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Whitehouse et al.

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[54] **DISCRETE TRANSFORM SYSTEMS USING PERMUTER MEMORIES**

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[52] **U.S. Cl.** ..... **364/827;** 307/221 D; 328/167; 333/70 T; 364/825; 364/862

[58] **Field of Search** ..... 235/193, 181, 156, 152, 235/186; 307/221 D; 328/167; 333/18, 28 R, 70 T; 357/24; 358/133, 138

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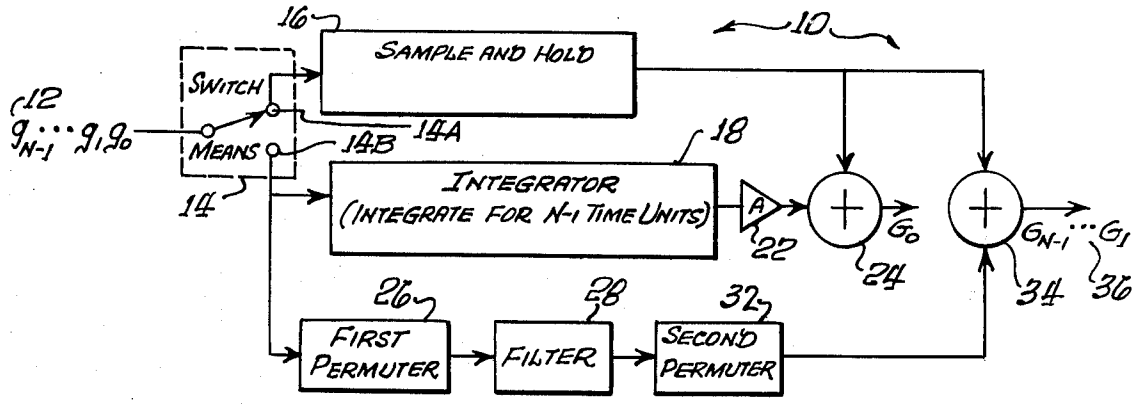
Primary Examiner—Joseph F. Ruggiero

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[57] **ABSTRACT**

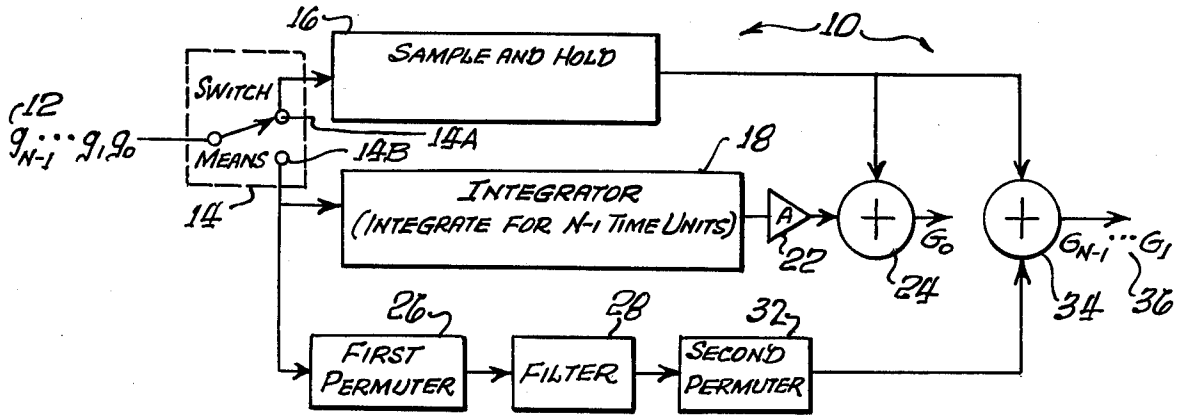
High-speed, high-accuracy, apparatus for performing discrete Fourier and discrete cosine transforms, suitable for signal processing applications. The apparatus includes a means for switching an input signal from a sample-and-hold circuit to a circuit which includes an integrator in cascade with an amplifier. The outputs of these two circuits are summed in a first summer to give one term, G_0, which is proportional to the d-c component of the input signal. In parallel with the integrator and amplifier is a cascade circuit comprising a first permuter, a filter and a second permuter. A permuter is a special type of memory. The outputs of the sample-and-hold circuit and of the second permuter are summed in a second summer, to result in an output comprising the other transformed terms, $G_1, \ldots, G_{N-1}$.

7 Claims, 2 Drawing Figures



NOTE: THE SWITCH IS IN THE UP POSITION FOR THE FIRST DATA SAMPLE, AND IS DOWN FOR THE REMAINING N-1 SAMPLES.

