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J. OSWALD

2,963,666

EIGHT-TERMINAL DIRECTIONAL FILTER NETWORK WITH CONSTANT
IMPEDANCE ON THE FOUR PAIRS OF TERMINALS

Filed Oct. 21, 1957

2 Sheets-Sheet 1

Fig. 1

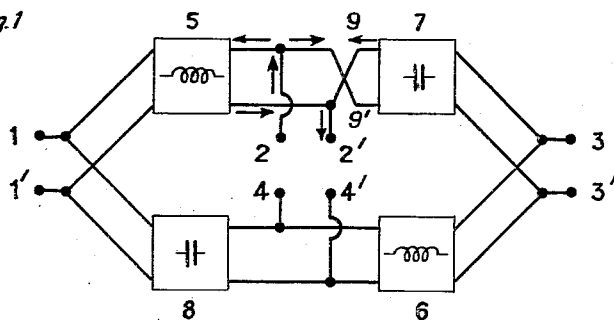


Fig. 2

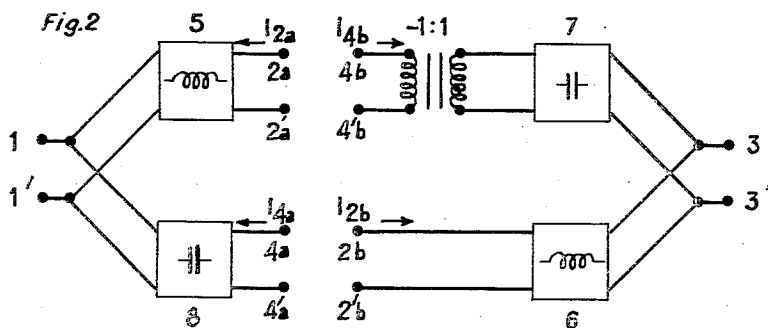
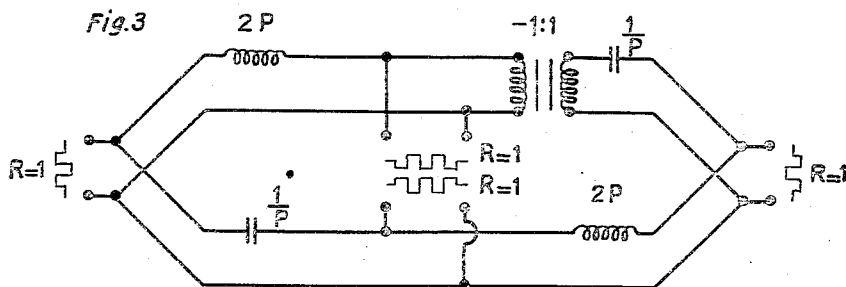


Fig. 3



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2 Sheets-Sheet 2

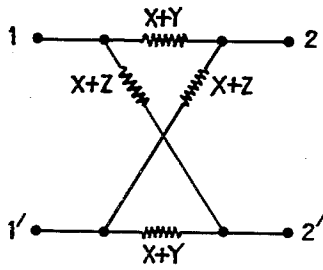


Fig. 4a

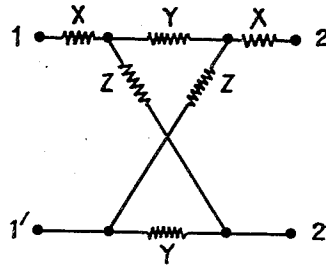


Fig. 4b

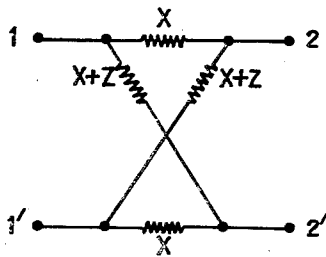


Fig. 5a

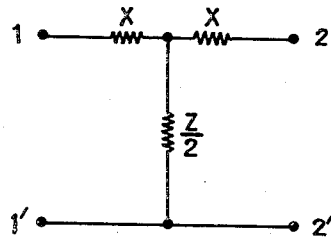


Fig. 5b

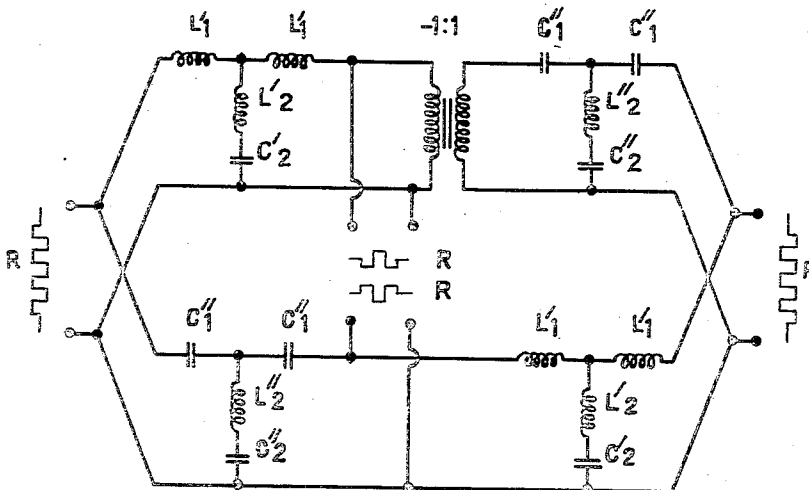


Fig. 6

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EIGHT-TERMINAL DIRECTIONAL FILTER NETWORK WITH CONSTANT IMPEDANCE ON THE FOUR PAIRS OF TERMINALS

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1 Claim. (Cl. 333—10)

The present invention relates to an eight-terminal directional filter of which the effective impedance on the four pairs of input terminals remains constantly equal to an ohmic resistance R whatever the frequencies of the voltages applied to the eight-terminal network.

