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(54) Title: MULTIPLIERLESS PYRAMID FILTER

(57) Abstract: Embodiments of a multiplierless pyramid filter are described.



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MULTIPLIERLESS PYRAMID FILTER

BACKGROUND

This disclosure is related to pyramid filters.

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In image processing it is often desirable to decompose an image, such as a scanned color image, into two or more separate image representations. In this context, these are referred to as background and foreground images. This descreening is also sometimes applied to remove halftone patterns that may exist
10 in an original scanned image. For example, these halftone patterns may cause objectionable artifacts for human eyes if not properly removed. The traditional approach for this decomposition or descreening is to filter the color image in order to blur it. These blurred results are then used to assist in determining how much to blur and sharpen the image in order to produce the decomposition. Typically
15 this blurring can be achieved using a "symmetric pyramid" filter. Symmetric pyramid finite impulse response (FIR) filters are well-known.

One disadvantage of this image processing technique, however, is that the complexity increases many fold when a number of pyramid filters of different sizes
20 are applied in parallel in order to generate multiple blurred images, to apply the technique as just described. A brute force approach for this multiple pyramid filtering approach is to use multiple FIR filters in parallel, as illustrated in FIG. 1. Such an approach demonstrates that the design and implementation of fast "symmetric pyramid filtering" architectures to generate different blurred images in
25 parallel from a single source image may be desirable.

The numbers provided in parenthesis for each FIR block in FIG. 1 represents the pyramid filter of corresponding length. For example, (1, 2, 1) are the filter coefficients for a symmetric pyramid finite impulse response (FIR) filter of order or length 3. Likewise, (1, 2, 3, 2, 1) are the coefficients for an FIR pyramid filter of order 5, and so forth.

Unfortunately, the approach demonstrated in FIG. 1 has disadvantages. For example, inefficiency may result from redundant computations. Likewise, FIR implementations frequently employ multiplier circuits. While implementations exist to reduce or avoid the use of multipliers, such as with shifting and summing circuitry, that may then result in increased clocking and, hence, may reduce circuit through-put. A need, therefore, exists for improving pyramid filtering implementations or architectures.

BRIEF DESCRIPTION OF THE DRAWINGS

The subject matter regarded as the invention is particularly pointed out and distinctly claimed in the concluding portion of the specification. The invention, however, both as to organization and method of operation, together with objects, features, and appendages thereof, may best be understood by reference of the following detailed description when read with the accompanying drawing in which:

FIG. 1 is a block diagram illustrating a brute force approach to implementing a finite impulse response (FIR) multiple pyramid filtering

architecture;

FIG. 2 is one embodiment of a multiplierless pyramid filter in accordance with the present invention;

5

FIG. 3 is one embodiment of a subcomponent of a pyramid filter in accordance with the present invention;

FIG. 4 is the embodiment of FIG. 2 employing the embodiment of FIG. 3;

10

FIG. 5 is a table showing a chronological sequence of state variable signal samples for one implementation of a pyramid filter in accordance with the present invention; and

15

FIG. 6 is a table showing a chronological sequence of filtered output signal samples for one implementation of a pyramid filter in accordance with the present invention.

20 DETAILED DESCRIPTION

In the following detailed description, numerous specific details are set forth in order to provide a thorough understanding of the invention. However, it will be understood by those skilled in the art that the present invention may be practiced without these specific details. In other instances, well-known methods,

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procedures, components and circuits have not been described in detail in order so as not to obscure the present invention.

As previously described, pyramid filtering, in particular, symmetric pyramid
5 filtering, may be employed in connection with color images or color image
processing in order to decompose or descreen the image, such as into a
background and foreground image, for example. Although the present invention
is not limited in scope in this respect, in such a context, pyramid filtering
architectures that reduce computational complexity or processing and/or
10 hardware cost are particularly desirable. Likewise, implementations that are
multiplierless, that is do not specifically employ any multiplications in the
implementation, are also desirable usually because such implementations or
embodiments are cheaper to implement than those that employ or include
multiplier circuits.

15

Although the invention is not limited in scope in this respect, FIG. 2
illustrates one embodiment 200 of a pyramid filter in accordance with the present
invention. The embodiment 200 comprises a unified multiplierless cascaded
symmetric pyramid filtering architecture to generate a multiple number of filtered
20 output signal streams for a series or sequence of pyramid filters having different
orders, the generation of the output signal streams occurring in parallel. In this
particular embodiment, although the invention is not limited in scope in this
respect, a filtered output signal stream is produced on every clock cycle for each
pyramid filter of a different order being implemented. Therefore, in addition to
25 being computationally efficient, this particular embodiment produces good results

in terms of throughput.