APPARATUS FOR PERFORMING A DISCRETE FOURIER OR ODD COSINE TRANSFORM.



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FIG. 1. APPARATUS FOR PERFORMING A DISCRETE FOURIER OR ODD COSINE TRANSFORM.

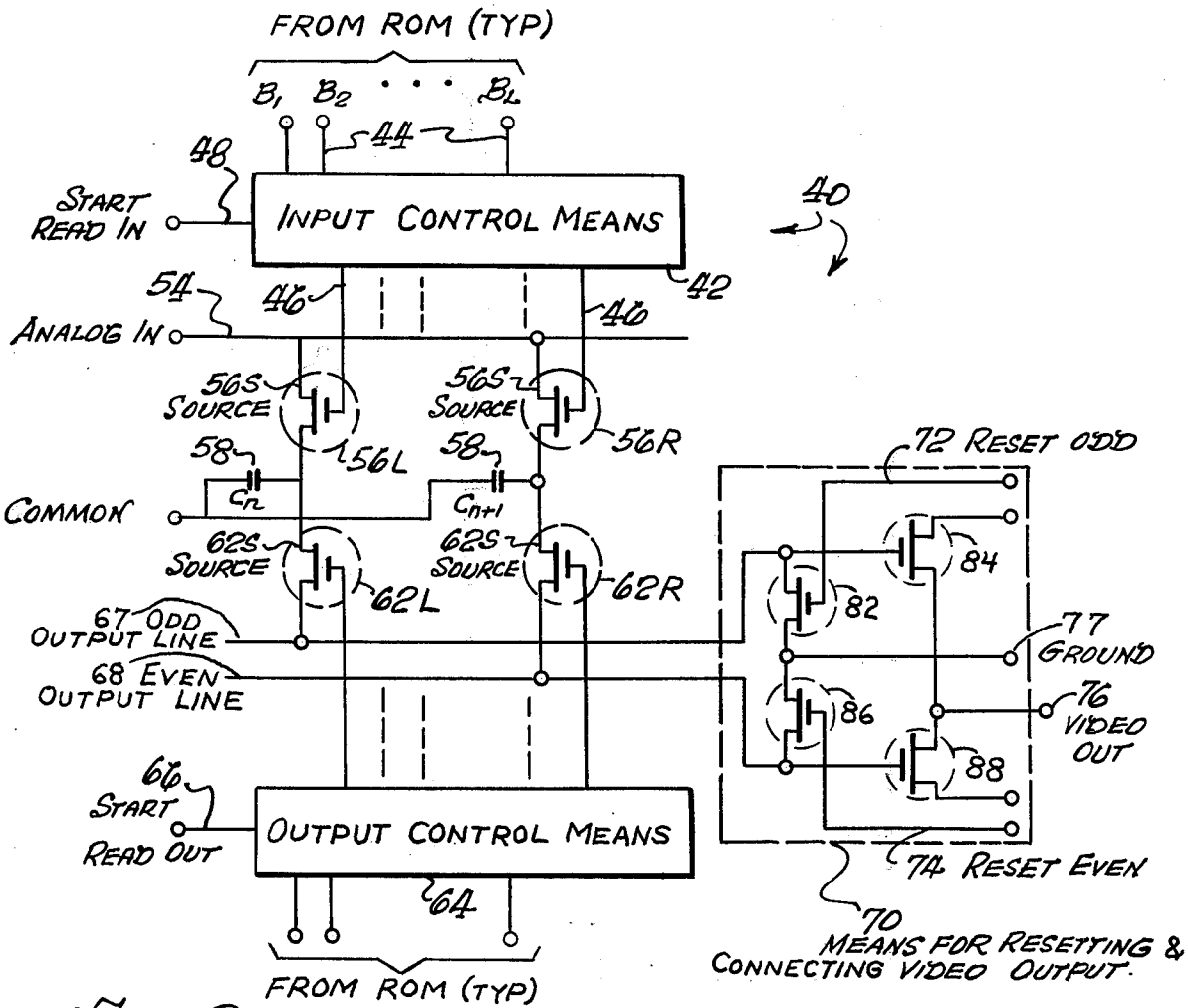


FIG. 2. ANALOG PERMUTATION MEMORY.

DISCRETE TRANSFORM SYSTEMS USING PERMUTER MEMORIES

STATEMENT OF GOVERNMENT INTEREST

The invention described herein may be manufactured and used by or for the Government of the United States of America for governmental purposes without the payment of any royalties thereon or therefor.

BACKGROUND OF THE INVENTION

This invention relates to high-speed, high-accuracy implementations of the discrete Fourier and discrete odd cosine transforms, suitable for signal processing applications such as sonar, radar, communication, beamforming, image data compression, etc.

