerable difficulty has been experienced in identifying the relationship between two signals. For example, one method for the measurement of an impedance function has been to provide an electrical bridge with accurately calibrated components in several of the bridge arms, two terminals to which an unknown two-terminal network can be connected, an oscillator to provide a single frequency, and a null detector. When the bridge is balanced, the impedance of the unknown two-terminal network can be read from the bridge dials as one complex number. This measurement is adequate only if the unknown contains only one energy storage element. If the unknown has several capacitors and inductors, the measurements must be repeated at several frequencies. If as many measurements as unknowns are made, the solution for the unknowns can be solved from a difficult set of equations. However, such solutions give very poor accuracy. To increase the accuracy, many more measurements can be made at many different frequencies but these must be combined statistically in an expensive high-speed digital computer. Another example arises in the measurement of the characteristics of amplifiers, servo mechanisms, three and four terminal networks, transmission lines, transducers, pneumatic and hydraulic information and power transmission devices, transistors, process controls, regulators, feedback control systems, and multivariate systems. One method has been to excite the input to a system with a frequency of known amplitude and to measure the output amplitude and phase with respect to the input. This measurement is repeated at many frequencies. Plots of gain and phase versus frequency, or a plot of gain versus phase are descriptions of the unknown system, but to obtain the equation of this plot, it is difficult and has been done in the past by trial and error or by the fitting of templates. One method for measuring the characteristics of a network or system is to excite the system with an impulse or a step function and measure the impulse response or step response as a function of time. The analysis of such measured functions has been very difficult. Digital computers can be used to find a summation of exponentials that equals the measured time function or to find the Fourier transform of the measured time function. In the measuring of the characteristics of a random signal, it has been the practice to calculate the autocorrelation function of the signal. To utilize this function in system synthesis often required converting it into a power-density spectrum by taking the Fourier transform of the autocorrelation function. In general,

the recorded autocorrelation has to be multiplied by a sine and cosine wave and integrated over the entire function. It has been necessary to repeat this process many times for all significant frequencies. The results obtained are tabular or graphical in form and do not provide an equation of the power spectrum. From the foregoing, it can be seen that there is a need for a new and improved method, where and system for the identification of the relationship between two signals.

In general, it is an object of the present invention to provide a method, apparatus and system for identification of the relationship between two signals which overcomes the above named disadvantages.

Another object of the invention is to provide a method, apparatus and system of the above character which can be utilized for reading more than one complex number or more than two parameters of an unknown two-terminal network simultaneously.

Another object of the invention is to provide a method, apparatus and system of the above character in which more than one excitation frequency can be used simultaneously.

Another object of the invention is to provide a method, and apparatus of the above character in which gain and phase can be automatically measured utilizing only the signals existing in an operating system and without the introduction of a signal into the system.

Another object of the invention is to provide a method, and apparatus of the above character for determining the coefficients in the differential equation describing an unknown process.

Additional objects and features of the invention will appear from the following description in which the preferred embodiments are set forth in detail in conjunction with the accompanying drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a block diagram of the system and apparatus for the identification of the relationship between two signals incorporating my invention.

FIG. 2 is a circuit diagram, partially in block form, of one type of identification machine, apparatus or system incorporating my invention.

FIG. 3 shows a circuit diagram for typical state variable generators.

FIG. 4 is a circuit diagram, partially in block form, of a computer for use in my identification machine.

FIG. 5 is a circuit diagram showing typical state variable networks for generating state variables.

FIG. 6 is a circuit diagram, partially in block form, of a computer similar to that shown in FIG. 4.

FIGS. 7 and 8 are circuits which receive excitation signals from FIG. 9 and generate three excitation state variables and multiplies each by its corresponding state variable coefficient.

FIG. 9 is a circuit diagram of a computer, partially in block form, which adds weighted combinations of the state variables and minimizes the sum by changing the state variable coefficients.

FIG. 10 is a circuit diagram which will be required in place of the circuit diagram shown in FIG. 10 when the identification machine is designed for a fourth order polynomial operation on X or for test circuits requiring the fourth derivative of X to represent the differential equation or impedance function.

FIG. 11 is a network for generating state variables by weighted additions.

FIG. 12 is a network for generating state variables which can be used in place of the network shown in FIG. 7.

FIG. 13 is a circuit diagram of a state variable generator.

FIG. 14 is a network consisting of a combination of state variable computers and coefficient multipliers.

FIG. 15 is a state variable transformation analog computer to be used in conjunction with the embodiments shown in FIGS. 3 and 4.

FIG. 16 is a state variable transformation analog computer also to be used in conjunction with the embodiments shown in FIGS. 3 and 4.

In general, the present invention for determining the relationship between two signals is characterized by the provision of novel means for generating state variables from the two signals which are functions of time and the provision of novel means for forming a trial differential equation relating the two signals. Means is provided for forming an error measure of the error between the trial differential equation and the actual differential equation, minimizing the error measure by summing the weighted state variables, and using parametric feedback to correct the trial differential equation until the sum of the weighted state variables is minimized.

More specifically, the apparatus and system utilized for identifying the relationship between two signals comprises means for generating a plurality of first derived signals linearly related to the first of the two signals and means for generating a plurality of second signals linearly related to the second of the two signals. Means is also provided for determining a plurality of coefficients and multiplying the same with the first and second derived signals. Means is provided for forming a sum of the products and amplifying the same. Means is provided for generating a signal containing a component related to the product of the amplified sum and one of the state variables. Means is also provided for adding the last named signal to the predetermined coefficient which was previously multiplied as a factor times said one of the state variables.

More in particular, there is shown in FIG. 1 a system and apparatus for identifying the relationship between two signals. This apparatus may also be called an identification machine. This apparatus or machine consists of a signal generator 21 which delivers an excitation signal 22 to excite an unknown process 23. The output from the signal generator 21 is also supplied on a circuit 24 to an excitation state variable network 26 which produces a plurality of excitation state variables on a plurality of output circuits 27 which are linearly related to the excitation signal from the signal generator 21. The response of the unknown process 23 to the excitation signal from the signal generator 21 is supplied on an output circuit 28 to a response state variable network 29 which produces a plurality of response state variables on a plurality of output circuits 31 which are linearly related to the output signal from the unknown process 23.

In general, it can be stated that the excitation state variables on the circuits 27 are related to one another by the mathematical operation of either integration or differentiation, and similarly the response state variables on the circuits 31 are related to one another by the mathematical operation of integration or differentiation. The state variable networks 26 and 29, are here-

inafter explained, can each consist of one network with a plurality of taps so that different voltages are available or can consist of a plurality of different networks, each one of which produces a separate voltage. The networks 26 and 29 can be of any suitable type so long as a linear relationship is created between the input to the network and the output of the network. Preferably, the relationship between the state variable and the excitation signal supplied to the network are such that the input signal has a denominator term in the transfer function which is the same as the denominator term in the transfer function for the relationship between each state variable and the input to the network or between each response state variable and the response signal.

As can be seen in FIG. 1, the excitation state variables on the circuits 27 and the response state variables on the circuits 31 are supplied to a computer 32. The purpose of the computer 32 is to form a trial differential equation. The differential equation consists of a constant times one of the state variables plus a different constant times a different one of the state variables, etc., so that a sum of all of the constants each times its respective state variable is equal to zero. If the constants are all of the proper values, the sum of the equation will be zero and the constants will properly represent the differential equation of the unknown process 23. However, if the constants are of the wrong values, then the sum of the products of each constant times its corresponding state variable will add up to provide an error function which is different than zero. The computer 32 analyzes the error function to bring it to zero by determining which constant is in error and in which direction it is in error. This analysis of the decomposition of the error function into its components due to the errors in the different constants is performed by multiplying the error function times one of the state variables. This product will have an average essentially equal to zero if the constant corresponding to that state variable is correct; an conversely, if the constant corresponding to the state variable is wrong, then this product will have a non-zero mean or an average value which has a magnitude proportional to the error in the constant and a polarity proportional to the polarity of the error in the constant. To correct the constant, then, the product of the state variable times the error function is integrated and the integral is added to the constant corresponding to that state variable. The polarity of the feedback loop is chosen to reduce the error to a minimum. With a feedback loop on all but one of the constants associated with the excitation and response state variables, and each feedback loop arranged to minimize the error due to the constant which it is controlling, the error function will be driven to an average of zero.

The useful information which this computer 32 derives is the set of constants which are changed until the sum of the products of these constants times the corresponding state variables is continuously equal to zero. This useful information is read out by a display means 33. For example, if the constants are obtained as the output voltages of a bank of integrators, then a meter can be switched to any one of the integrators to read its output voltage. The unknown process 23, as shown in FIG. 1, is intended to represent any unknown device which can receive an excitation and have a response which is related to the excitation by a linear differential equation. For example, the process could be an audio

amplifier in which the excitation is the microphone input and the response is the loudspeaker output, or the process could be a dynamo electric amplifier in which the excitation is a field voltage and the response is an armature voltage, or the process could be a two-terminal filter in which the excitation is the voltage impressed across the two terminals and the response is the current which flows into one terminal and out of the other. Or, the process could be a hydraulic transmission system in which the excitation is the displacement of a hydraulic valve and the response is the force on a hydraulic cylinder.

The signal generator 21 in FIG. 1 should not be a fixed single frequency oscillator. Preferably, it is a random noise or signal generator such as a low frequency gaussian noise generator manufactured by Automation Laboratories, Inc. of 179 Liberty Ave., Mineola, L. I., N. Y.; but it may be a square-wave generator, or a triangle wave generator, or a repetitive pulse generator, or a random pulse generator. The signal generator 21 in FIG. 1 may be a sweeping oscillator or an FM modulated oscillator whose rate of change of frequency is very large compared to the rate of convergence of the computer 32 in FIG. 1 to the values of the constants. The reason for this is that during the time of convergence, a large number of different frequencies should have passed through the process. The reason that the signal generator should vary its frequency rapidly is that it must deliver a wide variety of different frequencies for the excitation of the process during the time that the computer is changing the constants which it is evaluating.

The coefficients of the differential equation read out by the display means 33 can be designated as $a_0, a_1, a_2, \dots, a_k$ and $b_0, b_1, b_2, \dots, b_k$. Within the computer 32, there is provided means for forming the sum e of the products of the state variables times the corresponding coefficients. The state variables on the circuits 27 can be designated as $X_0, X_1, \dots, X_k$. The state variables on the circuits 31 can be designated as $Y_0, Y_1, \dots, Y_k$. The equation for the sum e can be written as follows:

$$e = \sum_{n=0}^{n=k} (a_n X_n + b_n Y_n) \quad (1)$$

The computer 32 is also provided with means for forming the plurality of products of each state variable times the function of the sum e . If the function of the sum e is designated $f(e)$, then this plurality of products is

$X_0 f(e), X_1 f(e), \dots, X_n f(e), \dots, X_k f(e), Y_0 f(e), Y_1 f(e), \dots, Y_n f(e), \dots, Y_k f(e)$. The computer 32 is also provided with means for integrating each of the above products and changing each of the coefficients but one in response to a time integration of one of the products in the plurality of products above. Specifically, the time integrations are:

$$\begin{aligned} a_0 &= -K \int_{-\infty}^0 X_0 f(e) dt \\ &\dots \\ a_n &= -K \int_{-\infty}^0 X_n f(e) dt \\ &\dots \\ a_k &= -K \int_{-\infty}^0 X_k f(e) dt \end{aligned} \quad (2)$$

$$\begin{aligned} b_0 &= -K \int_{-\infty}^0 Y_0 f(e) dt \\ &\dots \\ b_n &= -K \int_{-\infty}^0 Y_n f(e) dt \\ &\dots \\ b_k &= -K \int_{-\infty}^0 Y_k f(e) dt \end{aligned} \quad (3)$$

The meaning of the integral notation used above with time limits from minus infinity to zero is that the integration has been carried out from the time when the equipment was turned on until the present time. No initial value of the coefficient is shown in each equation above because in normal operation the integration is continued for an amount of time sufficient to destroy the initial values of the coefficients at the time that the equipment was turned on.