FIG. 2 is understood in the context of specific notation. For example, an input source signal, X , may be designated as follows:

5

$$X = (x_0, x_1, \dots, x_{i-2}, x_{i-1}, x_i, x_{i+1}, x_{i+2}, \dots)$$

In digital or discrete signal processing, filtering may be expressed as a convolution, $\square$, of the input signal, X , and a filter, F , in this context a digital filter of finite length, referred to here as a finite impulse response (FIR) filter. Therefore, the filtered output signal stream is indicated as follows:

10

$$Y = X \square F$$

15

As previously described, this particular embodiment of the invention employs pyramid filters. These filters are typically implemented using digital filters of lengths or orders that are odd, such as 3, 5, 7, 9, etc. Some examples of such digital filters are as follows:

20

$$F_3 = (1, 2, 1)$$

$$F_5 = (1, 2, 3, 2, 1)$$

$$F_7 = (1, 2, 3, 4, 3, 2, 1)$$

$$F_9 = (1, 2, 3, 4, 5, 4, 3, 2, 1)$$

25

...

$$F_N = (1, 2, 3, \dots, [N-1]/2, \dots, 3, 2, 1)$$

5

For the foregoing filters, the filtered output signals or output signal streams may be represented as follows:

10 $B^3 = X \square F_3 = (b_0^3, b_1^3, \dots, b_{i-1}^3, b_i^3, b_{i+1}^3, \dots)$ result of input signal X
filtered by F_3

$B^5 = X \square F_5 = (b_0^5, b_1^5, \dots, b_{i-1}^5, b_i^5, b_{i+1}^5, \dots)$ result of input signal X
filtered by F_5

$B^7 = X \square F_7 = (b_0^7, b_1^7, \dots, b_{i-1}^7, b_i^7, b_{i+1}^7, \dots)$ result of input signal X
15 filtered by F_7

$B^9 = X \square F_9 = (b_0^9, b_1^9, \dots, b_{i-1}^9, b_i^9, b_{i+1}^9, \dots)$ result of input signal X
filtered by F_9

.....

20

$B^N = X \square F_N = (b_0^N, b_1^N, \dots, b_{i-1}^N, b_i^N, b_{i+1}^N, \dots)$ result of input signal X
filtered by F_N

25 An alternate way to empirically represent these filtered output signal samples is

as follows:

$$b_i^3 = x_{i-1} + 2x_i + x_{i+1}$$

5

$$b_i^5 = x_{i-2} + 2x_{i-1} + 3x_i + 2x_{i+1} + x_{i+2}$$

$$b_i^7 = x_{i-3} + 2x_{i-2} + 3x_{i-1} + 4x_i + 3x_{i+1} + 2x_{i+2} + x_{i+3}$$

10

$$b_i^9 = x_{i-4} + 2x_{i-3} + 3x_{i-2} + 4x_{i-1} + 5x_i + 4x_{i+1} + 3x_{i+2} + 2x_{i+3} + x_{i+4}$$

Likewise, by introducing what is referred to, in this context, as state variables, the above expressions may be re-expressed as follows:

15

$$b_i^3 = x_i + s_i^3, \text{ where } s_i^3 = x_{i-1} + x_i + x_{i+1}$$

$$b_i^5 = b_i^3 + s_i^5, \text{ where } s_i^5 = x_{i-2} + x_{i-1} + x_i + x_{i+1} + x_{i+2}$$

$$b_i^7 = b_i^5 + s_i^7, \text{ where } s_i^7 = x_{i-3} + x_{i-2} + x_{i-1} + x_i + x_{i+1} + x_{i+2} + x_{i+3}$$

20

$$b_i^9 = b_i^7 + s_i^9, \text{ where } s_i^9 = x_{i-4} + x_{i-3} + x_{i-2} + x_{i-1} + x_i + x_{i+1} + x_{i+2} + x_{i+3} +$$

x_{i+4}

Hence, the desired pyramid filter may be expressed as follows:

25

$$B^3 = X + S_3, \text{ where } S_3 = (s_0^3, s_1^3, s_2^3, \dots, s_{i-1}^3, s_i^3, s_{i+1}^3, \dots)$$

$$B^5 = B^3 + S_5, \text{ where } S_5 = (s_0^5, s_1^5, s_2^5, \dots, s_{i-1}^5, s_i^5, s_{i+1}^5, \dots)$$

$$5 \quad B^7 = B^5 + S_7, \text{ where } S_7 = (s_0^7, s_1^7, s_2^7, \dots, s_{i-1}^7, s_i^7, s_{i+1}^7, \dots)$$

$$B^9 = B^7 + S_9, \text{ where } S_9 = (s_0^9, s_1^9, s_2^9, \dots, s_{i-1}^9, s_i^9, s_{i+1}^9, \dots)$$

A study of FIG. 2 illustrates that the computed output signal streams, B_3 ,
 10 B_5 , B_7 , B_9 , etc. of the pyramid filters shown in FIG. 2 are produced by the
 embodiment illustrated.

FIG. 5 is a table illustrating a chronological sequence of state variable
 signals or state variable signal streams, S_2 , S_3 , S_4 , ..., S_7 generated respectively by
 15 adders, 210, 220, ..., 260, illustrated in FIG. 2. Likewise, FIG. 6 is a table showing
 a chronological sequence of filtered output signal streams, B_3 , B_5 , B_7 , etc. As
 illustrated in FIG. 2, these output signal streams are produced by adders 270, 280
 and 290.