Present analog high-speed transform implementations are largely based on the chirp-Z transform architecture, and require a premultiplication, a convolution, and a postmultiplication. The two most useful filter technologies for the convolution are surface wave devices and charge transfer devices. When surface acoustic wave devices are used, balanced mixers are generally used as the multipliers, and the mixers limit the attainable transform accuracy. When charge transfer devices are used as the filters, baseband multipliers are required. At present it is necessary to build these multipliers as discrete components, off the charge-coupled device (CCD) chip. The multipliers prevent the fabrication of the transform device as a single, large-scale integrated (LSI) chip and contribute greatly to the cost of the transform system, as well as limiting the transform accuracy.

In the implementation of the odd discrete cosine transform, each permuter need permute only one real sequence, and each filter only need be one real filter.

This is distinct from the Fourier transform case where there is complex input data present, as well as a complex transform kernel.

This makes a prime cosine transform implementation much simpler than a chirp-Z cosine transform implementation, since the latter uses four real multipliers and four real filters.

SUMMARY OF THE INVENTION

The invention relates to apparatus for taking the discrete transform, which may be either the Fourier or the odd cosine transform, of a sampled analog input signal, consisting of N data samples, $g_0, g_1, \ldots, g_{N-1}$.

The apparatus comprises means, connected to the input signal, for connecting the input signal to either of two connections. Means are connected to one of the connections of the connecting means, for sampling the first data sampler, g_0, and holding it for N time units, $2 \leqq N \leqq 256$, a time unit being one inter tap delay, which may be as short as 0.2 microseconds, with present CCD transversal filters. Means are connected to the other connection of the connecting means, for integrating the input signal, excluding the g_0 sample, for N-1 time units.

Means, whose input is connected to the output of the integrating means, amplify the integrated signal. A first means, whose two inputs are connected to the outputs of the means for sampling and holding and of the amplifying means, sums its two input signals, the output signal being G_0, which is proportional to the direct-current component of the input signal, $g_0, \ldots, g_{N-1}$.

A first means is connected to the other connection of the connecting means, for performing a permutation on its input signal. Means, whose input is connected to the output of the fist permuting means, filters the permuted signal. A second means, whose input is connected to the output of the filtering means, permutes its input signal. A second means, having inputs connected to the outputs of the sampling and holding means and of the second permuting means, has an output which comprises the transformed terms $G_1, \ldots, G_{N-1}$.

STATEMENT OF THE OBJECTS OF THE INVENTION

An object of the invention is to provide apparatus which can perform a prime Fourier transform or a prime cosine transform on an input signal.

Another object of the invention is to provide apparatus which eliminates the multipliers required in prior art implementations.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a block diagram of apparatus capable of performing a prime discrete Fourier transform, or a prime odd discrete cosine transform (ODCT).

FIG. 2 is a block diagram of an analog permutation memory, of the type used in the embodiment shown in FIG. 1.

DESCRIPTION OF THE PREFERRED EMBODIMENT

Referring now to FIG. 1, this figure shows an apparatus **10** for taking the discrete transform of a sampled analog input signal **12** consisting of N data samples, $g_0, g_1, \ldots, g_{N-1}$.

Means **14** are connected to the input signal **12**, for connecting the input signal to either of two connections, **14A** or **14B**.

Means **16** are connected to one of the connections **14A** of the connecting means **14**, for sampling the first data sample, g_0, and holding it for N time units, $2 \leqq N \leqq 256$, a time unit being one transversal filter intertap delay time. The limitation $N \leqq 256$ is not a fundamental limitation; it results from the fact that present CCD transversal filters are limited to about **512** taps and a filter with **2N-3** taps is required to realize a prime transform of length N.

Means **18** are connected to the other connection **14B** of the connecting means **14**, for integrating the input signal **12**, excluding the g_0 sample, for **N-1** time units.

Means **22**, whose input is connected to the output of the integrating means **18**, amplify the integrated signal.

A first means **24**, whose two inputs are connected to the outputs of the means for sampling and holding **16** and of the amplifying means **22**, sums its two input signals, the output signal being G_0.

A first means **26**, connected to the other connection **14B** of the connecting means **14**, performs a permutation on its input signal.

Means **28**, whose input is connected to the output of the first permuting means **26**, filter the permuted signal.

In the prior art, whether the data is real or not, generally a premultiplication by a complex chirp is required. Therefore, at the output of the premultiplier appears a complex function, which enters a complex filter, which generally comprises four real filters.