With the proper choice of the function $f(e)$, e will be driven towards a minimum, and the coefficients a_k and b_k will change due to the action of the integrators until they reach final steady state values, which values will satisfy the following differential equation for the unknown process 23 giving the relationship between the excitation on circuit 22 called X and the response on circuit 28 called Y .

$$\sum_{n=0}^k (a_n T^n p^n X + b_n T^n p^n Y) = 0 \quad (4)$$

The function $f(e)$ can be linearly proportional to e , such as would be obtained from an amplifier whose input is e . Alternatively, the function $f(e)$ can be the polarity of e only, i.e., $e/|e|$. In this case, the function can be generated by the motion of the armature of a relay whose coil current is driven by the output of an amplifier whose input is e . The plurality of products above can then be obtained by connecting each state variable to reversing contacts mounted on the armature of the relay.

It should be appreciated that the present invention is not limited to the function $f(e)$ enumerated above. For example, the function $e/|e|$ may be used, or the function e^3 may be used. As another example, for m equal to 2 or greater, the function may be

$$f(e) = (X_n^{1-1/m} + e X_n^{-1/m})^m - (X_n^{1-1/m} - e X_n^{-1/m})^m \quad (5)$$

FIG. 2 shows a wiring diagram partially in block form of an identification apparatus or machine of the type shown in FIG. 1. Analog computer notation is used in FIG. 2 and analog computer terminology will be used in describing the operation of FIG. 2. The signal X to the unknown process 23 is derived from the signal generator 21 as explained in FIG. 1. The signal Y is the response from the unknown process 23. Signal X passes on circuit 24 through two different filter networks 36 and 37 which make up the state variable network 26 and generate two different voltages X_0 and X_1 , respectively. The voltage X_0 is generated by the network 36 which is a lag filter formed by a series resistor R and a shunt capacitor C so that the output voltage is read across the

capacitor and is related to the signal X by the transfer function $1/1 + sT$. The signal X_1 is produced by the filter network 37 which consists of a series capacitor C and a shunt resistor R so that the voltage is read across the resistor R . The signal X_1 is related by the transfer function $sT/1 + sT$ so that the signal X_1 is the pure derivative of the signal X_0 times the constant T . The signals X_0 and X_1 correspond to the excitation state variables appearing on the circuits 27 in FIG. 1.

In a similar manner, the output signal Y is supplied to a pair of filter networks 38 and 39 which form the response state variable network 29 to produce two different voltages Y_0 and Y_1 which correspond to the state variables appearing on the circuits 31 of FIG. 1. The signal Y_0 is produced by a lag filter network 38 which is formed in the same manner as filter network 36 so that the output signal Y_0 is related by the transfer function $1/1+sT$ to the input signal Y . Similarly, the output signal Y_1 is produced by a lead filter network 39 so that the output signal y_1 is related by the transfer function $sT/1+sT$ to the input signal Y . More generally stated, the state variable networks 26 and 29 are preferably chosen so that the network producing the state variable X_0 is identical to the network producing the state variable Y_0 . The same is true for X_1 and Y_1 .

The state variables X_0 , X_1 , Y_0 and Y_1 are supplied to the computer 32 which, in the embodiment shown in FIG. 2, consists of a stepping switch 41 which is provided with two banks 42 and 43 of stationary contacts adapted to be engaged by a pair of movable contacts 44 and 46, respectively. The state variable signals are connected to the stationary contacts of bank 42 so that one of the state variables can be selected at a time to be supplied to the input of a multiplier 48 which also can be identified as an analyzer. The contacts of bank 42 are, therefore, identified as X_0 , X_1 , Y_0 and Y_1 . The second bank of stationary contacts 43 of the stepping switch 41 are identified as a_0 , a_1 , b_0 and b_1 which are adapted to be contacted by the movable contact 46 and to be supplied with energy from the analyzer 48.

The feedback loop 55 shown in FIG. 2 when the contact 46 is in engagement with the a_0 contact will produce continuously a rate of change of a_0 in the correct direction until the error function entering the analyzer multiplier 48 is minimized. The normal operation of the apparatus shown in FIG. 2 is not to permit the constant a_0 to be adjusted for a sufficient length of time to minimize the error function but only to permit the constant to change a small amount and then the stepping switch steps to the next contact and receives the state variable X_1 , analyzes the error function utilizing the state variable X_1 and uses this analysis to correct the constant a_1 , and making only a small part of the total correction necessary in the constant a_1 . Then, the stepping switch continues to step through the other contacts and when it has cycled through all positions, it returns to the position as shown in FIG. 2 and makes an additional correction in the constant a_0 and repeats this cycling in a periodic manner until the necessary correction is obtained for all of the constants.

In accordance with conventional analog computer notation, all of the multipliers and the summing amplifiers and the integrators in FIG. 2 are assumed to be of the inverting type such that the output is the negative of the operation on the input which the device is intended to perform.

A clamping switch 61 is provided which is connected to the parameter b_0 and holds it at the constant value of +1 irrespective of the output of the integrator connected to b_0 . The purpose for this clamping switch 61 is so that a differential equation utilizing two terms in the numerator and two terms in the denominator has four coefficients but only three of the coefficients are independent, that is, one can select any one coefficient and divide all the others by it and obtain a correct differential equation. One would simply have changed all the numerator terms and all the denominator terms up or down by some constant factor. In order to set the scale of these factors and have only as many converging operations in the identification machine as the number of independent variables, one can, therefore, solve only three of the four coefficients. By setting the parameter b_0 equal to one, then the other three coefficients can be solved for. This is satisfactory if the unknown process contains either differentiation or gain at zero frequency but this is not satisfactory if the unknown process contains pure integration. In that case, the parameter b_0 should be zero. With the clamping switch 61 in the position shown in FIG. 2, the parameter b_0 is held at unity and this will cause all the other parameters to tend to increase to very large numbers. For measuring an unknown process containing pure integration, the clamping switch 61 of FIG. 2 is thrown to the upper position shown in FIG. 2 in which the parameter a_0 is held at unity. Then, during the normal operation of the machine, the parameter b_1 will reach a finite number and the parameter b_0 will reach zero. Thus, when the clamping switch 61 is in the upper position, the machine is satisfactory for measuring unknown processes which have either unity d-c gain or infinite d-c gain due to one or more integrations.

The state variable networks shown in FIG. 2 are of a unique type. They are chosen such that the network producing the state variable X_0 is identical to the network producing the state variable Y_0 . The reason for this is that the poles of this state variable network are, therefore, removed from the differential equation and permit a representation of the unknown process by a set of parameters closely related to the coefficients of the conventional differential equation. In a similar manner, the poles of the network to produce the state variable X_1 are the same as the poles of the network to produce the state variable Y_1 . This also results in a simplification of the interpretation of the parameters which are obtained by the identification procedure and also result in a larger number of independent variables which can be evaluated by the identification machine for a given quantity of equipment. In addition, in FIG. 2, a further improvement has been made by setting the poles of the state variable network to produce the state variable X_0 also equal to the poles of the state variable network to produce the state variable X_1 . In other words, not only do the state variable networks appear in pairs which are identical, i.e., the pair for X_0 , Y_0 and the pair for X_1 , Y_1 which is one important requirement, but, in addition, the poles of one pair are equal to the poles of the other pair. This, then causes these pole terms to completely cancel out of the relationship which is fulfilled by the identification machine so that the relationship which is fulfilled by the identification machine has the same parameters in it as the conventional differential equation, i.e., the parameters a_0 , a_1 , b_0 and b_1 are the coefficients of the first order differ-

tial equation representing the unknown process. If the unknown process contains second or third order terms, then the identification machine must contain additional state variables to analyze for these higher order terms. In general, the state variable machine will contain many of these state variable networks and many parameters like a_1 and b_1 , but for the purposes of illustration, FIG. 2 has been shown sufficient to identify in the unknown process one numerator zero, one denominator pole and one gain term.

Previous investigators who have attempted to build identification machines of this type have tried to generate the state variable X_1 by calculating the derivative of the signal X and have tried to generate corresponding state variable X_2 by generating the second derivative of the signal X . Now it is well known to those skilled in the art that pure first derivatives and pure second derivatives cannot in fact be calculated, and consequently, previous attempts to build identification machines have resulted in state variables with errors that are related to the dynamic mistakes made in attempting to generate state variables with unrealizable networks.

The unique networks used in FIG. 2 are realizable networks as hereinafter described such that the derivative is generated with its corresponding pole which cannot be eliminated and then the state variable X_0 is generated by using the pole alone. This produces a relationship between state variables which contain no error, that is, the state variable X_1 is the pure derivative of the state variable X_0 and nothing need be said about its relationship to the signal X . Or stated another way, the state variable X_1 is not the derivative of the signal X . It is the derivative of the signal X with a significant amount of filtering. Computer 32 as shown in FIG. 2 is more fully described in U.S. Pat. No. 3,526,761.

From the foregoing, it can be seen that the network 26 comprises means for generating a plurality of first derived signals which are linearly related to the first of two signals, that is, the input signal X . The network 29 consists of means for generating a plurality of second derived signals which are linearly related to the second of two signals, that is, the output signal Y , from the unknown process. The coefficients a_1, a_0, b_1, b_0 which are determined can be called weighting factors. These weighting factors are multiplied by multipliers 51 with the corresponding linearly derived signal. The products obtained are added in the summing amplifier 52.

In FIG. 3, there is shown circuitry for typical state variable generators 26 and 29. The input to the unknown system or process 23 is designated by X and the output from the unknown system is designated by Y . As shown in FIG. 3, the input signal X is decomposed into the derived state variables designated by X_0, X_1, X_2, X_3 and X_4 through an RC network, and similarly the negatives of these same state variables are derived through the use of an inverting amplifier indicated by $-X$ through a similar RC network as shown in FIG. 3. Similarly, the output signal Y is decomposed into derived state variables designated Y_0, Y_1, Y_2, Y_3 and Y_4 , through an RC network, and the negatives of these same signals are derived by the use of an inverting amplifier $-Y$ and through a similar RC network as shown in FIG. 3.

FIG. 4 is a circuit diagram, partially in block form, of the computer 32 which receives the plurality of state variables generated by the circuitry as shown in FIG. 3. The state variables are connected to the terminals indicated in FIG. 4. These state variables are supplied to

upper and lower contacts, to a movable contact of ganged reversing switches 84, and to integrators 86 which are identified as K/p which perform integrations with respect to time. The outputs of the integrators are supplied to multipliers 87 which are indicated by x and the outputs from the multipliers are added in a summing device 88 indicated by $+$. The output of the summing device 88 is indicated as θ_c and is supplied to a relay 89. When θ_c is a positive, the relay 89 is operated so that the movable contacts of the reversing switches 84 engage the upper contacts, and conversely when θ_c is negative, the movable contacts engage the lower contacts.

