Nov. 9, 1948. L. E. NORTON 2,453,453

FREQUENCY MODULATION SYSTEM

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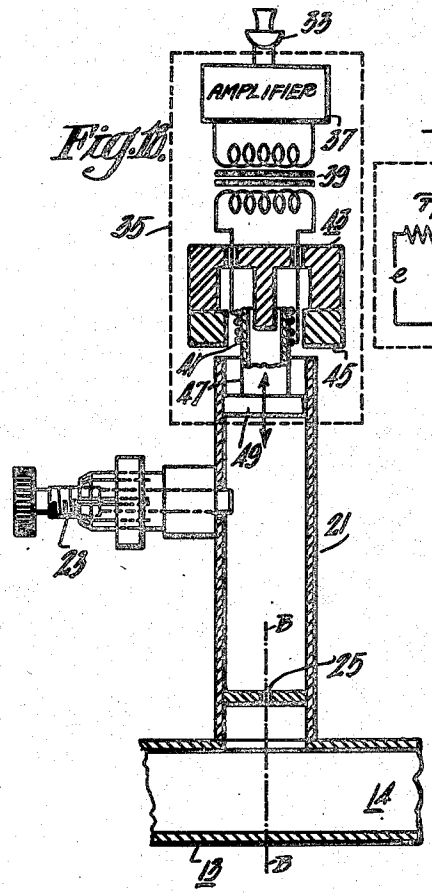


Fig. 1a.

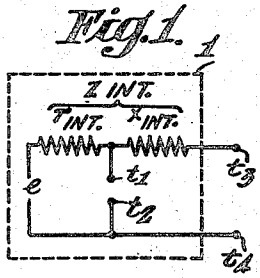


Fig. 1.

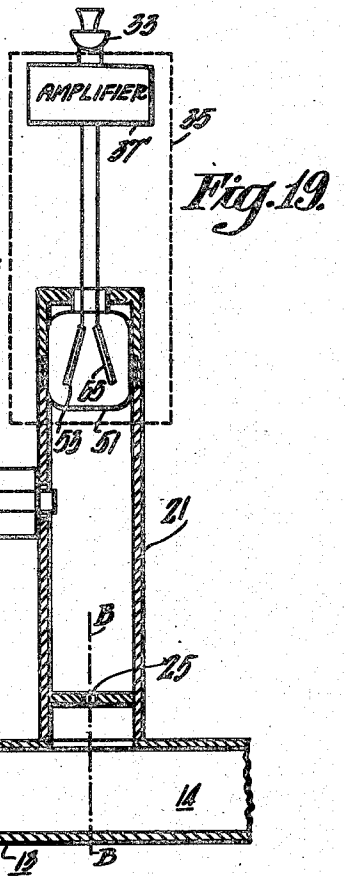


Fig. 1b.

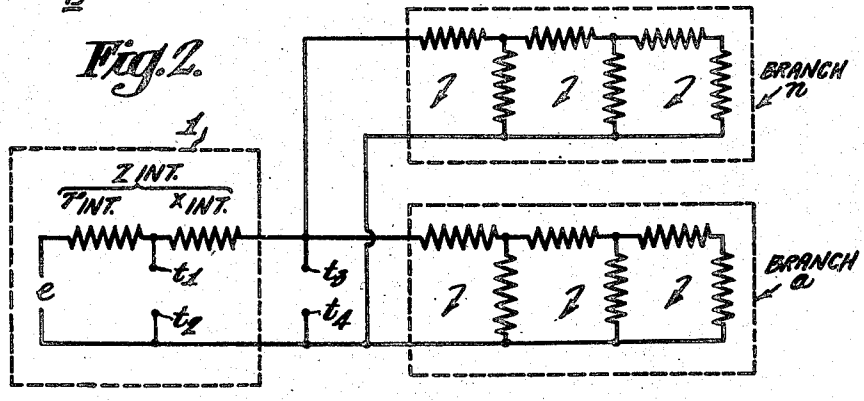


Fig. 2.

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FREQUENCY MODULATION SYSTEM

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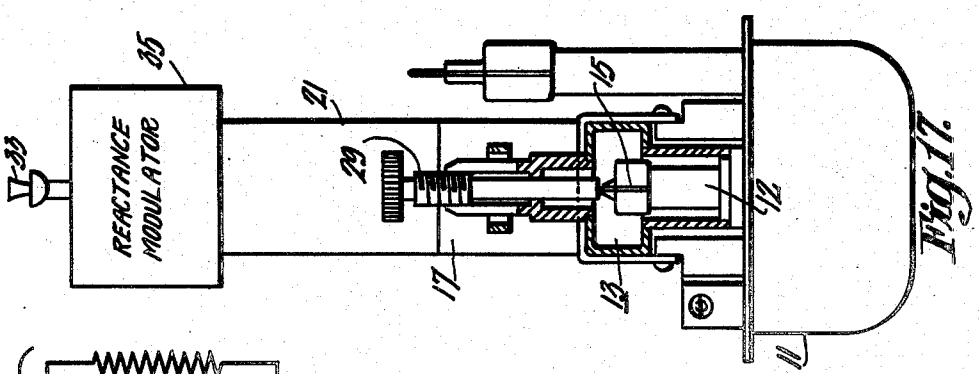


Fig. 17.

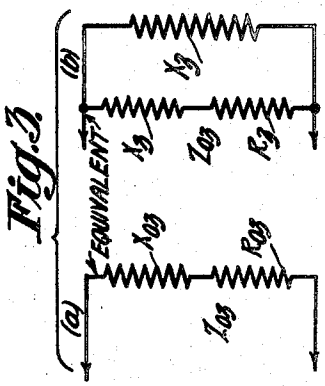


Fig. 3.

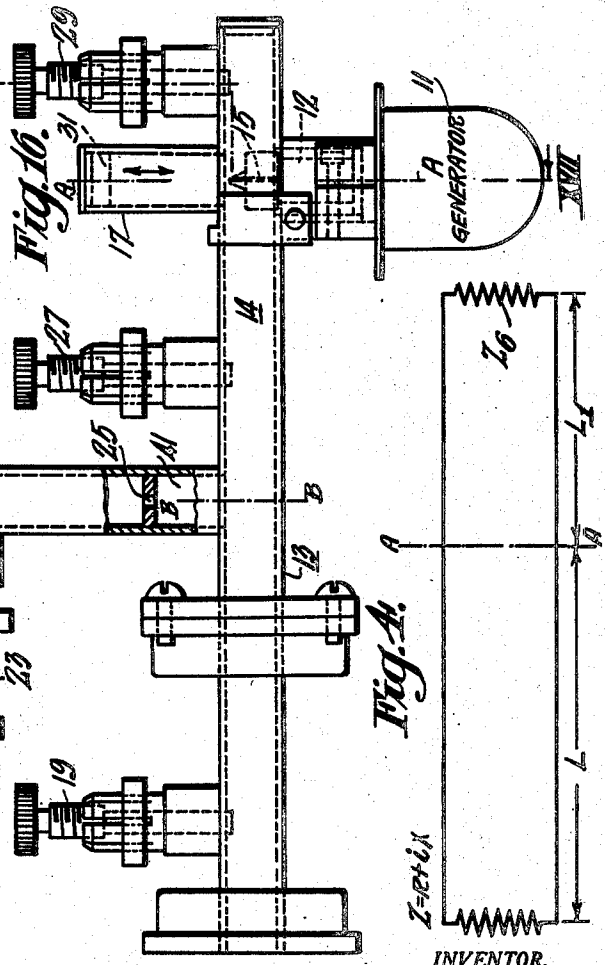


Fig. 16.

Fig. 4.

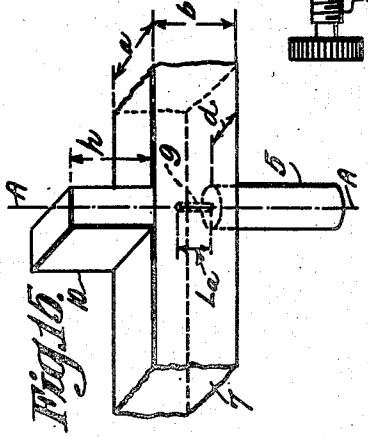


Fig. 15.

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FREQUENCY MODULATION SYSTEM

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Fig. 5.

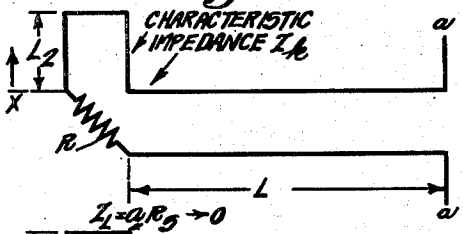


Fig. 6.

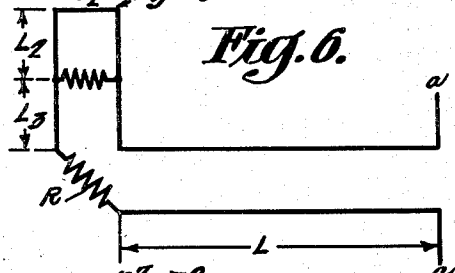


Fig. 7.