For the case of the prime transform, where the first step in the algorithm is not a multiplication, but a permutation, if the input data is real, after permutation it is still real. If the prime Fourier transform is being per-

formed, (1) with a complex input signal four real filters are required, two to handle the real part of the signal and two to handle the imaginary part of the signal, whereas (2) with a real input signal, two filters are required. It is analogous to multiplying a real number by a complex number, which would require two real multipliers. If the prime cosine transform is performed where ordinarily the input data is real, with a real kernel for the transform, only one real filter is required.

A second means **32**, whose input is connected to the output of the filtering means **32** permutes its input signal, and thereby forms a cross-correlated output in permuted order.

A second means **34**, having inputs connected to the outputs of the sampling and holding means **16** and of the second permuting means **32**, has an output **36** comprising the transformed terms $G_1, \ldots, G_{N-1}$.

If the purpose of apparatus **10** is to take the prime Fourier transform of a complex input signal **12**, the means for sampling and holding **16** would be a complex means, and the means for integrating **18** would be a complex integrator comprising two real integrators, one each for the real and imaginary parts.

The means for amplifying **22** which is optional for the Fourier transform would be a complex amplifier (i.e., a pair of real amplifiers) having a gain of one. The first and second permuting means, **26** and **32**, would be complex permuters, that is, comprise a real channel permuter and an imaginary channel permuter.

The means for filtering **28** would be a complex filter, and the two summing means, **24** and **34**, would be complex summers. The complex means **28** for filtering would comprise four transversal filters and two summing means or three transversal filters and three summing means, as described in Bond et al, "Complex Filters, Convolvers and Multipliers", U.S. Pat. No. 3,926,367, dated Dec. 16, 1975.

On the other hand, if the apparatus **10** is to be capable of taking the prime odd discrete cosine transform (ODCT) of an input signal **12** which is real, the means for sampling and holding **16** would be a real means, and the means for integrating **18** would be a real integrator. In this case, the means **22** for amplifying is a real amplifier having a gain of two. The first and second permuting means **26** and **32**, are real permuters; the means **28** for filtering is a real filter; and the two summing means **24** and **34**, are real summers. The real means for filtering may comprise a single transversal filter.

Referring now to FIG. **2**, this figure shows a permutation memory **40** comprising an input control means **42**, having a plurality of inputs **44** for an L-bit binary number, $B_1, B_2 \ldots B_L$, and a plurality 2^L of outputs **46**. The B_n comprise an L-bit binary control word, controlling 2^L samples to be stored. In an embodiment actually built, the control word comprised six bits, controlling $2^6 = 64$ samples, the control word designating any of **64** addresses. Means **48**, connected to the decoding means **42**, initiate the read in of the L-bit number. The binary decoder **42** takes a binary word and puts an ON signal on one of the lines, a line which corresponds to the value of the binary word. The value may be designated as

$$\sum_{k=1}^{L} B_k 2^{k-1}.$$

When the line is turned on, the analog input channel is connected to only one of the capacitors, long enough to charge up the capacitor **58**. All other switches **56** remain open, except the one charged up.

Means **54** are provided for applying an input signal, generally an analog signal. A first plurality 2^L of switching means **56** are connected to the 2^L outputs **46** of the decoding means **42** and to the signal applying means **54**, each of the switching means being in a normally open condition. A plurality 2^L of means **58** are connected to the switching means **56**, for storing a charge when a specific switching means, connected to a corresponding charge storing means **58**, is in a closed condition. A second plurality 2^L of switching means **62** is connected to the first plurality of switching means **56** and to the charge storing means **58**.

An output control means **64** is connected to the second plurality of switching means **62**, for reading out the states of the 2^L charge-storing means **58**, generally a capacitor, as to the charge on each.

Means **66** are connected to the output control means **64**, for initiating the read out.

The input control means **42** and the output control means **64** may be a binary decoder and a shift register, respectively. Alternately, a shift register may be used as the input control means **42** and a binary decoder may be used as the output control means **64**. Another alternative would be to use binary decoders for both means **42** and **64**. The prior art uses shift registers for both means **42** and **64**, and this permits only serial read-in and read-out, and cannot perform permutations.

If the read out uses a shift register **64**, it is loaded with all zeroes except a single ONE. Normally, the non-zero ONE would be at the left-hand position, and at successive clock times it would move to the right. The output line is connected to the first capacitor **58** during the first clock time. The next clock time, the one moves to the right in the shift register **64** and is connected to the next charged capacitor. Eventually, the output line **76** reads through all the capacitors, in a manner somewhat similar to a rotary switch.

Means **70** are connected to second plurality of switching means **62**, for discharging the charge storing means **58**.

