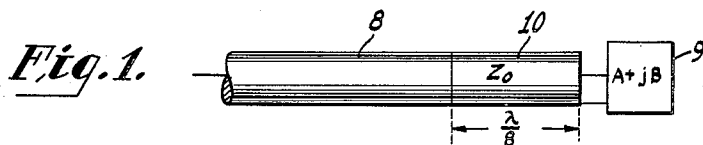
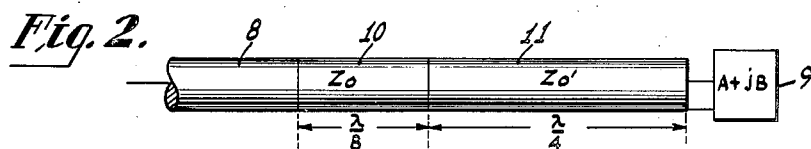
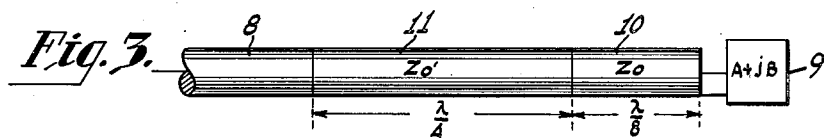
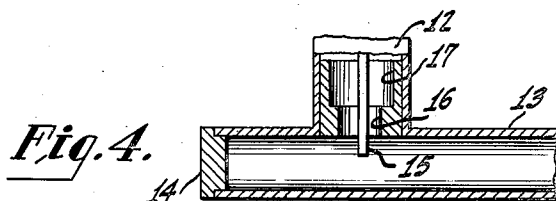
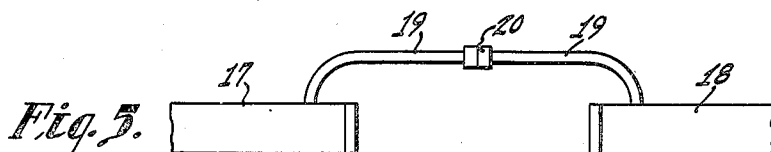
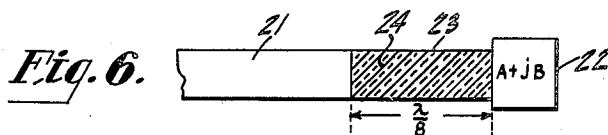
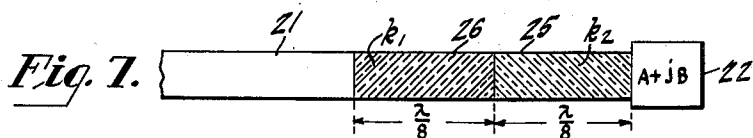


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J. COLLARD  
HIGH-FREQUENCY IMPEDANCE TRANSFORMER  
FOR TRANSMISSION LINES  
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2,507,692



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## UNITED STATES PATENT OFFICE

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## HIGH-FREQUENCY IMPEDANCE TRANSFORMER FOR TRANSMISSION LINES

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1

This invention relates to high frequency impedance transformers.

For many purposes it may be desired to convert a complex impedance into a substantially non-reactive or pure resistance. For example, in the resistance pad referred to in the British patent application No. 16,507/42 it may be desired to remove an unwanted reactance from the pad so as to leave a substantially pure resistance. Also, in cases where it is desired to provide an impedance match between a coaxial line circuit and a waveguide, difficulties are sometimes encountered in obtaining the desired impedance match owing to the presence of an unwanted reactance. Furthermore, it is sometimes necessary to connect a waveguide to a hollow cavity resonator which has a complex impedance and in such a case it may be necessary to make the complex impedance of the resonator appear as a substantially non-reactive or pure resistance.

The object of the present invention therefore is to provide an improved impedance transformer for the purpose of converting a complex impedance into a substantially non-reactive or pure resistance.