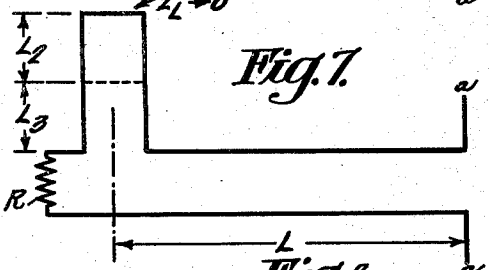


Fig. 8.

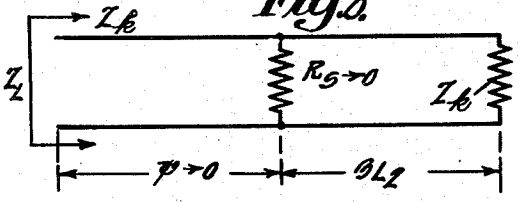


Fig. 9.

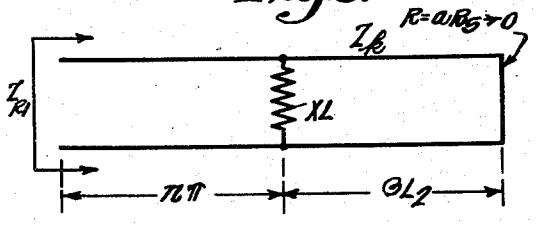


Fig. 10.

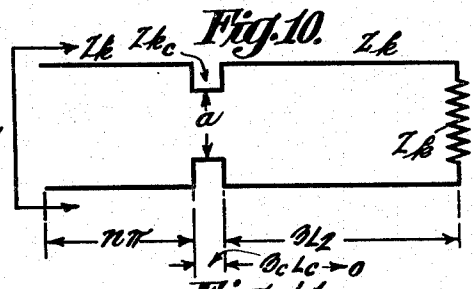


Fig. 11.

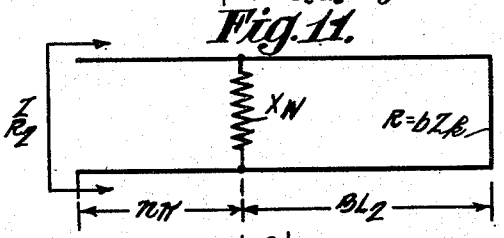
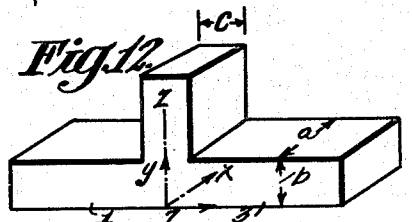


Fig. 12.



a=λ. WHERE λ. IS FREE SPACE WAVE LENGTH
b≤a AND SPECIAL CASE WHERE C=b

Fig. 13.

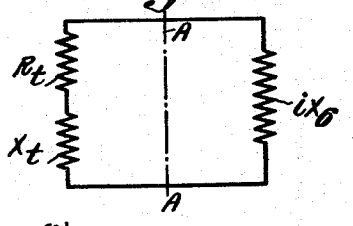
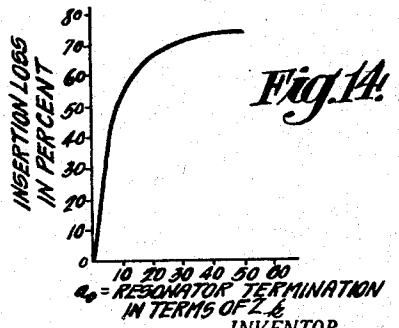


Fig. 14.



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FREQUENCY MODULATION SYSTEM

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Fig. 20.

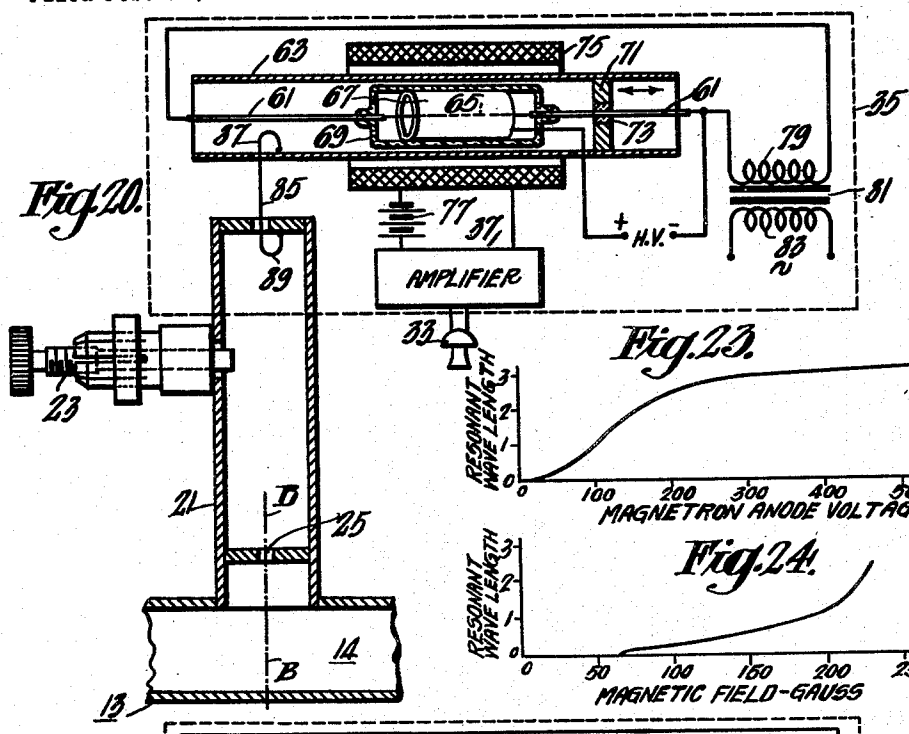


Fig. 23.

Fig. 24.

Fig. 21.

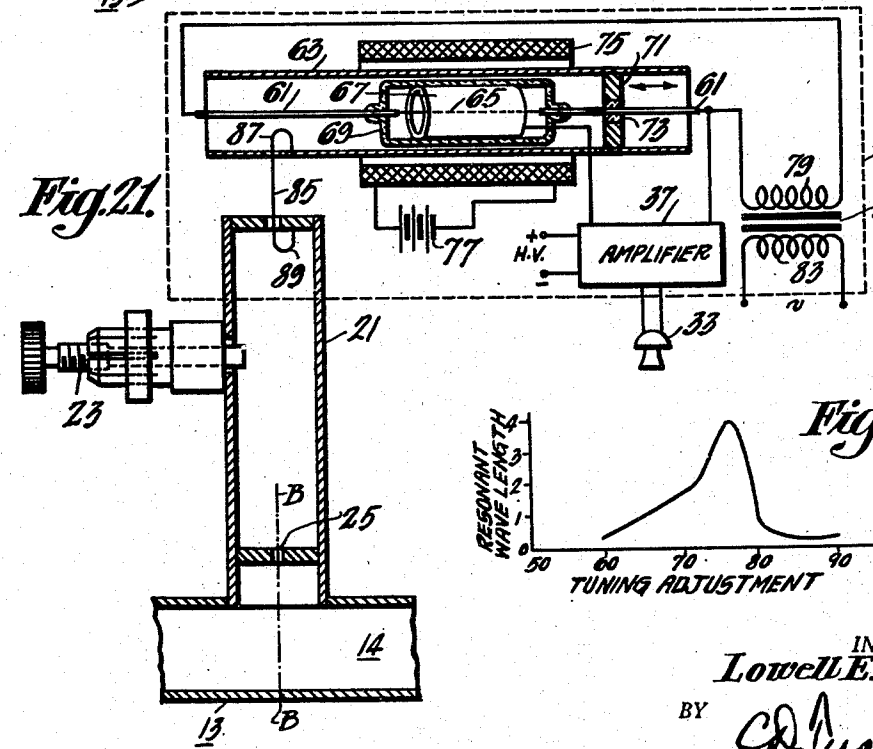


Fig. 25.

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FREQUENCY MODULATION SYSTEM

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Fig. 22.

