PCT WORLD INTELLECTUAL PROPERTY ORGANIZATION
International Bureau

INTERNATIONAL APPLICATION PUBLISHED UNDER THE PATENT COOPERATION TREATY (PCT)

(51) International Patent Classification 5 :		(11) International Publication Number:	WO 91/03795
G06F 15/68	A1	(43) International Publication Date:	21 March 1991 (21.03.91)

(21) International Application Number: PCT/US90/04727

(22) International Filing Date: 21 August 1990 (21.08.90)

(30) Priority data:
399,134 28 August 1989 (28.08.89) US

(71) Applicant: EASTMAN KODAK COMPANY [US/US]; 343 State Street, Rochester, NY 14650 (US).

(72) Inventors: KAPLAN, Martin, Charles ; 126 Greystone Lane, Rochester, NY 14618 (US). KWON, Heemin ; 49 Hunters Run, Pittsford, NY 14534 (US).

(74) Agent: DUGAS, Edward; 343 State Street, Rochester, NY 14650-2201 (US).

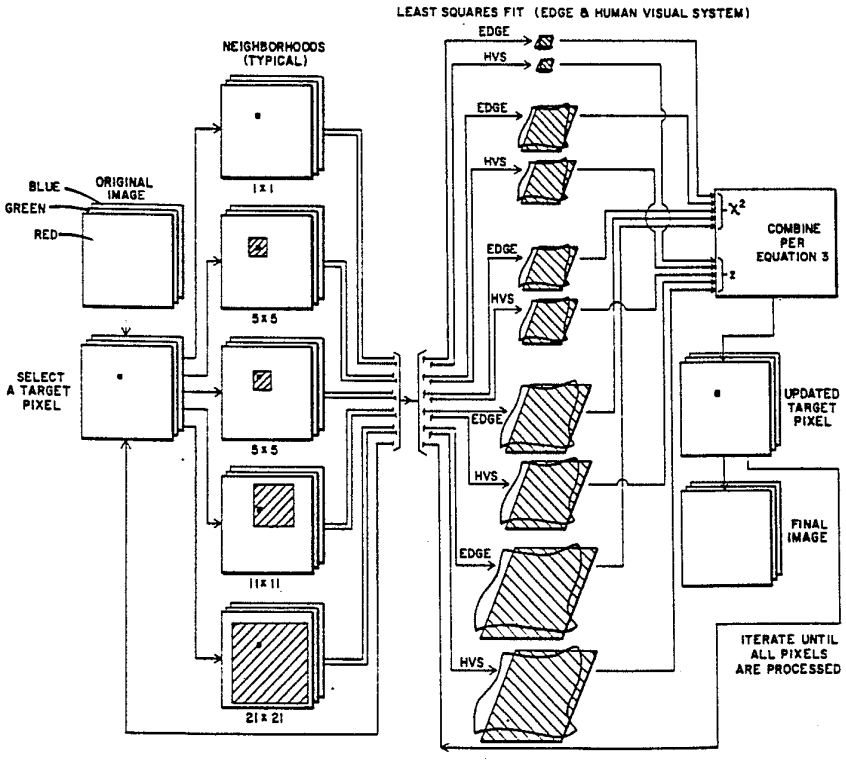
(81) Designated States: AT (European patent), BE (European patent), CH (European patent), DE (European patent)*, DK (European patent), ES (European patent), FR (European patent), GB (European patent), IT (European patent), JP, LU (European patent), NL (European patent), SE (European patent).

Published
With international search report.
Before the expiration of the time limit for amending the claims and to be republished in the event of the receipt of amendments.

(54) Title: DIGITAL IMAGE NOISE REDUCTION OF LUMINANCE AND CHROMINANCE BASED ON OVERLAPPING PLANAR APPROXIMATION

(57) Abstract

The present invention reduces noise in digital photographic images based on the assumption that images may be decomposed into two types of regions, smooth regions and edge regions. Smooth regions are areas of the image lacking any sharp detail, such as blue sky. Edge regions are regions containing sharp detail, such as edges and textured regions (such as grass). The present method reduces noise in the smooth regions by a mathematical blurring technique based on least squares regression. The blurring does not degrade the sharpness of the image, because there are no sharp details in the smooth regions. Edge regions are left undisturbed to maintain sharpness, but the noise is less noticeable in those regions than in the smooth regions. The method operates upon the luminance and chrominance component of a digital image.



* See back of page

DESIGNATIONS OF "DE"

Until further notice, any designation of "DE" in any international application whose international filing date is prior to October 3, 1990, shall have effect in the territory of the Federal Republic of Germany with the exception of the territory of the former German Democratic Republic.

FOR THE PURPOSES OF INFORMATION ONLY

Codes used to identify States party to the PCT on the front pages of pamphlets publishing international applications under the PCT.

AT	Austria	ES	Spain	MC	Monaco
AU	Australia	FI	Finland	MG	Madagascar
BB	Barbados	FR	France	ML	Mali
BE	Belgium	GA	Gabon	MR	Mauritania
BF	Burkina Fasso	GB	United Kingdom	MW	Malawi
BG	Bulgaria	GR	Greece	NL	Netherlands
BJ	Benin	HU	Hungary	NO	Norway
BR	Brazil	IT	Italy	PL	Poland
CA	Canada	JP	Japan	RO	Romania
CF	Central African Republic	KP	Democratic People's Republic of Korea	SD	Sudan
CG	Congo			SE	Sweden
CH	Switzerland	KR	Republic of Korea	SN	Senegal
CM	Cameroon	LI	Liechtenstein	SU	Soviet Union
DE	Germany	LK	Sri Lanka	TD	Chad
DK	Denmark	LU	Luxembourg	TG	Togo
				US	United States of America

DIGITAL IMAGE NOISE REDUCTION OF LUMINANCE AND CHROMINANCE BASED ON OVERLAPPING PLANAR APPROXIMATION

Cross-Reference To Related Application

The present application is related to U.S. Patent Application Serial No. ____,____filed on even date with the present application and entitled "A Computer Based Digital Image Noise Reduction Method Based on Overlapping Planar Approximation" by M. Kaplan et al. The related application bears the Assignee's Docket No. 54,891.

Technical Field of the Invention

The present invention relates to an image data processing method and, more particularly, to a method for reducing noise in digital photographic images based on smoothing, by a blurring technique, the smooth regions of a digital image while maintaining the edge regions undisturbed.

BACKGROUND OF THE INVENTION

To transform a digital image to a finished print of high quality requires that noise components, introduced because of the transformation of the image from an image medium to electrical signals be reduced and/or eliminated such that the noise does not become discernible to the human eye.

One of the methods receiving widespread use in the prior art is related to smoothing the differences between the values of the gray levels of pixels located in neighborhoods. A difficulty

associated with this smoothing process is that it not only removes the noise components, but it also blurs the edges. The edges exist when there is a transition region such as a region containing sharp detail, for example, grass which may define or outline a neighboring region which may be either smooth or sharply detailed.

A patent of interest for its teaching of noise reduction in a data processed image is U.S. Patent No. 4,734,770 entitled "Image Data Processing Method and Device Therefor" by I. Matsuba. The method of that patent treats the first image data set, consisting of a plurality of pixels having values corresponding to their gray level some of which contain noise components, by transforming them into second image data to reduce the noise components appearing in the second image data. The method operates by selecting pixels to be treated and obtaining proposed values (candidate values) which are given to the object pixels based on a stated relationship. A probability function is used to determine whether a pixel in question should be raised to the candidate value or should be maintained at its present value. The probability formula includes the image energy component as one of its terms. A recognition is made for edges of patterns based on energy level comparisons, wherein a high energy level difference between a central and an adjoining pixel would have a high probability of defining an edge.

In the present invention a solution to the edge blurring, when performing the smoothing operation, approaches the problem by determining edge regions and by leaving those regions undisturbed or unoperated upon and by applying a smoothing function to the smooth regions so as to reduce noise in those areas. The method has two responsibilities, one to identify whether a pixel belongs to an edge or to a

smooth region and, two, to compute a smoothing code value for the pixel if it belongs to a smooth region. Within this application, these two operations are called "segmentation" and "smoothing," respectively. Both of the aforementioned tasks are accomplished by a least squares fit of a plane to the image, in a neighborhood around the pixel of interest. If the goodness of the fit is small, then in that neighborhood the image is well approximated by a plane, and thus must be a smooth region. Further, the fit provides a smooth estimate for the code value at the pixel of interest which is the value of the fitted plane at the pixel of interest.

SUMMARY OF THE INVENTION

The present invention method reduces noise (graininess) in digital photographic images by operating on the luminance component of an image and the chrominance components. Generally speaking, this is accomplished by a mathematical generalization from a scalar field to a vector field, with special consideration being given to the characteristics of the human visual system.

One preferred method of the present invention for reducing digital image noise is comprised of the steps of forming a signal matrix of pixel values for each primary color of an original image; determining if a pixel belongs to an edge region and if not, computing a smoothing code value from the pixel so as to reduce image noise.

To determine if a pixel of interest belongs to an edge region a neighborhood is selected around the pixel and a least squares fit is performed to provide a goodness of fit factor. Next, another least squares fitting plane operation is performed using a matrix to provide an estimated value of the pixel of

interest. The above is continued for all neighborhoods around the pixel. All of the estimated values are combined with weights based on the goodness of fit to provide a pixel value. The operation is continued for all pixels in the original image and the pixel values derived therefrom are used to form an output image with reduced digital noise.