According to one feature of the invention a high frequency impedance transformer is provided comprising a length of transmission line, said transmission line having at one end a complex impedance  $A + jB$  and wherein, in order to make said complex impedance appear from the other end of said line as a substantially non-reactive or pure resistance, the length and the characteristic impedance  $Z_0$  of the transmission line and the complex impedance are so related to one another that

$$Z_0 = \frac{-kB + \sqrt{k^2 B^2 + 4(A^2 + B^2)}}{2}$$

where

$$k = \frac{1 - \tan^2 \theta}{\tan \theta}$$

According to another feature of the invention a high frequency impedance transformer is provided comprising a length of waveguide, said waveguide having at one end a complex impedance  $A + jB$  and wherein, in order to make said complex impedance appear from the other end of said waveguide as a substantially non-reactive

2

or pure resistance, the length and characteristic impedance  $Z_0$  of the waveguide and the complex impedance are so related to one another that

$$Z_0 = \frac{-kB + \sqrt{k^2 B^2 + 4(A^2 + B^2)}}{2}$$

where

$$k = \frac{1 - \tan^2 \theta}{\tan \theta}$$

In the preferred forms of the invention the length of transmission line or waveguide is made effectively equal to one-eighth of the operating frequency and the characteristic impedance of the transmission line or waveguide and the modulus of said complex impedance are made substantially equal to one another in order to make said complex impedance appear as a substantially non-reactive or pure resistance. Thus, if the length of transmission line or waveguide is one-eighth of a wavelength long or any odd multiple of one-eighth of a wavelength, then the impedance  $Z_0$  of the transmission line or waveguide is made equal to the modulus

$$(\sqrt{A^2 + B^2})$$

of the complex impedance. The value of said pure resistance is given by the expression

$$M \sqrt{\frac{M+B}{M-B}}$$

where  $M$  is the modulus of the complex impedance.

If the length of transmission line or waveguide employed is equal to or is in the neighborhood of a quarter of a wavelength of the operating frequency or integral multiples thereof, then in order to make the complex impedance appear as a substantially non-reactive or pure resistance it is necessary to make the characteristic impedance of the transmission line or waveguide very small which is inconvenient in practice and hence such lengths of transmission line or waveguide should be avoided.

If the value of the substantially pure resistance afforded by the transformer is not of the required value, then it is possible to employ, in conjunction with the transformer, a further section of transmission line or waveguide effectively equal to one-quarter of a wavelength long or odd inte-

gral multiples thereof so as to convert the pure resistance afforded by the transformer in accordance with the invention to some other desired value.

In order that the invention may be clearly understood and readily carried into effect it will now be more fully described with reference to the accompanying drawings, in which:

Figure 1 illustrates diagrammatically a transformer in accordance with one form of the invention,

Figures 2 and 3 illustrate diagrammatically transformers in accordance with the form of the invention shown in Figure 1 associated with further sections of transmission line for converting the pure resistance to a desired value,

Figure 4 illustrates the application of the invention to the coupling of a coaxial line circuit to a wave guide,

Figure 5 illustrates diagrammatically the coupling of two waveguides by a coaxial line circuit,

Figure 6 illustrates diagrammatically an impedance transformer comprising a section of waveguide, and

Figure 7 illustrates diagrammatically a modification of the arrangement shown in Figure 6.

Referring now to Figure 1 of the drawings which illustrates the invention as applied to a transformer comprising a section of coaxial line, the reference numeral 8 indicates a section of coaxial line feeding a complex impedance  $A+jB$  indicated by the reference numeral 9. In order to make the complex impedance appear to be a substantially non-reactive or pure resistance at the end of the transmission line 8 an impedance transformer is inserted between the line 8 and the impedance 9 indicated at 10 composed of a further section of coaxial transmission line the length and the characteristic impedance  $Z_0$  of the section 10 and the complex impedance 9 are so related to one another that

$$Z_0 = \frac{-kB + \sqrt{k^2 B^2 + 4(A^2 + B^2)}}{2} \quad (1)$$

where

$$k = \frac{1 - \tan^2 \theta}{\tan \theta}$$

It is preferred to make the length of the section 10 effectively equal to one-eighth of the wavelength of the operating frequency, the characteristic impedance of the section 10 and the modulus of the complex impedance 9 being made substantially equal to one another. Thus, if the length of the section 10 is one-eighth of a wavelength long or any odd multiple of one-eighth of a wavelength, then the impedance  $Z_0$  of the transmission line is made equal to the modulus

$$(\sqrt{A^2 + B^2})$$

of the complex impedance. Where the length of the section 10 is equal to  $n$  odd eighth-wavelengths then when  $n$  is 1, 5, 9 etc., the value of the pure resistance is given by the expression

$$M \sqrt{\frac{M+B}{M-B}}$$

and when  $n$  is 3, 7, 11 etc., the value of the pure resistance is given by the expression

$$M \sqrt{\frac{M-B}{M+B}}$$

where  $M$  is the modulus of the complex impedance.