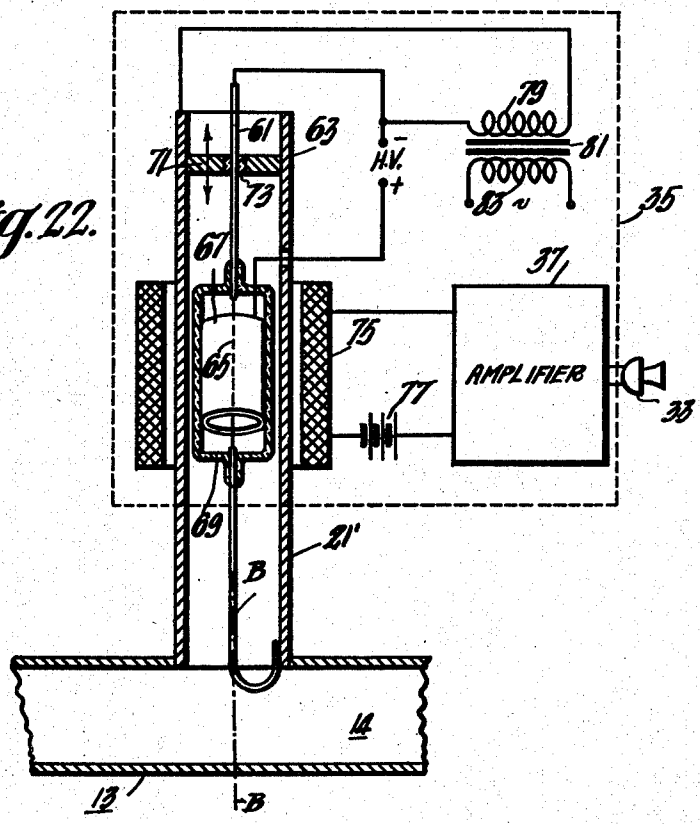
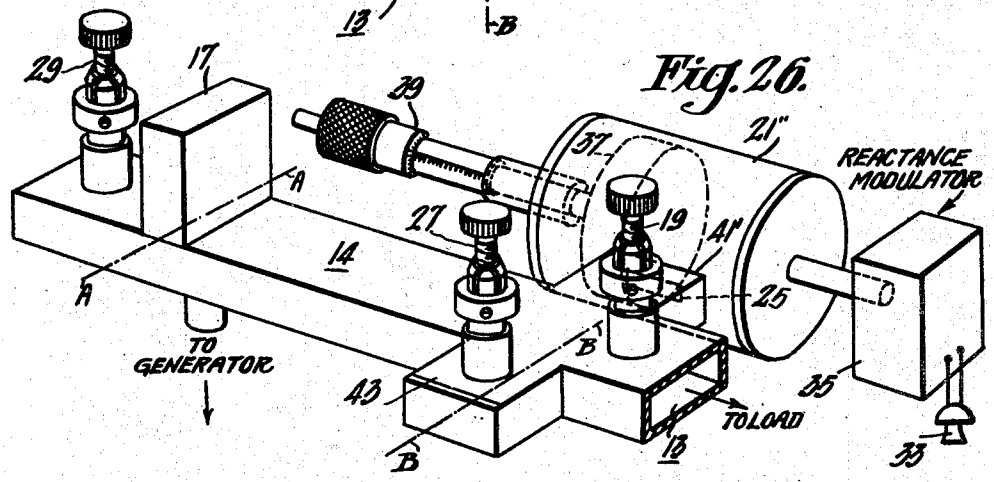


Fig. 26.



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Patented Nov. 9, 1948

2,453,453

UNITED STATES PATENT OFFICE

2,453,453

FREQUENCY MODULATION SYSTEM

Lowell E. Norton, Princeton Junction, N. J., assignor to Radio Corporation of America, a corporation of Delaware

Application February 26, 1945, Serial No. 579,767

19 Claims. (Cl. 179—171.5)

1

This invention relates generally to signal transmission systems and more particularly to improved methods of and means for modulating the frequency of signal generating systems by means of control resonator networks, the characteristics of which are varied in response to signal intelligence.

Substantially linear frequency modulation of magnetrons or other ultra-high frequency generators has become of increasing importance due to the widespread use of such devices as sources of ultra-high frequency energy. Stable and linear frequency modulation is particularly necessary when the magnetron or other high frequency source must operate unattended for comparatively long intervals during which ambient temperatures, terminal impedances, operating potentials and currents and magnetic field characteristics may fluctuate or vary between wide limits.

The methods of and means for frequency modulating such devices, to be described in detail hereinafter, may be employed at any frequency. They are particularly convenient when the frequency is sufficiently high so that the use of waveguide or transmission line sections having distributed reactance characteristics are feasible. At lower frequencies, artificial lines comprising lumped circuit components may be substituted for lines having distributed constants. At any frequency, equivalent circuits comprising combinations of suitable mutual reactances, lumped or distributed reactances and resistances, may be employed.

It is well known that the frequency of a generator which includes two resonant circuits having relatively widely different "Q" values will be controlled by the resonant circuit having the higher "Q" value, and more effectively controlled if the higher "Q" circuit has the lower resonant resistance. It also is well known that other parameters being equal, higher "Q" resonant circuits, provide improved frequency stability in such generators.

The instant invention provides improved, substantially linear frequency modulation over that attainable with prior systems, and although somewhat unorthodox in theory and application, it is readily adaptable to the frequency modulation of signal generators at any frequency. The frequency modulating network comprises a high "Q" variable impedance of a complex nature which is transformed to a new impedance with a higher "Q" and lower absolute value, and which is then parallel-resonated with another

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reactance of opposite sign. The variable "synthetic" control resonator thus provided is parallel-connected to the generator and load, and the impedance of the control resonator is varied in response to signal intelligence to modulate the frequency of the generator.

Circuit elements having rapid rates of change of phase angle with frequency and critical optimum line lengths may be determined, as described in detail hereinafter, to provide practical circuit configurations. It is emphasized that the instant system differs basically from previous practice wherein the reactive elements of a high "Q" resonator are first combined, and then transformed to new values, the transformed resonator impedance being shunt-connected with the generator and load.

The present invention is an improvement over the system disclosed in my copending U. S. application, Serial No. 579,766, filed February 26, 1945, entitled "Method of and means for frequency stabilizing frequency generators," assigned to the same assignee as the instant application.

The system is effective in reducing undesirable frequency shifts due to variations in load impedance. For any terminal impedance of random phase and magnitude which will provide a standing wave ratio of not more than 1.5 in a waveguide system connecting the load and the stabilizer system, it is possible, in the absence of modulating signals, to limit the frequency shift to one part in fifteen thousand over the entire range of terminal impedances represented by these conditions. The improved operation provided by the instant system is due to the fact that the theoretical limit of the resultant effective "Q" of the control impedance may be shown to be one-half of the squared value

$$\left(\frac{Q^2}{2}\right)$$

of the "Q" of the control impedance. The limiting factor is the loss in the impedance transformation. The resultant high "Q" network combined with great phase sensitivity with respect to modulating potentials provides high sensitivity and fidelity of frequency modulation.

Among the objects of the invention are to provide improved methods of and means for modulating the output frequency of signal generators. Another object is to provide improved methods of and means for synthesizing control resonant circuits for modulating the frequency of carrier wave generators. An additional ob-

ject of the invention is to provide improved methods of and means for synthesizing a variable high "Q" resonant circuit, the impedance of which is varied in response to signal intelligence. A further object is to provide improved methods of and means for effectively raising the "Q" of a variable resonant circuit, the reactance of which is responsive to signal intelligence. Another object is to provide improved methods of and means for frequency-modulating carrier signal generators by electromagnetic means. A further object is to provide improved methods of and means for frequency modulating carrier signal generators by ionizable means. An additional object is to provide improved methods of and means for frequency modulating carrier signal generators by electronic variable reactance means responsive to signal intelligence. A still further object of the invention is to provide improved methods of and means for providing a high "Q" resonant circuit wherein a high "Q" variable impedance responsive to signal intelligence is transformed to a lower variable impedance value having a higher "Q", and then a predetermined value of said transformed variable impedance is parallel-resonated with a reactance of opposite sign.

The prior art and the instant invention, both as to theory and practical circuit configurations, will be described in greater detail by reference to the accompanying drawings of which Figures 1 to 13 are schematic circuit diagrams illustrating the mathematical discussion of the features of the invention; Figure 14 is a graph illustrating said mathematical treatment of the invention; Figure 15 is a fragmentary perspective view of the coupling between the carrier signal generator and the frequency modulation network; Figure 16 is a side elevational view of a basic embodiment of the invention; Figure 17 is a cross-sectional end elevational view of said basic embodiment of the invention taken along the section line XVII; Figure 18 is a fragmentary, partially schematic, side elevational view of a first or electromagnetic embodiment of the invention; Figure 19 is a fragmentary, partially schematic, side elevational cross-sectional view of a second or ionizable embodiment of the invention; Figure 20 is a fragmentary, partially schematic, side elevational, cross-sectional view of a third or electronic embodiment of the invention; Figure 21 is a fragmentary, partially schematic, side elevational, cross-sectional view of a first modification of said third embodiment of the invention; Figure 22 is a fragmentary, partially schematic, side elevational, cross-sectional view of a second modification of said third embodiment of the invention; Figures 23, 24 and 25 are graphs illustrating the operating characteristics of said third embodiment of the invention and said modifications thereof; and Figure 26 is a fragmentary perspective view of a modification of said basic embodiment of the invention illustrated more completely in Figs. 16 and 17. Similar reference characters are applied to similar elements throughout the drawing.

Any oscillator or generator used as a source of energy has certain characteristics which for convenience may be expressed in terms of impedance concepts. The resulting equivalent circuits may contain many elements which may or may not be linear with certain parameters such as frequency. However, under any one set of operating conditions, this complicated equivalent circuit may always be replaced by a "characteristicless" potential, e, which is independent of frequency or external impedances, and a single impedance, Z_{int}, as shown in Figure 1. If the operating parameters are changed even slightly it is possible for the new, simple equivalent circuit to contain a radically different potential and impedance.

The generator 1, may be considered as having load terminals t_3 and t_4. It would be advantageous to have terminals t_1 and t_2 available, but terminals t_3 and t_4 are the only ones which are accessible. However, if Z_{int} is of the form

$$Z_{int}=r_{int}+ix_{int} \quad (1)$$

it will always be possible, by proper choice of an external circuit, to tune out the reactive term with an external series reactance x_{ext} which is

$$x_{ext}=-x_{int} \quad (2)$$

so that the oscillator may then be represented by the potential, e, in series with the resistance, r_{int}.