20 In addition to providing the filtered output signal streams, B_3 , B_5 , B_7 , the
 table in FIG. 6 illustrates the generation of these filtered output signal streams in
 chronological order of clocking as applied to the pyramid filter architecture
 embodiment shown in FIG. 2. As previously illustrated, output signal streams
 may also be produced from signal samples, such as x_i and s_i , that is the input
 25 signal samples and the state variable signal samples. However, the table shown

in FIG. 6 illustrates that b_i^3 is generated one clock cycle ahead of s_i^5 . Therefore, by delaying the signal sample b_i^3 by a clock cycle, it may be added or summed with s_i^5 to generate b_i^5 in accordance with the equations provided previously. This is accomplished, for example, by delay element or digital delay unit 295 in FIG. 2.

5 Therefore, output signal sample B_3 delayed by one clock cycle is summed with state variable signal sample S_5 to generate output signal samples B_5 . Likewise, digital delay units 285 and 275 may be employed to generate output signal sample streams B_5 and B_7 , respectively. Likewise, the input signal sample stream, X , may be delayed and summed with S_3 to generate pyramid filter output

10 signal sample stream B_3 .

It is noted that the embodiment in accordance with the invention shown in FIG. 2 may be reorganized or redrawn to take into account an embodiment of a component of a pyramid filter in accordance with the invention, as shown in FIG.

15 3. Embodiment 300 shown in FIG. 3 comprises a pyramid filter component including a delay unit 310 and three adders 320, 330 and 340. For high speed and throughput performance, fast adders may be employed, such as proposed in the literature. A "Carry Lookahead Adder" is one such example, although the invention is not restricted in scope in this respect. In this particular embodiment,

20 the delay unit and adders are coupled to produce higher order pyramidally output signal samples or signal sample streams and pyramidally filtered state variable signal samples or signal sample streams from input signal samples or signal sample streams and lower order pyramidally filtered output signal samples or signal sample streams. For example, referring to the embodiment shown in FIG.

25 3, X comprises the input signal samples or signal sample stream, B_i comprises

the high order of pyramidally filtered output signal samples or signal sample stream, and S_i represents the pyramidally filtered state variable signal samples or signal sample stream. Likewise, B_{i-2} comprises or represents the lower order pyramidally filtered output signal samples or signal sample stream and S_{i-2} represents the lower order pyramidally filtered state variable signal samples or signal sample stream. Therefore, in this particular embodiment, the difference in order between the higher and lower order pyramidally filtered output signal samples and state variable signal samples or signal sample streams is two, although, of course, the invention is not limited in scope in this respect.

10

FIG. 4 shows the embodiment of FIG. 2 applying the embodiment of FIG. 3 as a component or subcomponent. Therefore, a component 410 is illustrated in FIG. 4 comprising delay unit 295 and adders 280, 230 and 240 from FIG. 2. Likewise, for the embodiment shown in FIG. 4, the pyramid filter 200 is shown implemented on an integrated circuit 400. Likewise, as illustrated in FIG. 4, pyramid filter 200 comprises a sequence of scalable cascaded multiplierless units, such as 410, 420, etc. Likewise, each of these scalable cascaded multiplierless units produces a different order of pyramidally filtered output signal samples or a different order output signal stream. For example, unit 410 produces output signal samples or signal sample stream B_5 , unit 420 produces output signal samples or signal sample stream B_7 , etc.

It will, of course, be understood that, although particular embodiments have just been described, the invention is not limited in scope to a particular embodiment or implementation. For example, one embodiment may be in

25

hardware, whereas another embodiment may be in software. Likewise, an embodiment may be in firmware, or any combination of hardware, software, or firmware, for example. Likewise, although the invention is not limited in scope in this respect, one embodiment may comprise an article, such as a storage
5 medium. Such a storage medium, such as, for example, a CD-ROM, or a disk, may have stored thereon instructions, which when executed by a system, such as a computer system or platform, or an imaging system, for example, may result in an embodiment of a method in accordance with the present invention being executed, such as an embodiment of a method of filtering or processing an image
10 or video, for example, as previously described. For example, an image processing platform or an imaging processing system may include an image processing unit, a video or image input/output device and/or memory.

While certain features of the invention have been illustrated and described
15 herein, many modifications, substitutions, changes and equivalents will now occur to those skilled in the art. It is, therefore, to be understood that the appended claims are intended to cover all such modifications and changes as fall within the true spirit of the invention.

Claims:

1. An integrated circuit comprising:
5 a pyramid filter;
said pyramid filter comprising a sequence of scalable cascaded multiplierless units, each of said units producing a different order pyramidally filtered output signal sample stream.
- 10 2. The integrated circuit of claim 1, wherein at least one of said multiplierless units comprises a delay unit and three adders, said delay unit and adders being coupled to produce a higher order pyramidally filtered output signal sample stream and state variable signal sample stream from an input signal sample stream and a lower order pyramidally filtered output signal sample stream and
15 state variable signal sample stream.
3. The integrated circuit of claim 2, wherein the difference in order between the higher and lower order pyramidally filtered output signal sample stream and state variable signal sample stream is two.
- 20 4. The integrated circuit of claim 3, wherein the adders comprise carry lookahead adders.
5. A pyramid filter component comprising:
25 a delay unit and three adders, said delay unit and adders being coupled to

produce a higher order pyramidally filtered output signal sample stream and state variable signal sample stream from an input signal sample stream and a lower order pyramidally filtered output signal sample stream and state variable signal sample stream.