A directional filter of this kind, which will hereinafter be called "strict directional filtering," is particularly suitable for the repeater stations of carrier current telecommunication systems in which different frequency bands and a common amplifier member are used for the transmission of signals in the two directions.

Devices of this type have already been suggested, which consist of bridged circuits comprising reciprocal four-terminal networks. These devices, however, require very delicate balancing.

There is also a description in German Patent No. 673,336, and the U.S. counterpart No. 2,115,138 of eight-terminal networks in which each of the pairs of terminals is connected to the adjacent pairs of terminals by networks with complementary pass-bands, with phase inversion on one of the pairs of terminals. Although, however, the condition necessary for the unit constituted in this way to offer a constant impedance to all the pairs of terminals is indicated in this patent, the means given do not make it possible for this condition to be strictly satisfied.

Now, a strict realization of the constancy of the impedance on the inputs is of the greatest practical interest; in particular it makes it possible to reduce to a very low value the irregularities of attenuation on a cable comprising repeater stations, even when the number of these is high, since such irregularities become very troublesome on a long cable if this condition is not satisfied; it is also well known that in a reactive network, a very low value of the attenuation of the filters in their pass-bands cannot be obtained without reducing the impedance deviations which determine the reflection losses.

The eight-terminal directional filter network according to the invention consists of two channels placed in parallel between their opposite inputs, and each comprising a symmetrical low-pass filter and a high-pass filter, having complementary pass-bands, the two low-pass filters and the two high-pass filters respectively being identical, said filters being arranged in inverse direction on the two channels with phase inversion on one of the channels, said eight-terminal network being characterized in this that, in order to ensure on each of the inputs the same effective impedance strictly constant as a function of the frequency of the voltage applied to said inputs, the filters of which it is composed satisfy the specifications which will hereinafter be set forth.

The attached drawings represent:

Fig. 1, a general diagram of the eight-terminal directional device.

Fig. 2, the splitting up of the eight-terminal network according to Fig. 1 into two six-terminal networks.

Figs. 3 and 6, two particular embodiments of a strict eight-terminal directional filter network according to the invention.

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Figs. 4a and 4b, 5a and 5b, various filter sections of which the equivalence is utilised for the realization of the eight-terminal network according to Fig. 6.

Fig. 1 represents the known diagram of a low-pass-high-pass eight-terminal directional filter of which the four pairs of access or input terminals are respectively designated by 1, 1', 2, 2', 3, 3', 4, 4'. The term "input" is used herein in designating the terminals, but it is intended that this terminology describe all terminals, any pair of which may assume a similar role. This directional filter comprises two identical low-pass filters 5 and 6, and two identical high-pass filters 7 and 8. These filters are connected in parallel, but they could equally well be connected in series.

The output connections of the filter 7 are crossed between the terminals 9 and 2', 9' and 2. A device of this kind is used in many two-wire carrier current systems, the inputs 1, 1' and 3, 3', for example, being connected to the input and the output of an amplifier. It will be shown that it is possible to determine the elements of the filters of a directional filter of this kind in such a way that its effective impedance, seen from each of the inputs, is constant at all frequencies, and that there is no interaction between the opposite inputs, 1, 1' and 3, 3' on the one hand, 2, 2' and 4, 4' on the other.

It will be assumed that the filters of the directional device are symmetrical and the form of its matrix of admittances will be determined.

In order to obtain this matrix it is convenient to split up the eight-terminal network into two hexapoles, as seen in Figure 2, with respective inputs 1, 1', 2a, 2'a, 4a, 4'a, and 3, 3', 2b, 2'b, 4b, 4'b, a transformer of ratio -1 being inserted on the output of the filter 7 of the second hexapole, so as to obtain the arrangement of Fig. 1.

As will be seen from an examination of the admittances, this transformer ensures the suppression of any interaction between the groups of terminals constituting opposite inputs.

We shall designate as y'_{11} , y'_{12} , y'_{22} , the elements of the matrices of admittances Y' of the low-pass filters 5 and 6, and by y''_{11} , y''_{12} , y''_{22} , the elements of the matrices of admittances Y'' of the high-pass filters 7 and 8, the prime notation (') identifying the elements of the matrices of admittances of the low-pass filters and the double prime (") identifying the elements of the matrices of admittances of the high-pass filters, whereas subscripts 11 and 12 identify admittances at the access terminals 1, 1' and 2, 2', respectively, and the subscript 22 designates mutual admittances. It is thus possible to write:

$$(1) \quad Y' = \begin{vmatrix} y'_{11} & y'_{12} \\ y'_{12} & y'_{22} \end{vmatrix} \quad (1') \quad Y'' = \begin{vmatrix} y''_{11} & y''_{12} \\ y''_{12} & y''_{22} \end{vmatrix}$$

with the conditions of symmetry:

$$(2) \quad y'_{22} = y'_{11} \quad (2') \quad y''_{22} = y''_{11}$$

According to the well known properties of matrices of derivation directional filtering it will be seen that the matrix Y_a of the left hand hexapole of Fig. 2, of which the terminals are 1, 1', 2a, 2'a, 4a, 4'a, is written:

$$(3) \quad Y_a = \begin{vmatrix} y'_{11} + y''_{11} & y'_{12} & y''_{12} \\ y'_{12} & y'_{11} & 0 \\ y'_{12} & 0 & y''_{11} \end{vmatrix}$$

and that the matrix Y_b of the right hand hexapole, of which the terminals are 3, 3', 4b, 4'b, 2b, 2'b, is expressed:

$$(4) \quad Y_b = \begin{vmatrix} y'_{11} + y''_{11} & -y''_{12} & y'_{12} \\ -y'_{12} & y''_{11} & 0 \\ y'_{12} & 0 & y''_{11} \end{vmatrix}$$

by reason of the presence of the transformer of ratio -1 .