The image is a mathematical field because it is a function of spatial variables, namely the x and y coordinates of the pixel position in the image. The luminance image is a scalar field, because it is a function having a single value (luminance) at each spatial position (pixel). The full 3-color image (luminance and chrominance) is a vector field, because it has 3 values (red, green, blue) at each spatial position. To provide a mathematical generalization from a scalar field to a vector field, the scalar equations need to be rewritten in the proper form for a vector field.

From the foregoing, it can be seen that it is a primary object of the present invention to provide an improved method for removing noise components from the luminance and chrominance of digital image signals without degrading edge values.

It is a further object of the present invention to provide an improved smoothing of the pixel values of a digital image in smooth regions while not disturbing edge regions.

It is an additional object of the present invention to provide a method for reducing noise in the smooth regions of a digital image by a blurring technique based on a least squares regression applied to luminance and chrominance signals.

These and other objects of the present invention will become more apparent from the following description when taken in conjunction with the

attached drawings wherein like characters indicate like parts and which drawings form a part of the present description.

BRIEF DESCRIPTION OF THE DRAWINGS

Figure 1 is a line diagram illustrating the smoothing of a near edge pixel by a convolution technique.

Figure 2 is a line diagram illustrating the smoothing of a near edge pixel by a least squares technique.

Figure 3 illustrates a square neighborhood of pixel elements with the target pixel being near an edge.

Figure 4 illustrates a matrix of pixels with the smoothed pixels being distanced from an edge on both sides so as to create a channel of unsmoothed pixels surrounding each edge.

Figure 5 illustrates four different blocks of (5 x 5) neighboring pixels about one target pixel (four quadrant neighborhoods).

Figure 6 illustrates a matrix of pixel elements representing selected neighborhoods for two target pixels on opposite sides of an edge.

Figure 7 represents an overlapping of corresponding neighborhoods for two adjacent target pixels.

Figures 8A and 8B illustrate a flow chart depicting the method steps of the present invention.

Figure 9 is a verbal flow chart verbally depicting the method steps of the flow chart of Figures 8A and 8B.

DESCRIPTION OF THE PREFERRED EMBODIMENT

As previously mentioned, the method of the present invention has two major chores. One is to identify whether a target pixel belongs to an edge or a smooth region and the other is to compute a smoothed code value for the pixel if it belongs to the smooth region. The smoothing process can be done by convolution with a smoothing kernel, as is common practice in digital signal processing. But doing the smoothing by least squares provides an advantage. Referring to Figure 1, wherein the smoothing of a pixel by a convolution is shown requiring the kernel "neighborhood" to be symmetric and centered on the target pixel, to provide an unbiased estimate for the smoothed target pixel. When the target pixel approaches an edge, the neighborhood surrounding the target pixel will project pixel members onto and past the edge surface. A least squares method has no such requirement.

- Let $N_{kl,\alpha}$ be a neighborhood of type α, centered at pixel location (k,l).

- Let the 3-component vector $\vec{Z}_{ij}$ be the color values (red, green, blue vector) and Σ_{ij} be the 3 x 3 noise covariance matrix at pixel (i,j).

- Let $X^2_{kl,\alpha}$ be the goodness of fit for the linear least squares regression of a hyper-plane on the neighborhood $N_{kl,\alpha}$ and let $\vec{Z}^{kl,\alpha}_{ij}$ be the estimate of the regression at pixel location (i,j), where pixel (i,j) is some pixel in the neighborhood $N_{kl,\alpha}$. A multi-variate linear least squares

regression (which is a standard mathematical technique) of a hyper-plane finds the vector coefficients $\vec{a}$, $\vec{b}$, $\vec{c}$ which minimize the fitting error $X^2_{kl,\alpha}$ defined by:

$$\chi^2_{kl,\alpha} = \sum_{(i,j)\in N_{kl,\alpha}} \left[\vec{z}_{ij} - \left(\vec{a}_{kl} + \vec{b}_{kl}\, i + \vec{c}_{kl}\, j\right)\right]^{\dagger} \Sigma_{ij}^{-1} \left[\vec{z}_{ij} - \left(\vec{a}_{kl} + \vec{b}_{kl}\, i + \vec{c}_{kl}\, j\right)\right] \tag{1}$$

and the estimated code value at pixel (i,j) is:

$$\vec{z}_{ij}^{\,kl,\alpha} = \vec{a}_{kl} + \vec{b}_{kl}\, i + \vec{c}_{kl}\, j \tag{2}$$

- Compute the noise reduced code value at pixel (i,j) as:

$$\vec{z}_{ij}' = \frac{\sum_{kl,\alpha} w\left(\alpha, \chi^2_{kl,\alpha}\right) \vec{z}_{ij}^{\,kl,\alpha}}{\sum_{kl,\alpha} w\left(\alpha, \chi^2_{kl,\alpha}\right)} \tag{3}$$

where the sums are all kl,α such that pixel $(i,j) \in N_{kl,\alpha}$.

- Apply the weighting function,

$$w\left(\alpha, \chi^2_{kl,\alpha}\right) = \frac{W_\alpha}{n_\alpha} \epsilon^{-\frac{\chi^2_{kl,\alpha}}{\nu_\alpha} r_\alpha} \tag{4}$$

where W_α is an empirically chosen weight for each neighborhood type, n_α is the number of pixels in the neighborhoods ν_α is the degrees of freedom for the least squares fit (which is the number of pixels in the neighborhood α minues 9, because the vector coefficients $\vec{a}$, $\vec{b}$, $\vec{c}$ have a total of 9 components) and r_α is an empirically chosen rolloff parameter.

To further generalize the method, equations 1-4 are rewritten. Two least squares fits must be performed, once for segmentation and again for smoothing, so we must minimize two X^2's.

$$\chi^2_{Ekl,\alpha} = \sum_{(i,j)\in N_{kl,\alpha}} \left[\vec{z}_{ij} - \left(\vec{a}^E_{kl} + \vec{b}^E_{kl}\, i + \vec{c}^E_{kl}\, j\right)\right]^{\dagger} M^E_{ij} \left[\vec{z}_{ij} - \left(\vec{a}^E_{kl} + \vec{b}^E_{kl}\, i + \vec{c}^E_{kl}\, j\right)\right] \tag{5}$$

$$\chi^2_{Hkl,\alpha} = \sum_{(i,j)\in N_{kl,\alpha}} \left[\vec{z}_{ij} - \left(\vec{a}^H_{kl} + \vec{b}^H_{kl}\, i + \vec{c}^H_{kl}\, j\right)\right]^{\dagger} M^H_{ij} \left[\vec{z}_{ij} - \left(\vec{a}^H_{kl} + \vec{b}^H_{kl}\, i + \vec{c}^H_{kl}\, j\right)\right] \tag{6}$$

where the matrices M^E_{ij}, M^H_{ij} are related to the noise covariance matrix as explained later. (Note: in the notation, E stands for "edge," and H stands for "human visual system," representing the segmentation and smoothing steps, respectively.) The weights are computed based on the segmentation fit,

$$w\left(\alpha, \chi^2_{Ekl,\alpha}\right) = \frac{W_\alpha}{n_\alpha} e^{-\frac{\chi^2_{Ekl,\alpha}}{\nu_\alpha} r_\alpha} \tag{7}$$

and the least squares estimated code values are based on the smoothing fit,

$$\vec{z}^{\,kl,\alpha}_{ij} = \vec{a}^H_{kl} + \vec{b}^H_{kl}\, i + \vec{c}^H_{kl}\, j \tag{8}$$

The final result is the noise-reduced image code value,

$$\vec{z}'_{ij} = \frac{\sum_{kl,\alpha} w\left(\alpha, \chi^2_{Ekl,\alpha}\right) \vec{z}^{\,kl,\alpha}_{ij}}{\sum_{kl,\alpha} w\left(\alpha, \chi^2_{Ekl,\alpha}\right)} \tag{9}$$

The most likely application of this further generalization would be to define the two weighting matrices as follows:

$$M_{ij}^{E} = \left(U_E^{\dagger} D_E U_E\right)^{\dagger} \Sigma_{ij}^{-1} \left(U_E^{\dagger} D_E U_E\right) \tag{10}$$

$$M_{ij}^{H} = \left(U_H^{\dagger} D_H U_H\right)^{\dagger} \Sigma_{ij}^{-1} \left(U_H^{\dagger} D_H U_H\right) \tag{11}$$

where U_E, U_H are unitary matrices, D_E, D_H are diagonal matrices, and Σ_{ij} is the noise covariance matrix. The unitary matrices U_E, U_H perform a rotation in vector space, and the diagonal matrices D_E, D_H provide different weights along the different axes in the rotated vector space. Thus, a likely choice would be to select a U_H which rotates into a luminance type space (placing one vector space axis along luminance Y, and another vector space axis along the more important of the two chrominance directions), and D_H which weights the luminance most heavily, and weights the less important of the two chrominance components least. U_E, D_E are selected to rotate into a space which is good for locating edges, with appropriate weights, which may be a luminance space, a principal components space, or some other space. Note that in a principal component space, by definition U diagonalizes the covariance matrix Σ_{ij}, which simplifies the computation. Also, if U_E, D_E are chosen identical to U_H, D_H, then equations 5 and 6 become identical and the computational burden is reduced. If D_E, D_H are both the identity matrix, then equations 5–10 reduce back to equations 1–4, regardless of the values of

U_E, U_H. Thus, rotating the vector space accomplishes nothing, unless the different axes are weighted differently.