5

6. The pyramid filter component of claim 5, wherein the difference in order between the higher and lower order pyramidally filtered output signal sample stream and state variable signal sample stream is two.

10 7. The pyramid filter component of claim 6, wherein the adders comprise carry lookahead adders.

8. A method of filtering an image comprising:

delaying a filtered output signal sample stream of a first order;

15 summing the delayed filter output signal sample stream with a state variable signal sample stream of a second order.

9. The method of claim 8, wherein the difference between the first and second order is two.

20

10. The method of claim 8, wherein the filtered output signal sample stream comprises a pyramidally filtered output signal sample stream and the state variable signal sample stream comprises a pyramidally filtered state variable signal sample stream.

25

11. The method of claim 10, wherein summing the delayed filter output signal sample stream with a state variable signal sample stream of a second order comprises applying carry lookahead addition to produce the sums.

5 12. An article comprising: a storage medium, said storage medium having stored thereon instructions, that, when executed result in image filtering by:

delaying a filtered output signal sample stream of a first order;

summing the delayed filter output signal sample stream with a state variable signal sample stream of a second order.

10

13. The article of claim 12, wherein the difference between the first and second order is two.

14. The article of claim 12, wherein the filtered output signal sample stream
15 comprises a pyramidally filtered output signal sample stream and the state variable signal sample stream comprises a pyramidally filtered state variable signal sample stream.

15. The article of claim 14, wherein said instructions, when executed, further
20 result in the summing of the delayed filter output signal sample stream with a state variable signal sample stream of a second order comprises applying carry lookahead addition to produce the sums.

16. An image processing system comprising:
25 an image processing unit to filter scanned color images;

said image processing unit including at least one pyramid filter;

said at least one pyramid filter comprising a sequence of scalable cascaded multiplierless units, each of said units producing a different order pyramidally filtered output signal sample stream.

5

17. The image processing system of claim 16, wherein at least one of said multiplierless units comprises a delay unit and three adders, said delay unit and adders being coupled to produce a higher order pyramidally filtered output signal sample stream and state variable signal sample stream from an input signal sample stream and a lower order pyramidally filtered output signal sample stream and state variable signal sample stream.

10

18. The image processing system of claim 17, wherein the difference in order between the higher and lower order pyramidally filtered output signal sample stream and state variable signal sample stream is two.