If the value of the substantially pure resistance

afforded by the transformer is not of the required value, then it is possible to employ, in conjunction with the transformer, a further section of a transmission line effectively equal to one-quarter of a wavelength or odd integral multiples thereof so as to convert the pure resistance afforded by the transformer in accordance with the invention to some other desired value. Figure 2 of the drawings illustrates the introduction of such a quarter wavelength section 11 between the section 10 and the complex impedance 9, whilst Figure 3 of the drawings illustrates the introduction of the quarter wavelength section 11 between the line 8 and the section 10. In the latter case, assuming that the complex impedance  $A+jB$  is transformed by the transformer 10 to a pure resistance  $R$  and it is desired to convert the pure resistance  $R$  to another pure resistance  $R'$ , for example, in order to provide an impedance match with the line 8, then the characteristic impedance  $Z'_0$  of the quarter wavelength section 11 is given by

$$Z'_0 = \sqrt{RR'}$$

Whilst the invention is capable of a wide variety of uses, it is of particular importance in matching a coaxial cable to a wave guide. Heretofore when it has been desired to effect an impedance match between a cable and a waveguide, such has been accomplished by connecting the coaxial cable to a probe projecting into the waveguide. To effect an impedance match two requirements have to be satisfied. Firstly, the resistive component of the guide impedance as seen from the end of the coaxial cable must be made equal to the characteristic impedance of the cable, and, secondly, the reactive component of the guide impedance as seen from the end of the coaxial circuit must be made zero. Usually, in order to satisfy these requirements the length of the portion of the probe which projects into the guide is adjusted so that the resistive component is of the required value which usually leaves a large reactive term which is then tuned out by varying the position of a tuning piston associated with the waveguide. It has been found that it is even sometimes necessary to employ additional tuning plungers. On account of the number of adjustable elements that it is necessary to employ in such an arrangement the latter is not only complicated to adjust but the impedance match is liable to be disturbed by mechanical shock. Furthermore, the matching of a cable to a waveguide in this manner necessitates an accurate balance to be made between two relatively large reactances and hence any small change in the operative wavelength which changes either or both of the two opposing reactances usually results in the two reactances becoming unequal giving rise to a relatively large residual reactance. Thus the arrangement described affords only a satisfactory impedance match over a relatively narrow frequency band.

Figure 4 of the drawings illustrates the invention as employed to obtain an impedance match between a coaxial cable 12 and a waveguide 13. One end of the waveguide 13 is closed by a conductor 14 which may be permanently fixed in position as distinct from the adjustable piston employed in the prior arrangement above referred to and the coaxial cable is connected to a probe 15 projecting into the waveguide, the probe 15 also being rigidly fixed in position in any suitable manner and disposed at a desired position with reference to the closed end of the guide. The probe 15 may comprise a projecting portion of

5

the centre conductor of the coaxial cable the outer conductor of which is connected to the waveguide, as shown. The impedance at the point where the probe passes into the waveguide is determined by any of the well known means and, in general, this impedance will be complex and not equal to the characteristic impedance of the coaxial cable. In order to convert the complex impedance into a substantially pure resistance a transformer in accordance with the invention is employed which, as shown, comprises a sleeve 16 surrounding the probe 15 or centre conductor and insulated therefrom, said sleeve being preferably one-eighth of a wavelength long. The characteristic impedance of the transmission line formed by the sleeve 16 and the portion of the probe 15 or centre conductor surrounded thereby is made equal to the modulus of the complex impedance. This transformer will convert the complex impedance to a pure resistance  $R$  and if it is necessary to convert the pure resistance  $R$  to a value equal to the characteristic impedance of the coaxial cable in order properly to terminate the coaxial cable, then a further transformer effectively equal in length to one-quarter of a wavelength is then inserted between the cable 13 and the transformer in accordance with the invention, the characteristic impedance of the quarter-wave transformer being made equal to

$$\sqrt{Z' Z_0} R$$

where  $Z'$  is the characteristic impedance of the coaxial cable. The quarter-wave transformer, as shown, comprises a sleeve 17 surrounding and insulated from the centre conductor of the coaxial cable and it may be convenient in practice to form the sleeves 16 and 17 integrally with one another, the sleeves fitting within and being connected to the outer conductor of the cable, as shown. The construction described will provide an impedance match between a coaxial cable and a waveguide which is of a rigid nature and requires no adjustment after being initially correctly assembled. If desired the space between the centre conductor and said sleeve may be filled with solid insulation and it will therefore be appreciated that the physical lengths of the transformers may be different from their electrical lengths depending upon the dielectric constant of the insulation employed.