In Figure 2 the oscillator is shown as connected to any external terminal impedance composed of m branches each of which contains m_n branches. If all of the elements in the external circuit of Figure 2 are pure reactances, with no real terms and if $r_{int}\rightarrow 0$, then the actual external circuit configuration is unimportant since the rate of phase change, with frequency, of the external circuit at terminals t_3—t_4 no longer depends upon the actual circuit configuration.

Actually a circuit with this limitation is of no value since by specifying that the elements are all reactive, it is assumed that the system will not be allowed to radiate or to supply power to a load. To be useful, at least one of the impedances of the external circuit must be complex and therefore contain a real term. By introducing only one real term in the external circuit of Figure 2 the actual circuit configuration becomes important since the rate of phase change, with frequency, at terminals t_3—t_4 will depend upon the particular configuration.

If Z_{int}, is controllable, then it must be so chosen that a large rate of phase change with frequency at t_3—t_4 will also produce a large rate of phase change at t_1—t_2. If there is no control of x_{int}, then the external circuit must produce a large rate of phase change with frequency at t_1—t_2 and not necessarily at t_3—t_4.

One other practical restriction must be imposed on the external circuits. At very high frequencies it is difficult to measure impedances and other quantities related to potentials and currents in composite circuits. Therefore the final circuit must not be so complex as to preclude the possibility of accurate adjustments of the circuit elements. Also, if possible, critical and radical changes in circuit operation which will be easily detectable in monitoring equipment used during adjustment should occur near optimum adjustment of critical circuit values, so that proper initial adjustment may be made.

Obviously, an infinite number of circuit configurations is possible. To insure a correct approach to the problem without becoming quite hopelessly involved in too many possible circuit forms, it is desirable to make certain reasonable aproximations which transform the problem to a much simpler one. The validity of these initial approximations may be examined when a tentative frequency modulation network has been found.

The basic circuit of the instant invention is shown in Figure 4. The impedance Z_1 will be the parallel impedance of the two branches in

the plane A—A. The lines are of length L_1 and L_2 and may be either wave guides, transmission lines, artificial lines, or equivalent circuits containing mutual reactances and impedances.

Referring to Figure 4, it is convenient to let the "characteristic impedances" of lines L and L_1 be alike and represented by Z_k, and the propagation constant by $P=\alpha+i\beta$.

Actually, of course, the lines will have some attenuation so that transmission equations will in general have exponential attenuating factors, and the line equations may be written conveniently with the use of the usual hyperbolic functions. However, to simplify the expressions greatly, it may be assumed that the distributed series resistance of the line per unit length approaches zero and the distributed shunt resistance of the line per unit length approaches infinity. Thus the hyperbolic functions may be replaced by their corresponding trigonometric functions. Accordingly, the impedance Z_1 in the plane A—A is

$$Z_1=\frac{\dfrac{R+ix}{Z_k}\cos\beta L+i\sin\beta L}{\cos\beta L+i\dfrac{R+ix}{Z_k}\sin\beta L}\cdot\frac{\dfrac{R_6+ix_6}{Z_k}\cos\beta L_1+i\sin\beta L_1}{\cos\beta L_1+\dfrac{R_6+ix_6}{Z_k}\sin\beta L_1}}{\dfrac{\dfrac{R+ix}{Z_k}\cos\beta L+i\sin\beta L}{\cos\beta L+i\dfrac{R+ix}{Z_k}\sin\beta L}+\dfrac{\dfrac{R_6+ix_6}{Z_k}\cos\beta L_1+i\sin\beta L_1}{\cos\beta L_1+i\dfrac{R_6+ix_6}{Z_k}\sin\beta L_1}}Z_k \tag{3}$$

Rationalizing, an exceedingly cumbersome expression for Z_1 is obtained, which may be written in simplified form

$$Z_1=\rho+i\rho' \tag{4}$$

in which ρ and ρ' are each functions of R, x, R_6, x_6, Z_k, βL, and βL_1.

The various circuit constants must be selected and related so that the maximum rate of phase change with frequency in Z_1 at A—A is obtained. This may be accomplished by making

$$\frac{\gamma^2\left(\frac{\rho'}{\rho}\right)}{\gamma R^2}=0 \text{ for a maximum} \tag{5}$$

$$\frac{\gamma^2\left(\frac{\rho'}{\rho}\right)}{\gamma x^2}=0 \tag{6}$$

$$\frac{\gamma^2\left(\frac{\rho'}{\rho}\right)}{\gamma R_6^2}=0 \tag{7}$$

$$\frac{\gamma^2\left(\frac{\rho'}{\rho}\right)}{\gamma x_6^2}=0 \tag{8}$$

$$\frac{\gamma^2\left(\frac{\rho'}{\rho}\right)}{\gamma(\beta L)^2}=0 \tag{9}$$

$$\frac{\gamma^2\left(\frac{\rho'}{\rho}\right)}{\gamma(\beta L_1)^2}=0 \tag{10}$$

where x and x_6' are functions of frequency, and R is practically independent of frequency for small frequency changes Δf near f, but it is apparent that the solution of the simultaneous Equations 5 to 10 is impractical. However, consideration of the desired result in view of the circuit of Fig. 4 permits certain assumptions to be made which simplify the solution.

In Figure 4 let $L=0$, and let R be the load resistance directly (or some transformed load resistance). By connecting in series with R a

reactance x which is a function of frequency, one element of a parallel (referred to a plane through A—A) resonant circuit is provided. The other element is R_6+ix_6. For maximum rate of phase change with frequency at A—A, impedance $R+ix$ and R_6+ix_6 should have the highest possible Q values. By using a shorted section of low attenuation line it is seen that

$$\left.\begin{aligned}R_6&\to 0\\ Z_6&=ix_6\end{aligned}\right\} \tag{11}$$

The branch Z contains the load resistance R, which limits the upper value for x since, if x is too large, the resonant resistance at A—A will be too high. The attendant difficulties for a high resonant resistance have already been discussed. The result is that the Q of the branch $R+ix$ will be limited quite severely.

If, a line length L is selected which will transform the impedance $Z=R+ix$ to $Z_t=R_t+ix_t$ in which

$$\left.\begin{aligned}&\frac{x_t}{R_t}\gg\frac{x}{R}\\ \text{and}\quad&R^2+x^2\gg R_t^2+x_t^2\end{aligned}\right\} \tag{12}$$

The absolute value of impedance $|Z|$ can be transformed to a smaller numerical absolute value $|Z_t|$ and at the same time the ratio

$$\left|\frac{x}{R}\right|$$

is transformed to a larger ratio

$$\left|\frac{x_t}{R_t}\right|$$

This feature is very important.

At the plane A—A, looking to the left, the impedance Z_t is

$$Z_t=Z_k\frac{\dfrac{R+ix}{Z_k}\cos\beta L+i\sin\beta L}{\cos\beta L+i\dfrac{R+ix}{Z_k}\sin\beta L} \tag{13}$$

which reduces to a more convenient form

$$Z_t=Z_k\frac{\dfrac{R}{Z_k}+i\left\{\dfrac{x}{Z_k}\cos 2\beta L+\left[1-\left(\dfrac{x}{Z_k}\right)^2-\left(\dfrac{R}{Z_k}\right)^2\right]\dfrac{\sin 2\beta L}{2}\right\}}{\dfrac{1}{2}\left[1+\left(\dfrac{x}{Z_k}\right)^2+\left(\dfrac{R}{Z_k}\right)^2\right]-\dfrac{x}{Z_k}\sin 2\beta L+\left[1-\left(\dfrac{x}{Z_k}\right)^2-\left(\dfrac{R}{Z_k}\right)^2\right]\dfrac{\cos 2\beta L}{2}} \tag{14}$$

Instead of the inequalities of (12), it is convenient to write

$$t=\frac{\dfrac{x_t}{R_t}}{\dfrac{x}{R}} \tag{15}$$

where

$$t>1,\ \frac{x}{R}\gg 1,\text{ and }\frac{x_t}{R_t}\gg 1.$$

Thus t is the Q ratio of the transformed impedance to the untransformed impedance.

$$U^2=\frac{R^2+x^2}{R_t^2+x_t^2} \tag{16}$$

where $U>1$. The product Ut may be maximized where the line length βL will become the variable.

Considering frequency changes of the form

$$f=f_0(1+\delta)$$

where f_0 is the initial frequency, δ may be either positive or negative, and $\delta \ll 1$. Then

$$\frac{x^2}{R^2}=Q^2 \tag{17}$$

and

$$\frac{x_t^2}{R_t^2}=Q_t^2 \tag{18}$$

$$x^2+R^2=R^2(Q^2+1) \tag{19}$$

and

$$x_t^2+R_t^2=R_t^2(Q_t^2+1) \tag{20}$$

Using Equations 19 and 20 in 16, it is seen that

$$U=\frac{R}{R_t}\sqrt{\frac{Q^2+1}{Q_t^2+1}} \text{ and if } Q\gg 1,\ Q_t\gg 1$$

$$U=\frac{R}{R_t}=\frac{Q}{Q_t}=\frac{x}{x_t} \tag{21}$$

$$R_t^2=\frac{R^2}{U^2t^2} \tag{22}$$

From Equation 21

$$x_t^2=\frac{x^2}{U^2} \tag{23}$$

Since

$$|Z_t|^2=x_t^2+R_t^2$$

then

$$|Z_t|^2=\frac{1}{U^2}\left[\frac{R^2}{t^2}+x^2\right] \tag{24}$$

Since it is desirable to make the inequalities of (12) as great as possible, it follows that the conditions which make the product Ut a maximum are desirable. The conditions which make U and t reach their maximum values independently should be checked to be sure that the required conditions are not opposing.