Operation of the computer shown in FIG. 4 as a part of the identification machine may be briefly described as follows. Let h_i and k_i be the machine outputs. The machine forms the trial integro-differential equation:

$$\sum_{i=0}^n (h_i X_i - k_i Y_i) = -9. \quad (6)$$

The purpose of the identification machine is to find the unknown differential equation representing the true relationship between the state variables, which is

$$\sum_{i=0}^n (h_i X_i - k_i Y_i) = 0 \quad (7)$$

The machine will have found the correct values for h_i and k_i which relate the state variables of the unknown system in the manner shown by equation 7, when it has been running long enough, so that the value θ_c from equation 7, or the value of the input to the relay shown in FIG. 6, is zero continuously in time. Any error in a coefficient h_i or k_i will cause the variable θ_c to deviate from zero during transients and during the progress of random variations of the state variables.

In FIG. 4, each coefficient will converge to its correct value because of the following mechanism: Assume that all of the coefficients are correct except h_1 ; assume further that X_1 is positive and that $-X_1$ is negative; then the output of the multiplier connected to h_1 will be too negative if h_1 is too positive or will be too positive if h_1 is too negative. This output will contribute an error component to θ_c such that the relay will move to the top contact if h_1 is too negative and the relay will move to the bottom contact if h_1 is too positive. With the relay on the top contact the integral connected to the state variable X_1 will integrate a positive number and will therefore increase the variable h_1 bringing it from its too negative value towards its correct value. In a similar manner when the relay is on its bottom contact, the input to the integrator for h_1 will integrate a negative number and drive h_1 from its too positive value again towards its correct value. This negative feedback iterative convergence of the assumed value of h_1 in the machine towards the correct value as determined by the relationship between the state variables X_i and Y_i will always have the correct direction of change for either polarity of X_1 .

In a similar manner, each h_i and k_i in FIG. 4 will converge towards its correct value independently of the polarity of the associated state variable. The rate of convergence of each coefficient is proportional to the magnitude of the associated state variable. For exam-

ple, if state variable Y_2 should be always very small, then the rate of convergence of k_2 will always be small. Therefore, those coefficients that are most important in the integro-differential equation will assume their correct values most rapidly, and those coefficients that are least important will take the longest time before their values become significant. When the identification machine is first turned on, it calculates a first order linear approximation to the unknown process, and then gradually improves upon this by calculating a second order approximation and then a third order approximation and continuously expands the complexity of the model used to represent the unknown process. The accuracy is proportional to the running time. If all of the state variables are stationary random variables, and the engineer does not know how much data is necessary in order to obtain a valid model of the unknown process, then the identification machine can be operated in real time until the rates of change of the identified parameters are negligible in the judgment of the engineer. There is no need to record the input and output signals from the unknown process.

Note that one parameter, h_0 , has been chosen arbitrarily in FIG. 4. This sets the scale factor for the other parameters. For example, if the unknown process has a dc gain of 100, then k_0 will equal 0.01 for the usual designation of state variables. If the unknown process has an integral, then k_0 will equal 0 in FIG. 4.

The computer shown in FIG. 4 will work satisfactorily for a wide variety of selections of state variables. A convenient selection of state variables is shown in FIG. 3. In this case, the input to the unknown system is the variable X . The state variable X_0 is chosen as the random variable X filtered with a four-time-constant low-pass filter. This provides a desirable smoothing of the fluctuations, particularly for those coefficients calculating the low pass characteristics of the unknown system. The state variable X_1 is related to the input X by the same four-time-constant filtering network, but in addition, there is one zero factor in the numerator of the transfer function from X to X_1 . X_2 has a second order numerator term and the same four-time-constant filtering poles. X_3 has a third order numerator term. All of the resistors have the same value R , and all of the capacitors have the same value C . The transfer functions relating this particular selection of state variables to the random input X are

$$p^n = d^n/dt^n$$

(8)

$$T = RC$$

(9)

$$D = 1 + 10 pT + 15 (pT)^2 + 7 (pT)^3 + (pT)^4$$

(10)

$$X_0 = X/D$$

(11)

$$X_1 = (1 + pT) X/D$$

(12)

$$X_2 = (1 + 3 pT + (pT)^2) X/D$$

(13)

$$X_3 = (1 + 6 pT + 5 (pT)^2 + (pT)^3) X/D \quad (14)$$

$$X_4 = X$$

(15)

In a similar manner Y_i state variables are related to the random output Y by the following equations

$$Y_0 = Y/D$$

(16)

$$Y_1 = (1 + pT) Y/D$$

(17)

$$Y_2 = (1 + 3 pT + (pT)^2) Y/D$$

(18)

$$Y_3 = (1 + 6 pT + 5 (pT)^2 + (pT)^3) Y/D$$

(19)

$$Y_4 = Y$$

(20)

The identification machine solves for the values of h_i and k_i in equation 7. If equations 11 – 20 inclusive are substituted into equation 7, one has the following

$$\begin{aligned} & h_0 X/D + h_1 (1 + pT) X/D + h_2 (1 + 3pT + (pT)^2) X/D \\ & + h_3 (1 + 6pT + 5 (pT)^2 + (pT)^3) X/D + h_4 X - k_0 Y/D \\ & - k_1 (1 + pT) Y/D - k_2 (1 + 3pT + (pT)^2) Y/D \\ & - k_3 (1 + 6pT + 5 (pT)^2 + (pT)^3) Y/D - k_4 Y = 0 \end{aligned} \quad (21)$$

Multiplying through by the denominator polynomial D yields

$$\begin{aligned} & (h_0 + h_1 + h_2 + h_3 + h_4)X + (h_1 + 3h_2 + 6h_3 + 10h_4) TpX \\ & + (h_2 + 5h_3 + 15h_4) T^2 p^2 X + (h_3 + 7h_4) T^3 p^3 X + T^4 p^4 X \\ & - (k_0 + k_1 + k_2 + k_3 + k_4) Y - (k_1 + 3k_2 + 6k_3 + 10k_4) TpY \\ & - (k_2 + 5k_3 + 15k_4) T^2 p^2 Y - (k_3 + 7k_4) T^3 p^3 Y \\ & - k_4 T^4 p^4 Y = 0 \end{aligned} \quad (22)$$

The usual method of writing the differential equations for the unknown system in FIG. 3 in terms of the random input X and the random output Y would be

$$\sum_{i=0}^n (a_i p^i X - b_i p^i Y) = 0 \quad (23)$$

This is a shorthand method of writing

$$\begin{aligned} & a_0 X + a_1 pX + a_2 p^2 X + a_3 p^3 X + a_4 p^4 X - b_0 Y - b_1 pY \\ & - b_2 p^2 Y - b_3 p^3 Y - b_4 p^4 Y = 0 \end{aligned} \quad (24)$$

It can be seen from a comparison of equation 22 and equation 24, term by term, that the parameter transformation between the conventional differential equation

notation and the transformed state variables, which are used in FIG. 3, is

$$a_0 = h_0 + h_1 + h_2 + h_3 + h_4$$

(25) 5

$$a = (h_1 + 3h_2 + 6h_3 + 10h_4) T$$

(26)

$$a_2 = (h_2 + 5h_3 + 15h_4) T^2$$

(27)

$$a_3 = (h_3 + 7h_4) T^3$$

(28)

$$a_4 = h_4 T^4$$

(29)

$$b_0 = k_0 + k_1 + k_2 + k_3 + k_4$$

(30) 20

$$b_1 = (k_1 + 3k_2 + 6k_3 + 10k_4) T$$

(31)

$$b_2 = (k_2 + 5k_3 + 15k_4) T^2$$

(32) 25

$$b_3 = (k_3 + 7k_4) T^3$$

(33)

$$b_4 = k_4 T^4$$

(34)

From the foregoing, it can be seen that the identification machine, apparatus or system identifies an integro-differential equation. Since the identification machine is connected to the input and the output of the unknown system, it observes these signals as they vary with time. There is no need to perturb the input when the unknown system is operating in its usual manner, and there exists commands and other variations in the input signal.

The differential equation is formed as an integro-differential operation on the input variable plus a different integro-differential operation on the output variable summing to zero. The identifier starts with assumed integro-differential parameters or coefficients and forms the function which should equal zero. Errors are represented by deviations from zero, and these are used to slowly correct each of the parameters in the assumed integro-differential operations.

The convergence rate of the machine is adjustable and can be chosen in accordance with the spectra of the input and output signals. For a variable-parameter system, if the parameter variation is slow, the identification machine will track the parameter variations and will deliver the value of the parameter as a function of time. For a variable parameter system where the parameter variation is quite rapid or where the parameter is a function of one of the signals in a non-linear manner, then the identification machine delivers a statistical best estimate of the linearized parameter, smoothing out the variations.

In FIG. 5, there is shown another type of circuitry for generating state variables in which the state variables are closely related to the derivatives of the input and output of the unknown system. By comparison of FIGS. 3 and 5 with FIGS. 2, 12, 13 and 14, it can be seen that

the embodiments in FIG. 3 and 5 are similar to those shown in FIGS. 2, 12, 13 and 14 with the exception that in FIGS. 3 and 5 the differentiations are performed by bilateral loaded networks rather than by isolated differentiators as in FIGS. 2, 16, 19 and 22. The four energy storage networks shown in FIG. 5 will have denominators of their transfer functions of fourth order. The state variable X_4 is, therefore, the fourth derivative of the random variable X , but with an additional filtering produced by four poles. This additional filtering is not undesirable. The state variable X_1 has a composite transference with a derivative filtered by three zeros and four poles. The equations for all of the state variables in FIG. 5 are given by

$$D = 1 + 7(pT) + 15(pT)^2 + 10(pT)^3 + (pT)^4 \quad (35)$$

$$X_0 = X$$

(36)

$$X_1 = (pT + 5(pT)^2 + 6(pT)^3 + (pT)^4) X/D$$

(37)

$$X_2 = ((pT)^2 + 3(pT)^3 + (pT)^4) X/D$$

(38)

$$X_3 = ((pT)^3 + (pT)^4) X/D$$

(39)

$$X_4 = (pT)^4 X/D$$

(40)

$$Y_0 = Y$$

(41)

$$Y_1 = (pT + 5(pT)^2 + 6(pT)^3 + (pT)^4) Y/D$$

(42)

$$Y_2 = ((pT)^2 + 3(pT)^3 + (pT)^4) Y/D$$

(43)

$$Y_3 = ((pT)^3 + (pT)^4) Y/D$$

(44)

$$Y_4 = (pT)^4 Y/D$$

(45)

These state variables are inserted into the integro-differential equation 6 for the identification machine, and the entire equation operated on by the operator D from equation 35. Then it can be seen that the parameter transformation for FIG. 7 is

$$a_0 = h_0$$

(46)

$$a_1 = (7h_0 + h_1)T$$

(47)

$$a_2 = (15h_0 + 5h_1 + h_2)T^2$$

(48)

$$a_3 = (10h_0 + 6h_1 + 3h_2 + h_3)T^3$$

(49)

$$a_4 = (h_0 + h_1 + h_2 + h_3 + h_4)T^4 \quad (50)$$

$$b_0 = k_0 \quad (51)$$

$$b_1 = (7k_0 + k_1)T \quad (52)$$

$$b_2 = (15k_0 + 5k_1 + k_2)T^2 \quad (53)$$

$$b_3 = (10k_0 + 6k_1 + 3k_2 + k_3)T^3 \quad (54)$$

$$b_4 = (k_0 + k_1 + k_2 + k_3 + k_4)T^4 \quad (55)$$

The parameters above are applicable for the differential equation in the form of equation 23 or equation 24.