Figure 2 illustrates the advantage of utilizing a least squares technique. The target pixel can be located with an off-center neighborhood, thereby permitting the target pixel to approach closer to an edge region. As Figure 1 shows, a convolution smoothing tends to blur edges, because they "leak" into neighboring pixels. A least squares method can avoid blurring edges by choosing neighborhoods which avoid the edges.

Referring to Figure 3, an algorithm based on the above takes a 5 x 5 neighborhood centered on each target pixel and performs a least squares fit and, if the value X^2 (X^2 is a sum of squared errors normalized for the standard deviation at each pixel) is below a threshold value, the algorithm replaces the target pixel's code value by the least squares estimate. This approach is acceptable in the interior of a smooth region but fails for target pixels which are near the border of a smooth region. This failure is caused by the 5 x 5 neighborhood overlapping the edge or border of the smooth region, which in turn results in a large X^2 that exceeds the threshold value and therefore the pixel of interest is not replaced with the pixel code value or in other words is not smoothed. The result of continuing that process around an edge is shown in Figure 4 such that a channel or roadway of unsmoothed pixels around an edge line will result.

An improvement is achieved by considering several neighborhoods for each target pixel and selecting one of the neighborhoods which results in the most preferred fit. For example, Figures 5A through 5D illustrate a target pixel being a corner pixel for four 5 x 5 neighborhoods. These

neighborhoods can be called quadrant neighborhoods and are used to determine the value of X^2 which has the smallest value. This value can then be compared against the threshold value to determine if the pixel value should be replaced with a code value. If none of the neighborhood values is below the threshold then it is assumed that the pixel is truly an edge pixel and a smoothing is not applied to it. If at least one neighborhood value is below the threshold, then the neighborhood with the lowest value is used to compute the smoothed value.

In Figure 6, the selected target pixel neighborhoods, for two target pixels, are shown positioned on opposite sides of an edge. The result of this is that for pixels near an edge, the pixels will be smoothed by incorporating them into neighborhoods entirely on the correct side of the edge. This method provides proper noise reduction right up to the edges (as opposed to leaving bands of noise surrounding the edges), but it also produces an artifact which makes the picture look "blocky". The artifact is caused because the algorithm has a tendency to use one type of quadrant neighborhood "such as the upper left" for several adjacent target pixels, and then to switch to another type of quadrant neighborhood (such as a lower right) for the next several target pixels, and so on.

To eliminate the "blocky" look, all of the neighborhoods containing the target pixel were used. For 5 x 5 neighborhoods, for each target pixel there are 25 neighborhoods containing the target pixel. This is intended to provide a more continuous behavior, to reduce blocking, because of the larger number of neighborhoods used.

Unfortunately, the "blocking" persists even after using all neighborhoods. This was attributed to the fact that for two adjacent target pixels, there

are two sets of 25 neighborhoods (one set for each target pixel), and each neighborhood in one set contains mostly the same pixels as several neighborhoods in the other set (for example see Figure 7). Thus the two sets of least squares fits produce similar results, and when the algorithm selects the neighborhood with the lowest X^2 for each target pixel, it tends to pick nearly identical neighborhoods from each of the two sets of neighborhoods. As the algorithm scans the picture selecting new target pixels, it continues to select similar neighborhoods. Occasionally the selection of neighborhoods flips to a radically different choice, and then this new selection persists over may target pixels. A noticeable blocking artifact is produced where the selection flips. This is basically the same blocking artifact as described above for quadrant neighborhoods.

Since the blocking artifact appears to be caused by the instability in the selection of the single best neighborhood for each target pixel, the algorithm was modified to avoid the need to make a selection. Instead of selecting one neighborhood, the algorithm uses all neighborhoods of a target pixel, weighting them according to "goodness," applying large weight for small X^2 (good fits) and small weight for large X^2 (poor fits). This eliminates the blocking artifact.

The weighting function $w(\alpha,X^2)$ (equation 4) was originally based on the X^2 probability distribution, which is statistically appropriate when the model surface (a plane) correctly describes the data surface (image), with the fitting error X^2 resulting only from noise and errors in choosing the fitting parameters. However, analysis showed that the fitting errors X^2 were statistically distributed very differently from the X^2 distribution, even in

the smooth regions. This indicates that the model surface (plane) is not an adequate description of the data surface (image). This is not surprising, since the data surface has a very curved and irregular shape, even within the smooth regions. To deal with this difficulty, the ad hoc weighting function of equation 4 was substituted for the original X^2 distribution based weighting function.

Neighborhood size selection is a remaining factor. The larger the neighborhood, the larger the noise reduction, because many pixels contribute to the smoothing. However, a large neighborhood size is unable to deal with details in the picture smaller than the neighborhood size. This is because for small details there is no neighborhood which fits entirely within the detail, and thus the algorithm considers the detail to be an edge region, and provides no smoothing. This difficulty is dealt with by using neighborhoods of several sizes. The algorithm weights different sized neighborhoods differently, with the result being that large smooth regions of the picture are smoothed primarily by large neighborhoods. Smaller smooth regions are smoothed primarily by smaller neighborhoods, because the large neighborhoods don't fit within the region, thus have large X^2 and small weight.

Finally, it should be noted that the algorithm includes neighborhoods of size 1 x 1. Although this can be mathematically described simply as 1 x 1 neighborhoods, the intent and result are somewhat different than for larger neighborhoods — the 1 x 1 neighborhoods provide the mechanism by which edge regions are permitted to remain unsmoothed. In an edge region, all neighborhoods larger than 1 x 1 yield large X^2 when the least squares fit is performed. Strictly speaking, it is not possible to fit a 1 x 1 neighborhood with a plane, because there

is only one 3-component data point and nine fitting parameters (making the fit mathematically underdetermined). Instead, a perfect fit is assumed (i.e. the fit estimate of the code value at the single pixel is exactly equal to the original code value, and $X^2 = 0$). The result is that in edge regions, only the 1 x 1 neighborhoods have small X^2 and are heavily weighted, and they pass the original picture information unsmoothed.

Referring now to Figures 8A, 8B and 9 in conjunction, from a multi-band original digital image (Blue, Green and Red) a target pixel is selected. For each target pixel, a neighborhood is selected. The total number of neighborhoods for the preferred embodiment is 588. For each neighborhood selection the least squares best fitting plane is computed using the "Edge" matrix (equation 5) to yield the quality of fit X^2. Additionally, for each neighborhood selection the least squares best fitting plan is computed using the "Human Visual System" matrix (equation 6) to yield the estimated value (z) at the target pixel. The process is iterated for all 588 selected neighborhoods. All 588 estimates (z) are combined with weights based on the quality of fit (X^2), according to equation 3 to yield the output image pixel value at the target position. This process is iterated until all pixels in the original image have been processed.

This approach generalizes the approach of Assignee's Docket No. 54,891 as follows.

First, a full noise covariance matrix (Σ_{ij}) is used, and thus it takes into account correlations of the noise in the different channels. Second, the weights $w(\alpha,X^2)$ are computed for all 3 channels together, not independently, and the same weight is used in each of the 3 channels. Computing

the weights by combining the channels enhances the ability to recognize edges and textures. Third, it takes into consideration the human visual system.

It is desirable to further generalize the equations, to take into consideration the characteristics of the human visual system. The present method is a two step method. First, it segments the image by computing the weights (equations 5,7). Second, it smooths the image code values (equations 6,8,9). There is a significant difference between these two steps that must be considered. The smoothing step computes new image code values, which will be directly observed by the human eye, so special attention is paid to the luminance channel because of its greater significance to the human visual system.

On the other hand, the segmentation step represents an effort to locate edges and textures in the image, and does not compute new image code values, and thus luminance is less relevant in the segmentation step. If luminance is significant in the segmentation step, it is for a more subtle reason. The human visual system has evolved into a highly efficient system for understanding the visual world, and thus perhaps its emphasis on the luminance channel is a commentary that when trying to understand a scene, luminance is given extra consideration.

While there has been shown what is considered to be the preferred embodiment of the invention, it will be manifest that many changes and modifications may be made therein without departing from the essential spirit of the invention. It is intended, therefore, in the annexed claims, to cover all such changes and modifications that may fall within the true scope of the invention.

CLAIMS:

1. An overlapping planar approximation method for reducing digital image noise in multi-band digital images comprising the steps of:

(a) selecting a target pixel (which is a vector with a number of components equal to the number of bands in the image) from a set of pixels representing an image of interest;

(b) selecting a neighborhood of the target pixel;

(c) computing a least squares best fit plane of pixel vectors for the selected neighborhood of step b to provide a goodness of fit (X^2) and an estimated vector value of the target pixel;

(d) iterating steps b and c for various types (sizes) and positions of neighborhoods of the target pixel;

(e) summing all estimated values of the target pixel with weights based on the goodness of fit (X^2) to provide an output image pixel for the target position; and

(f) iterating steps a through e for all pixels from the image of interest, so as to form a set of reduced noise image pixels.