In the following we shall designate respectively by U_a ,

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U_3, U_4 , the voltages between the terminals 2, 2', 3, 3', 4, 4', and by $U_{2a}, U_{2b}, U_{4a}, U_{4b}$ the voltages between the terminals 2a and 2'a, 2b and 2'b, 4a and 4'a, 4b and 4'b. Similarly we shall designate by I_2, I_3, I_4 the current intensities of 2' towards 2, 3' towards 3 and 4' towards 4, in the resistances connected to the input, and by $I_{2a}, I_{2b}, I_{4a}, I_{4b}$ the intensities of the currents entering by the corresponding terminals in the filters of the eight-terminal network, or, for I_{4b} , in the transformer.

It is then obvious that:

$$\begin{aligned} U_2 &= U_{2a} = U_{2b} & U_4 &= U_{4a} = U_{4b} \\ I_2 &= I_{2a} + I_{2b} & I_4 &= I_{4a} + I_{4b} \end{aligned}$$

by taking these relations into consideration it will be seen that the matrix of the eight-terminal network is expressed as:

$$Y = \begin{bmatrix} y'_{11} + y''_{11} & y'_{12} & 0 & y''_{12} \\ y'_{12} & y'_{11} + y''_{11} - y''_{12} & 0 & 0 \\ 0 & -y''_{12} & y'_{11} + y''_{11} & y'_{12} \\ y''_{12} & 0 & y'_{12} & y'_{11} + y''_{11} \end{bmatrix} \quad (5)$$

If an electromotive force U_1 is applied to the terminals 1, 1' of the eight-terminal network, all the other inputs being closed on resistances of value R , which can be taken as unity, we have:

$$(6) \quad U_2 = -J_2 \quad U_3 = -J_3 \quad U_4 = -J_4$$

J_2, J_3, J_4 , designating the current intensities in resistances respectively connected to the second, third and fourth pairs of terminals, and the relations can be written:

$$(7) \quad \begin{cases} J_2 = y'_{12} U_1 & - (y'_{11} + y''_{11}) J_2 & + y''_{12} J_3 & - y'_{12} J_4 \\ J_3 = y''_{12} J_2 & - (y'_{11} + y''_{11}) J_3 & - y'_{12} J_4 & - (y'_{11} + y''_{11}) J_4 \\ J_4 = y'_{12} U_1 & & & \end{cases}$$

which make it possible to determine J_2, J_3, J_4 , as a function of U_1 .

The absence of interaction will be expressed by

$$(8) \quad J_3 = 0$$

which is verified, since the determinant D_3 appearing in the numerator of the fraction which represents J_3 is expressed as:

$$D_3 = \begin{vmatrix} - (1 + y'_{11} + y''_{11}) & - y'_{12} U_1 & 0 \\ y''_{12} & 0 & - y'_{12} \\ 0 & - y''_{12} U_1 & - (1 + y'_{11} + y''_{11}) \end{vmatrix} \quad (9)$$

and is identically zero.

In order to express the adaptation of the network on the terminals 1, 1', it is necessary to make:

$$(10) \quad U_1 = J_1$$

in the Equations 7, J_1 being the input current in the directional filter, and to add to it:

$$(11) \quad J_1 = (y'_{11} + y''_{11}) J_1 - y'_{12} J_2 - y''_{12} J_4$$

A system (11) is thus obtained of four linear and homogeneous equations in J_1, J_2, J_3, J_4 :

$$(12) \quad \begin{cases} (y'_{11} + y''_{11} - 1) J_1 & - y'_{12} J_2 & & - y''_{12} J_4 & = 0 \\ - y'_{12} J_1 & + (y'_{11} + y''_{11} + 1) J_2 - y''_{12} J_3 & & & = 0 \\ - y''_{12} J_1 & - y''_{12} J_2 & + (y'_{11} + y''_{11} + 1) J_3 + y'_{12} J_4 & & = 0 \\ & & + y'_{12} J_3 & + (y'_{11} + y''_{11} + 1) J_4 & = 0 \end{cases}$$

By cancelling the determinant H of this system, the condition of adaptation can be obtained on the terminals 1, 1', and consequently on the other terminals, given the structure of the directional filter.