From Equations 21 and 15

$$Ut=\frac{R}{R_t} \tag{25}$$

From Equations 25 and 14

$$Ut=\frac{1}{2}\left\{1-\frac{2x\sin 2\beta L}{Z_k}+\left(\frac{x}{Z_k}\right)^2+\left(\frac{R}{Z_k}\right)^2+\left[1-\left(\frac{x}{Z_k}\right)^2-\left(\frac{R}{Z_k}\right)^2\right]\cos 2\beta L\right\} \tag{26}$$

$$\frac{\gamma(Ut)}{\gamma\beta L}= \tag{27}$$

$$\left\{-\frac{2x}{Z_k}-\left[1-\left(\frac{x}{Z_k}\right)^2-\left(\frac{R}{Z_k}\right)^2\right]\tan 2\beta L\cos 2\beta L=0\right.$$

The roots are

$$\cos 2\beta L=0 \quad \beta L=\frac{\pi}{4},\ \frac{3\pi}{4}\ \ldots\ \frac{2n-1}{4}\pi \tag{28}$$

where n is any integer, and

$$\tan 2\beta L=-\frac{2x}{Z_k\left[1-\left(\frac{x}{Z_k}\right)^2-\left(\frac{R}{Z_k}\right)^2\right]} \tag{29}$$

If a matched load condition is assumed for the terminal impedance

$$R=Z_k \text{ but } x\neq 0$$

Equation 29 reduces to

$$\tan 2\beta L=\frac{2Z_k}{x}$$

and to fulfill Equation 17

$$\frac{Z_k}{x}\to 0$$

$$\tan 2\beta L\to 0 \text{ and } \beta L=\frac{(2n-1)}{2}\pi \tag{30}$$

n is any integer.

The conditions to be investigated occur, therefore, at

$$\left.\begin{matrix}\cos 2\beta L=0\\ \sin 2\beta L=1\end{matrix}\right\} \tag{31}$$

$$\left.\begin{matrix}\cos 2\beta L=0\\ \sin 2\beta L=-1\end{matrix}\right\} \tag{32}$$

$$\left.\begin{matrix}\cos 2\beta L=1\\ \sin 2\beta L=0\end{matrix}\right\} \tag{33}$$

and

$$\left.\begin{matrix}\cos 2\beta L=-1\\ \sin 2\beta L=0\end{matrix}\right\} \tag{34}$$

In each case R_t and x_t may be obtained by the use of Equation 14 and these quantities may be compared with R and x respectively. In each case assume

$$\left.\begin{matrix}R=Z_k\\ x>>Z_k\end{matrix}\right\} \tag{35}$$

Then for the first condition, Equation 31

$$Z_t=R_t+ix_t=\frac{2Z_k^3}{x^3}-iZ_k \tag{36}$$

or

$$R_t=\frac{2Z_k^3}{x^2}$$

and

$$x_t=-Z_k$$

The initial impedance Z was

$$Z=R+ix=Z_k+ix$$

so that the quantities

$$U,\ t,\ Q=\frac{x}{R},\text{ and } Q_t=\frac{x_t}{R_t}$$

may be compared

$$|U|=\frac{x}{x_t}=\frac{x}{Z_k} \tag{37}$$

$$|t|=\frac{x_tR}{R_tx}=\frac{x}{2Z_k} \tag{38}$$

$$Q=\frac{x}{R}=\frac{x}{Z_k} \tag{39}$$

$$Q_t=\frac{x_t}{R_t}=\frac{x^2}{2Z_k^2} \tag{40}$$

The interesting result is that

$$Q_t=\frac{Q^2}{2} \tag{41}$$

and at the same time

$$\sqrt{R_t^2+x_t^2}=\frac{\sqrt{R^2+x^2}}{Q}$$

It has been assumed that $Q\gg 1$, so if $Z=R+ix$ is parallel resonated with a resistance $x'\cong -x$ the resonant resistance is

$$R_{res}=xQ \tag{42}$$

while if

$$Z_t=\frac{2Z_k^3}{x^3}-iZ_k$$

is parallel resonated with a reactance $x_t'\cong -x_t$, the resonant resistance is

$$R_{t\ res}=x_tQ_t=\frac{xQ}{2}=\frac{R_{res}}{2} \tag{43}$$

It therefore appears that the initial Q has been increased to

$$\frac{Q^2}{2}$$

at the same time the parallel resonant resistance has been decreased from

$$R_{res} \text{ to } \frac{R_{res}}{2}$$

For the second condition of Equation 32

$$Z_t=R_t+ix_t=\frac{2Z_k^2}{x^2}+iZ_k \tag{44}$$

Correspondingly,

$$U=\frac{x}{x_t}=\frac{x}{Z_k} \tag{45}$$

$$t=\frac{x_tR}{R_tx}=\frac{x}{2Z_k} \tag{46}$$

$$Q=\frac{x}{R}=\frac{x}{Z_k} \tag{47}$$

$$Q_t=\frac{x_t}{R_t}=\frac{x^2}{2Z_k^2}=\frac{Q^2}{2} \tag{48}$$

$$\sqrt{R_t^2+x_t^2}=\frac{\sqrt{R^2+x^2}}{Q} \tag{49}$$

The initial ($Z=R+ix$ in parallel with $-ix$) resonant resistance is

$$R_{res}=\frac{xQ}{2} \tag{50}$$

while the transformed parallel resonant resistance is

$$R_{tres}=\frac{xQ}{2}=\frac{R_{res}}{2} \tag{51}$$

Apparently the only difference is that the reactance x_t changes sign.

For the third condition of Equation 33

$$Z_t=R_t+ix_t=Z_k+ix \tag{52}$$

and in similar fashion

$$U=\frac{x}{x_t}=1 \tag{53}$$

$$t=\frac{x_tR}{R_tx}=1 \tag{54}$$

$$Q=\frac{x}{R}=\frac{x}{Z_k} \tag{55}$$

$$Q_t=\frac{x_t}{R_t}=Q \tag{56}$$

$$\sqrt{R_t^2+x_t^2}=\sqrt{R^2+x^2} \tag{57}$$

For this condition nothing has been changed or gained by the transformation. For the fourth condition of Equation 34,

$$Z_t=\frac{Z_k^2}{x^2}-i\frac{Z_k^2}{x} \tag{58}$$

Continuing as before

$$U=\frac{x}{x_t}=\frac{x^2}{Z_k^2}=Q^2 \tag{59}$$

$$t=\frac{x_tR}{R_tx}=1 \tag{60}$$

$$Q=\frac{x}{R}=\frac{x}{Z_k} \tag{61}$$

$$Q_t=\frac{x_t}{R_t}=\frac{x}{Z_k} \tag{62}$$

or

$$Q_t=Q \tag{63}$$

$$\sqrt{R_t^2+x_t^2}=\frac{\sqrt{R^2+x^2}}{Q^2} \tag{64}$$

The initial ($Z=R+ix$ in parallel with $-ix$) resonant resistance is

$$R_{res}=xQ \tag{65}$$

while the transformed parallel resonant resistance is

$$R_{tres}=Z_k=\frac{R_{res}}{Q^2} \tag{66}$$

Therefore it may be concluded that the first two conditions are identical except for change of sign of the reactance x_t and both are very useful. Condition three is of no value and condition four must be compared with the first two.

The physical circuit configuration which will provide the arrangement of Figure 4 to the left of the plane A—A must be determined. This may be done with sections of transmission lines or with sections of wave guides having the necessary modifications. The circuit of Figure 5 will provide the arrangement of Figure 4 to the left of A—A. The circuit is composed of R, the shorted length of line L_2 and the line L. However, if the reactance x, where $x=Z_k \tan \beta L_2$, is large as compared to R, as is necessary, it will be difficult to deliver sufficient power to the resistance R. Therefore, it is convenient to substitute the circuit of Figure 6. The line L_2 initially is selected to be approximately $n\pi$ wavelengths long and the wavelength is varied from λ to $\lambda(1+\delta)$ where it is assumed that $\delta << 1$. Thus

$$\beta L_2=n\pi+\phi \tag{67}$$

and βL_2 changes in the following manner

$$\beta L_2=(n\pi+\phi)(1+\delta) \tag{68}$$

The resistance Rs, where $Rs \to 0$, is inserted to provide a decoupled resonator. The equivalent circuit, using sections of wave guides, is shown in Figure 7.

Accordingly, a rapid rate of reactance change in x with frequency is obtained. This condition will be as satisfactory as if $x>>R$ and the difficulty in delivering power to R is avoided. In Figure 6 the effect of the discontinuity introduced by inserting Rs as shown must be determined. This may be done by referring to the auxiliary circuit of Figure 8. It may be shown that the resonator above Rs, but including Rs, in Figure 6 reduces to the circuit of Figure 9.