15

19. The image processing system of claim 18, wherein the adders comprise carry lookahead adders.

20

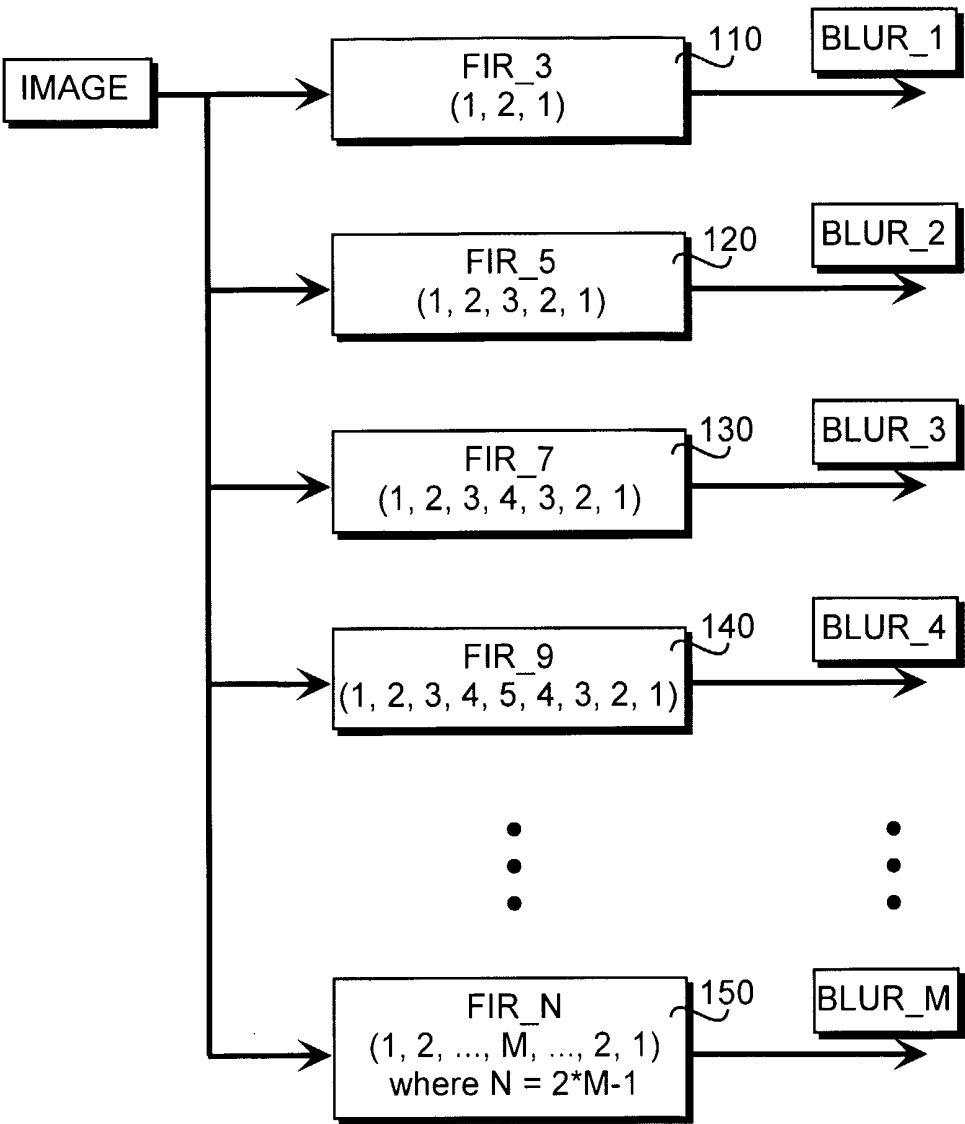


FIG. 1

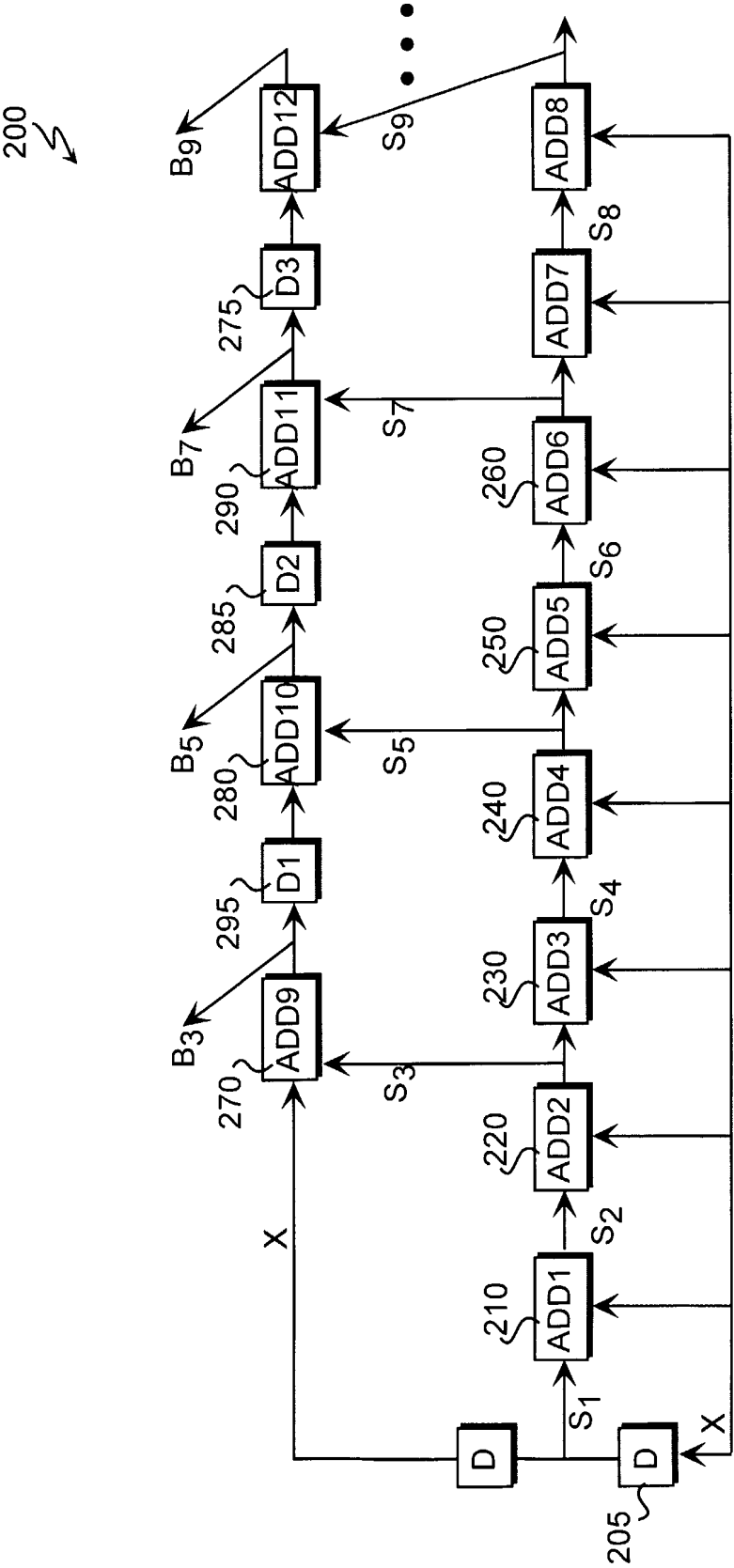


FIG. 2

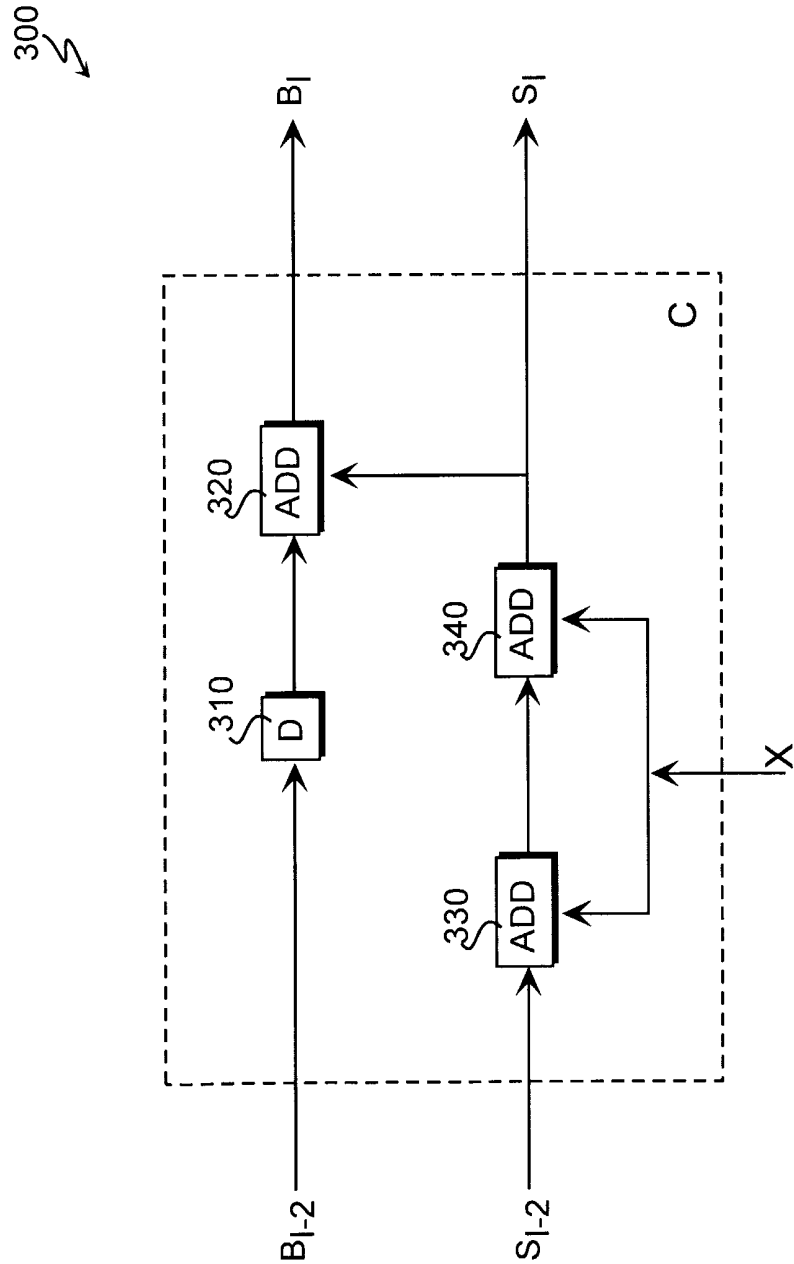


FIG. 3

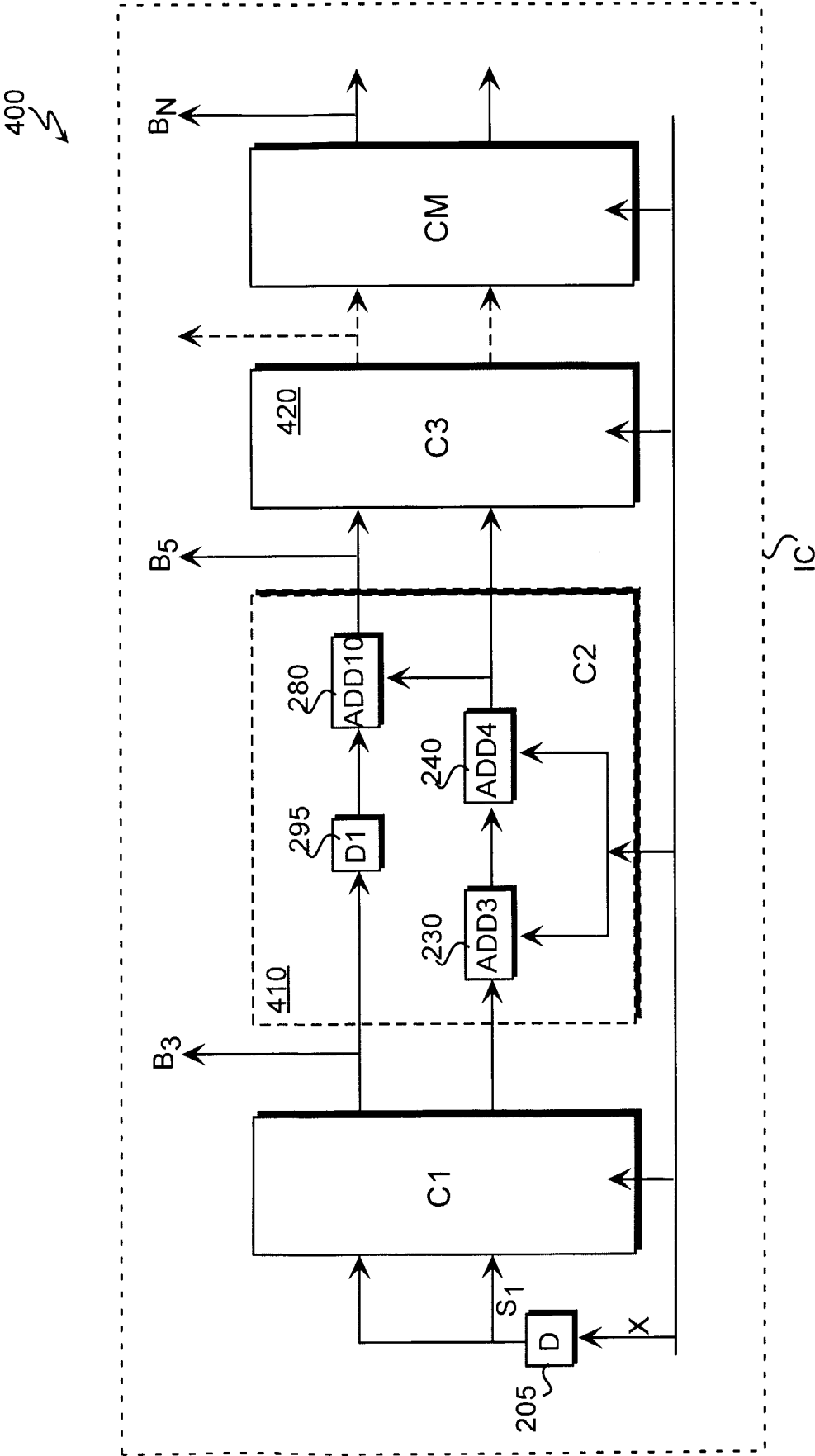


FIG. 4

5/6

$X = S_1$	S_3 (B_3)	S_5 (B_5)	S_7 (B_7)
x_0	x_0	x_0	x_0
x_1	$s_0^3 = x_0 + x_1$ ($b_0^3 = x_0 + s_0^3 = 2x_0 + x_1$)	$x_0 + x_1$ (?)	$x_0 + x_1$ (?)
x_2	$s_1^3 = x_0 + x_1 + x_2$ ($b_1^3 = x_1 + s_1^3 = x_0 + 2x_1 + x_2$)	$s_0^5 = x_0 + x_1 + x_2$ ($b_0^5 = b_0^3 = s_0^5 = 3x_0 + 2x_1 + x_2$)	$x_0 + x_1 + x_2$ (?)
x_3	$s_2^3 = x_1 + x_2 + x_3$ ($b_2^3 = x_2 + s_2^3 = x_1 + 2x_2 + x_3$)	$s_1^5 = x_0 + x_1 + x_2 + x_3$ ($b_1^5 = b_1^3 = s_1^5 = 2x_0 + 3x_1 + 2x_2 + x_3$)	$s_0^7 = x_0 + x_1 + x_2 + x_3$ ($b_0^7 = b_0^5 + s_0^7$)