One arrives at the result more simply by taking into account the condition (8), expressing the absence of interaction between opposed access (admittances). By

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making J_3 equal to zero in the second and fourth Equation 12, one obtains J_2 and J_4 in the function of J_1 :

$$(13) \quad J_2 = \frac{y'_{12} J_1}{y'_{11} + y''_{11} + 1}$$

$$(13') \quad J_4 = \frac{y''_{12} J_1}{y'_{11} + y''_{11} + 1}$$

By applying these values in the first relation (12), one arrives at the desired condition of adaptation:

$$(14) \quad (y'_{11} + y''_{11})^2 - y'_{12}^2 - y''_{12}^2 = 1$$

By designating by $\bar{J}_1, \bar{J}_2, \bar{J}_3, \bar{J}_4$ the conjugated quantities of J_1, J_2, J_3, J_4 one obtains:

$$(15) \quad \bar{J}_2 = \frac{-y'_{12} \bar{J}_1}{1 - y'_{11} - y''_{11}} = \frac{y'_{12} \bar{J}_1}{y'_{11} + y''_{11} - 1}$$

$$(15') \quad \bar{J}_4 = \frac{-y''_{12} \bar{J}_1}{1 - y'_{11} - y''_{11}} = \frac{y''_{12} \bar{J}_1}{y'_{11} + y''_{11} - 1}$$

One can see that the relation (14) is equivalent (equal) to:

$$(16) \quad J_2 \bar{J}_2 + J_4 \bar{J}_4 = J_1 \bar{J}_1$$

which expresses the preservation of the real power.

The relation (14) which ensures the constancy of the effective impedance seen from the inputs, was certainly indicated in the aforementioned German Patent No. 673,336, which gives it a form in which the reactances of the branches of the equivalent lattice-type filter elements appear, but the method given in order to satisfy it is a method of approximation, based on a choice of

the poles and the zeros of the mutual admittances, based on the theory of elliptical functions.

The approximation of this condition is all the more satisfactory as the class of the functions increases and consequently the number of elements of the filters.

The match is rigorous at the frequencies of zero and infinity, and the frequency of separation of the network for which the attenuations are equal on the two paths.

On the other hand, it should be noted that, in the network according to the U.S. Patent 2,115,138, the effective attenuations of the bands to be eliminated on the two paths represent, for inverse pulsations (of the product $\omega^2 c$), equal minimum values. This is not the same in the networks of the present invention, where the filter networks can be determined in such a manner as to impart to undesired frequencies effective attenuations of different minimum values on the two paths, which can be advantageous for certain applications.

The elements of the directional filter which is the object of the present invention on the contrary strictly satisfy the condition (14), so that the directional filter according to the invention will have for all frequencies, an impedance exactly equal to 1 on the inputs, even for the directional filters of lowest degree, as is demonstrated by the first of the examples which will be given further on.

Let α_{22} and α_{14} be the respective effective attenuations between the inputs 1 and 2, 1 and 4. By taking into consideration (14), we have

$$(17) \quad e^{2\alpha_{12}} = 1 + \frac{y'^2_{12}}{y'^2_{12}} \quad e^{2\alpha_{14}} = 1 + \frac{y'^2_{12}}{y'^2_{12}}$$

For the low-pass high-pass directional filter which is

the object of the present invention, y'_{12} , and consequently y'_{11} offers, for the zero frequency, one pole; y''_{12} , and consequently y''_{11} , offers a zero (for y''_{11} this results from (14) and from the symmetry of the filters).

It follows that we can put:

$$(18) \quad y'_{11} + y''_{11} = \frac{G}{U} y'_{12} = -\frac{f}{U} y''_{12} = -\frac{h}{U}$$

in which G and U represent the even and odd parts of a Hurwitz polynomial in relation to the variable $p=j\omega$, ω being the pulsation (classic property of reactance or admittance functions) and f and h represent even polynomials.

The matrix (5) therefore takes the form (19)

$$(19) \quad Y = \begin{vmatrix} \frac{G}{U} & -\frac{f}{U} & 0 & -\frac{h}{U} \\ -\frac{f}{U} & \frac{G}{U} & \frac{h}{U} & 0 \\ 0 & \frac{h}{U} & \frac{G}{U} & -\frac{f}{U} \\ -\frac{h}{U} & 0 & -\frac{f}{U} & \frac{G}{U} \end{vmatrix}$$

in which G , U , f and h are bound by the relation:

$$(20) \quad G^2 - U^2 = f^2 + h^2$$

which is the expression of (14) in relation to the new functions introduced.

Matrices of similar form have been given by Cauer to represent strict hexapolar directional filters, i.e. having a constant effective impedance on the terminals where the mixture of frequencies to be separated is applied.

This matrix (19) cannot, however, be considered as an obvious extension of the hexapolar matrix of Cauer, by reason of an additional condition which must be satisfied by the elements of (19), a condition without analogue in the hexapolar matrices.

It is, in fact, necessary for

$$\frac{G}{U}$$

to represent the sum of the admittances of two symmetrical four-terminal networks of which

$$-\frac{f}{U} \text{ and } -\frac{h}{U}$$

respectively are the mutual admittances. In the Cauer hexapoles, of which the filters are simply "tight-coupled," the condition (20) is sufficient for realization.

It will be assumed that the mutual admittances admit, for an angular frequency ω_k , a common pole, with the respective polar portions:

$$\frac{C'_k p}{p^2 + \omega_k^2} \quad \frac{C''_k p}{p^2 + \omega_k^2}$$

The admittance represented by

$$\frac{G}{U}$$

will admit, for this same pole, a polar portion:

$$\frac{\gamma_k p}{p^2 + \omega_k^2}$$

γ_k being, according to (14), given by the relation:

$$(21) \quad \gamma_k^2 = C'^2_k + C''^2_k$$

as is shown by the cancellation of the polar portion of the second order. According to the known properties of the matrices of admittances of symmetrical quadripoles, how-

ever, γ_k must be the sum of two positive terms γ'_k , γ''_k such as:

$$\gamma'^2_k \geq C'^2_k \quad \gamma''^2_k \geq C''^2_k$$

which leads to:

$$(22) \quad \gamma_k^2 \geq \{C'_k\} + \{C''_k\}^2$$

A comparison of (21) and (22) shows that one of the coefficients C'_k , C''_k is necessarily nil; the poles of the mutual admittances are therefore separate and, if C'_k alone is different from zero, we have:

$$\gamma_k = |C'_k|$$

It is therefore necessary that, in the matrix (19) U shall be a divisor of fh . This condition is sufficient, as can easily be seen.