2. The overlapping planar approximation method, according to Claim 1, wherein the goodness of fit of step c is defined as:

$$\chi^2_{kl,\alpha} = \sum_{(i,j)\in N_{kl,\alpha}} \left[\vec{z}_{ij} - \left(\vec{a}_{kl} + \vec{b}_{kl}\, i + \vec{c}_{kl}\, j\right)\right]^{\dagger} \Sigma_{ij}^{-1} \left[\vec{z}_{ij} - \left(\vec{a}_{kl} + \vec{b}_{kl}\, i + \vec{c}_{kl}\, j\right)\right] \qquad (1)$$

and the estimated code value vector of a target pixel positioned at (i,j) is:

$$\vec{z}_{ij}^{\,kl,\alpha} = \vec{a}_{kl} + \vec{b}_{kl}\, i + \vec{c}_{kl}\, j \qquad (2)$$

and the computed noise reduced code value vector of the target pixel at position (i,j) is:

$$\vec{z}_{ij}' = \frac{\sum_{kl,\alpha} w\left(\alpha, \chi^2_{kl,\alpha}\right) \vec{z}_{ij}^{\,kl,\alpha}}{\sum_{kl,\alpha} w\left(\alpha, \chi^2_{kl,\alpha}\right)} \tag{3}$$

where the sums are over all kl,α such that pixel $(i,j) \in N_{kl,\alpha}$ and where the weighting function is:

$$w\left(\alpha, \chi^2_{kl,\alpha}\right) = \frac{W_\alpha}{n_\alpha} \epsilon^{-\frac{\chi^2_{kl,\alpha}}{\nu_\alpha} r_\alpha} \tag{4}$$

wherein $N_{kl,\alpha}$ is a neighborhood of type α centered at pixel (k,l), W_α is an empirically chosen weight for each neighborhood type, n_α is the number of pixels in the neighborhood, ν_α is the degrees of freedom for the lease squares fit, r_α is an empirically chosen rolloff parameter of order unity, $\vec{z}_{ij}$ is a code value vector (having one component per band of the multi-band image), Σ_{ij} is the noise covariance matrix at pixel (i,j), $\chi^2_{kl,\alpha}$ is the goodness of fit for the linear least squares regression of a plane on neighborhood $N_{kl,\alpha}$, and $\vec{z}_{ij}^{\,kl,\alpha}$ is the estimate of the regression at pixel location (i,j), where pixel (i,j) is some pixel in the neighborhood $N_{kl,\alpha}$.

3. The overlapping planar approximation method, according to Claim 2, for the case of the noise covariance matrix Σ_{ij} being diagonal to thereby partially decouple the equations for the various bands.

4. The overlapping planar approximation method, according to Claim 2, wherein two least squares fits are substituted in place of the single least squares fit by replacing equations 1 through 4 by the following equations:

$$\chi^2_{Ekl,\alpha} = \sum_{(i,j)\in N_{kl,\alpha}} \left[\vec{z}_{ij} - \left(\vec{a}^E_{kl} + \vec{b}^E_{kl}\, i + \vec{c}^E_{kl}\, j\right)\right]^{\dagger} M^E_{ij} \left[\vec{z}_{ij} - \left(\vec{a}^E_{kl} + \vec{b}^E_{kl}\, i + \vec{c}^E_{kl}\, j\right)\right] \tag{5}$$

$$\chi^2_{Hkl,\alpha} = \sum_{(i,j)\in N_{kl,\alpha}} \left[\vec{z}_{ij} - \left(\vec{a}^H_{kl} + \vec{b}^H_{kl}\, i + \vec{c}^H_{kl}\, j\right)\right]^{\dagger} M^H_{ij} \left[\vec{z}_{ij} - \left(\vec{a}^H_{kl} + \vec{b}^H_{kl}\, i + \vec{c}^H_{kl}\, j\right)\right] \tag{6}$$

and the estimated code value of a target pixel positioned at (i,j) is:

$$\vec{z}^{\,kl,\alpha}_{ij} = \vec{a}^H_{kl} + \vec{b}^H_{kl}\, i + \vec{c}^H_{kl}\, j \tag{7}$$

and the computed noise reduced code value of the target pixel at position (i,j) is:

$$\vec{z}'_{ij} = \frac{\sum_{kl,\alpha} w\left(\alpha, \chi^2_{Ekl,\alpha}\right) \vec{z}^{\,kl,\alpha}_{ij}}{\sum_{kl,\alpha} w\left(\alpha, \chi^2_{Ekl,\alpha}\right)} \tag{8}$$

where the sums are over all kl,α such that pixel $(i,j) \in N_{kl,\alpha}$ and where the weighting function is:

$$w\left(\alpha, \chi^2_{Ekl,\alpha}\right) = \frac{W_\alpha}{n_\alpha} e^{-\frac{\chi^2_{Ekl,\alpha}}{\nu_\alpha} r_\alpha} \tag{9}$$

where M^E_{ij} and M^H_{ij} are arbitrary matrices, chosen to optimize segmentation and smoothing, respectively.

5. The overlapping planar approximation method, according to Claim 4, wherein the matrices M_{ij}^{E} and M_{ij}^{H} are defined as follows:

$$M_{ij}^{E} = \left(U_E^{\dagger} D_E U_E\right)^{\dagger} \Sigma_{ij}^{-1} \left(U_E^{\dagger} D_E U_E\right) \tag{10}$$

$$M_{ij}^{H} = \left(U_H^{\dagger} D_H U_H\right)^{\dagger} \Sigma_{ij}^{-1} \left(U_H^{\dagger} D_H U_H\right) \tag{11}$$

where U_E, U_H are unitary matrices, D_E, D_H are diagonal matrices, and Σ_{ij} is the noise covariance matrix such that the unitary matrices provide rotation of the vector space axes and the diagonal matrices attach different weight to the different axes in the rotated vector spaces.

6. The overlapping planar approximation, method, according to Claim 5, with the unitary matrices chosen to rotate into a luminance/chrominance vector space, and the diagonal matrices chosen to assign greater weight to the luminance axis in the rotated spaces.

7. The overlapping planar approximation method, according to Claim 1, wherein said goodness of fit (X^2) is used as a measure of edge detection, texture detection, and image segmentation.

8. The overlapping planar approximation method, according to Claim 4, wherein said goodness of fit (X^2) is used as a measure of edge detection, texture detection, and image segmentation.

9. The overlapping planar approximation method, according to Claim 5, wherein said goodness of fit (X^2) is used as a measure of edge detection, texture detection, and image segmentation.

10. A method of image smoothing and noise reduction by least squares fits, comprising the steps of:

(a) selecting a target pixel from a set of pixels representing a multi-band image of interest;

(b) selecting a neighborhood containing the target pixel;

(c) computing the least squares best fit plane for the selected neighborhood of step b; and

(d) using the least squares fit estimated value of step c as the smoothed output value for the target pixel.

11. The method of image smoothing and noise reduction, according to Claim 10, wherein two least squares fits are substituted in place of the least squares best fit of step c by the use of the following equations:

$$\chi^2_{Ekl,\alpha} = \sum_{(i,j)\in N_{kl,\alpha}} \left[\vec{z}_{ij} - \left(\vec{a}^E_{kl} + \vec{b}^E_{kl}\, i + \vec{c}^E_{kl}\, j\right)\right]^{\dagger} M^E_{ij} \left[\vec{z}_{ij} - \left(\vec{a}^E_{kl} + \vec{b}^E_{kl}\, i + \vec{c}^E_{kl}\, j\right)\right] \tag{5}$$

$$\chi^2_{Hkl,\alpha} = \sum_{(i,j)\in N_{kl,\alpha}} \left[\vec{z}_{ij} - \left(\vec{a}^H_{kl} + \vec{b}^H_{kl}\, i + \vec{c}^H_{kl}\, j\right)\right]^{\dagger} M^H_{ij} \left[\vec{z}_{ij} - \left(\vec{a}^H_{kl} + \vec{b}^H_{kl}\, i + \vec{c}^H_{kl}\, j\right)\right] \tag{6}$$

and the estimated code value of a target pixel positioned at (i,j) is:

$$\vec{z}^{\,kl,\alpha}_{ij} = \vec{a}^H_{kl} + \vec{b}^H_{kl}\, i + \vec{c}^H_{kl}\, j \tag{7}$$

and the computed noise reduced code value of the target pixel at position (i,j) is:

$$\vec{z}'_{ij} = \frac{\sum_{kl,\alpha} w\left(\alpha, \chi^2_{Ekl,\alpha}\right) \vec{z}^{\,kl,\alpha}_{ij}}{\sum_{kl,\alpha} w\left(\alpha, \chi^2_{Ekl,\alpha}\right)} \tag{8}$$

where the sums are over all kl,α such that pixel $(i,j) \in N_{kl,\alpha}$ and where the weighting function is:

$$w\left(\alpha, \chi^2_{Ekl,\alpha}\right) = \frac{W_\alpha}{n_\alpha} e^{-\frac{\chi^2_{Ekl,\alpha}}{\nu_\alpha} r_\alpha} \tag{9}$$

where M^E_{ij} and M^H_{ij} are arbitrary matrices, chosen to optimize segmentation and smoothing, respectively.

12. The method of image smoothing and noise reduction, according to Claim 11, wherein the matrices M^E_{ij} and M^H_{ij} are defined as follows:

$$M^E_{ij} = \left(U^\dagger_E D_E U_E\right)^\dagger \Sigma^{-1}_{ij} \left(U^\dagger_E D_E U_E\right) \tag{10}$$

$$M^H_{ij} = \left(U^\dagger_H D_H U_H\right)^\dagger \Sigma^{-1}_{ij} \left(U^\dagger_H D_H U_H\right) \tag{11}$$

where U_E, U_H are unitary matrices, D_E, D_H are diagonal matrices, and Σ_{ij} is the noise covariance matrix such that the unitary matrices provide rotation of the vector space axes and the diagonal matrices attach different weight to the different axes in the rotated vector spaces.