It will be appreciated from the above that an impedance match can be obtained whatever is the length of the portion of the probe 15 which projects into the waveguide and furthermore, the match can be obtained whatever the position of the probe relatively to the closed end of the waveguide. This is advantageous since it is possible thereby to make the arrangement less susceptible to changes of operating frequency. As stated above, the magnitude of the complex impedance as seen through the transformer according to the invention when employing a transformer effectively equal to one-eighth of a wavelength long, is equal to

$$M \sqrt{\frac{M+B}{M-B}}$$

and if  $B$  is small compared with  $A$ , that is to say, about one-fifth or less, the expression reduces approximately to  $A+B$ . Hence, if the length of the probe is adjusted together with the position of the closed end of the guide so that the waveguide impedance has its two components  $A$  and  $B$  varying with frequency in such a way that the sum of the two is constant, then the impedance

6

seen through the transformer section will also be constant with frequency. Thus, since it is possible to vary the length of the probe and the position of the closed end of the waveguide independently of one another, it is possible to obtain a condition in which the component  $A$  falls with frequency and the component  $B$  rises with frequency over a relatively wide range of frequencies in such a way that the sum of the two components is constant. Hence, such an arrangement will afford an impedance match over a relatively wide frequency range.

If desired, two sections of waveguide may be connected together by a length of coaxial cable each end of which is matched to its associated waveguide by the construction described with reference to Figure 4. Figure 5 illustrates such an arrangement, the reference numerals 17 and 18 indicating two sections of waveguide and the numeral 19 a length of coaxial cable. The coaxial cable is provided with a rotatable joint 20, such as the plug and socket joint described in the specification of British patent application No. 11,862/42, the provision of such a joint enabling the two sections of waveguide to be rotated with respect to one another without the necessity of providing specially constructed joints in the waveguide sections. Heretofore, when it has been desired to employ two sections of waveguide mounted for relative rotational movement it has been usually necessary to provide special tuning plungers and to maintain the distance between the two waveguide sections very small and of a fixed dimension. If the two sections of waveguide are coupled by means of a coaxial cable as shown in Figure 5, the distance between the two waveguide sections can be made of any desired length, since the coupling is afforded by a coaxial cable which can be of any appropriate length without disturbing the impedance match. There is also an added advantage that the coaxial cable joining the two waveguides can be bent so as to negotiate corners or other obstructions. The dimensions of the coaxial cable should be chosen to afford a minimum loss but the dimensions must not be such as to permit the propagation of unwanted modes.

The invention can also be applied to impedance transformers composed of sections of waveguide.

It is known that the theory of two-wire transmission lines, such as coaxial cables, can be applied to waveguides providing that in any given problem the guides have substantially the same physical dimensions and energy of the same mode is transmitted through the guides. In the case of a two-wire transmission line the impedance at any point is defined as the ratio of voltage to the current at that point. In the case of a waveguide, however, the impedance at any point is defined as the ratio of the transverse electric field to the transverse magnetic field at that point.

Suppose that in any guide of any form in cross-section there are three axes,  $x$ ,  $y$  and  $z$  mutually at right-angles and that  $z$  is the axis along which propagation occurs, then the characteristic impedance  $Z_0$  of the guide is

$$\frac{E_x}{H_y} \text{ or } -\frac{E_y}{H_x}$$

these two ratios having the same value. This ratio gives the same value at every point in the guide but is different for each mode of the energy transmitted.

For H waves (or transverse electric waves) the ratio becomes  $\lambda_t/\lambda_0$ , where  $\lambda_0$  is the wavelength in

air and  $\lambda_t$  is the corresponding value in the guide.

For E waves (or transverse magnetic waves) the ratio is

$$\frac{\lambda_0}{k\lambda_t}$$

where  $k$  is the dielectric constant of the material in the guide and the permeability of this material is assumed to be unity. Since  $\lambda_t$  depends on the particular E or H mode being transmitted and also on the dimensions of the guide, it follows that the characteristic impedance of the guide is also a function of these quantities. In practice, however, energy of only one mode is propagated and since, as stated above, the two-wire theory only applies to waveguides when the physical dimensions of the latter remain substantially constant, the problem of applying the present invention to waveguides is considerably simplified.