In similar fashion the effect of the cut-off window between L_3 and the resonator L_2 in Figure 7 also must be determined. It is useful to introduce the auxiliary circuit of Figure 10 for this purpose. It may be shown that the resonator above the cut-off window, but including the window, in Figure 7 reduces to the circuit of Figure 11.

Apparently it is not generally appreciated that the resonators of Figure 9 or Figure 11 when viewed from Rs or the cut-off coupling window respectively, or at a plane $n\pi$ removed may be made to operate either as series type or parallel type resonators.

In the wave guide case this is effected by choice of window size as compared to that window size which matches the resonator to the wave guide. This fact is especially important. For simplicity it may be assumed that the lines to the right of x_L in Figure 9 and to the right of window x_W in Figure 11 have no attenuation. Then for series type operation as viewed from Z_{R1} and Z_{R2} respectively, x and cut-off guide $\beta_c L_c$, which is represented in Figure 11 by x_W, may be omitted for all practical purposes. At resonance in Figure 9 and Figure 11, β_{L2} will always be $n\pi$. In both cases the resonators will display all the characteristics of series resonators, and at resonance the resonant resistance will be low and approach 0 as $R \to 0$.

In many cases it is desirable to have Z_L and Z_W in Figures 9 and 11, respectively, exhibit the characteristics of parallel resonators with their char-

acteristic high resonant resistance. This may be done as follows:

In Figure 8, a line of characteristic impedance Z_k is shunted with a resistance $R_s \to 0$, and the termination Z_k is provided as shown. Then

$$Z_L = Z_k \frac{\frac{R_s}{Z_k+R_s}(1+\tan^2\psi)+i\tan\psi\left[1-\left(\frac{R_s}{Z_k+R_s}\right)^2\right]}{1+\left(\frac{R_s}{Z_k+R_s}\right)^2\tan^2\psi} \quad (69)$$

which for $R_s \to 0$ reduces to

$$Z_L = Z_k \frac{\frac{R_s}{Z_k}+i\psi}{1+\left(\frac{R_s}{Z_k}\psi\right)^2} = Z_k\left[\frac{R_s}{Z_k}+i\psi\right] \quad (70)$$

Disregarding the real part of Equation 70, since that is a term which will affect only the Q, the discontinuity provided by R_s is equivalent to the reactance ix_L in Figure 9.

The reason for introducing ϕ in Equation 67 is apparent. If ϕ is large enough so that the input reactance of βL_2 is negative, the arrangement of Figure 9 is obtained where ix_L and the negative reactance of line βL_2 which is $iZ_k \tan \beta L_2$, are parallel resonated. The line βL_2 is never $n\pi$ at resonance but instead is always in the second or fourth quadrants. The impedance Z_{R1} in Figure 9 accordingly may be a large resistance at resonance and has all the characteristics of a parallel resonator when viewed $n\pi$ to the left of x_L.

The same situation exists when wave guides are used with a thin cut-off window. In Figure 10 a thin section of cut-off guide is terminated in a section of guide of characteristic impedance Z_k which is terminated in turn by Z_k. In this case the propagation constant of the section cut-off guide is

$$\gamma = i\frac{2\pi}{\lambda_0}\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2} \quad (71)$$

where λ_0 is the free space wavelength and a is the width of the guide. As usual, beyond cut-off

$$\frac{\lambda_0}{2a} > 1$$

so that Equation 71 becomes

$$\gamma = -\frac{2\pi}{\lambda_0}\sqrt{\left(\frac{\lambda_0}{2a}\right)^2-1} = -\beta_c \quad (72)$$

The "characteristic impedance" of the section of cut-off guide Z_{kc} must be a negative imaginary so that

$$Z_{kc} = -i\mu Z_k \quad (73)$$

Then, the input impedance of the thin section of cut-off guide which is terminated in Z_k is

$$Z_W = Z_{kc}\frac{\frac{Z_k}{Z_{kc}}\cosh(\beta_c L_c)-\sinh(\beta_c L_c)}{\cosh(\beta_c L_c)-\frac{Z_k}{Z_{kc}}\sinh(\beta_c L_c)} \quad (74)$$

which reduces to

$$Z_W = Z_k \frac{1-\tanh^2(\beta_c L_c)+i\left(\mu+\frac{1}{\mu}\right)\tanh(\beta_c L_c)}{1+\frac{\tanh^2(\beta_c L_c)}{\mu^2}} \quad (75)$$

and for $\beta_c L_c$ small

$$Z_W = Z_k \frac{1-(\beta_c L_c)^2+i\left(\mu+\frac{1}{\mu}\right)(\beta_c L_c)}{1+\left(\frac{\beta_c L_c}{\mu}\right)^2} \quad (76)$$

For the special case where $\beta_c L_c << 1$

$$Z_W = 1+i\left(\mu+\frac{1}{\mu}\right)\beta_c L_c \quad (77)$$

so that it appears that the thin cut-off section of wave guide serves as a lumped reactance ix, which is

$$ix_W = i\left(\mu+\frac{1}{\mu}\right)\beta_c L_c \quad (78)$$

Again in Equation 67 the value of ϕ is chosen so that βL_2 in Figure 11 is in the second or fourth quadrant. The reactance ix_W is resonated with the negative reactance $Z_k \tan \beta L_2$ at the desired frequency. The resonant resistance is a function of

$$\frac{x_W}{Z_k \tan \beta L_2}$$

and increases with increasing

$$\frac{x_W}{Z_k \tan \beta L_2}$$

At any one frequency the magnitude of the parallel resonance resistance is controlled by proper choice of μ, $\beta_c L_c$, and βL_2. Again at resonance $\beta L_2 \neq n\pi$.

Since, for some applications, it may be advantageous to operate the resonators of Figures 9 and 11 as series resonators, the condtiions for this operation must be determined. The right hand termination in each case becomes $R \to 0$.

The impedance Z_{R1} must be determined. This time the line is $n\pi$ long initially at resonance, and changes from $n\pi$ to $n\pi(1+\delta)$ as the wavelength in the guide is changed. Again $R_s << 1$, and

$$\frac{n\pi}{1+\delta} = n\pi - \frac{n\pi\,\delta}{1+\delta} \quad (79)$$

Then

$$Z_{R1} = iZ_k \frac{\frac{aR_s}{Z_k}\left[\cos^2\left(n\pi-\frac{n\pi\delta}{1+\delta}\right)+\sin^2\left(\frac{n\pi\delta}{1+\delta}\right)\right]+ i\sin\left(\frac{n\pi\delta}{1+\delta}\right)\cos\left(n\pi-\frac{n\pi\delta}{1+\delta}\right)\left[\left(\frac{aR_s}{Z_k}\right)^2-1\right]}{\cos^2\left(n\pi-\frac{n\pi\delta}{1+\delta}\right)+\left[\frac{aR_s}{Z_k}\sin\left(\frac{n\pi\delta}{1+\delta}\right)\right]^2} \quad (80)$$

And when Z_{R1} is connected to parallel with R_s,

$$Z' = \frac{R_s Z_k\left\{\frac{aR_s}{Z_k}\left[\cos^2\left(n\pi-\frac{n\pi\delta}{1+\delta}\right)+\sin^2\left(\frac{n\pi\delta}{1+\delta}\right)\right]+i\sin\left(\frac{n\pi\delta}{1+\delta}\right)\cos\left(n\pi-\frac{n\pi\delta}{1+\delta}\right)\left[\left(\frac{aR_s}{Z_k}\right)^2-1\right]\right\}}{(1+a)R_s\cos^2\left(n\pi-\frac{n\pi\delta}{1+\delta}\right)+R_s\left[\frac{aR_s}{Z_k}\sin\left(\frac{n\pi\delta}{1+\delta}\right)\right]^2+aR_s\sin\left(\frac{n\pi\delta}{1+\delta}\right)+iZ_k\sin\left(\frac{n\pi\delta}{1+\delta}\right)\cos\left(n\pi-\frac{n\pi\delta}{1+\delta}\right)\left[\left(\frac{aR_s}{Z_k}\right)^2-1\right]} \quad (81)$$

For the symmetrical case $a=1$, and n even, Equation 81 reduces to

$$Z' = \frac{R_s}{2}\frac{1-i\frac{Z_k n\pi\delta}{2R_s(1+\delta)}}{1+\left[\frac{Z_k n\pi\delta}{2R_s(1+\delta)}\right]^2} \quad (82)$$

and is for $\delta << 1$

$$Z' = \frac{R_s}{2}\frac{1-\frac{iZ_k n\pi\delta}{2R_s}}{1+\left[\left(\frac{Z_k n\pi\delta}{2R_s}\right)\right]^2} \quad (83)$$

As would be expected Z' has the characteristic form of impedance variation of a resonator.

If line L_2 is considered to be the resonator and if the equivalent Q is expressed as

$$Q_r=\frac{Z_k n\pi}{4R_S} \tag{84}$$

then Equation 71 converts to the characteristic form

$$Z'=\frac{R_S}{2}\frac{1-i2Q_r\delta}{1+4Q_r\delta^2} \tag{85}$$

and the phase change of Z' with frequency is proportional to Q_r as would be expected.