13. The overlapping planar approximation method of Claim 1 applied to multi-channel digital signals other than multi-band digital images.

14. The overlapping planar approximation method, according to Claim 2, wherein the weighting function is selected to provide the desired result.

15. The overlapping planar approximation method, according to Claim 2, wherein the selected neighborhood sizes of the target pixel are 21 x 21,

11 x 11, 5 x 5, 1 x 1, the empirical weights are:
$W_{21\times21} = 64$, $W_{11\times11} = 16$, $W_{5\times5} = 16$, $W_{1\times1} = 1$,
and the rolloff parameters are:
$r_{21\times21} = r_{11\times11} = r_{5\times5} = r_{1\times1} = 1$.

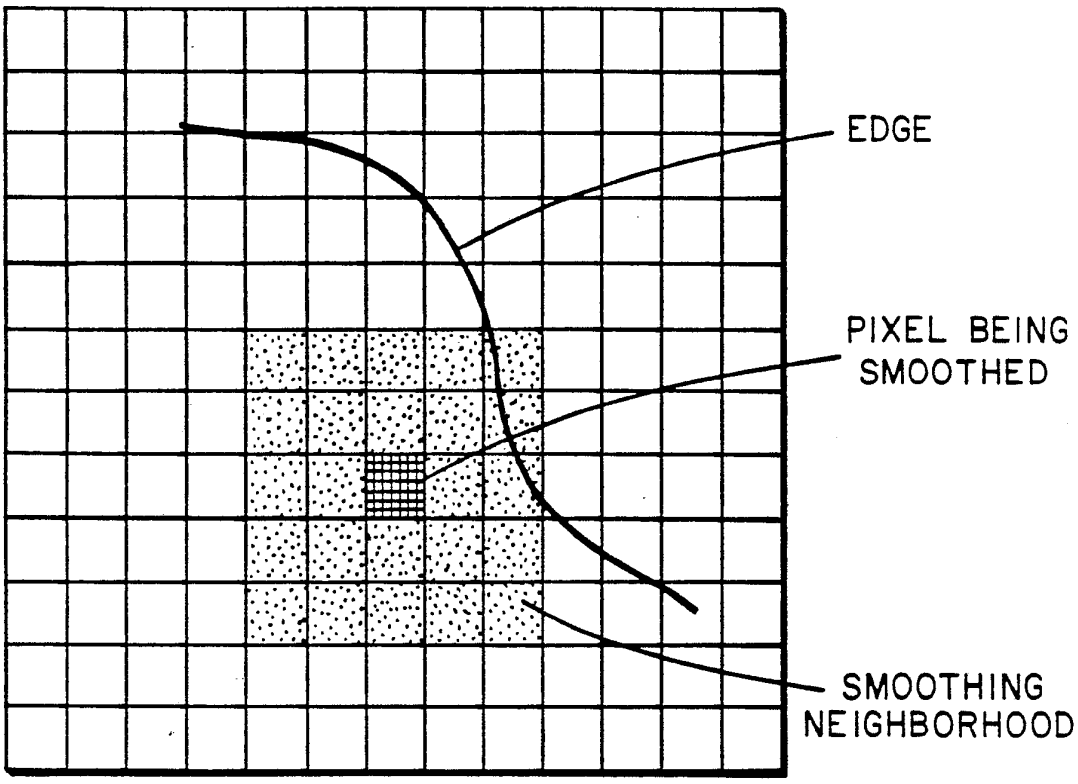

EDGE
PIXEL BEING SMOOTHED
SMOOTHING NEIGHBORHOOD

FIG. 1

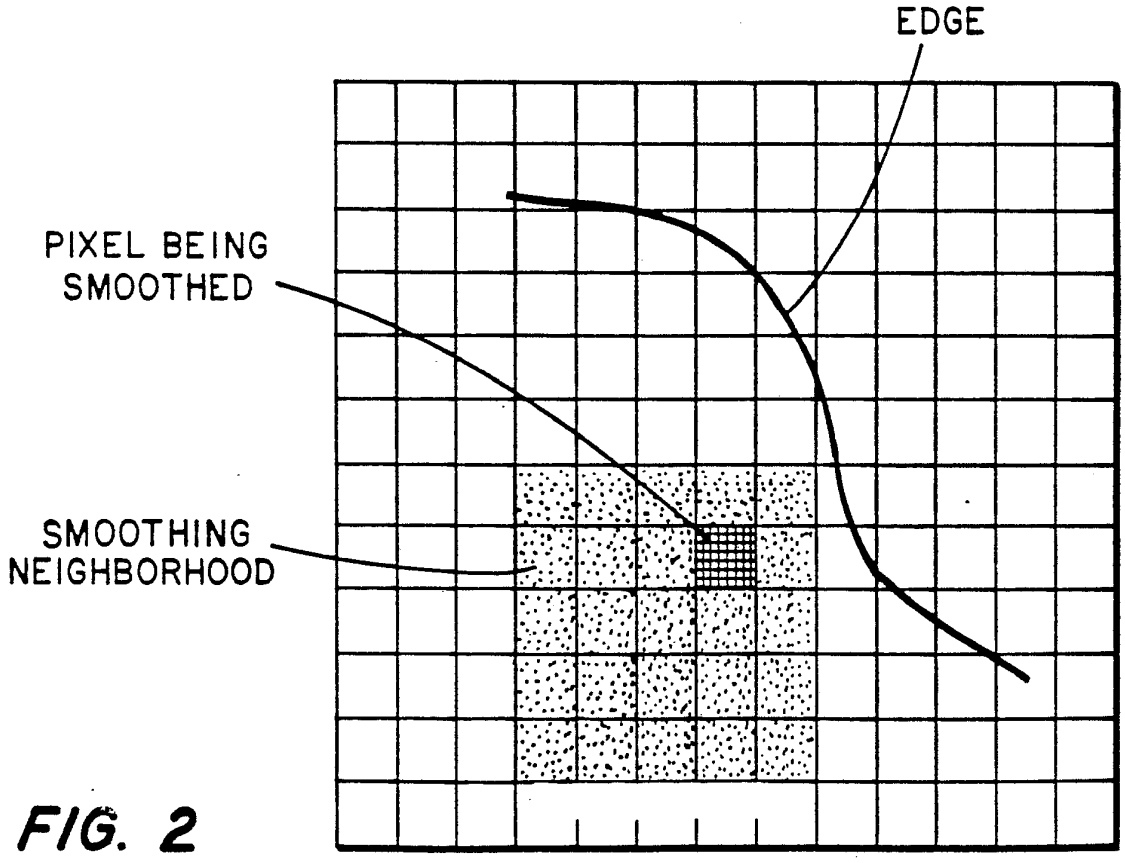

EDGE
PIXEL BEING SMOOTHED
SMOOTHING NEIGHBORHOOD

FIG. 2

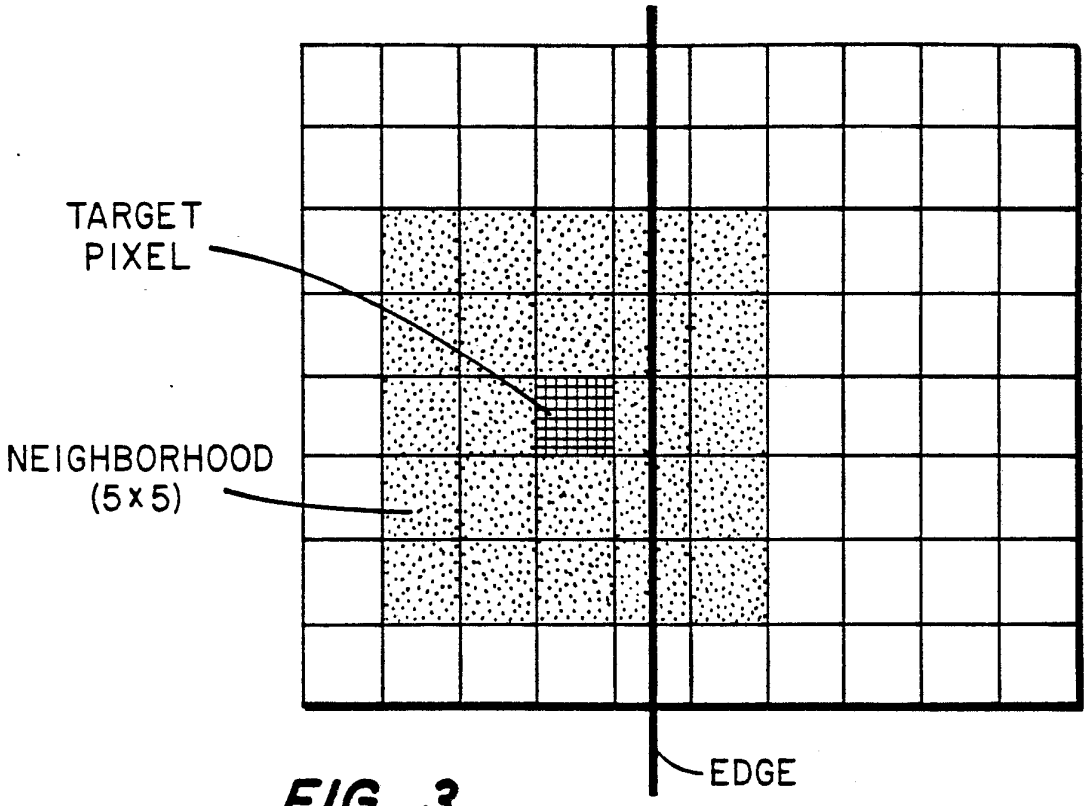
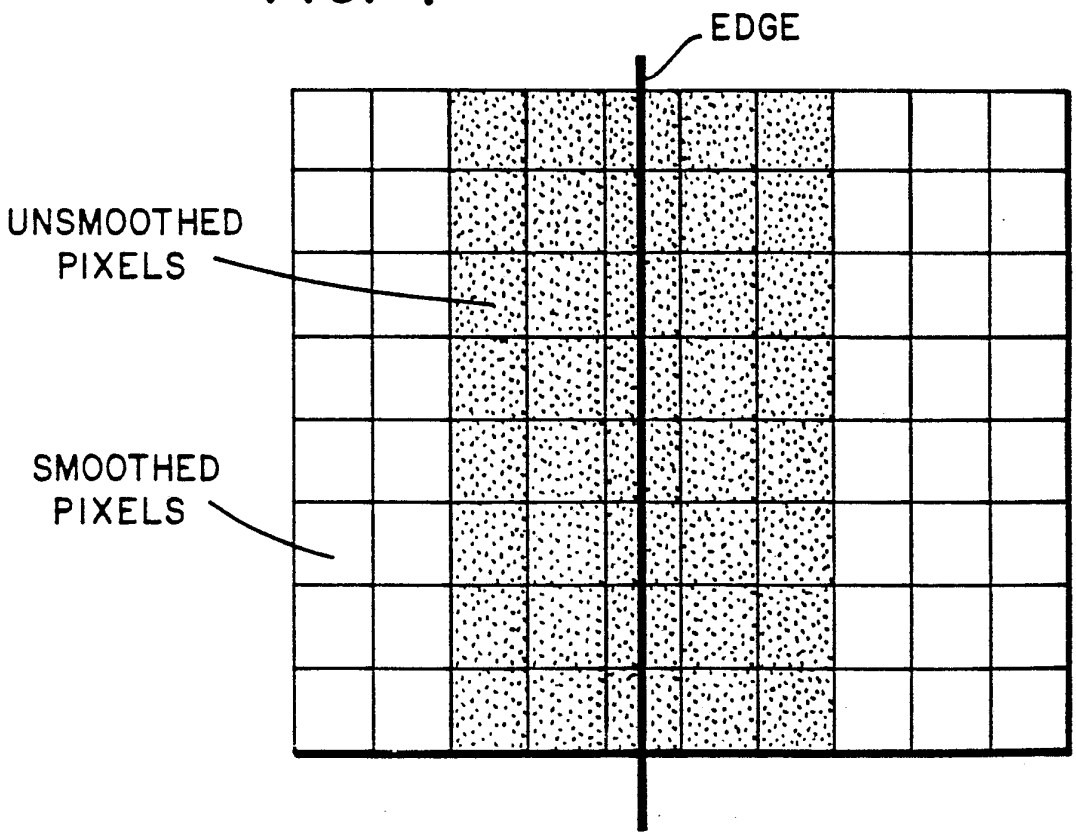