Resuming the Equations 17', 17'' which can be written:

$$(23') \quad e^{2\alpha_{12}} = 1 + \frac{h^2}{f^2} \quad (23'') \quad e^{2\alpha_{14}} = 1 + \frac{f^2}{h^2}$$

it will be seen that the condition indicated entails a certain restriction in the choice of the even function

$$\frac{h}{f}$$

which is simply the function φ determining the attenuations in the two channels of the directional filter by:

$$(24') \quad e^{2\alpha_{12}} = 1 + \varphi^2 \quad (24'') \quad e^{2\alpha_{14}} = 1 + \frac{1}{\varphi^2}$$

The choice of f and h , from which Cauer determines the directional filter matrices, cannot therefore be made here, and an entirely new method has to be given in order to deal with the case of the eight-pole (or eight-terminal) network.

In the matrix (19) the sub-matrix of the second order composed of the elements of the two first lines and the two first columns is the matrix of the low-pass filter obtained by leaving connected on the terminals 1, 1' and 2, 2' of the first low-pass filter of the eight-pole network, the input terminals of the adjacent high-pass filters, respectively short-circuited on their opposite inputs 3, 3' and 4, 4'.

This low-pass filter, which will be designated by S , is obviously symmetrical and we shall call q_2 its characteristic admittance function and q_1 its characteristic attenuation function, defined by:

$$(25) \quad q_1 = \coth \Gamma$$

Γ being the transfer exponent of the filter, of which exponent the real and imaginary parts respectively represent the characteristic attenuation and phase-shift. As is well known, we shall have:

$$(26) \quad y'_{11} + y''_{11} = \frac{G}{U} = q_1, q_2$$

$$(27) \quad y'_{12} = -\frac{f}{U} = -q_2 \sqrt{q_1^2 - 1}$$

and the relation (20) shows that:

$$(28) \quad y''_{12} = -\frac{h}{U} = -\sqrt{q_2^2 - 1}$$

The representation of admittances by means of functions q_1 , q_2 permits one to satisfy identically the relation (14) in such a manner that the match is rigorously realized (achieved) at all frequencies.

Functions q_1 and q_2 belong to the class of the "Q functions" introduced by Cauer, and for the properties of which we can refer to his work "Theorie der linearen Wechselstromschaltungen" (Theory of linear alternating current circuits) second edition, 1954, Akademie Verlag, Berlin, VI.3: "Properties of Q-functions," pages 216 to 220. It will be recalled that these are "positive functions" of $p=j\omega$, ω being the pulsation, i.e. of positive real

part for the values of p on the right of the imaginary axis, and of which functions, moreover, the square is a rational function, real on the imaginary axis (real frequencies). At the real frequencies the "Q functions" can therefore be either real or purely imaginary, and their determination, starting from their square, is made so that they assume for the real frequencies positive values in their domain of reality. In the domain in which they are imaginary they have the peculiarities of a reactance.

Two functions q_1, q_2 are said to be "conjugated" if the frequency domains for which the one is real coincide with those for which the other is imaginary. The product and the quotient of two such functions always define reactance (or admittance) functions.

One example of conjugated functions is given by the characteristic functions of attenuation and impedance (or admittance) of a symmetrical filter, the attenuation function being real in the attenuated bands and the admittance function being real in the pass bands.

In the present case, in which the functions q_1, q_2 relate to a low-pass filter (the same would obviously apply to a high-pass filter) these functions only present a single branching point at the cut-off frequency; as variable the "standardised frequency" Ω will be taken, ratio of the frequency to the cut-off frequency

$$f_0 = \frac{\omega_0}{2\pi}$$

in which the complex variable $P = j\Omega$.

The "class" of a "Q function" is defined as the half-degree in Ω^2 , or P^2 , of its square. The choice of the half-degree to define the class is made with the object of assigning the class 1 to the elementary filter element, and class $\frac{1}{2}$ to the half-element.

The characteristic admittance function q_2 is itself standardised, i.e. related to the value $1/R$ of the constant effective admittances on the inputs.

In accordance with the relation (28), $\sqrt{q_2^2 - 1}$ represents a mutual admittance; the function q_2 , characteristic admittance function of symmetrical low-pass, therefore also belongs to the class of high-pass antimetric filter attenuation functions. They necessarily have one pole at infinity, a branching zero at the standardised frequency 1, a value 1 at the zero frequency, and are of the form, if $n + \frac{1}{2}$ is their class:

$$(29) \quad q_2 = \sqrt{P^2 + 1} \frac{Ln(P^2)}{Mn(P^2)}$$

$Ln(P^2)$ and $Mn(P^2)$ designating polynomials in P^2 of n degree, of which the constant term is equal to 1, and of which the zeros, all negative and higher than 1 in modulus, are overlapping, the smallest in modulus belonging to $Mn(P^2)$.

The functions q_2 employed offer, for n frequencies $\sigma_1, \sigma_2, \dots, \sigma_n$ less than 1, maxima equal to 1, and the knowledge of said frequencies, which can be arbitrarily fixed, and which will be chosen from those for which it is desired to obtain a point of infinite attenuation on the high-pass side, completely determines q_2 (see, for this determination, Cauer op. cit., VI. 5. VI. 6, pages 226-231).