Similarly, for Figure 11, βL_2 will be $n\pi$ at resonance and change to $n\pi(1+\delta)$ with changes in wavelength in the guide. The termination for the thin cut-off guide is

$$Z_S=Z_k\frac{b+i\tan[n\pi(1+\delta)]}{1+ib\tan[n\pi(1+\delta)]} \tag{86}$$

$$Z_S=\frac{b\{1+\tan[n\pi(1+\delta)]\}+i(1-b^2)\tan[n\pi(1+\delta)]}{1+b^2\tan^2[n\pi(1+\delta)]} \tag{87}$$

or $Z_S=C+iD$ where

$$C=\frac{b\{1+\tan[n\pi(1+\delta)]\}}{1+b^2\tan^2[n\pi(1+\delta)]} \tag{88}$$

$$D=\frac{(1-b^2)\tan[n\pi(1+\delta)]}{1+b^2\tan^2[n\pi(1+\delta)]} \tag{89}$$

$$Z_{R2}=\frac{\frac{C}{\mu Z_k}\cos(2\beta_c L_c)+\left[1+\frac{(D^2-C^2)}{(\mu Z_k)^2}\right]\frac{\sin(2\beta_c L_c)}{2}+i\left[\frac{D\cos(2\beta_c L_c)}{(\mu Z_k)}-\frac{CD\sin(2\beta_c L_c)}{(\mu Z_k)^2}\right]}{\frac{1}{2}+\frac{\cos(2\beta_c L_c)}{2}-\frac{C\sin\beta_c L_c}{\mu Z_k}+\frac{1-\cos(2\beta_c L_c)}{2(\mu Z_k)^2}(C^2+D^2)}Z_k \tag{90}$$

Since the impedance change near resonance is to be used for frequency control purposes, it is desirable to operate the cavity resonator near cut off. Accordingly the change in wavelength in a rectangular guide (also used as a resonator in this application because of convenience) λ_g, either with frequency or its inverse, λ_0, the free space wavelength is important.

The guide has cross section ab and $a>b$. The wavelength in the guide is

$$\lambda_g=\frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}} \tag{91}$$

Then

$$\frac{d\lambda_g}{d_0}=\frac{1-\left(\frac{\lambda_0}{2a}\right)+\left(\frac{\lambda_0}{2a}\right)^2}{\left[1-\left(\frac{\lambda_0}{2a}\right)^2\right]^{3/2}} \tag{92}$$

or

$$d\lambda_g=\frac{d\lambda_0}{\left[1-\left(\frac{\lambda_0}{2a}\right)^2\right]^{3/2}}$$

For standard X-band guide $2a=1.80$ inches and at 9310 megacycles per second $\lambda_0=1.27$ inches, so that

$$d\lambda_g=2.82\,d\lambda_0 \tag{93}$$

which indicates that the rate of change of wavelength in the guide is 2.82 times the rate of change of frequency. The factor 2.82 would, of course, be greater by operating nearer cut-off.

Considering the circuit of Figure 7 with the T junction shown in Figure 12, under conditions wherein

$$a>\frac{\lambda_0}{2}$$

where λ_0 is the free space wavelength, $b<a$, and the special case where $c=b$.

Y_1 is the normalized admittance extrapolated to $Z=0$ for the left leg. $Z>0$. Y_3 is the normalized admittance extrapolated to $Z=0$ for the right leg. $Z>0$.

Y_2 is the normalized admittance extrapolated to $y=b$ for the vertical leg. $y>b$.

$Y_1=Y_3=1$ for a travelling wave propagated in the +Z direction.

$Y_1=Y_3=-1$ for a travelling wave propagated in the −Z direction.

For the circuit configuration of Figure 6, Y_1, Y_2, and Y_3 are related by the following two equations.

$$Y_1=\frac{[1+a_2(-a_1+ia_3Y_2)]Y_3+2(ia_1+a_3Y_2)}{1+a_2(-a_1+ia_3Y_2+i2Y_3)} \tag{94}$$

$$Y_3=\frac{[1+a_2(-a_1+ia_3Y_2)]Y_1-2(ia_1+a_3Y_2)}{1+a_2(-a_1+ia_3Y_2-2iY_1)} \tag{95}$$

where a_1, a_2, and a_3 are real positive numbers which are functions of b/λ_g and c/b, where λ_g is the wavelength of the longest TE wave propagated in the guide. For this case assume $b=c$. Then $a_1=.368$, $a_2=10.4$, and $a_3=2.49$ for a standard x-band wave guide.

Considering the case where wave propagation is from the left, where the vertical branch has an admittance Y_2, and where the right hand branch has a matched termination

$$Y_3=1 \tag{96}$$

and Equation 95 becomes

$$Y_3=\frac{[1+10.4(-.368+i2.49Y_2)]Y_1-2(i.368+2.49Y_2)}{1+10.4(-.368+i2.49Y_2-2iY_1)}=1 \tag{97}$$

or

$$Y_1=\frac{-2.82+4.98Y_2+i(25.9Y_2+.735)}{-2.82+i(25.9Y_2+20.8)} \tag{98}$$

It is sometimes easier to use a rationalized form of Equation 98. If Y_2 is of the form

$$Y_2=y(\cos\theta+i\sin\theta) \tag{99}$$

$$Y_1=\frac{23.25+249.8y\sin\theta+544y\cos\theta+671y^2+i(56.52+505y\sin\theta-103.8y\cos\theta-129\,y^2)}{438.9+146y\sin\theta+1080y\cos\theta+671y^2} \tag{100}$$

In Figure 7 the resonant resistance of the resonator may be expressed in terms of the characteristic impedance of the line L_3 and may be considered as a termination for L_3. The impedance of the resonator off resonance as a termination for L_3 should be considered to determine the rate of change of phase angle and reactance with frequency. It is convenient to consider the resonant resistance and impedance values ½Q above and below the resonant frequency. The resonant resistance is a_0Z_k where Z_k is the characteristic impedance of L_3 and a_0Z_k is either greater than, or less than Z_k, as has been shown. The impedance at a frequency ½Q higher than the resonant frequency will be

$$\frac{a_0Z_k}{2}(1-i)$$

and the impedance at a frequency ½Q lower than the resonant frequency will be

$$\frac{a_0Z_k}{2}(1+i)$$

To complete the frequency control circuit of Figure 4, it is only necessary to consider various terminations for L_3 in Figure 7 which involve

various values of a_0, and the transforming action of L_3. This terminating admittance is the Y_2 of Equation 98. The admittance Y_1 is then the termination for line L in Figure 4. Line L transforms Y_1 to a new admittance at the plane A—A. The corresponding impedance at A—A is then parallel resonated with the reactance provided by line L_1.

This final resonator is then connected directly to the oscillator, or to the oscillator through a coupling line or circuit.

It is necessary first to determine the line length L_3 in Figure 7 which will give maximum rate of phase change at the junction with line L, all for a change in phase at the resonator, which is then the termination for L_3. The resonator termination in terms of Z_k is a_0Z_k. At resonance a_0 is a real positive number, and off resonance it is complex with a positive real part, so that

$$a_0=a_1+ia_2 \tag{101}$$

Accordingly, the normalized input impedance of line L_3 as viewed from line L is

$$Z_c=a_1(1+\tan^2 L_3)+\frac{i[a_2(1-\tan^2 \beta L_3)+(1-a_1^2-a_2^2)\tan \beta L_3]}{1-2a_2 \tan \beta L_3+(a_1^2+a_2^2)\tan \beta L_3} \tag{102}$$

and the phase angle is measured by

$$P=k\frac{a_2(1-\tan^2 \beta L_3)+(1-a_1^2-a_2^2)\tan \beta L_3}{a_1(1+\tan^2 \beta L_3)} \tag{103}$$

$$\frac{1}{Z_c}$$

in Equation 102 is the Y_2 of Equation 98.

Rewriting Equation 103

$$P=k\frac{a_2(\cos^2 \beta L_3-\sin^2 \beta L_3)+(1-a_1^2-a_2^2)\sin \beta L_3 \cos \beta L_3}{a_1} \tag{104}$$

or

$$P=k\frac{a_2 \cos^2 \beta L_3+\frac{1-a_1^2-a_2^2}{2}\sin \beta L_3}{a_1} \tag{105}$$

To obtain the maximum rate of change of P with βL_3 and a_2

$$\frac{\gamma^2_\rho}{\gamma \beta L_3^2}=\frac{k}{a_1}[-2(1-a_1^2-a_2^2)\sin 2\beta L_3-4a_2 \cos 2\beta L_3]=0 \tag{106}$$

$$\frac{\gamma^2_\rho}{\gamma a_2^2}=\frac{k}{a_1}[-2a_2 \cos 2\beta L_3-\sin 2\beta L_3]=0 \tag{107}$$

The region where a_2 is zero initially and then assumes small positive and negative values is of primary interest. Equations 106 and 107 give

$$\beta L_3=0, \pi/2, \pi. \ldots \tag{108}$$

Obviously, from Equations 108, 102 and 98, if Y_1 in Equation 98 is to undergo maximum rate of phase change with small changes in a_2, from zero, there are two choices for βL_3.