TARGET
PIXEL
NEIGHBORHOOD
(5x5)
EDGE

FIG. 3

FIG. 4


EDGE
UNSMOOTHED
PIXELS
SMOOTHED
PIXELS

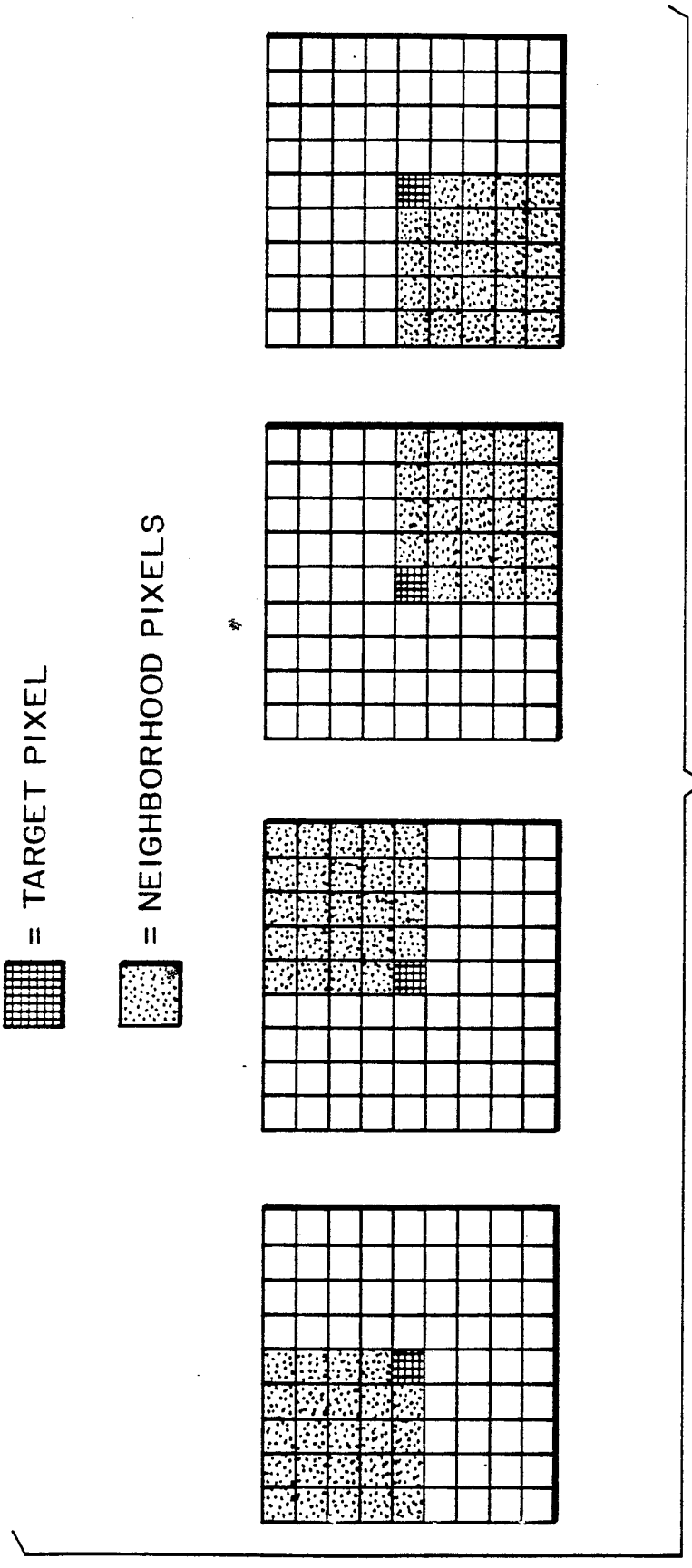


FIG. 5

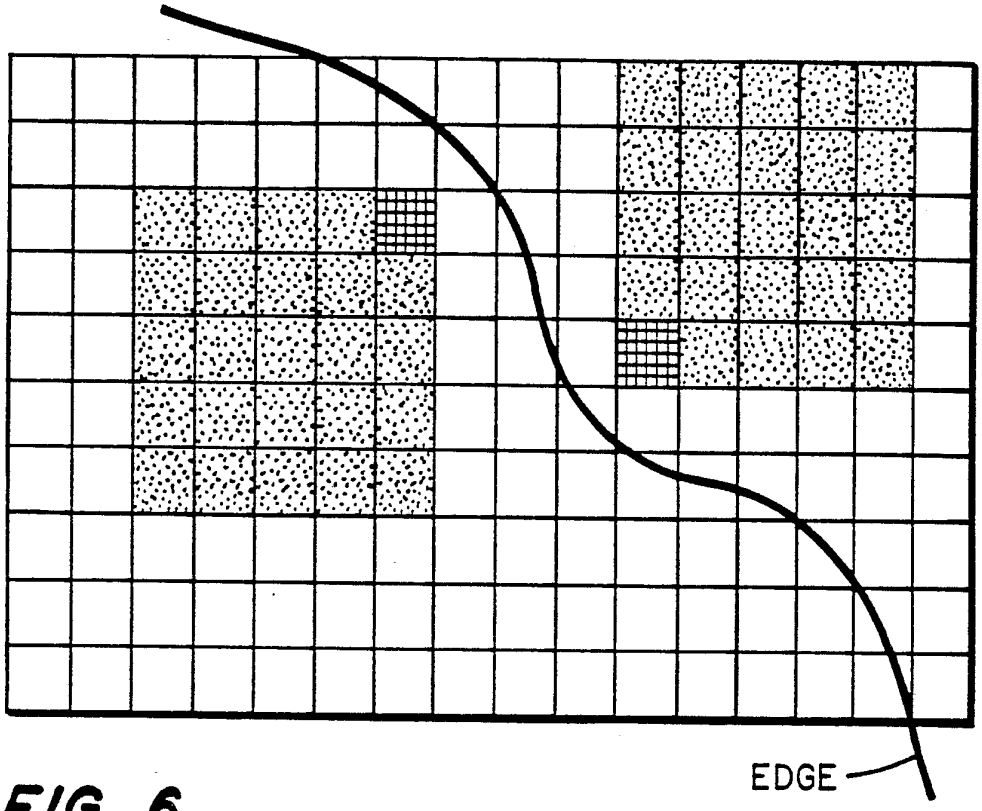

EDGE

FIG. 6

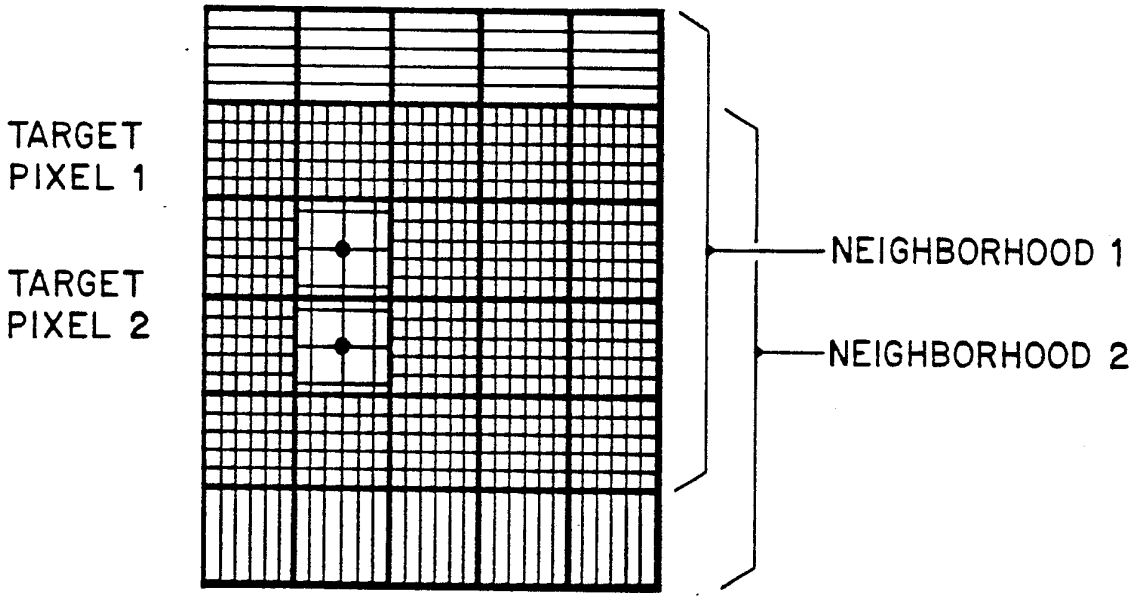

TARGET PIXEL 1
TARGET PIXEL 2
NEIGHBORHOOD 1
NEIGHBORHOOD 2

FIG. 7

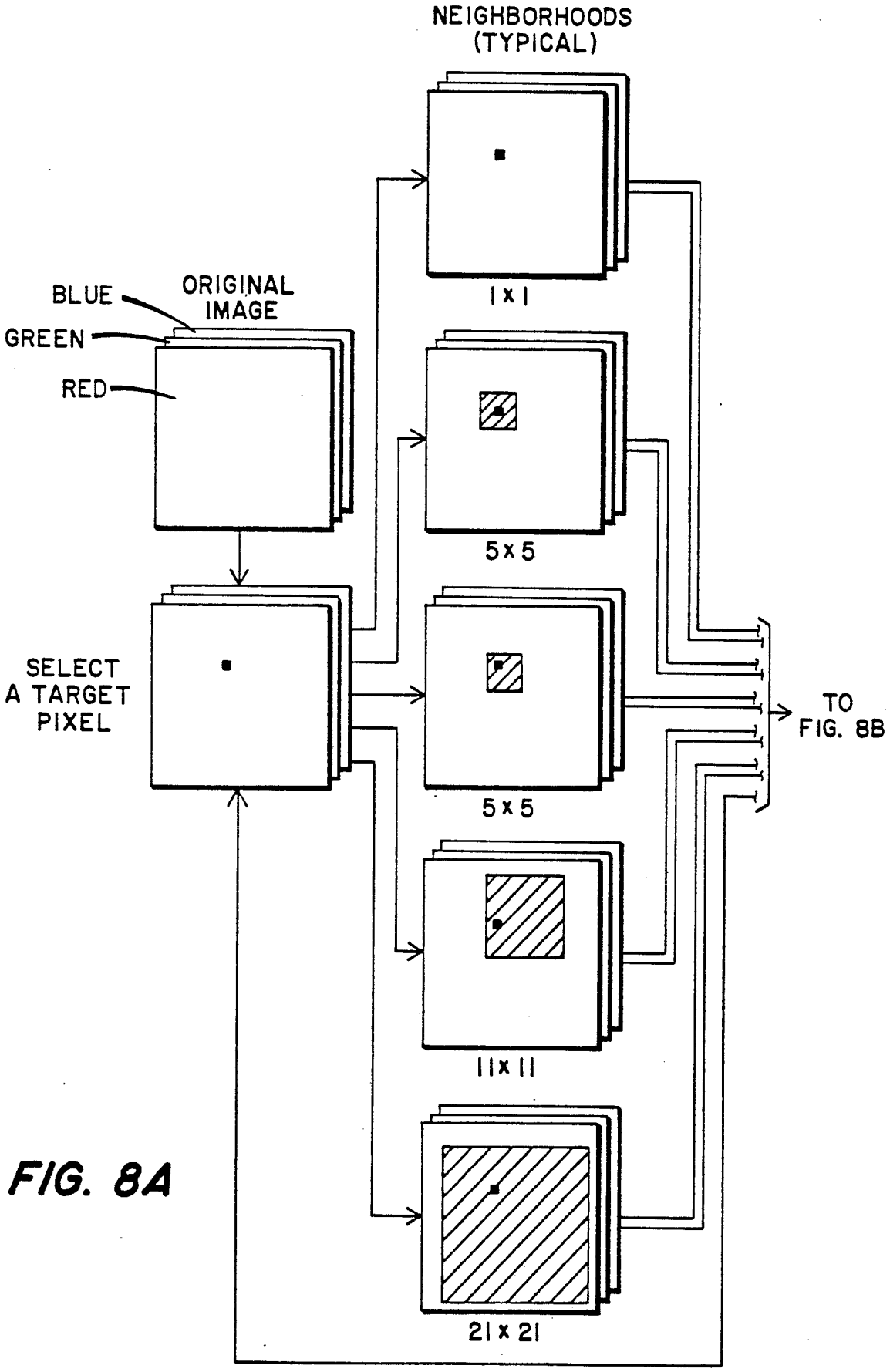

NEIGHBORHOODS