Equation 28 of which the second term is certainly a "mutual admittance" function then gives the expression of y''_{12} , which obviously has the same poles as q_2 .

Then we take, as characteristic attenuation function q_1 of the filter S, a conjugate function of q_2 , which will be chosen so that $q_1 - 1$ has double zeros in P^2 for all the polar frequencies of q_2 , including the frequency at infinity; in this way the condition is ensured, which recognised as necessary and sufficient, that y'_{12} and y''_{12} have no common pole. The four-pole S therefore has this peculiarity of admitting the peaks of infinite attenuation

for all the poles of the characteristic admittance. If m be the class of q_1 , this function will be in the form:

$$(30) \quad q_1 = \frac{R_m(P^2)}{P\sqrt{P^2 + 1}S_{m-1}(P^2)}$$

$R_m(P^2)$ and $S_{m-1}(P^2)$ representing polynomials in P^2 of degrees m and $m-1$, of which the terms of highest degree have as coefficient 1, of which the zeros, all negative and less than 1 in modulus, are overlapping, that of the highest modulus belonging to $R_m(P^2)$. The condition indicated, that $q_1 - 1$ presents double zeros in P^2 for the zeros of $Mn(P^2)$, as also for the infinite frequency is expressed by $2n+1$ relations which perfectly determine q_1 , if m equals $n+1$. The preferable mode of determination was indicated by Cauer in the passages of his work already mentioned.

If m is higher than $n+1$, the determination of q_1 will be made for an arbitrary choice of $(m-n-1)$ additional double zeros of $q_1 - 1$. For these zeros, the admittance y'_{12} will be zero and there will be a peak off infinite attenuation, on the low-pass side of the directional filter.

As q_1 and q_2 have been chosen in the manner just explained, $y'_{12} = -q_2\sqrt{q_1^2 - 1}$ will only admit the poles of q_1 , with the same moduli of residues as q_1, q_2 , and y''_{12} the poles of q_2 with the same moduli of residues as q_1, q_2 , which makes it possible, merely by the development of the mutual admittances along their polar parts, to obtain y'_{11} and y''_{11} .

The low-pass filters of the eight-terminal network, 5 and 6, and the high-pass filters 7 and 8, are thus perfectly determined. It will be noted that the admittances of the branches of the equivalent lattices, having as their respective values $y'_{11} \pm y'_{12}$ for the low-pass filters, $y''_{11} \pm y''_{12}$ for the high-pass filters, admit for polar portions, for each of the filters, the double of the polar portions with positive residues of the mutual admittance, and the double change of sign, of the polar portions with negative residues of the same admittance.

It is in no way meant to say that the lattice-type diagram should preferably be used in producing filters. The peaks of infinite attenuation, on the high-pass side of the directional filter, are given by the values 1 of q_2 and the poles of q_1 ; the peaks of attenuation on the low-pass side by the poles of q_2 , and the additional values 1 of q_1 .

It would equally well have been possible to extract from the matrix 19 the sub-matrix composed of the elements of the first and fourth lines and the first and fourth columns, which represent a high-pass symmetrical four-terminal network S', which could be made to play the same part as S; it would then have been necessary to introduce a function q_2 of low-pass antimetric filter attenuation.

Finally it should be noted that by substituting matrices of impedances for the matrices of admittances, one would make the synthesis of an eight-terminal network of "strict directional filtering" of the "series" diagram on all its inputs.

Two embodiments of directional filter eight-pole networks according to the invention will now be given.

The eight-pole networks will be designated by the classes of the functions q_1 and q_2 ; q_1 is of integral class and q_2 of semi-odd class, as has been seen.

The eight-pole network ($m; n + \frac{1}{2}$) is an eight-pole network for which the function q_1 is of class m , the function q_2 of class $n + \frac{1}{2}$.

First example.—Eight-pole network of the type (1; 0.5)

We are given:

$$q_2 = \sqrt{P^2 + 1}$$

which leads to

$$y'_{12} = -P; y'_{11} = P$$

q_1 will be an odd function, with pole at zero fre-

quency, and taking the value 1 for the pole at infinity of q_2 .

q_1 is of the form:

$$\frac{P^2 + \alpha^2}{P\sqrt{P^2 + 1}}$$

It is necessary to take $\alpha^2 = 1/2$ to obtain the double zero at infinity of $q_1 - 1$, considered as a function of P^2 .

Therefore:

$$q_1 = \frac{2P+1}{2P\sqrt{P^2+1}}; \sqrt{q_1^2-1} = \frac{1}{2P\sqrt{P^2+1}}$$

and consequently:

$$y'_{12} = -\frac{1}{2P}; y''_{11} = \frac{1}{2P}$$

The filters are reduced to series reactances, inductances for the low-pass filters, capacities for the high-pass filters. Fig. 3 represents the diagram of the eight-terminal network obtained in this way. The function φ in this case equals $2P^2$.

Second example.—Eight-pole network of the type (2; 1.5)

We take a function q_2 of class 1.5; this is a high-pass antismetric filter attenuation function, taking the value 1 at the zero frequency and for a pulsation $\Omega'_{1\infty}$ which will be defined by:

$$(31) \quad \Omega'^2_{1\infty} = 1 - m_2^2 (m_2 < 1)$$

q_2 has the form

$$(32) \quad q_2 = \sqrt{P^2 + 1} \frac{P^2 + \alpha^2}{\beta^2 P^2 + \alpha^2}$$

(in which $\alpha^2 > \beta^2$, $\alpha^2 > 1$).