x_4	$s_3^3 = x_2 + x_3 + x_4$ ($b_3^3 = x_3 + s_3^3 = x_2 + 2x_3 + x_4$)	$x_0 + x_1 + x_2 + x_3 + x_4$ ($b_2^5 = b_2^3 = s_2^5 = x_0 + 2x_1 + 3x_2 + 2x_3 + x_4$)	$s_1^7 = x_0 + x_1 + x_2 + x_3 + x_4$ ($b_1^7 = b_1^5 + s_1^7$)
x_5	$s_4^3 = x_3 + x_4 + x_5$ ($b_4^3 = x_4 + s_4^3 = x_3 + 2x_4 + x_5$)	$x_1 + x_2 + x_3 + x_4 + x_5$ ($b_3^5 = b_3^3 = s_3^5 = x_1 + 2x_2 + 3x_3 + 2x_4 + x_5$)	$s_2^7 = x_0 + x_1 + x_2 + x_3 + x_4 + x_5$ ($b_2^7 = b_2^5 + s_2^7$)
x_6	$s_5^3 = x_4 + x_5 + x_6$ ($b_5^3 = x_5 + s_5^3 = x_4 + 2x_5 + x_6$)	$x_2 + x_3 + x_4 + x_5 + x_6$ ($b_4^5 = b_4^3 = s_4^5 = x_2 + 2x_3 + 3x_4 + 2x_5 + x_6$)	$x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6$ ($b_3^7 = b_3^5 + s_3^7$)
x_7	$s_6^3 = x_5 + x_6 + x_7$ ($b_6^3 = x_6 + s_6^3 = x_5 + 2x_6 + x_7$)	$x_3 + x_4 + x_5 + x_6 + x_7$ ($b_5^5 = b_5^3 = s_5^5 = x_3 + 2x_4 + 3x_5 + 2x_6 + x_7$)	$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$ ($b_4^7 = b_4^5 + s_4^7$)