It can be seen that the computer shown in FIG. 6 is identical to the one shown in FIG. 4 with the exception that the state variables are supplied to different terminals.

In general, it can be stated that the circuitry as shown in FIG. 4 is for a form of identification machine which is suitable when the unknown has integrals. On the other hand, the embodiment shown in FIG. 6 is suitable for an identification machine when the unknown is high-pass only.

Expressed in another way, if the output of the unknown process has zero mean, even though the input variable is non-zero mean, then the identification machine using circuitry of the type as shown in FIG. 4 will have difficulties. The ratio of h_0/k_0 should be zero. Since h_0 has been arbitrarily chosen, the machine tries to drive all the k_i 's to infinity. This is a non-convergent mode of operation. To avoid this difficulty, the coefficient k_0 should be chosen arbitrarily and the machine should solve for the coefficient h_0 . The arrangement shown in FIG. 8 makes this possible. In this embodiment, the coefficients will be automatically determined satisfactorily for an unknown system which contains derivatives, but not integrals, in cascade with the information flow from X to Y .

FIG. 7 is a network to derive state variables and weighted state variables from the excitation signal X in FIG. 1. As can be seen from FIG. 7, it contains potentiometers P1, P2 and P3 and a number of resistors and capacitors. The voltages created on the terminals in the network in FIG. 7 are designated as shown. The proportional setting of the wiper arm of the potentiometer P1 is called the parameter or coefficient a_0 . When the wiper arm is electrically the same as the X_0 terminal, a_0 is unity. When the wiper arm is electrically the same as the $-X_0$ terminal, a_0 is -1 . When the arm is in its mid position, a_0 is zero. The voltage picked off by the wiper arm of the potentiometer P1 is a product of the state variable X_0 appearing on the X_0 terminal and the coefficient a_0 . The proportional setting of the wiper arm of the potentiometer P2 is the parameter or coefficient a_1 . The voltage picked up by the wiper of the potentiometer P2 is the product a_1X_1 . The proportional setting of the wiper arm of the potentiometer P3 is the parameter coefficient a_2 . The voltage picked off by the wiper of the potentiometer P3 is product a_2X_2 .

The circuitry shown in FIG. 7 discloses a new, unique and improved circuitry for obtaining state variables. Heretofore, the time derivative of a function has been obtained with an operational amplifier and an RC circuit, connected as a high-pass filter. This has had the disadvantage that the amplifier is a dynamic device whose characteristics change. The RC circuit passes high frequencies, and accentuates the noise components of a signal whose derivative is being taken. There is always an upper limiting frequency above which the circuit does not calculate a true derivative, but is a flat unity gain network. In the past, the time integral of a function has been obtained with an operational amplifier and an RC circuit, connected as a low pass filter. This has had the disadvantage that the amplifier is a dynamic device whose gain changes with time. The circuit has a lower limiting frequency below which it does not calculate a pure integral, but is a flat unity gain network. A perfect differentiator had to have infinite gain at infinite frequency. A perfect integrator had to have infinite gain at zero frequency.

The circuitry in FIG. 7 makes possible an improved method for obtaining derived state variables X_0 , X_1 and X_2 related to the input variable X . The state variable X_2 is the exact time derivative of the state variable X_1 . The state variable X_1 is the exact time derivative of the state variable X_0 . The upper limiting frequency is not a function of the circuit in FIG. 7, but is a function of the inductance of the resistors and the conductors to the capacitors. The lower limiting frequency is a function of the leakage and the soaking effect of the capacitors. The useful frequency band of the circuit shown in FIG. 7 is many decades wider than that of conventional RC differentiators. Since it is a passive circuit, its stability is superior to that of amplifiers. The state variable X_0 is related to the signal X by attenuation and phase lag, and there is no requirement that the gain exceed unity. The transmission from X to X_0 is a low pass transmission.

The state variable X_1 is related to the signal X by attenuation and phase shift, and there is no requirement that the gain exceed unity. The transmission from X to X_1 is a band-pass transmission. The state variable X_2 is related to the signal X by attenuation and phase lead. There is no requirement that the gain exceed unity. The transmission from X to X_2 is a high-pass transmission.

The circuitry shown in FIG. 7 can also be interpreted in terms of integration. The state variable X_1 is the exact time integral of the state variable X_2 . The state variable X_0 is the exact time integral of the state variable X_1 . The state variable X_0 is the exact time double integral of the state variable X_2 .

The components in FIG. 7 have the following unique relationship to each other:

Capacitors C1-C5 have a value of C, whereas capacitors C6 and C7 have a value of 4C.

The resistors have the following values:

Resistors	
R1	0.3303 R
R2	0.5 R
R3	0.5 R
R4	0.3303 R
R5	0.0472 R
R6	0.0472 R
R7	3.075 R
P1, P2, P3	1.000 R

A typical value of R is 20,000 ohms. A typical maximum value for C is 5 microfarads. C should be adjust-

able in decade or binary values for changing the range of the time scale or frequency band in the impedance function for the unknown process or system 23, FIGS. 1 and 3.

Other unique advantages of the circuitry shown in FIG. 7 can be seen from examination of the Table above. The resistances of the loading potentiometers are all alike. The values of the capacitors are related one to the other by integers which simplifies the decade switching.

The circuitry shown in FIG. 7 can be used with impulse excitation, square wave excitation, triangular wave excitation, random excitation or frequency sweeping sinusoidal excitation.

As pointed out above, FIG. 8 is a circuit identical to that of FIG. 10 but driven by the signal Y to produce state variables Y_0 , Y_1 and Y_2 . The products of the state variable and the coefficient are produced at the wiper arms of the potentiometers P4, P5 and P6 as indicated.

The potentiometers P1-P6 shown in FIGS. 7 and 8 are preferably of a helipot type which have dials that can be calibrated to read directly the parameters a_2 , a_1 , a_0 , b_2 , b_1 and b_0 , respectively. The potentiometers also may have shaft extensions to which can be connected electro-mechanical transducers which can be used to provide torque to turn the shafts to adjust the various parameters automatically.

In FIG. 9, there is shown a computer which is utilized for minimizing the error function of a weighted state variable sum and is connected to the networks shown in FIG. 7 and 11 so that the terminals identified in the same manner are connected to each other. The products $a_0 X_0$, $a_1 X_1$, etc., are each connected through resistors 140 connected to a low level error bus 141 identified as "e bus". The resistors 140 form an adder network and have equal ohmic values so that the voltage on the low level error bus 141 is proportional to the sum of the signals to which the resistors are connected. The low level "e bus" is connected to the input of a high gain amplifier 142 which has input overload protection so that the input signal to the high gain amplifier 142 consists of the voltage between the low level error bus 141 and ground. Preferably, the ground terminal in FIG. 9 connected to the high gain amplifier 142 should have the same potential as the center tap between X and $-X$ in FIG. 7 which is the same potential as the center tap between Y and $-Y$ in FIG. 8.

The output of the amplifiers 142 is a pair of push-pull voltages with opposite polarities with respect to ground. These two output voltages appear on the E bus 143 and the -E bus 144. The +E bus 143 and the -E bus 144 are connected to a bank of 12 vacuum tubes VT1 - VT12 with two vacuum tubes included in each envelope 146. Each of the tubes consists of a cathode, a plate and five grids as shown. The plate of each of the tubes is supplied with a suitable voltage such as 300 volts from a power supply 147 through a load resistor 148. The cathodes of the tubes are interconnected and are connected through a resistor 149 to ground. The first grids of the tubes VT1, VT3, VT5, VT7, VT9 and VT11 are all connected to the +E bus 143, whereas the first grids of the tubes VT2, VT4, VT6, VT8, VT10 and VT12 are all connected to the -E bus 144. The third grid of the tubes VT1, VT3, VT5, VT7, VT9 and VT11 are connected to terminals which are identified as X_0 , X_1 , X_2 , Y_0 , Y_1 and Y_2 , respectively. The third grids of tubes VT1 and VT2 are interconnected. The third grids

of tubes VT3 and VT4 are interconnected. The third grids of tubes VT5 and VT6 are interconnected. The third grids of tubes VT7 and VT8 are interconnected. The third grids of tubes VT9 and VT10 are interconnected. The third grids of tubes VT11 and VT12 are interconnected. The fifth grids of all of the tubes are connected to the interconnected cathodes within each envelope 146. The second and fourth grids of all of the tubes are interconnected and are connected to the plate voltage supply 147. Six DC motors 151 which are provided with permanent magnet fields are associated with the tubes and have their armatures connected between the plates of the tubes. Thus, one of the DC motors 151 has its armature connected between the plates 15 of the tubes VT1 and VT2. The other motors 151 are connected in a similar manner to the other tubes. A normally open disabling switch 152 is provided in the circuit connecting the armature to the plate of the tube VT1. The switch 152, for reasons hereinafter explained, can be called a disabling switch for a_0 . Another normally open switch 153 is provided in the circuitry connecting the armature of a motor 151 to the plate of the tube VT7. The switch, as hereinafter explained, can be termed disabling switch for b_0 . The motors 151 are provided with shafts 156 which are mechanically connected to the wiper arms of Helipot potentiometers P1 - P6 which are also shown in FIGS. 7 and 8. The circuit shown in FIG. 10 generates the state variables X_0 , X_1 and X_2 which are impressed across the potentiometers P1, P2, P3, P4, P5 and P6 are connected to the DC motors 151 so that voltage difference is impressed across the armatures of these motors and will cause them to turn at velocities proportional to the respective armature voltages, rotating the respective shafts and changing the positions of the wiper arms of the potentiometers P1, P2 and P3, respectively. Each voltage $a_0 X_0$, $a_1 X_1$ and $a_2 X_2$ in FIG. 7 is, therefore, responsive to the time integral of the respective armature voltage of the motor through a shaft. The respective potentiometer is mechanically connected to the shaft, and its wiper arm is positioned as the shaft is rotated.

The circuit shown in FIG. 8 generates the state variables Y_0 , Y_1 and Y_2 which are impressed across the potentiometers P4, P5 and P6, respectively. The voltage differences impressed across the armatures of the motors will cause them to turn in velocity proportional to the respective armature voltages, rotating the respective shafts, and changing the position of the wiper arms of the potentiometers. Each voltage $b_0 Y_0$, $b_1 Y_1$, $b_2 Y_2$ in FIG. 8 is, therefore, responsive to the time integral of the respective armature voltage of the motor through whose shaft the respective potentiometer is mechanically connected.

The purpose of the high gain amplifiers 142, the vacuum tubes VT1-VT12 and the motors 151 is to cause the motor shafts 156 to turn thereby varying the values of the potentiometer settings until the magnitude of the low level error bus voltage is minimized. This condition is achieved through the following sequence: Assume that all of the coefficients have their correct, final steady state values except a_2 . Let the error in a_2 be called α_2 , equal to the actual value less the correct value. The e bus 141 will then have a non-zero voltage proportional to the error in a_2 times X_2 , which is $\alpha_2 X_2$. This voltage will be amplified by the value K by the amplifiers 142 and a much larger value will appear on the E bus 143 with an equal voltage of opposite polarity on

the -E bus 144. In the tube VT5, the +E voltage is multiplied by the X_2 state variable so that the current in this tube has a component proportional to $K \alpha_2 (X_2)^2$. In the tube VT6, the -E voltage is multiplied by the X_2 state variable, so that the current in this tube has a component proportional to $-K \alpha_2 (X_2)^2$. If a_2 is too positive and too great, then the current in tube VT5 will be too large and the current in the tube VT6 will be too small. The potential of the plate of the tube VT5 will be too low, and the potential of the plate of the tube VT6 will be too high. The DC motor 151 will, therefore, have a potential impressed across it whose polarity is determined by the polarity of α_2 , the error in the coefficient a_2 . The shaft 156 will, therefore, turn at a velocity proportional to α_2 , times $(X_2)^2$ and in a direction dependent upon the polarity of the error in a_2 . The mechanical connection will be chosen so that the direction of turning allows reduction in error in a_2 and always reduction in the magnitude of α_2 . This is a negative parametric feedback loop.