(TYPICAL)
BLUE
GREEN
RED
ORIGINAL
IMAGE
1 x 1
5 x 5
SELECT
A TARGET
PIXEL
5 x 5
TO
FIG. 8B
11 x 11
21 x 21

FIG. 8A

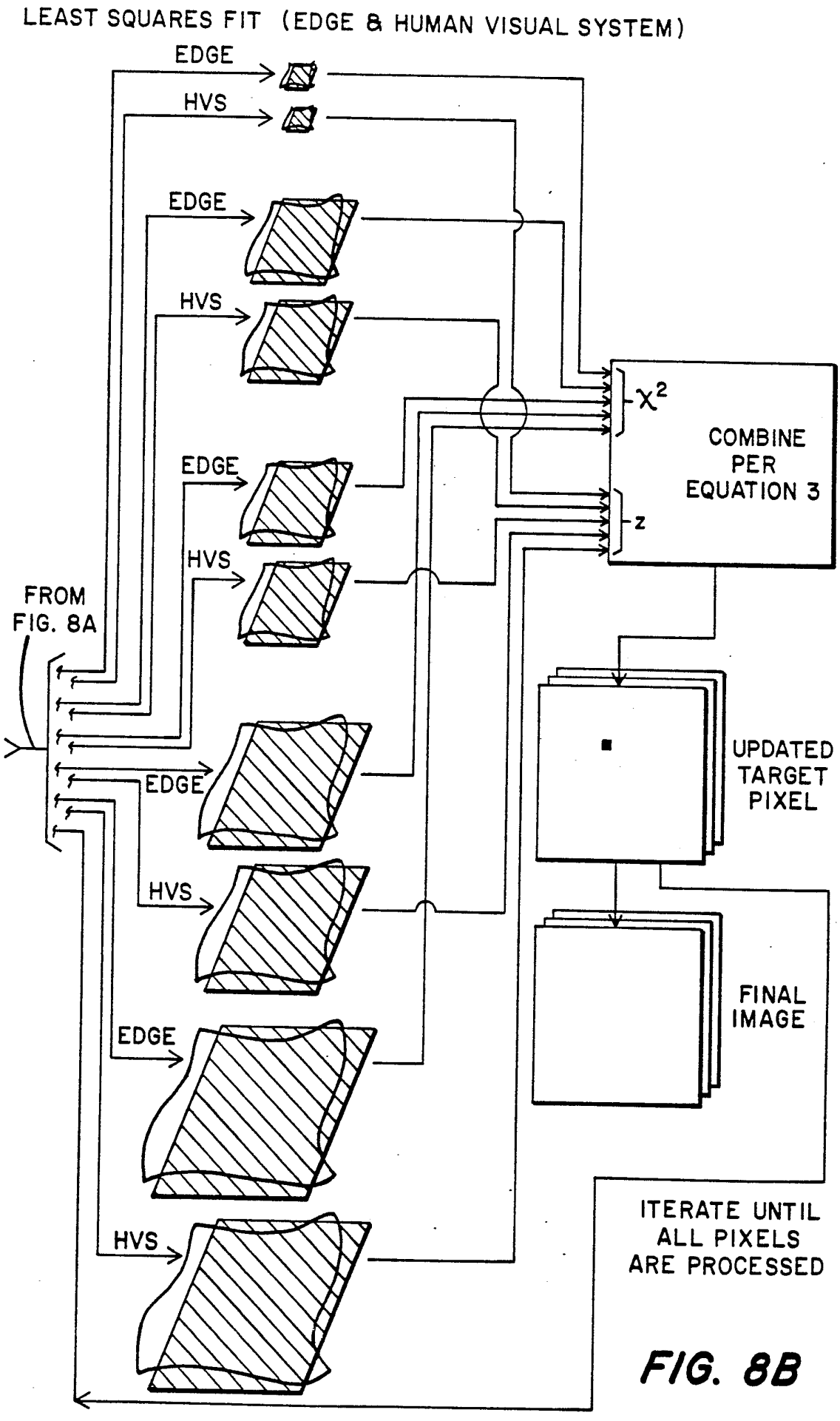

LEAST SQUARES FIT (EDGE & HUMAN VISUAL SYSTEM)
EDGE
HVS
EDGE
HVS
EDGE
HVS
FROM FIG. 8A
EDGE
HVS
EDGE
HVS
χ^2
z
COMBINE PER EQUATION 3
UPDATED TARGET PIXEL
FINAL IMAGE
ITERATE UNTIL ALL PIXELS ARE PROCESSED