α^2 and β^2 are defined by the condition that $q_2 - 1$ admits a double zero for: $P^2 = m_2^2 - 1$, which gives:

$$\begin{aligned} -m_2^2 + \epsilon m_2 (\beta^2 - 1) &= \alpha^2 - \beta^2 \\ -2m_2 + \epsilon (\beta^2 - 1) &= 0 \\ [\epsilon = \pm 1] \end{aligned}$$

whence it follows that:

$$\beta^2 = 1 + 2\epsilon m_2, \quad \alpha^2 = m_2^2 + \beta^2 = (m_2 + \epsilon)^2$$

It is obviously necessary to take $\epsilon = +1$, hence:

$$\alpha^2 = (m_2 + 1)^2; \beta^2 = 2m_2 + 1$$

so that:

$$(33) \quad q_2 = \sqrt{P^2 + 1} \frac{P^2 + (1 + m_2)^2}{(1 + 2m_2)P^2 + (1 + m_2)^2}$$

The function q_1 is determined by its values 1, for the frequencies corresponding to the poles of q_2 , i.e. the infinite frequency and the frequency $\Omega'_{1\infty}$ defined by:

$$(34) \quad \Omega'^2_{1\infty} = \frac{(1 + m_2)^2}{1 + 2m_2} = \frac{1}{1 - m_1^2}$$

with

$$m_1 = \frac{m_2}{1 + m_2}$$

It is convenient to take as intermediary in the calculation the function

$$q'_1 = \coth \frac{\Gamma}{2}$$

to which q_1 is bound by the relation:

$$(35) \quad q_1 = \frac{q'^2_1 + 1}{2q'_1}$$

q'_1 , taking the value 1 at the infinite frequency and at the frequency $\Omega'_{1\infty}$, has the form:

$$\frac{P^2 + \alpha^2}{P\sqrt{P^2 + 1}}$$

α^2 being defined by:

$$\alpha^2 - \Omega'^2_{1\infty} = \Omega'_{1\infty} \sqrt{\Omega'^2_{1\infty} - 1}$$

that is:

$$\alpha^2 = \frac{1}{1 \pm m_1}$$

The sign + must be chosen, as α^2 is less than 1; therefore, expressing α^2 as a function of m_2 :

$$(36) \quad \alpha^2 = \frac{1 + m_2}{1 + 2m_2}$$

and taking (35) into consideration:

$$(37) \quad q_1 = \frac{2(1 + 2m_2)^2 P^4 + (8m_2^2 + 10m_2 + 3)P^2 + (1 + m_2)^2}{2(1 + 2m_2)P\sqrt{P^2 + 1}[(1 + 2m_2)P^2 + 1 + m_2]}$$

Knowing q_2 and q_1 by (33) and (37), we have:

$$\begin{aligned} (38) \quad y''_{12} &= -\sqrt{q_2^2 - 1} = -\frac{P(P^2 + 1 - m_2^2)}{(1 + 2m_2)P^2 + (1 + m_2)^2} \\ &= \frac{-P}{1 + 2m_2} + \frac{2m_2^2(1 + m_2)P}{(1 + 2m_2)^2 \left[P^2 + \frac{(1 + m_2)^2}{1 + 2m_2} \right]} \end{aligned}$$

$$\begin{aligned} (39) \quad y'_{12} &= -q_2 \sqrt{q_1^2 - 1} = -\frac{P^2 + (1 + m_2)^2}{2(1 + 2m_2)P[(1 + 2m_2)P^2 + 1 + m_2]} \\ &= -\frac{1 + m_2}{2(1 + 2m_2)P} + \frac{m_2^2(3 + 2m_2)P}{2(1 + 2m_2)^2 \left(P^2 + \frac{1 + m_2}{1 + 2m_2} \right)} \end{aligned}$$

and consequently, given the relation between the residues of the admittances and the mutual admittances:

$$(40) \quad y''_{11} = \frac{P}{1 + 2m_2} + \frac{2m_2^2(1 + m_2)P}{(1 + 2m_2)^2 \left[P^2 + \frac{(1 + m_2)^2}{1 + 2m_2} \right]}$$

$$(41) \quad y'_{11} = \frac{1 + m_2}{2(1 + 2m_2)P} + \frac{m_2(3 + 2m_2)P}{2(1 + 2m_2)^2 \left(P^2 + \frac{1 + m_2}{1 + 2m_2} \right)}$$

We shall put:

$$(42) \quad \begin{cases} \frac{1 + m_2}{2(1 + 2m_2)} = H'_1, & \frac{m_2(3 + 2m_2)}{2(1 + 2m_2)^2} = H'_2 \\ \frac{1}{1 + 2m_2} = H''_1, & \frac{2m_2^2(1 + m_2)}{(1 + 2m_2)^2} = H''_2 \end{cases}$$

As has previously been recalled in a symmetrical four-terminal network of admittances y_{11} , y_{12} , the admittances

$$\frac{1}{\xi}, \frac{1}{\eta}$$

of the branches of the equivalent lattice-type network are respectively equal to $y_{11} + y_{12}$ and $y_{11} - y_{12}$.