In a similar manner, an error in the coefficient a_1 will contribute a component of voltage to the e bus 141 proportional to the error times X_1 , and this amplified component is separated out of the +E and -E signals by the tubes VT3 and VT4, which cause the shaft 156 to turn with a velocity proportional to the magnitude of the error in a_1 times the average $(X_1)^2$ and in a direction to reduce the magnitude of the error in a_1 .

Likewise, tubes VT1 and VT2 will select out of the error a component due to the error in a_0 , will turn the shaft 156 until the component disappears. The other tubes will select out of the error, components due to the errors in the associated coefficients of the differential equation.

When all the coefficients have the correct values necessary to keep the low level e bus 141 at zero voltage, then the potentials on the +E and -E buses will also be zero. The currents in tubes VT5 and VT6 will, therefore, be equal; there will be no voltage on the DC motor 151; and the shaft 156 will be stationary. For this same condition, the current in tube VT1 will equal the current in VT2, etc.

After the computer in FIG. 1 or 9 has been operating for a sufficient time so that the state variable weightings a_n and b_n , being the potentiometer setting in FIGS. 1 and 9, have reached their steady-state values and are not changing, then the signal on the e bus in FIG. 9 has been minimized and the following equation has been satisfied in a statistical sense.

$$a_0 X_0 + a_1 X_1 + a_2 X_2 + \dots + b_0 Y_0 + b_1 Y_1 + b_2 Y_2 + \dots = 0 \quad (56)$$

$$\sum_{n=0}^k [a_n X_n(t) + b_n Y_n(t)] = 0 \quad (57)$$

Because of the design and construction of the state variable decomposition networks in FIGS. 8 and 7, all of the X_n state variables can be expressed in terms of $X_0(t)$, and all of the Y_n state variables can be expressed in terms of $Y_0(t)$.

Therefore, an equation which is satisfied by the machine is

$$\sum_{n=0}^k [a_n p^n X_0(t) + b_n p^n Y_0(t)] = 0 \quad (58)$$

wherein

$$p^n Z(t) = d^n Z(t) / dt^n \quad (59)$$

In the steady-state, the Laplace transform of this equation is

$$\sum_{n=0}^k [a_n s^n X_0(s) + b_n s^n Y_0(s)] = 0 \quad (60)$$

Because of the identical construction of the networks in FIGS. 8 and 7, the poles of the transfer function from X to X_0 are equal to the poles of the transfer function from Y to Y_0 .

In terms of the variables X and Y appearing on circuits identified as X and Y respectively, this transform equation becomes

$$\sum_{n=0}^k [a_n s^n \frac{X(s)}{D(s)} + b_n s^n \frac{Y(s)}{D(s)}] = 0 \quad (61)$$

Multiplying through by $D(s)$, and substituting $-K_2$ for $X(s)$ and $K_1 G$ for $Y(s)$, yields

$$\sum_{n=0}^k [-K_2 a_n s^n + K_1 b_n s^n G(s)] = 0 \quad (62)$$

Solving for G yields

$$G(s) = \frac{K_2 \sum_{n=0}^k a_n s^n}{K_1 \sum_{n=0}^k b_n s^n} \quad (63)$$

In terms of the Fourier transform,

$$G(j\lambda) = \frac{K_2 \sum_{n=0}^k a_n (j\lambda)^n}{K_1 \sum_{n=0}^k b_n (j\lambda)^n} \quad (64)$$

This is in the form desired.

The calibrations on the Helipot potentiometers P1, P2, P3, P4, P5 and P6 read the transfer function of the unknown system 23.

From the signals X and Y are derived state variables which are used in the analysis of the relationship between X and Y . The state variables are obtained from the networks shown in FIGS. 7 and 8.

The voltage X_0 in FIG. 10 is given by

$$X_0/X = 1/2 + 6(pT) + (pT)^2 \quad (65)$$

where p stands for the time derivative operation, d/dt ,

$$pX_0 = dX_0/dt \quad (66)$$

$$T = RC \quad (67)$$

and the capacitors and resistors have the relative values previously tabulated.

The input voltage X is related to the state variable X_0 10 by

$$X = (2 + 6Tp + T^2 p^2) X_0 \quad (68)$$

The voltage X_1 in FIG. 10 is given by

$$X_1 = (pT) X/2 + 6(pT) + (pT)^2 \quad (68a)$$

The voltage X_2 in FIG. 10 is given by

$$X_2 = (pT)^2 X/2 + 6(pT) + (pT)^2 \quad (69)$$

The three state variables are, therefore, related to each other as follows:

$$X_1 = dX_0/dt \quad (70)$$

$$X_2 = dX_1/dt = d^2X_0/dt^2 \quad (71)$$

Note that the state variable X_1 is related to the state variable X_0 by being the derivative of the state variable X_0 . This is not an approximate derivative of the state variable X_0 , but is an exact derivative for all the frequency components in X_0 . These networks are particularly chosen to have this unique relationship so that the output of one is exactly the derivative of the output of the other. It is necessary that the poles of these two networks, therefore, produce identical filtering actions. This can be seen by examination of the previous equations. This unique relationship has been achieved in a passive, attenuative network with resistive loading. The voltage X_0 is obtained as a push-pull voltage across a 20,000 ohm potentiometer P1 whose output can be adjusted anywhere from $+X_0$ to $-X_0$. The setting of the potentiometer P1 is the coefficient a_0 and the output of the potentiometer P1 is a voltage a_0X_0 . In the network in FIG. 7, R is chosen as 20,000 ohms. The parameter $T = RC$ appears in the filtering equation. T is a variable which is set by the operator of the machine, and should be so chosen that the coefficients in the differential equation for the network have reasonably large on-scale values. Or stated another way, the coefficient ωT should be near unity for the middle frequency for which the impedance function is desired. The scale of T is changed by ganging all of the capacitors together and changing them simultaneously in half decade steps. The values of C can be: 10 microfarads, 5 microfarads, 1 microfarad, 0.5 microfarad, 0.1 microfarad, 0.05 microfarad, 0.01 microfarad, 0.005 microfarad, 0.001 microfarad, and 0.0005 microfarad. These correspond to T values of 200 milliseconds, 100 milliseconds, 20 milliseconds, 1 millisecond, 2 milliseconds, 200 microsec-

onds, 100 microseconds, 20 microseconds, and 10 microseconds.

The potentiometer P2 between terminals X_1 and $-X_1$ was designed to have the identical output load impedance as the potentiometer P3 between terminals X_0 and $-X_0$, so that a second 20,000 ohm helipot could be placed across the X_1 output for the purpose of representing coefficient a_1 to be multiplied by the state variable X_1 . This coefficient is obtained by the position of the shaft of the potentiometer P2 and will be one of the unknowns which is solved for by the identification computer.

The network was designed so that it could be loaded with a 20,000 ohm potentiometer P1 between terminals $+X_2$ and $-X_2$. The setting of the potentiometer shaft is the coefficient a_2 and the output voltage of the potentiometer is the voltage a_2X_2 .

Since the network in FIG. 8 is identical to the network in FIG. 7, the analogous relations for the Y variables are

$$Y_0 = Y/2 + 6(pT) + (pT)^2 \quad (72)$$

$$Y = (2 + 6Tp + T^2 p^2) Y_0 \quad (73)$$

$$Y_1 = (pT)Y/2 + 6(pT) + (pT)^2 \quad (74)$$

$$Y_2 = (pT)^2 Y/2 + 6(pT) + (pT)^2 \quad (75)$$

$$Y_1 = dY_0/dt \quad (76)$$

$$Y_2 = dY_1/dt = d^2Y_0/dt^2 \quad (77)$$

FIG. 8 is a network identical to that shown in FIG. 7. It calculates the state variables Y_0 , Y_1 and Y_2 from the input voltage Y .

With these state variables the differential equation

$$a_0X_0 + a_1X_1 + a_2X_2 + b_0Y_0 + b_1Y_1 + b_2Y_2 = e \quad (78a)$$

can be formed. FIG. 9 shows the resistive network 140 which is used as an adder to form the above equation. The input to the high gain amplifier 142 is proportional to the low level error signal e in equation 78a.

What is desired to identify the circuit under test is the differential equation obtained by setting

$$e = 0 \quad (78b)$$

The control system shown in FIG. 9 does this. When the control system has completed its operation, and the settings on all of the helical potentiometers are their final steady state correct values, then the differential equation which has been formulated is

$$\frac{[a_0 + a_1(pT) + a_2(pT)^2]X + [b_0 + b_1(pT) + b_2(pT)^2]Y}{2 + 6(pT) + (pT)^2} = 0 \quad (79a)$$

This equation is satisfied by the same coefficients

$$[a_0 = a_1(pT) + a_2(pT)^2] X + [b_0 + b_1(pT) + b_2(pT)^2] Y = 0 \quad (79b)$$

The differential equation 101b formed by zero equal to the sum of the products of the coefficients times the state variables can be written

$$\sum_{n=0}^{n=2} (a_n X_n + b_n Y_n) = 0 \quad (78c)$$

The coefficients that satisfy this equation are identically the same as the coefficients that satisfy the classical differential equation relating the variables X and Y by virtue of the dynamics of the circuit under test. For $\tau = t/T$, and $p = d/dt$, the classical equation is

$$a_0 X + a_1 (dX/d\tau) + a_2 (d^2X/d\tau^2) + b_0 Y + b_1 (dY/d\tau) + b_2 (d^2Y/d\tau^2) = 0 \quad (79a)$$

$$a_0 X + a_1 T p X + a_2 T^2 p^2 X + b_0 Y + b_1 T p Y + b_2 T^2 p^2 Y = 0 \quad (79b)$$

$$\sum_{n=0}^{n=2} a_n (T p)^n X + b_n (T p)^n Y = 0 \quad (79c)$$

Since the classical equation 79b for the test circuit is identical to the weighted state variable equation 79b for the identification machine, the coefficients derived by the machine are the correct coefficients for the impedance function. The impedance function as obtained internally in the machine is therefore

$$Y/X = -[a_0 + a_1(pT) + a_2(pT)^2]/[b_0 + b_1(pT) + b_2(pT)^2] \quad (80)$$

The impedance function of unknown two-terminal network 93 is given by

$$Z = (K_X R_s / K_Y) (a_0 + a_1(pT) + a_2(pT)^2/b_0 + b_1(pT) + b_2(pT)^2) \quad (81)$$

Note that the impedance in equation 81 is positive although Y/X is negative. This change of sign is due to the fact that the current in R_s in phase with Y produces a negative X .

Any one of the coefficients, a_0, a_1, a_2, b_0, b_1 or b_2 , may be chosen arbitrarily by the operator of the machine. To do this conveniently, in FIG. 9 there is shown a disabling switch 152 on the d-c motor 151 which drives the shaft 156 to the potentiometer P1 for a_0 . If this switch 152 is opened, then the coefficient a_0 may be set arbitrarily by the operator. When testing impedance or transfer functions which do not have pure derivative action, a typical setting for a_0 is unity. When testing impedance or transfer functions which do not have a pure derivative action, then it is more desirable to permit a_0 to be found by the converging action of the machine, and to select the coefficient b_0 and set it equal to unity. For this purpose, a disabling switch 153 has been shown in FIG. 9 in series with the d-c motor 151 which drives the shaft 156 of the potentiometer P4 for b_0 .