x_8	$s_7^3 = x_6 + x_7 + x_8$ ($b_7^3 = x_7 + s_7^3 = x_6 + 2x_7 + x_8$)	$x_4 + x_5 + x_6 + x_7 + x_8$ ($b_6^5 = b_6^3 = s_6^5 = x_4 + 2x_5 + 3x_6 + 2x_7 + x_8$)	$x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$ ($b_5^7 = b_5^5 + s_5^7$)
x_9	$s_8^3 = x_7 + x_8 + x_9$ ($b_8^3 = x_8 + s_8^3 = x_7 + 2x_8 + x_9$)	$x_5 + x_6 + x_7 + x_8 + x_9$ ($b_7^5 = b_7^3 = s_7^5 = x_5 + 2x_6 + 3x_7 + 2x_8 + x_9$)	$x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$ ($b_6^7 = b_6^5 + s_6^7$)
x_{10}	$s_9^3 = x_8 + x_9 + x_{10}$ ($b_9^3 = x_9 + s_9^3 = x_8 + 2x_9 + x_{10}$)	$x_6 + x_7 + x_8 + x_9 + x_{10}$ ($b_8^5 = b_8^3 = s_8^5 = x_6 + 2x_7 + 3x_8 + 2x_9 + x_{10}$)	$x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$ ($b_7^7 = b_7^5 + s_7^7$)