The guide most commonly employed is one of rectangular cross-section and in such a guide energy of the  $H_{10}$  mode is usually employed. With such a form of guide and energy of the stated mode, the characteristic impedance of the guide is

$$Z_0 = \frac{\lambda_t}{\lambda_0} = \frac{377}{\sqrt{k - \left(\frac{\lambda_0}{2a}\right)^2}} \quad (2)$$

where  $a$  is the longer side of the cross-section of the guide. The ratio

$$\frac{\lambda_0}{2a}$$

is constant since the guide has substantially the same physical dimensions throughout its length. Thus, in a transformer in accordance with the invention and composed of a section of waveguide the characteristic impedance of the waveguide can be given an appropriate value to transform a complex impedance into a pure resistance by suitably choosing the dielectric medium of the guide.

Figure 6 of the drawings illustrates a waveguide 21 feeding a hollow resonant cavity 22 which presents to the waveguide a complex impedance  $A+jB$ . In order to convert the complex impedance into a substantially pure resistance a waveguide transformer 23 composed of a section of waveguide is inserted between the waveguide 21 and the cavity 22, the length and the characteristic impedance  $Z_0$  of the section 23 and the complex impedance 22 being related to one another in accordance with Formula 1 heretofore referred to. The desired characteristic impedance of the section 23 is imparted by providing the section 23 with a suitable dielectric medium indicated at 24. Preferably, as in the transformer shown in Figure 1, the length of the section 23 is equal to one-eighth of the operating wavelength, in which case the impedance  $Z_0$  of the section 23 is made equal to the modulus

$$(\sqrt{A^2+B^2})$$

of the complex impedance.

Theoretically, the desired impedance of the waveguide section can be attained by suitably choosing the dielectric medium within the guide section, but in practice, however, any material which is sufficiently loss free to be used in a guide has substantially unit permeability with the result that a complication may arise due to the fact that whereas a dielectric constant of unity is obtainable by using air and dielectric constants from about 2.5 to 100 can be obtained by using,

for example, polystyrene suitably loaded with titanium dioxide, there is no available suitable substance which affords a constant between unity and 2.5. If therefore any given problem requires a dielectric constant within the range of 1 to 2.5 or between zero and unity, the difficulty can be overcome by employing two sections in series with dielectrics having constants which can be obtained with available dielectrics.

To simplify this problem the term

$$\left(\sqrt{\frac{\lambda_0}{2a}}\right)$$

can be assumed to be small compared with  $k$  so that it is possible to write

$$Z_0 = \frac{377}{\sqrt{k}} \quad (3)$$

If in any particular problem this simplification cannot be employed, then the Formula 2 referred to above must be used. Assuming that the simplified formula can be employed, then it may be required to convert the complex impedance  $280+j100$  to a pure resistance. The modulus of this complex impedance is 297 ohms which requires  $k$  to be 1.61 which is within the range of dielectric constants which cannot conveniently be obtained. To overcome this difficulty the transformer may employ two eighth wavelength sections arranged in series as shown in Figure 7, the first section 25 having a dielectric medium of dielectric constant  $k_1$  and the second section 26 a constant  $k_2$ . If the first section is terminated by the complex impedance, then the impedance seen through the two sections will be a pure resistance providing that

$$Z_2 = Z_1 \sqrt{\frac{(Z_1+B)^2 + A^2}{(Z_1-B)^2 + A^2}}$$

where  $Z_2$  is the characteristic impedance of one of the eighth wavelength sections and  $Z_1$  is the characteristic impedance of the other eighth wavelength section. It is thus possible by employing available dielectric mediums to provide eighth wavelength sections of waveguide having characteristic impedances of the required values to satisfy the Equation 3. For instance, in the example referred to above  $k_1$  may be equal to 4.6 giving a characteristic impedance  $Z_1$  of 175 ohms in which case  $k_2$  will be required to be 2.5 so as to afford a characteristic impedance  $Z_2$  equal to 238 ohms.

Alternatively, instead of employing two eighth wavelength sections, it is possible to employ a quarter wavelength section and then an eighth wavelength section, the quarter wavelength section transforming the complex impedance to a suitable value to enable an available dielectric medium to be employed for the eighth wavelength section. In this case if  $Z_1$  is the characteristic impedance of the quarter wavelength section and  $A+jB$  is the complex impedance, then the impedance seen through the quarter wavelength section is

$$\frac{Z_1^2(A-jB)}{A^2+B^2}$$

The modulus of which is:

$$\frac{Z_1^2}{\sqrt{A^2+B^2}}$$

With such an arrangement it is then necessary to make the impedance of the eighth wavelength section equal to this modulus. It is preferred, however, to employ two eighth wavelength sec-

tions since, in this case, the overall length of the transformer is shorter and is less susceptible to changes in the wavelength of the transmitted energy.