The action of the circuits in FIG. 9 can be described as follows: The input to the high gain amplifier 142 when the machine is first energized, may be a value of e of 100 volts on the bus 141. Consequently, the high gain amplifier 142 is saturated and operates like a re-

lay. The output voltage on the E-bus 143 will change infrequently between its maximum positive and negative values. As the machine drives the potentiometers towards their correct seatings, the voltage on the e-bus 141 drops to millivolts. For high accuracy then, it is desirable to amplify this small error signal and amplification of the high gain amplifier 142 is at least 40 db and preferably 60 db. The error signal now appears on the E bus 143 as a proportional rather than a clipped signal. This error signal is mixed or cross-correlated with the state variable X_0 in the first pair of multigrid converter tubes VT1, VT2. The net result is that the voltage impressed across the D-C motor 151 which is connected between the two plates of the push-pull pair of converter tubes has two components. One of these components is proportional to the product of X_0 and E . This component has an average or mean value which will turn the d-c motor slowly in the direction to correct an error in the coefficient a_0 .

Typical multigrid converter tubes which may be used for VT1 and VT2 are the 6SB7-Y and the 5915.

Other means for multiplying the voltage on the E bus 119 by the state variable X_0 on terminal 94 may be used to obtain a signal to drive the d-c motor 151. One method is to produce the equation

$$\text{Voltage} = X_0 \frac{E}{|E|}$$

which is the state variable voltage between terminals X_0 and $-X_0$ in FIG. 7 impressed directly on the d-c motor 151 but with a polarity which reverses as the error voltage on 143 reverses. The error voltage, therefore, operates a switching gate in a polarity sensitive rectifier or phase sensitive demodulator. It is also possible to drive the d-c motor by the direct multiplication from a four quadrant multiplier. In this case,

$$\text{Motor voltage} = X_0 E \quad (82)$$

FIG. 10 shows a network required in place of the network shown in FIG. 7 in the identification machine shown in FIGS. 7, 8 and 9 in case the identification machine is designed for a fourth order polynomial operation on X , or for test circuits requiring the fourth derivative of X to represent the differential equation or impedance function. In the network shown in FIG. 14, the X and $-X$ terminals are connected to the corresponding terminals in FIG. 7. The network shown in FIG. 10 as shown. Also included are potentiometers P7, P8, P9, P10 and P11, of which the wipers are designated with an $a_0 X_0, a_1 X_1, a_2 X_2, a_3 X_3$ and $a_4 X_4$. The potential on the wiper of the potentiometer with respect to ground represents the corresponding weighted state variable. The plurality of additional terminals are provided which are identified as X_0, X_1, X_2, X_3 and X_4 . The potentials on these terminals with respect to ground represent the corresponding state variable.

It should be pointed out to maintain the symmetry of the voltages with respect to ground in FIG. 10, the components must be symmetrical with respect to the input terminals X and $-X$. For this reason, the corresponding components have values which are equal to each other in the network. The values of the resistors and capacitors in FIG. 10, are chosen so that

$$X_4 = T p X_3 \quad (83)$$

$$X_3 = TpX_2 \quad (84)$$

$$X_2 = Tpx_1 \quad (85)$$

$$X_1 = TpX_0 \quad (86)$$

In addition to replacing the network of FIG. 7 with the network shown in FIG. 10, it is necessary to replace the network shown in FIG. 8 with a network which is identical in parameter values with those shown in FIG. 10. This new circuit, which can be called the Y states variable generator, has input terminals Y and $-Y$ which are connected to the corresponding terminals in FIG. 8. The network generates the state variables Y_0, Y_1, Y_2, Y_3 and Y_4 and the weighted state variables $b_0Y_0, b_1Y_1, b_2Y_2, b_3Y_3$ and b_4Y_4 , respectively.

The adder network shown in FIG. 9 including the resistors 140 must be expanded by providing four more adder input terminals, each with resistance between the input terminal and the e bus 141 equal to the other equal adder input resistances. These four additional input terminals are connected to receive the four additional weighted state variables a_3Y_3, a_4Y_4, b_3Y_3 and b_4Y_4 . Corresponding to these four additional inputs, there must be eight additional multi-grid vacuum tubes driving four additional DC motors connected to the four additional potentiometers whose shaft settings are proportional to a_3, a_4, b_3 and b_4 .

This complete augmented machine will solve the state variable equation whose coefficients are equal to those in the differential equation of the unknown test circuit.

$$\sum_{n=0}^{n=4} (a_n X_n + b_n Y_n) = 0 \quad (87)$$

$$\sum_{n=0}^{n=4} (a_n (Tp)^n X + b_n (Tp)^n Y) = 0 \quad (88)$$

The network shown in FIG. 7 is a new, unique and useful generator of multiple output voltages that have a prescribed relationship one to the other. This desirable relationship is achieved by the unique topological arrangement of the components of the network. The network shown in FIG. 10 is also a new, unique and useful generator of multiple output voltages that have a prescribed relationship one to the other. This desirable relationship is achieved by the unique topological arrangement of the components of the network.

There are other networks which may be used for the purpose of generating state variables from an input signal, with the provision that the state variables appear in conjunction with a variable parameter in such a manner that the weighted state variables with controllable weightings can also be generated by the network. FIG. 11 is an example of a state variable generating network receiving the input signals $+X$ and $-X$ and generating the state variables P_0, P_1, P_2, P_3 and P_4 , and also generating the weighted state variables $a_0P_0, a_1P_1, a_2P_2, a_3P_3$

and a_4P_4 . Through proper selection of component values, the following conditions can be fulfilled:

$$P_4 = TpP_3 \quad (89)$$

$$P_3 = TpP_2 \quad (90)$$

$$P_2 = TpP_1 \quad (91)$$

$$P_1 = TpP_0$$

The network in FIG. 11 has the advantage that the number of decade capacitors required is fewer than for the network in FIG. 10. In FIG. 11, the state variables are formed by weighted additions. The derived state variables P are related to each other by $P_n = (Tp)^n - mP_m$

In FIG. 12, there is shown another network whose derived state variables fulfill the condition

$$X_1 = T p X_0 \quad (92)$$

$$X_2 = T p X_1 \quad (93)$$

This network in FIG. 12 may be used in place of the network shown in FIG. 7 for the identification machine of FIGS. 12, another network identical to that in FIG. 12 is also used instead of the network in FIG. 11.

If the identification machine and apparatus is connected to two signals in a unilateral feedback system, it will not, in general, read either the forward or the feedback transference. What it reads is a function of the location of the commands or random load disturbances that enter the system. Consider the case where there exists only one adder in the entire unilateral feedback loop and to which can enter a signal as a function of time. In a servo mechanism, said signal can be a command signal, but if there is a command signal, there can be no load disturbances. Or, if the command is zero, the signal can be random load forces which are added. In a process regulator, the disturbances may be added in the process. Consider the identification machine connected to points A and B within the feedback loop. If the adder into which the single permitted signal enters is in the branch from A to B going in the direction of unilateral information and power flow, then the identification machine reads the transference of the unilateral branch from B to A.

To summarize certain portions of the foregoing, the output signal from an unknown process is related to the input signal to the process by the transfer function of the process. In addition, the output signal may contain components due to noise generated within the process, and other components due to additional process input signals of different types. In a multivariable system, there is a different transfer function for the relationship between each input and each output. Selecting a particular input and a particular output, the transference is the equation for the portion of the output which correlates with the input, as a function of the input and the dynamic mechanism of the unknown process. If this function is written as a differential equation, then the transference is the ratio of the correlative output to the

input expressed as a ratio of polynomials in the differential operator. If the transference is a frequency response, then it is the ratio of the correlative output to the input expressed as a complex phasor. if the transference is a Laplace transform, then is is the ratio of the Laplace transform of the correlative output divided by the Laplace transform of the input expressed as a function of the Laplace variable s .

As pointed out previously, the purpose of my identification machine is to calculate the differential equation coefficients in numerical form. The identification machine uses two inputs: one proportional to the input to the unknown process, and the other proportional to the output from the unknown process. For the purposes of explanation, the following notation will be used.

$$\text{Let } p^n = d^n/dt^n \quad (94a)$$

X is the process input and Y is the correlative component of the process output. The dynamics of the unknown system is defined by the differential equation

$$B(p) Y = A(p) X \quad (94b)$$

where

$$A(p) = \sum_{n=0}^g a_n p^n \quad (94c)$$

and

$$B(p) = \sum_{n=0}^h b_n p^n \quad (94d)$$

Written in transference form, the differential equation becomes

$$Y = [A(p)/B(p)] X \quad (94e)$$

The order of the numerator of this transference is g , and the order of the denominator is h . In general,

$$g < h \quad (94f)$$

but this is not a restriction on the operation of the machine.

There are $g + h + 2$ coefficients in the polynomials in equations 94c and 94d, but there are only $g + h + 1$ independent variables. For this reason, one of the coefficients may be arbitrarily selected. If the unknown does not contain a cascade integration, b_0 may be arbitrarily set at unity. If the unknown does not contain a cascade differentiation, a_0 may be arbitrarily set at unity. Both a_0 and b_0 may not be chosen arbitrarily.

An example of another form of state variable generators is shown in FIG. 13. The uppermost ladder network 206 is the state variable generator G_0 which produces the signal x_0 . The second ladder network 207 is the state variable generator G_1 , the third ladder network 208 is the state variable generator G_2 , and the bottom ladder network 209 is the state variable generator G_3 .

The values of the components in these ladder networks can be selected by conventional network techniques to fulfill the conditions imposed by equation 148. The basic feature of these ladder networks is that the poles of each ladder network are identical to the poles of each other ladder network. The zeros of each ladder network lie either at the origin of the Laplace frequency plane or at infinity. The first ladder network has no zero at the origin. The second ladder network for G_1 has only one zero at the origin. The third ladder network has two zeros at the origin, and the fourth ladder network has three zeros at the origin.

FIG. 14 shows a combination of state variable computers and coefficient multipliers. A preferred form for the integrators to be used in conjunction with state variable computers and is a small direct current motor whose direction of rotation is proportional to the polarity of the input electrical signal, and whose shaft is connected to a rotary potentiometer, so that the position of the wiper arm on the potentiometer is linearly proportional to the angle of the motor shaft as in the embodiment shown in FIG. 9. The angular position of the wiper arm of the potentiometer is the preferred form of parameter read-out for FIGS. 14. The preferred form of each multiplier to be used in FIG. 14 is to connect the non-movable terminals of each potentiometer across the output terminals of one of the state variable computers in FIGS. 13 or 14 such that the potential between the wiper arm and a grounded terminal is proportional to the product of the state variable times the corresponding parameter coefficient.

An identification machine, apparatus or system based on the principles set forth above can also be used with a sampled data system to obtain the z -transform of an unknown operation. For example, an unknown system may have the transfer function from x to y described by a ratio of polynomials in the frequency variable

$$z = e^{+sT} \quad (95)$$

The most convenient state variables to use in such a case are the successive values of X and Y delayed by 1, 2, 3, etc., units of the sample time T . The identification machine, therefore, includes provision for successive delays of the input and output signals of the unknown system and the multiplication of these delayed signals by the transfer function parameters a_h and b_h , in such a manner that these parameters slowly converge to the values specified by the unknown system.