FIG. 5

X	S ₂	S ₃	S ₄	S ₅	S ₆	S ₇
x ₀	x ₀	x ₀	x ₀	x ₀	x ₀	x ₀
x ₁	x ₀ + x ₁	x ₀ + x ₁	x ₀ + x ₁	x ₀ + x ₁	x ₀ + x ₁	x ₀ + x ₁
x ₂	x ₁ + x ₂	x ₀ + x ₁ + x ₂	x ₀ + x ₁ + x ₂	x ₀ + x ₁ + x ₂	x ₀ + x ₁ + x ₂	x ₀ + x ₁ + x ₂
x ₃	x ₂ + x ₃	x ₁ + x ₂ + x ₃	x ₀ + ... + x ₃	x ₀ + x ₁ + x ₂ + x ₃	x ₀ + ... + x ₃	x ₀ + x ₁ + x ₂ + x ₃
x ₄	x ₃ + x ₄	x ₂ + x ₃ + x ₄	x ₁ + ... + x ₄	x ₀ + x ₁ + x ₂ + x ₃ + x ₄	x ₀ + ... + x ₄	x ₀ + x ₁ + x ₂ + x ₃ + x ₄
x ₅	x ₄ + x ₅	x ₃ + x ₄ + x ₅	x ₂ + ... + x ₅	x ₁ + x ₂ + x ₃ + x ₄ + x ₅	x ₀ + ... + x ₅	x ₀ + x ₁ + x ₂ + x ₃ + x ₄ + x ₅
x ₆	x ₅ + x ₆	x ₄ + x ₅ + x ₆	x ₃ + ... + x ₆	x ₂ + x ₃ + x ₄ + x ₅ + x ₆	x ₁ + ... + x ₆	x ₀ + x ₁ + x ₂ + x ₃ + x ₄ + x ₅ + x ₆
x ₇	x ₆ + x ₇	x ₅ + x ₆ + x ₇	x ₄ + ... + x ₇	x ₃ + x ₄ + x ₅ + x ₆ + x ₇	x ₂ + ... + x ₇	x ₁ + x ₂ + x ₃ + x ₄ + x ₅ + x ₆ + x ₇
x ₈	x ₇ + x ₈	x ₆ + x ₇ + x ₈	x ₅ + ... + x ₈	x ₄ + x ₅ + x ₆ + x ₇ + x ₈	x ₃ + ... + x ₈	x ₂ + x ₃ + x ₄ + x ₅ + x ₆ + x ₇ + x ₈
x ₉	x ₈ + x ₉	x ₇ + x ₈ + x ₉	x ₆ + ... + x ₉	x ₅ + x ₆ + x ₇ + x ₈ + x ₉	x ₄ + ... + x ₉	x ₃ + x ₄ + x ₅ + x ₆ + x ₇ + x ₈ + x ₉
x ₁₀	x ₉ + x ₁₀	x ₈ + x ₉ + x ₁₀	x ₇ + ... + x ₁₀	x ₆ + x ₇ + x ₈ + x ₉ + x ₁₀	x ₅ + ... + x ₁₀	x ₄ + x ₅ + x ₆ + x ₇ + x ₈ + x ₉ + x ₁₀

FIG. 6