FIG. 8B

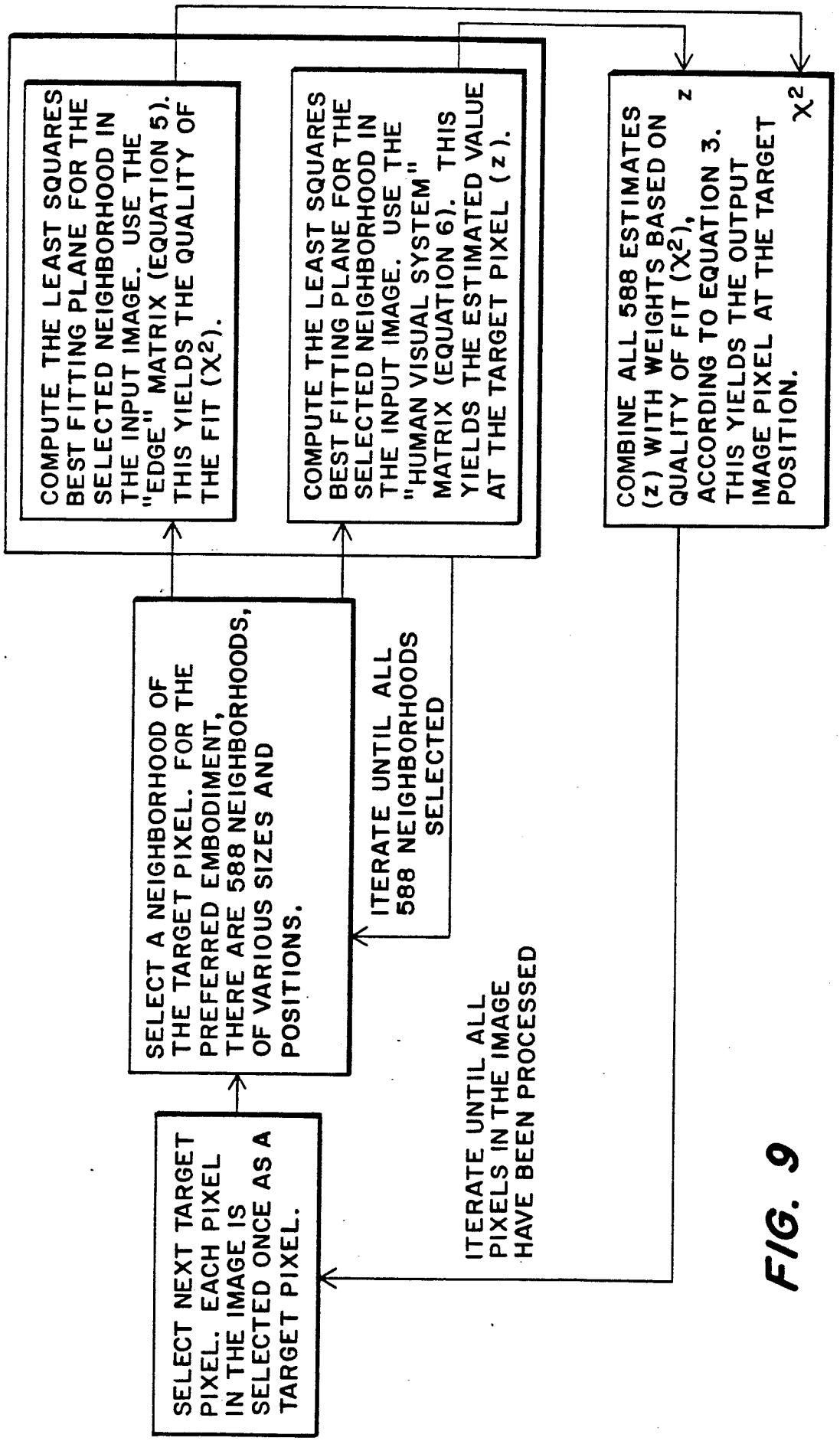

SELECT NEXT TARGET PIXEL. EACH PIXEL IN THE IMAGE IS SELECTED ONCE AS A TARGET PIXEL.
SELECT A NEIGHBORHOOD OF THE TARGET PIXEL. FOR THE PREFERRED EMBODIMENT, THERE ARE 588 NEIGHBORHOODS, OF VARIOUS SIZES AND POSITIONS.
COMPUTE THE LEAST SQUARES BEST FITTING PLANE FOR THE SELECTED NEIGHBORHOOD IN THE INPUT IMAGE. USE THE "EDGE" MATRIX (EQUATION 5). THIS YIELDS THE QUALITY OF THE FIT (X^2).
COMPUTE THE LEAST SQUARES BEST FITTING PLANE FOR THE SELECTED NEIGHBORHOOD IN THE INPUT IMAGE. USE THE "HUMAN VISUAL SYSTEM" MATRIX (EQUATION 6). THIS YIELDS THE ESTIMATED VALUE AT THE TARGET PIXEL (z).
COMBINE ALL 588 ESTIMATES (z) WITH WEIGHTS BASED ON QUALITY OF FIT (X^2), ACCORDING TO EQUATION 3. THIS YIELDS THE OUTPUT IMAGE PIXEL AT THE TARGET POSITION. z X^2
ITERATE UNTIL ALL 588 NEIGHBORHOODS SELECTED
ITERATE UNTIL ALL PIXELS IN THE IMAGE HAVE BEEN PROCESSED

FIG. 9

INTERNATIONAL SEARCH REPORT

International Application No PCT/US 90/04727

I. CLASSIFICATION OF SUBJECT MATTER (if several classification symbols apply, indicate all) [6]

According to International Patent Classification (IPC) or to both National Classification and IPC

IPC[5]: G 06 F 15/68

II. FIELDS SEARCHED

Minimum Documentation Searched [7]

Classification System	Classification Symbols
IPC[5]	G 06 F 15/68

Documentation Searched other than Minimum Documentation to the Extent that such Documents are Included in the Fields Searched [8]

III. DOCUMENTS CONSIDERED TO BE RELEVANT [9]

Category *	Citation of Document, [11] with indication, where appropriate, of the relevant passages [12]	Relevant to Claim No. [13]
A	Journal of the Optical Society of America A, volume 5, no. 1m January 1988, Optical Society of America, (Woodbury, NY, US), N. Ohyama et al.: "Least-squares filter for color-image restoration", pages 19-24 see page 19, right-hand-column, lines 1-33; page 20, paragraph B: "Least-Squares Filter"	1-3,10
A	WO, A, 89/07799 (EASTMAN KODAK CO.) 24 August 1989 see figures 8-11; page 7, lines 29-36; page 8, lines 1-12; claims 3,6-8	1-5,10,11

./.

* Special categories of cited documents: [10]

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

IV. CERTIFICATION

Date of the Actual Completion of the International Search	Date of Mailing of this International Search Report
18th December 1990	21. 01. 91
International Searching Authority	**Signature of Authorized Officer**
EUROPEAN PATENT OFFICE	M. Pegs

PEIS

III. DOCUMENTS CONSIDERED TO BE RELEVANT (CONTINUED FROM THE SECOND SHEET)

Category *	Citation of Document, 11 with indication, where appropriate, of the relevant passages	Relevant to Claim No.
A	Proceedings of The IEEE Pacific Rim Conference on Communications, Computers and Signal Processing, 4-5 June 1987, Victoria B.C., Canada, IEEE, Zbi-Quiang Liu et al.: "Sequential Kalman filter for image restoration ", pages 436-439 see page 437, paragraph "Initialization of the Algorithm", paragraph "The Local Spatial Statistics Measurements"	1-4-10,11

ANNEX TO THE INTERNATIONAL SEARCH REPORT ON INTERNATIONAL PATENT APPLICATION NO. US 9004727 SA 40846

This annex lists the patent family members relating to the patent documents cited in the above-mentioned international search report. The members are as contained in the European Patent Office EDP file on 15/01/91
The European Patent Office is in no way liable for these particulars which are merely given for the purpose of information.

Patent document cited in search report	Publication date	Patent family member(s)	Publication date
WO-A- 8907799	24-08-89	EP-A- 0356432	07-03-90