It is apparent from the foregoing that I have provided a new and improved identification machine, and apparatus for testing or identifying an unknown process. For the identification machine to operate properly, the unknown process must be disturbed with a signal which is more complicated than the process that is, the disturbance signal must contain simultaneously a variety of frequencies. As explained above, this can be obtained by using the transients that occur due to turning on and off a disturbance signal. For example, a step function can serve as a very satisfactory disturbance to introduce into an unknown process. A sine wave which is turned on and off contains an adequate supply of frequencies for the satisfactory performance of an identification machine. A repeated squarewave or a repeated triangle wave are satisfactory input signals to the pro-

cess. Other satisfactory input disturbances are a frequency modulated sinewave or a swept sinewave. One of the most desirable disturbances is a random signal which looks like white Gaussian noise. If the unknown process itself during normal operation appears to have an input signal containing many frequency components or containing filters of random noise, then it is possible to monitor this input signal, and no additional disturbance need be introduced into the process. Thus, under certain conditions, there is no need to disturb the process by test or signals other than the random fluctuations which are normally present in the unknown process.

With the use of my identification machine, the length of time required to identify the unknown process is reduced to a minimum. This is true because there is no need to measure the frequency response of the process for all possible frequencies, but instead, only the most important coefficients in the integro-differential equation are solved for. The time required to identify the process is also reduced to a minimum because there is no necessity to record the input and output of the process for a very long time and apply statistical computation to this recording since the identification machine operates on line in real time and can be turned off as soon as the fluctuations in the identified parameters are less than a predetermined value. The time required for identification of the unknown process is also reduced to a minimum because there is no need to factor a Bode diagram into its poles and zeroes. Also, there is no need to calculate the auto-correlation of the input and the cross-correlation between the input and the output, nor to calculate the Fourier transforms of these in order to obtain the system transference.

The equation obtained by the identification machine, is exactly correct and does not have any errors in any of the coefficients, if the appropriate formula of disturbance function is present, if the machine operation time is long enough, and if the correct method of compensating for additive noise on the output signal or the effect of additional not-monitored input signals is employed.

In FIG. 15 there are ten input terminals marked X_0 through X_4 and $-X_0$ through $-X_4$ which should be connected to the ten corresponding marked terminals in circuit 26 of FIG. 3. In FIG. 23 there are 8 operational summers or adders designated 301 through 308 inclusive. In FIG. 15 there are 10 output circuits, eight of them from the analog operational adders and two which are directly connected from the input circuits. The ten output circuits are designated 310 through 319 inclusive. Circuit 310 which delivers signal V_0 is connected directly to input circuit with signal X_0 . Circuit 315 which is marked $-V_0$ is connected directly to input circuit marked for the signal $-X_0$. Circuit 311 is the output of adder 301 and delivers the signal $-V_1$. Circuit 312 is the output of operational adder 302 and delivers the signal V_2 . Circuit 313 is the output of operational adder 303 and delivers the signal $-V_3$. Circuit 314 is the output of operational adder 304 and delivers the signal V_4 . Circuit 316 is the output of operational adder 305 and delivers the signal V_1 . Circuit 317 is the output of operational adder 306 and delivers the signal $-V_2$. Circuit 318 is the output of operational adder 307 and delivers the signal V_3 . Circuit 319 is the output of operational adder 308 and delivers the signal $-V_4$. Operational adder 301 receives two input signals designated

X_0 and $-X_1$. Operational adder 305 receives two input signals designated $-X_0$ and X_1 . Operational adder 302 receives three input signals, one from X_2 , one from $-X_1$ through a gain block of 3 and one from X_0 through a gain block of 2. Operational adder 306 receives three input signals, one from $-X_2$, one from X_1 through a gain block with a gain of 3 and one from $-X_0$ through a gain block with a gain of 2. Operational adder 303 receives 4 input signal, one from $-X_3$, one from X_2 through a block with a gain of 5, one from $-X_1$ through a block with a gain of 9 and one from X_0 through a block with a gain of 5. Operational adder 307 also receives four input signals, one directly from X_3 , one from $-X_2$ through a gain block of 5, one from X_1 through a gain block of 9 and one from $-X_0$ through a gain block of 5. Operational adder 304 receives 5 input signals, one from X_4 , one from $-X_3$ through a gain block of 7, one from X_2 through a gain of 20, one from $-X_1$ through a gain of 28, and one from X_0 through a gain of 14. Operational adder 308 also receives five input signals, one from $-X_4$, one from X_3 through a gain of 7, one from $-X_2$ through a gain of 20, one from X_1 through a gain of 28, and one from $-X_0$ through a gain of 14.

The block diagram in FIG. 15 generates five signals and their negatives which satisfy the equations 96 through 100.

$$V_0 = X_0 \quad (96)$$

$$V_1 = X_1 - X_0 \quad (97)$$

$$V_2 = X_2 - 3X_1 + 2X_0 \quad (98)$$

$$V_3 = X_3 - 5X_2 + 9X_1 - 5X_0 \quad (99)$$

$$V_4 = X_4 - 7X_3 + 20X_2 - 28X_1 + 14X_0 \quad (100)$$

This set of equations is called the state variable transformation or alternatively called a linear transformation of state variable signals.

In the use of this state variable transformation in an adaptive estimator or an adaptive control system, it is usually desired to have two sets of state variable transformations one for the X variables in FIG. 3 and a second identically constructed transformation for the Y variables in FIG. 3. The Y diagram for the Y variable transformation would also be identical to that of FIG. 15. The equations for the Y variable transformation using a block diagram and circuit identical to FIG. 15 are equations 101 through 105.

$$W_0 = Y_0 \quad (101)$$

$$W_1 = Y_1 - Y_0 \quad (103)$$

$$W_2 = Y_2 - 3Y_1 + 2Y_0 \quad (104)$$

$$W_3 = Y_3 - 5Y_2 + 9Y_1 - 5Y_0 \quad (102)$$

$$W_4 = Y_4 - 7Y_3 + 20Y_2 - 28Y_1 + 14Y_0 \quad (105)$$

Equations 35 through 45 on page 31 of the specification give the relationship between the state variables in FIG. 3 due to the unique selection of parameters in the rc circuits. Using equations 37 through 45 for X_i and Y_i in terms of X and Y and inserting these or substituting these in equations 96 through 105 yields a set of equations for the output signals in FIG. 15 in terms of the input to the unknown system in FIG. 3. This set of relationships is equations 106 through 110 inclusive.

$$V_0 = X/D \quad (106)$$

$$V_1 = pT X/D \quad (107)$$

$$V_2 = (pT)^2 X/D \quad (108)$$

$$V_3 = (pT)^3 X/D \quad (109)$$

$$V_4 = (pT)^4 X/D \quad (110)$$

In a similar manner, eliminating Y_i in equations 101 through 105 using equations 41 through 45 yields equations 111 through 115 for the transformed state variables W_i in terms of the output of the unknown system 23 in FIG. 3 designated Y .

$$\begin{aligned} W_0 &= Y/D & (111) \\ W_1 &= pT Y/D & (112) \\ W_2 &= (pT)^2 Y/D & (113) \\ W_3 &= (pT)^3 Y/D & (114) \\ W_4 &= (pT)^4 Y/D & (115) \end{aligned}$$

It can be seen by an examination of equations 106 through 115 inclusive that the state variables generated in FIG. 23 are related one to another by pure integration or pure differentiation. These state variables are the input to the unknown system 23 in FIG. 5 and its derivatives operated on by a polynomial lag filter and a gain factor change. Equations 206 through 209 are for the output of the set of linear transformations similar to those in FIG. 23 but a duplicate set connected to the Y terminals in filter 29 in FIG. 5. This set of state variables also has the same properties as the state variables shown specifically in FIG. 23 in that each state variable is a pure derivative or pure integral of one of the others and one is the filtered output of the unknown system 23 and the others are successive derivatives of this filtered output.

The block diagram shown in FIG. 15 and the signals generated therein are intended to be connected to FIG. 4 in the following manner. Each V_i in FIG. 15 is intended to be connected to the corresponding X_i in FIG. 4. Each $-V_i$ in FIG. 15 is to be connected to the corresponding $-X_i$ in FIG. 4. Likewise the block diagrams for the Y signals and W signals which have been defined by equations 101 through 105 and 205 through 115 are intended to be connected also to FIG. 6 as follows. Each W_i signal is to be connected to the corresponding Y_i input to FIG. 4.

With these connections the adaptive estimator in FIG. 4 will operate until it reaches the convergent condition given by equation 116.

$$\sum_{i=0}^m (h_i V_i - k_i W_i) = 0 \quad (116)$$

The weighted sum of each h coefficient times the corresponding variable V_i minus the k_i coefficients times the corresponding state variables W_i equals zero at this convergent condition. With reference to equation 23 on page 29 of the specification, the differential equation of the unknown system 23 in FIG. 3 is

$$\sum_{i=0}^m (a_i p^i X - b_i p^i Y) = 0 \quad (117)$$

Substituting equations 106 through 115 for V and W into equation 116 yields the relationship between the computer read-out coefficients h and k and the differential equation coefficients A and B .

$$\sum_{i=0}^m (h_i (pT)^i X/D - k_i (pT)^i Y/D) = 0 \quad (118)$$

$$\sum_{i=0}^m h_i T^i p^i X - k_i T^i p^i Y = 0 \quad (119)$$

By comparison of equations 213 and 211 it can be seen from inspection that a_i equals $h_i T^i$ and b_i equals $k_i T^i$.

$$a_i = h_i T^i \quad (120)$$

$$b_i = k_i T^i \quad (121)$$

The estimator in FIG. 4 therefore delivers the coefficients in the differential equation for the unknown system 23 in FIG. 3 modified only by a scaling time constant. It is not necessary, therefore, to transform these coefficients as shown in equations 25 through 34, because FIG. 15 has provided a transformation of the state variables instead and has caused the coefficients to therefore read-out directly proportional to the coefficients in the differential equation.

FIG. 16 is a block diagram and circuit diagram for a parameter estimation scheme for an unknown system with particular emphasis on the generation of transformed state variables which are suitable for use in the estimator of FIG. 4 to produce read-out coefficients which are directly related to the coefficients in the differential equation for the unknown system. In FIG. 16 there is shown an unknown system 400 with an input excitation on terminal 401 and a useful output signal on terminal 402. The input is designated $X(t)$ and the output is designated $Y(t)$.

The input signal is also presented to a four element RC ladder lag filter 403 to generate four auxiliary signals each being a linear transformation of $X(t)$. The output signal is also presented to an identical four element RC ladder lag filter 404 to generate four additional derived signals. In FIG. 16 the signal across the first capacitor in the network 403 which is driven by the input of the unknown system is designated f_3 . The signal across the second capacitor is f_2 , across the third capacitor is f_1 , and across the output capacitor is f_0 . In the filter 404 whose input is provided by the output of the unknown system the signal across the first capacitor is g_3 , the signal across the second capacitor g_2 , the signal across the third capacitor g_1 and the signal across the fourth and last capacitor is g_0 . For simplicity in notation, let T equal R times C .

$$T = RC \quad (122)$$

The equations therefore for the signals generated by the two ladder networks 403 and 404 in FIG. 24 are

$$D = 1 + 10 pT + 15 (pT)^2 + 7 (pT)^3 + (pT)^4 \quad (123)$$

$$f_0 = X/D \quad (124)$$

$$f_1 = (1 + pT) X/D \quad (125)$$

$$f_2 = (1 + 3pT + (pT)^2) X/D \quad (126)$$

$$f_3 = (1 + 6pT + 5(pT)^2 + (pT)^3) X/D \quad (127)$$

$$f_4 = (1 + 10pT + 15 (pT)^2 + 7 (pT)^3 + (pT)^4) X/D = X \quad (128)$$

$$g_0 = Y/D \quad (129)$$

$$g_1 = (1 + pT) Y/D \quad (130)$$

$$g_2 = (1 + 3 pT + (pT)^2) Y/D \quad (131)$$

$$g_3 = (1 + 6pT + 5 (pT)^2 + (pT)^3) Y/D \quad (132)$$

$$g_4 = Y \quad (133)$$

These f and g signals are further transformed by analog computer linear operations in FIG. 16 to generate a set of signals designated X_i and Y_i . The computations performed by the block diagram in FIG. 16 are defined by equations 134 through 143 inclusive.