The permutation memory **40** may further comprise means **70**, connected to the second switching means **62**, for resetting the means for decoding **42**, so that the complete cycle, from read in to read out, may be processed again.

After each capacitor **58** has been read out, the signal is destroyed by shorting each capacitor to ground, through ground line **77**. Lines, or buses, **67** and **68**, are alternately being used as a line to the video output **76** and a line to ground **77**.

The video output line **76** is connected to upper bus **67** half the time and to bus **68** the other time. The same is true of the ground line **77**. One of the two buses, **67** or **68**, is connected to the output **76**, the other bus, **68** or **67**, is connected to the ground line **77**. Each capacitor **58** is used to charge an output line **67** or **68**. After read out of each sample, it is necessary to discharge the corresponding lines **67** or **68** via the corresponding reset switch **72** or **74**.

In the permutation memory **40** shown in FIG. **2**, the means **70** for resetting comprises a pair of means, one means **72** for resetting those means for storing charges **58** which are associated with the odd-numbered bits, $B_1, B_3, \ldots$, of the L-bit binary number, and another means **74** for resetting those means for storing charges which

are associated with the even-numbered bits, B_2, B_4, . . . , of the L-bit binary number.

The permutation memory **40** may further comprise a means **76**, connected to the second plurality of switching means **62**, for obtaining the permuted output in video form.

In the permutation memory **40** the first and second switching means, **56** and **62**, comprise first and second pluralities 2^L of field-effect transistors (FETs), respectively. The first plurality of FETs **56** have their gates connected to the outputs of the means for decoding **42**, their sources **56S** connected to the means **54** for applying an input signal, and their drains connected to the charge-storing means, **57** and **58**, and to the sources **62S** of the second plurality of FETs **62**.

The second plurality of FETs **62** have their drains connected to the means for resetting **70** and their gates connected to the inputs to the read out means the output control means **64**.

The upper left FET **56L** allows the left capacitor **58**, labeled C_n, to be charged by a signal on analog line **54**. Similarly, the upper right FET **56R** permits a capacitor **58**, which is labeled C_{n+1}, to be charged at another time.

As the input control means **42** turns on various FET's, **56L** and **56R**, it causes various capacitors **58** to be charged by an analog input signal on lead **54**, when the associated FET is turned on. When all desired capacitors **58** have been charged, the read out process may then begin, through output control means **64**. This permits specific capacitors **58** to be connected to an output line, **67** or **68**. An even numbered capacitor **58**, C_{n+1}, may be connected by a corresponding FET **62R** to the even output line **68**, or an odd numbered capacitor C_n may be connected by a corresponding FET **62L** to the odd output line **67**.

Either output line, **67** or **68**, itself, has some capacitance. Therefore, after the associated output line is read by connecting it to the video output port **76**, it is necessary to discharge it through reset control **72** or **74**. One reset control **72** discharges the odd numbered capacitor C_n, through odd output line **67**. The other reset control **74** is provided for discharging the even-numbered line **68**.

Discussing now the timing of the processing of the data samples of the apparatus **10**, in general, during N time units the data is being read into the first permuter **26**. During the next N successive time units, filter **28** is being loaded. During another N time units, the data samples are read out of the filter **28**, and read into the second permuter **32**. Then another N time units are required to read the data out of the second permuter **32**, so that it can be summed in summer **34**.

Therefore, the sample and hold circuit **16** must hold the data samples **12**, for 3N time units before discharging them into summer **34**. Also, it takes a total of 4N time units for the data samples **12** to be processed by apparatus **10**.

In a typical sample and hold circuit, such as **16**, a charge is transferred to a capacitor and held on it until it is ready to be read out by discharging it.

The output signal **36**, which is read out, is an analog signal, but could include, as a special case, digital signals. The capacitors **58** are not merely charged or not charged, but they may have a specific charge, which can be read out. The signals are analog signals throughout the apparatus **10**, although the controls are digital. It is a digital control as to which capacitors **58** are connected to which bus lines, **67** or **68**. However, the values of the charges on the capacitors **58** are analog, they can assume a continuum of values.

Discussing now the theory behind the invention, and beginning with very broad concepts, the equation

$$G_k = \sum_{n=0}^{P-1} g_n f(nk) \tag{1}$$

defines a transform which may be implemented using permuter memories when the block length P is prime and f is a function which is periodic, with period equal to P.

For the prime Fourier transform,

$$f(x) = e^{-\frac{i2\pi x}{P}} \tag{2}$$

$$G_k = \sum_{n=0}^{P-1} g_n e^{-\frac{i2\pi nk}{P}} \tag{3}$$

where P is prime.