It will therefore be seen that for the low-pass filter, the reactances A' , B' of the branches of the equivalent lattice are given by:

$$(43) \quad \frac{1}{A'} = \frac{2H'_1}{P}; \frac{1}{B'} = \frac{2H'_2 P}{P^2 + \frac{1 + m_2}{1 + 2m_2}}$$

hence:

$$(44) \quad A' = \frac{P}{2H'_1}$$

$$(45) \quad B' = \frac{P}{2H'_2} \frac{1 + m_2}{1 + 2m_2} \frac{1}{2H'_2 P}$$

The second relation can be written:

(45)

$$B' = A' + \frac{(1+2m_2)P}{m_2(1+m_2)(3+2m_2)} + \frac{1+m_2}{2(1+2m_2)H'_2} \frac{1}{P} = A' + G'$$

G' representing a reactance.

Now it is known that generally a lattice-type network of branches $X+Y$, $X+Z$, is equivalent to a lattice type network of branches Y and Z inserted between two impedances X arranged in series on the two inputs. Figs. 4a and 4b illustrate this well known equivalence.

If Y is nil, the lattice element degenerates into a shunt impedance, so that the lattice element of branches X , $Y+Z$ is equivalent to a T network of which the longitudinal branches are equal to X and the middle branch to

$$\frac{Z}{2}$$

Figs. 5a and 5b represent the equivalent four-terminal networks.

It follows from this that the low-pass filter can be carried out by means of a T network of longitudinal branches A' and vertical branches

$$\frac{C'}{2}$$

The longitudinal branches therefore have reactances equal to

$$\frac{P}{2H'_1} = \frac{P(1+2m_2)}{1+m_2}$$

and consist of inductances equal to

$$\frac{R}{\omega_0} \frac{1+2m_2}{1+m_2}$$

while the vertical branch consists of a resonant circuit of capacity:

$$C'_2 = \frac{2m_2(3+2m_2)}{(1+m_2)(1+2m_2)R\omega_0}$$

and inductance:

$$L'_2 = \frac{(1+2m_2)R}{2m_2(1+m_2)(3+2m_2)\omega_0}$$

It will be seen that $L'_2 C'_2 \omega_0^2 (1+m_2)^2 = 1$, i.e. the frequency of resonance of this branch, for which there is a pole of attenuation, corresponds well to the zero of the function q_2 as was foreseen.

The case of the high-pass filters will be treated in the same way and it is recognised that they are equivalent to symmetrical T -type networks of which the longitudinal branches consist of capacities C''_1 of value equal to

$$\frac{2}{(1+2m_2)R\omega_0}$$

and the vertical branch of a resonant circuit of inductance

$$L''_2 = \frac{(1+2m_2)^2 R}{8m_2(1+m_2)\omega_0}$$

and capacity:

$$C''_2 = \frac{8m_2}{(1-m_2)(1+2m_2)^2 R\omega_0}$$

we have:

$$L''_2 C''_2 \omega_0^2 (1-m_2^2) = 1$$

which shows that the resonance pulsation is equal to the pulsation Ω''_0 of the attenuation pole correspond to the value 1 of q_2 .

Fig. 6 gives the complete diagram of this eight-pole network.

What is claimed is:

An eight-terminal directional filter network comprising two groups of pairs of opposite access terminals 1, 1'-3, 3' and 2, 2'-4, 4' and two pairs of symmetrical identical filters, a first pair of filters consisting of two low-pass filters, a matrix of admittance of which receives y'_{11} as the admittance at the access and y'_{12} as the mutual admittance and the second pair of filters being constituted by two high-pass filters, having a passing band complementary to the band of the low-pass filters, the matrix of admittance of which receives y''_{11} as the admittance at the access and y''_{12} as the mutual admittance, the low-pass filters being respectively connected to the pairs of adjacent terminals 1, 1'-2, 2' and 3, 3'-4, 4' of the network and the high-pass filters being respectively connected to the other pairs of terminals 1, 1'-4, 4' and 2, 2'-3, 3', the phases being inverse along the two paths comprised between two pairs of opposite terminals, characterized in this, that in order to ensure, at each pair of access terminals, the same strictly constant effective impedance as a function of the frequency of the voltage applied to said terminals, the low-pass filter and the high-pass filter respectively have mutual admittances:

$$y'_{12} = -q_2 \sqrt{q_1^2 - 1}; y''_{12} = -\sqrt{q_2^2 - 1}$$

and admittances on the terminals y'_{11} and y''_{11} obtained by developing y'_{12} and y''_{12} along their polar portions and by taking as residues the moduli of the residues of the mutual admittances, the function q_2 having the form:

$$q_2 = \sqrt{P^2 + 1} \frac{L_n(P^2)}{M_n(P^2)}$$

$L_n(P^2)$ and $M_n(P^2)$ designating polynomials P^2 of degree n (with $P=j\Omega$, Ω being the ratio of the applied frequency to the cut-off frequency) and of constant term equal to 1, determined by means of n parameters $\sigma_1, \sigma_2, \dots, \sigma_n$, all less than 1, by the condition that $q_2^2 - 1$ has double zeros in P^2 for the values corresponding to $\Omega = \sigma_1, \sigma_2, \dots, \sigma_n$, the function of q_1 having the form:

$$q_1 = \frac{R_m(P^2)}{P\sqrt{P^2 + 1} S_{m-1}(P^2)}$$

$R_m(P^2)$ and $S_{m-1}(P^2)$ designating polynomials in P^2 of degree m and $m-1$, of which the terms of highest degree have 1 as coefficient, and of which the $2m-1$ parameters are chosen so that $q_2 - 1$ has double zeros in P^2 for the n zeros of $M_n(P^2)$ and for the infinite frequency.

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