Substituting equations 124 through 133 into equations 134 through 143 yields

$$X_0 = X/D \quad (134)$$

$$X_1 = (pT) X/D \quad (135)$$

$$X_2 = (pT)^2 X/D \quad (136)$$

$$X_3 = (pT)^3 X/D \quad (137)$$

$$X_4 = (pT)^4 X/D \quad (138)$$

$$Y_0 = Y/D \quad (139)$$

$$Y_1 = (pT) Y/D \quad (140)$$

$$Y_2 = (pT)^2 Y/D \quad (141)$$

$$Y_3 = (pT)^3 Y/D \quad (142)$$

$$Y_4 = (pT)^4 Y/D \quad (143)$$

The computational system whose block diagram is shown in FIG. 16 yields the set of state variables which can be summarized by

$$X_j = (pT)^j X/D \quad (144)$$

$$Y_j = (pT)^j Y/D \quad (145)$$

These X and Y state variables have the property that each one is a pure integral or pure derivative of the adjacent state variable; that is, each state variable is the integral of the state variable which has the next larger subscript and each state variable is the derivative of that state variable which has the next unit smaller subscript.

To illustrate the circuit connections corresponding to the equations which have been written above, one particular equation will be traced through. For example, consider equation 230. This equation computes the signal X_2 which is shown in FIG. 16 on circuit 405. This circuit is delivered by the inverting amplifier 406 whose input is the signal on circuit 407 marked $-X_2$. The signal on circuit 407 is produced in the inverting operational summer or inverting adder 408.

Adder 408 has three inputs. One input comes from the non-inverting gain block 409 which is supplied by circuit 410 marked $-X_1$. This input therefore contributes a signal $-3X_1$ into the adder 408. Another input to adder 408 is delivered by circuit 411 which has the signal $-X_0$ on it and this contributes the input of magnitude $-X_0$. The third input to adder 408 comes from circuit 412 which is connected to the second capacitor in the RC filter 403 and which has a voltage on its designated f_2 . These three inputs to the adder 408 are the three terms in equation 230, when it is written in the form $-3X_1 - X_0 + f_2$. In order to eliminate X_1 and X_0 one may either use equations 228 and 229 or may refer further to the block diagram in FIG. 16. $-X_1$ in FIG. 24 appears on circuit 410 which is the output of operational inverting adder 413. The input to 413 is $-X_0$ on circuit 414 and f_1 on circuit 415. Circuit 414 comes from another inverting amplifier whose input is f_0 . Therefore, X_1 equals $-f_0 + f_1$.

In a similar manner, all of the output signals in FIG. 24 may be traced through the operational adders, inverting amplifiers and non-inverting gain blocks to verify that the linear transformation represented by equa-

tions 228 through 237 corresponds exactly to the linear transformation of the signals in FIG. 16.

The output signals from FIG. 16 are intended to be connected directly to the correspondingly marked terminals in FIG. 4 such that the identifier in FIG. 4 will operate until the coefficients h and k equal the coefficients in the differential equation of the unknown system 400 in FIG. 16. Equations 147 and 148 are satisfied at the convergent condition of the identifier in FIG. 4.

$$\sum_{j=0}^m (h_j X_j - k_j Y_j) = 0 \quad (146)$$

$$\sum_{j=0}^m (h_j T^j p^j X - k_j T^j p^j Y) = 0 \quad (147)$$

The differential equation of the unknown system 400 is equation 149

$$\sum_{j=0}^m (a_j p^j X - b_j p^j Y) = 0 \quad (148)$$

The coefficients in the differential equation are therefore given by equations 150 and 151

$$a_j = h_j T^j \quad (149)$$

$$b_j = k_j T^j \quad (150)$$

wherein h and k are the readouts of the estimator in FIG. 4.

It is apparent that in the present invention there is provided a multiport network with one input port and three output ports. Means is provided for supplying a voltage excitation at said input port. The network includes coupling means to generate first, second, and third output signals responsive to said excitation at the three output ports respectively, such that when the excitation (after a preselected time t equals zero) produces a first output signal changing in magnitude proportional to

$$s_1(t) = T^2 (1 - (1+t/T) e^{-t/T})$$

then the corresponding second output signal for positive time is proportional to

$$s_2(t) = t e^{-t/T}$$

and the corresponding third output for positive time is proportional to

$$s_3(t) = (1 - t/T) e^{-t/T}$$

where T is a function of said coupling mean within said network and wherein $s_2(t)$ is proportional to the time derivative of $s_1(t)$ and $s_3(t)$ is proportional to the time derivative of $s_2(t)$.

It is also apparent from the foregoing that I have provided an identification machine, apparatus and system in which a high frequency model of the unknown system can be obtained directly from the identification machine by connecting the output of each integrator to

a potentiometer setting in the model structure. By such means, models operating with time scales of 100 and 1,000 times that of the unknown process can be used in iterative calculators for optimum bang-bang controls and in hill-climbing optimization techniques for minimizing cost functions or maximizing value functions.

It is also apparent from the foregoing that the identification machine, process and system can also be used for tracking variable parameter systems and continuously delivering the variations in the coefficients of the integro-differential equation. For example, such an identification machine might be useful to track the resonant frequency and damping in the pitch axis of a high performance airplane as a function of changes in altitude, mach number and pitch angle.

I claim:

1. A multiport linear passive lumped-parameter network consisting of only linear passive elements with one input port and at least four output ports, means for providing an input signal at said input port, passive coupling means within said network for producing four output signals at the four output ports, each frequency component of said signals being described by an envelope magnitude and phase the frequency response of each of said output signals with respect to said input signal being finite (including zero) at all real frequencies, the second output signal having a phase leading the phase of the first output signal by 90° and the second output signal envelope magnitude being proportional to W times the first output signal of radian frequency W , the third output signal for each input signal having a phase leading the phase of the first output signal by 180 degrees and the third output signal envelope magnitude being proportional to ω^2 times the first output signal envelope magnitude, the fourth output signal for each input signal having a phase leading the phase of the first output signal by 270° and the fourth output signal envelope magnitude being proportional to ω^3 times the first output signal envelope magnitude for each input signal of radian frequency ω .

2. A multiport network consisting of only linear passive elements with one input port and three output ports, means for providing a voltage excitation signal having a parameter T at said input port, coupling means within the network to generate first, second, and third output signals responsive to said excitation at the three output ports respectively, such that when the excitation (after a preselected time t equals zero) produces a first output signal changing in magnitude proportional to

$$s_1(t) = T^2 [1 - (1 + t/T) e^{-t/T}]$$

then the corresponding second output signal for positive time is proportional to

$$s_2(t) = t e^{116 t/T}$$

and the corresponding third output signal for positive time is proportional to

$$s_3(t) = (1 - t/T) e^{-t/T}$$

for all T

3. A claim as in claim 2 wherein $s_1(t)$ is proportional to the time integral of $s_2(t)$ and $s_2(t)$ is proportional to the time integral of $s_3(t)$.

4. A claim as in claim 2 wherein $s_2(t)$ is proportional to the time derivative of $s_1(t)$ and $s_3(t)$ is proportional to the time derivative of $s_2(t)$.

5. A linear passive multiport circuit consisting of only linear passive elements comprising a two-terminal input with a source of voltage supplies to said input, a first pair of two impedances with different phase angles in series between the said two terminals with a first tap between the said two impedances, a second pair of two impedances with different phase angles in series between said first tap and one of said input terminals, with a second tap between the two said impedances of said second pair, a third pair of two impedances with different phase angles in series between said second tap and one of said input terminals, with a third tap between the two said impedances of said third pair, passive said first pair of two impedances generating a second signal proportional to the voltage between said first tap and said one input terminal, said second pair of two impedances generating a third signal proportional to the voltage between said second tap and said one input terminal, said third pair of two impedances generating a fourth signal proportional to the voltage between said third tap and said one input terminal, and passive means responsive to an algebraically linearly weighted combination of said, second, third and fourth signals to generate a fifth signal.

6. A circuit as in claim 5 whereby said third signal is proportional to the time derivative of said second signal, and said fourth signal is proportional to the time derivative of said third signal.

7. A circuit as in claim 5 whereby said third signal is proportional to the time integral of said second signal, and said fourth signal is proportional to the time integral of said third signal.

8. A linear passive circuit consisting of only linear passive resistors and capacitors for producing a plurality of state variable signals from a measured signal, comprising means for performing a dynamic passive operation on said measured signal to produce a first output, means for performing a second passive dynamic operation on said measured signal to provide a second output, means for performing a third passive dynamic operation on said measured signal to provide a third output, means for performing a fourth passive dynamic operation on said measured signal to provide a fourth output, whereby said second output is the derivative with respect to time of said first output and said third output is the time derivative of said second output, and said fourth output is the time derivative of said third output, and whereby the roots of the denominators of the transferences from said measured signal to each of said outputs are equal

9. A multiport circuit consisting of linear passive resistors and capacitors for producing a plurality of state variable signals from an input signal, passive means for generating from said input signal a first state variable signal, passive means for generating from said input signal a second state variable signal which is a time derivative of the first state variable signal, and means for generating from said input signal a third state variable signal which is a time derivative of the second state variable signal, and passive means for generating from said input signal a fourth state variable signal which is a time derivative of the third state variable signal, each of said means for generating a state variable signal having a gain less than a predetermined finite value of gain, the roots of the denominator of the complex gain as a function of frequency from said input signal to said first state variable signal being equal to the roots of the de-

nominator of the complex gain from said input signal to said second state variable signal and being equal to the roots of the denominator of the complex gain from said input signal to said third state variable signal and being equal to the roots of the denominator of the complex gain from said input signal to said fourth state variable signal.

10. A linear passive dynamic circuit consisting of only linear passive elements for producing a plurality of state variable signals from an input signal, passive means for generating a first state variable signal from said input signal, passive means for generating second, third, and fourth state variable signals from said input signal, said state variable signals being related such that a polynomial in the frequency variable times the Laplace transform of the first state variable signal using the frequency variable equals the Laplace transform of said input signal and so that the fourth state variable signal is the time integral of the third state variable and

so that the third state variable signal is the time integral of the second state variable signal and the second state variable signal is the time integral of the first state variable signal, each of said passive means for generating a state variable signal having a gain from input to state variable signal less than a predetermined value.

11. A circuit as in claim 8 wherein said transference from measured signal to the said third output has a numerator polynomial order not greater than its denominator polynomial order.

12. A circuit as in claim 9 wherein each means for generating a state variable signal contains an R-C ladder network.

13. A circuit as in claim 10 wherein each of said gains is limited to less than said predetermined value only by the linear dynamic properties of each of said means for generating a state variable signal.

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