Each sample of the input signal **12** may consist of pulses obtained from sampling an analog signal. The pulses may be narrow or wide, even so wide as to touch each other. In any given signal, the pulses would generally have the same width. The signal **12** consists of a train of such samples. The term g_o refers to the first input sample, whereas the output sample G_o, is proportional to the average of d-c value of the input sequence.

In the use of the prime Fourier transform, in general the input signal may be complex and the output signal may be complex.

The term "permuter" is, of course, derived from the term "permutation", which has the following connection to the invention: The powers of primitive roots are examined. These powers give every non-zero number modulo P. The powers do not necessarily occur in the sequence **1, 2, . . . , P-1**, but may occur in some permutation of the sequence.

For example, if P=**5** and R=**2**, then the powers of the primitive root 1^o, R, R^2, . . . , R^{P-2}, become **1, 2, 4, 8**, which MOD P=**5** equals 1, 2, 4, 3, a permutation of **1, 2, 3, 4**. In general, all terms, **1, 2, . . . , P-1**, would appear in the permutation, each term appearing exactly once.

This means that every one of the non-zero numbers can be represented as some power. So, for every n, n can be made equal to $R^{n'}$ mod P. It is to be noted that, $R^{P-1} = 1$. The range may be either from **0** to **P-2**, in the exponents, or from **1** to **P-1**, they are equivalent.

Examining the function $f(R^s)$, f is a function of the integers mod P, but R^s mod P will repeat periodically as soon as s=**P-1**. Therefore $f(R^s)$ has a period **P-1** as a functon of s, as distinguished from the function $f(s)$, which has a period of P.

The sum

$$\sum_{n'=1}^{P-1} g_{R^{n'}} f(R^{n'+s'})$$

automatically becomes a cyclic cross-correlation.

From the above discussion, it follows that the structure **10** shown in FIG. **1**, can serve to implement transforms of the type discussed. At present, the only two known useful transforms are the discrete prime Fourier transform (DFT) and the odd discrete cosine transform (ODCT).

Any equation for G_k such as Eq. (1) can be implemented in the manner shown in FIG. **1**. Exactly the same permutation orders are used as for implementing the Fourier transform, and the filter weights are the permuted version of the function *f*.

The prime transform reduces the transform computation to a cross-convolution, or a cross-correlation, of two complex sequences, the permuted data sequence and the appropriately permuted sinusoid.

Therefore, in FIG. **1**, in detail, the filter **28** must be complex, it is accepting a complex input signal and convolving it with a complex function. The means for filtering **28** could be realized as four real filters, or as three real filters, as described in U.S. Pat. No. 3,926,367, entitled COMPLEX FILTERS, CONVOLVERS, AND MULTIPLIERS, By Jeffrey M. Speiser et al, which issued on Dec. 16, 1975.

The first and second permuters, **26** and **32**, permute a complex sequence, and each consists of a real channel permuter and an imaginary channel permuter. For the discrete Fourier transform, the prime P would equal the data block length, N.

Discussing now the theory behind the invention as it relates to the odd discrete cosine transform (ODCT), in this case, f(nk) in Equation (3) is equal

$$\text{to } \cos\frac{2\pi nk}{2N-1}, \text{ and the prime } P = 2N-1. \tag{4}$$

The cosine transform is usually used with image data, which are real. As a result, the required permutation is a permutation of real data, the required convolution is a convolution of real data, and the final permutation is a permutation of real data. For the purpose of transmitting compressed data over a channel with limited bandwidth, the second permutation may be omitted from the transform device. This holds true for any application which does not require access to the transform values in their normal order.

For many purposes, a single permuter memory is itself a useful device. For example, in data transmission through channels with burst errors, reordering the data is useful, because instead of all errors appearing in a bunch, they are now spread out throughout the data, and some error-correcting codes can now compensate for the errors, so that the data is now useful. This process is termed "interleaving" in the coding literature.

The high speed implementation, **10** in FIG. **1**, of the odd discrete cosine transform eliminates the multipliers required in earlier implementations discussed in the prior art, based on the chirp-Z transform. The discrete cosine transform (DCT) is useful for television data compression since its basis vectors closely approximate those of the optimum Karhunen-Loeve transform for exponentially correlated data, as has been shown in the prior art. (R. W. Means, H. J. Whitehouse, and J. M. Speiser, "Television Encoding Using A Hybrid Discrete Cosine Transform and a Differential Pulse Code Modulation in Real Time", National Telecommunication Conference, 1974 Record, P.P. 61–66).