If the pure resistance afforded by the section or sections of the transformer shown in Figure 5 or 6 is not of the required value for impedance matching purposes, then it will be necessary as in the arrangements shown in Figures 2 and 3 to employ a further section of waveguide effectively equal to a quarter of a wavelength long to convert the pure resistance to the required value. Thus, in some cases it may be necessary to employ one or two eighth wavelength sections in association with a quarter wavelength section or a one-eighth wavelength section in association with two sections each of a quarter of a wavelength long. It may even be necessary in some cases to employ two quarter wavelength sections in series to perform the functions of the quarter wavelength section shown in Figures 2 and 3. This may be necessary on account of the fact that the dielectric constant required to give the quarter wavelength section the correct impedance is not available. If the first quarter wavelength section has a dielectric constant  $k_1$  and the second section a dielectric constant of  $k_2$  and assuming that the characteristic impedance of the guide is inversely proportional to the square root of the dielectric constant as assumed above, then if the resistance produced by the eighth wavelength section is  $R$  and that this resistance is to be converted in order to match a guide impedance of  $Z_0$  it is necessary to arrange that

$$R \frac{k_1}{k_2} = Z_0$$

Providing the ratio of  $k_1$  to  $k_2$  satisfies this equation the individual values of the dielectric constants are immaterial so that it is possible to arrange for the constants to fall within the available range.

If desired, instead of matching a coaxial cable to a waveguide by means of a probe associated with a transformer as described with reference to Figure 4 of the drawings, the necessary impedance match can be obtained by the use of a transformer or transformers associated with the waveguide as described with reference to Figures 6 and 7. Furthermore, it is not necessary in connection with the matching arrangement shown in Figure 4 for both transformers 16 and 17 to be associated with the probe since, if desired, one of these transformers can be associated with the waveguide.

Although in the above description certain specific lengths of transmission lines and waveguide sections are referred to, it will be understood that equivalent multiple lengths can be employed.

What I claim is:

1. A wave transmission system including a wave guide, and an intercoupled coaxial cable, said guide and said cable being coupled by a probe connected to a conductor of said cable and projecting into said wave guide, at a fixed position along said guide, one end of said wave guide being closed, a sleeve having a length equal to one-eighth of the operating wavelength of said system, surrounding said probe and insulated therefrom, the characteristic impedance of the section of transmission line formed by said sleeve and said probe being equal to the modulus

of the complex impedance of said wave guide at the point of insertion of said probe.

2. A wave transmission system including a wave guide intercoupled with a coaxial cable the intercoupling being effected by an extension of the central conductor of said cable projecting into said wave guide at a fixed position along said guide, one end of said wave guide being closed, a sleeve having a length equal to one-eighth of the operating wavelength of said system surrounding said central conductor and insulated therefrom, the characteristic impedance of the section of transmission line formed by said sleeve and said central conductor being equal to the modulus of the complex impedance of said wave guide at the point of insertion of said extension of said central conductor, and a further one-quarter wavelength matching section interposed between said coaxial line and said extension of said central conductor to match the impedance of said section of transmission line to said coaxial cable.

3. A wave transmission system including a wave guide and a coaxial cable coupled thereto, said coupling being constituted by a probe connected to a conductor of said cable projecting into said wave guide at a predetermined position along said guide, one end of said wave guide being closed, the length of said probe and the predetermined position being such that the two components of the wave guide impedance vary with frequency in such a manner that the sum of the two is constant, a sleeve having a length equal to one-eighth of the main operating wavelength surrounding said probe and insulated therefrom, the characteristic impedance of the section of transmission line formed by said sleeve and said probe being equal to the modulus of the complex impedance of said wave guide at the point of insertion of said probe.

4. A wave transmission system including a wave guide, and a coaxial cable coupled thereto the coupling comprising a probe constituting an extension of the center conductor of said cable projecting into said wave guide at a predetermined position along said guide, one end of said wave guide being closed, the length of said probe and the predetermined position being such that the two components of the wave guide impedance vary with frequency in such a manner that the sum of the two is constant, a sleeve having a length equal to one-eighth of the main operating wavelength surrounding said probe and insulated therefrom, the characteristic impedance of the section of transmission line formed by said sleeve and said probe being equal to the modulus of the complex impedance of said wave guide at the point of insertion of said probe, and a further one-quarter wavelength matching section between said coaxial line and said probe to match the impedance of said section of transmission line to the impedance of said coaxial cable.

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