The odd discrete cosine transform is defined as the first N Fournier coefficients of the length **2N-1** even extension of the data, assuming that the data consists of N real values. This is shown in Equation (5):

$$G_k = \sum_{n=-(N-1)}^{N-1} g_n e^{-\frac{i2\pi kn}{(2N-1)}} \quad \text{for } k = 0,1,\ldots N-1 \tag{5}$$

where $g_{-n} = g_n$.

An alternative way of writing Equation (5) is:

$$G_k = g_o + \sum_{\substack{m=-(N-1)\\ m\neq 0}}^{N-1} g_m e^{-\frac{i2\pi kn}{2N-1}} \tag{6}$$

The reason that the g_o term is taken out is that it is advantageous to change the product nk of the transform kernel *f*(*nk*), of Esq. (1) and (3), into a sum, making use of powers of a primitive root. A zero is never obtained as a power of a primitive root.

In order to be able to use a variant of Rader's Prime Transform Algorithm it may be assumed that P= **2N-1** is a prime. Note that the data block length, N, need not be a prime, as shown in Table **1**. This algorithm is described by C. Rader, "Discrete Fourier Transforms When the Number of Data Samples is Prime", Proc. IEEE, Vol. 56, P. 1107–1108, 1968.

Table 1

Selected Primes and the Corresponding OCDT Lengths and Filter Lengths

P (prime)	N = (P + 1)/2 (data block length)	2N = 3 = P − 2 = filter length
31	16	29
61	31	59
127	64	125
251	126	249
257	129	255

Note:
The filter lengths shown assume that the data is not recirculated or reread into the filter.

It has been shown in theory that if P is a prime, the discrete Fourier transform (DFT) of length P can be implemented using a circular convolution of length P-**1** together with two analog permuter memories of length P. For more details on this implementation, reference is directed to the article by Whitehouse, H. J., Means, R. W., and Speiser, J. M., entitled "Signal Processing Architectures Using Transversal Filter Technology", which appeared in the Proceedings of the **1975** IEEE International Symposium on Circuits and Systems, Boston, Apr.21 – 23, 1975, Paper SC.**1**. It will be shown herein that the symmetry of the extended data in Eq. (1) permits the size of the circular convolution and the permuter memories to be reduced by a factor of two.

For each prime P, there is an integer R, called a primitive root of P, such that the residues of R, R^2, ... R^{P-1} are all distinct modulo P, and include every non-zero residue modulo P. This is discussed by Gauss, C. F., "Disquisitiones Arithmeticae", Lipsiae, 1801. This is also available in the English translation by Arthur A. Clarke, Yale University Press, New Haven, 1966. See especially Pp. 37–40. Although primitive roots are also discussed in most modern textbooks on number theory, Gauss' original exposition remains unrivalled for clarity.

Therefore, for each integer n not congruent to zero mod P, *n* can be represented uniquely as a power of R modulo P, say $m = R^{n'} \pmod P$. The integer *n'* is called the index of *n* (mod P) with respect to the primitive root R. In effect, R plays the role of the base of a system of logarithms in modulo P arithmetic, and *n'* is the logarithm of n. This representation is useful because it al-

lows us to replace multiplication by addition in the exponent of the DFT, and thus reduce the DFT to a circular correlation, as shown in equation (7).

$$G_{R^{k'}} = g_0 + \sum_{n' = -(N-2)}^{N-1} g_{R^{n'}} e^{-\frac{2\pi R^{n'+k'}}{2N-1}} \quad (7)$$

Since zero does not have an index (logarithm) with respect to the primitive root, the zero frequency point in the transform must be computed separately, as shown in equation (8).

$$G_0 = \sum_{n=-(N-1)}^{N-1} g_n = g_0 + 2\sum_{n=1}^{N-1} g_n \quad (8)$$

The interpretation of Eq. (7) is that the discrete Fourier Transform (DFT) coefficients in permuted order are obtained by adding g_0 to the circular cross-correlation of a permuted sinusoid with a permuted version of the data points excluding g_0. For the special case of the discrete cosine transform (DCT), the symmetry of the extended data allows us to replace the complex exponential by a cosine, as shown in equation (9).

$$G_{R^{k'}} = g_0 + \sum_{n' = -(N-2)}^{N-1} g_{R^{n'}} \cos\left(\frac{2\pi R^{k'+n'}}{(2N-1)}\right) \quad (9)$$

It will now be shown that the permuted data and the permuted cosine have periodicity N-1, so that the circular correlation of length P-1 = 2N-2 can be reduced to a circular correlation of length N-1. First it will be noted that both the extended data and the cosine function are even. Let h_n be any even sequence. It will be shown that h_{R^s} has period N-1, where the subscript R^s is reduced modulo P = 2N-1.

It is well known in number theory that $R^{(P-1)/2} = -1$ (mod P). This is shown in the reference by Gauss. In the case under discussion, (P-1)/2 = (2N-2)/2 = N-1. Therefore $h_{R^s} + (N-1) = h_{R^s R^{N-1}} = h_{-R^s} = h_{R^s}$.

Using this periodicity property applied to Eq. (9) permits writing the odd discrete cosine transform in shorter form, as shown in Eq. (10).

$$G_{R^{k'}} = g_0 + 2\sum_{n'=1}^{N-1} g_{R^{n'}} \cos\left(\frac{2\pi R^{n'+k'}}{(2N-1)}\right) \quad (10A)$$

$$G_o = g_0 + 2\sum_{n=1}^{N-1} g_n \quad (10B)$$

The circular correlation required for Eq. (10A) may be implemented by any of the alternative methods using a transversal filter. The most straightforward would be to use a transversal filter of length 2(N-1)-1 = 2N-3, with tap weights of cos $(2\pi R^s/(2N-1))$, for $s = N-2, \ldots 1, 0, 1, 2 \ldots N-2$.

The architecture of the transform is shown in FIG. 1, and is virtually identical to that of the prime Fourier transform, except that the prime cosine transform need only permute real data and filter the permuted real data with a filter having real weights. The prime cosine transform is thus considerably simpler to implement than a prime Fourier transform of the same block length.

The analog permuter memory **40** shown in FIG. 2 has been developed as a modification of a commercially available serial analog memory. The commercially available serial-access memory stores analog samples as charges in an array of metal oxide semiconductor (MOS) capacitors under the control of read-in and read-out shift registers. The permuter memory shown in FIG. 2 differs in the fact that one of the shift registers has been replaced by a binary decoder **42**, thus allowing the data to be reordered by an external control signal **54**.

Obviously, many modifications and variations of the present invention are possible in the light of the above teachings, and, it is therefore understood that within the scope of the disclosed inventive concept, the invention may be practiced otherwise than as specifically described.

What is claimed is:

1. Apparatus for taking the discrete transform of a sampled analog input signal, consisting of N data samples, $g_0, g_1, \ldots$, comprising:

means, connected to the input signal, for connecting the input signal to either of two connections;

means, connected to one of the connections of the connecting means, for sampling the first data sample, g_0, and holding it for 3N time units,

means, connected to the other connection of the connecting means, for integrating the input signal, excluding the g_0 sample, for N-1 time units;

means, whose input is connected to the output of the integrating means, for amplifying the integrated signal;

a first summing means, whose two inputs are connected to the outputs of the means for sampling and holding and of the amplifying means, for summing its two input signals, the output signal being G_0, the zero frequency term of the transform;

a first permuting means, connected to the said other connection of the connecting means, for performing a permutation on its input signal;

means, whose input is connected to the output of the first permuting means, for filtering the permuted signals; and

second means, having inputs connected to the outputs of the sampling and holding means and of the filtering means, for summing its input signals, and whose output comprises the transformed terms G_1....., G_{N-1}.

2. The apparatus according to claim **1**, further comprising:

a second permuting means, interconnected between the filtering means and the second summing means, for permuting its input signal.

3. The apparatus according to claim **1**, for processing a complex input signal, wherein:

the means for sampling and holding is a complex means;

the means for integrating is a complex integrator, comprising two real integrators, one each for processing the real and imaginary parts of a complex input signal;

the amplifying means is a complex amplifier, comprising a pair of real amplifiers, having a gain of one;

the first and second permuting means are complex permuters, that is, comprise a real channel permuter and an imaginary channel permuter;

the means for filtering is a complex filter; and

the two summing means are complex summers;

the apparatus thereby being capable of taking the prime discrete Fourier transform of a complex input signal.

4. The apparatus according to claim **3** wherein:

the complex means for filtering comprises:
two pairs of transversal filters, one pair for filtering the real part of a complex input signal and the other for filtering the imaginary part of the complex input signal; and
two real summers one connected to each of the pairs of summers.

5. The apparatus according to claim 3, wherein:
the complex means for filtering comprises:
three transversal filters; and
three summers, connected to the three filters.

6. The apparatus according to claim 1, wherein:
the means for sampling and holding is a real means;
the means for integrating is a real integrator;
the means for amplifying is a real amplifier having a gain of two;
the first and second permuting means are real permuters;
the means for filtering is a real filter; and
the two summing means are real summers;
the apparatus thereby being capable of taking the odd discrete cosine transform (ODCT) of an input signal which is real.

7. The apparatus according to claim 6, wherein:
the real means for filtering comprises a transversal filter.

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