1. If the resonator termination on L_3 (Figure 7) operates as a parallel resonant circuit $a_0>1$ and $L_2 \neq n\pi$

$$\beta L_3=0, \pi, 2\pi \ldots n\pi \tag{109}$$

2. If the resonator termination on L_3 (Figure 7) operates as a series resonant circuit $a_0<1$ and $L_2=n\pi$ initially, then from Equation 108

$$\beta L_3=\frac{\pi}{2}, \frac{3\pi}{2} \ldots \frac{2n-1}{2}\pi \tag{110}$$

Condition 1 is preferred since the high ratio, and necessarily low attenuation

$$\frac{2n-1}{2}\pi$$

line transformer which is automatically incorporated in condition 2 is avoided. However, if extreme care is taken to make the

$$\frac{2n-1}{2}\pi$$

line transformer very low loss, condition 2 is also capable of providing good results. Since condition 1 is the preferred condition, it may be assumed in Figure 7, βL_2 (at resonance) $\neq n\pi$, $\beta L_3=n\pi$, but it should be understood that the alternative condition

$$\beta L_2 \text{ (at resonance)}=n\pi, \beta L_3=\frac{2n-1}{2}\pi$$

may be substituted, although the results then are likely to be poorer mainly because of the difficulty in making the

$$\frac{2n-1}{2}\pi$$

transformer good enough.

All that remains is to determine line length L in Figure 4. Equations 31, 32, 33, and 34 which result in Equations 41, 42, 48, 49, 56, 57, 63, and 64 indicate that βL should be either

$$\beta L=\frac{2n-1}{2}\pi \quad \text{or} \quad \beta L=\frac{2n-1}{4}\pi$$

The better condition may be determined. For various resonator terminations a_0Z_k for line L_2, where $a_0>1$, $\beta L_2=n\pi$. Equations 98 or 99 provide the values of Y_1 for the various indicated conditions of table I. Conditions at resonance and off resonance by $\pm\frac{1}{2}Q$ in frequency are shown.

Table I

[Resonator terminations a_0Z_k for line L_2 in Figure 7.]

$\beta L_3=n\pi$ Y_2 (normalized) $=\frac{1}{a_0}$ Y_1 in Equation 98.

Y_2	Y_1	Y_1
.02	.0743+i. 1240	.1448 / 59° 06′
.02 (1+i)	.0847+i. 1365	.1609 / 52° 77′
.02 (1−i)	.0640+i. 0958	.1155 / 56° 19′
.04	.0955+i. 1085	.1445 / 48° 39′
.04 (1+i)	.1170+i. 1480	.1890 / 51° 41′
.04 (1−i)	.0792+i. 0848	.1160 / 46° 58′
.10	.1520+i. 0809	.1720 / 28° 03′
.10 (1+i)	.1506+i. 1650	.2240 / 47° 45′
.10 (1−i)	.1214+i. 0127	.1220 /−5 ° 58′
.20	.239+i. 0793	.252 / 18° 53′
.20 (1+i)	.298+i. 2140	.367 / 35° 43′
.20 (1−i)	.232+i. 0735	.244 /−17° 36′
1.0	.564−i. 0803	.570 /−8 ° 08′
1.0 (1+i)	.538+i.565	.780 / 46° 24′
1.0 (1−i)	.785 − i. 2155	.815 / 15° 26′
5.0	.868−i. 164	.884 /−10° 42′
5.0 (1+i)	.937−i. 1098	.945 /−6 ° 40′
5.0 (1−i)	.909−i. 323	.965 /−19° 36′

Consider first the case where

$$\beta L=\left(\frac{2n-1}{4}\right)$$

and where $n=1$. As indicated heretofore, larger values of n merely increase the phase sensitivity, as is desirable, especially if the wave guide is

operating fairly near cut-off. The terminations for line L are

$$\frac{1}{Y_1}=Z$$

taken from Table I. Then, for

$$\beta L=\left(\frac{2n-1}{4}\right)$$

the transformed value of Z is Z_t which is

$$Z_t=R_t+ix_t=\frac{R+i(x+1)}{1-x+iR}=\frac{2R+i(1-R^2-x^2)}{1-2x+x^2+R^2} \quad (111)$$

As shown in Figure 13, Z_t is to be parallel resonated with $-ix_6$

$$Y_t=\frac{1}{Z_t}=g_t+ib_t \quad (112)$$

and

$$Y_6=-\frac{1}{ix_6}=ib_6 \quad (113)$$

When Y_t and Y_6 are connected in parallel the resultant admittance is

$$Y=Y_t+Y_6 \quad (114)$$

or

$$Y=g_t+i(b_t+b_6) \quad (115)$$

where

$$\left.\begin{aligned} b_t&=\frac{x_t}{R_t^2+x_t^2}\\ g_t&=\frac{R_t}{R_t^2+x_t^2}\\ b_6&=\frac{1}{x_6}\end{aligned}\right\} \quad (116)$$

At resonance $b_t+b_6=0$, so that, from Equation 116

$$x_6=\frac{R_t^2+x_t^2}{x_t} \quad (117)$$

At any other frequency x_t and R_t become x and r, respectively, and the parallel impedance becomes

$$Z_1=\frac{\frac{R_t^2+x_t^2}{x_t}\left\{\frac{R_t^2+x_t^2}{x_t}r+i\left[(R_t^2+x_t^2)\frac{x}{x_t}-(r^2+x^2)\right]\right\}}{r^2+\left[x-\left(\frac{R_t^2+x_t^2}{x_t}\right)\right]^2} \quad (118)$$

The impedance Z_1 is the final impedance applied to the oscillator either directly or through a coupling line or circuit. Values of Z_t and Z_1 for various values of Y_2 are shown in Table II.

Table II

[Resonator terminations a_0Z_k for line L_3 in Figure 7.]

$\beta L=\frac{2n-1}{4}\pi$ $\beta L_3=n\pi$ Y_2 (normalized) $=\frac{1}{a_0}$ Y_1 in Equation 98

Y_2	Z_t	Z_1
.02	.1175− i. 773= .780 /−81° 20′	5.60 Z_k
.02 (1+i)	.1294− i. 751= .760 /−80° 14′	(4.46 − i. 656)Z_k
.02 (1−i)	.1056−i. 822 = .820 /−82° 42′	(5.22+i2. 77)Z_k
.04	.145 − i. 775= .790 /−79° 25′	3.28 Z_k
.04 (1+i)	.1755− i. 729= .750 /−76° 27′	(3.13 − i. 588)Z_k
.04 (1−i)	.1335− i. 838= .850 /−80° 57′	(4.94+i1. 89)Z_k
.10	.251 − i. 820= .860 /−73° 0′	2.935 Z_k
.10 (1+i)	.291 − i. 693= .726 /−72° 26′	(1. 896− i. 633)Z_k
.10 (1−i)	.247 −i1.0 =1.03 /−76° 08′	(2. 815+i2. 02)Z_k
.20	.377 −i. 765 = .875 /−63° 50′	1.92 Z_k
.20 (1+i)	.380 −i. 558 = .659 /−55° 47′	(1. 133− i. 442)Z_k
.20 (1−i)	.520 −i1.04 =1.162 /−63° 26′	(1. 895+ i. 893)Z_k
1.0	.969 −i. 579 =1.13 /−30° 56′	1.31 Z_k
1.0 (1+i)	.394 −i. 143 = .419 /−19° 57′	(. 436−i. 0699)Z_k
1.0 (1−i)	1. 276 −i. 279 =1.309 /−12° 20′	(1. 17+i. 450)Z_k
5.0	1. 195 −i. 153 =1.202 /−7 ° 18′	.69 Z_k
5.0 (1+i)	1. 125 −i. 0693=1. 129 /−3 ° 31′	(. 639−i. 0652)Z_k
5.0 (1−i)	1. 42 −i. 056 =1. 42 /−2 ° 15′	(. 619+i. 1565)Z_k

Comparing the initial phase angle of the resonator, Y_2, and the phase angle of the control impedance Z_1, it is evident that the resonance characteristic of Z_1 is not symmetrical about the resonant frequency, but becomes approximately so in the region Y_2=0.2 to 0.1. Also, the phase angle change of the loaded resonator Z_1 in the same region Y_2=0.2 to 0.1 is equal to, or greater than, the original change in phase angle of Y_2.

It is possible to improve the symmetry about resonance by operating with $\beta L_3 \cong 13°\ 30'$ for which $Y_1=1$, but only at the expense of reduced rate of change of phase angle of Z_1. Use of an iris to provide a lumped reactance in βL_3 to provide the equivalent of operating βL_3 at length $\beta L_3=n\pi+13°\ 30'$, is possible.

The entire control network may be considered as a network made up of passive elements, and the insertion loss, defined in usual fashion, may then be computed. For the insertion loss to be significant it is necessary for the controlled oscillator to operate on the same point on its characteristic, and at the same frequency, with and without stabilizer.

The maximum power output for the oscillator, with potential e, working into a matched termination R_0

$$P=\left(\frac{eR_0}{R_0+R_0}\right)^2\frac{1}{R_0}=\frac{e^2}{4R_0} \quad (119)$$

which may be employed as a reference condition. The power delivered to a matched termination when the stabilizer is inserted may be computed readily. The difference in the power is the insertion loss. The power delivered to a matched termination for various values of Y_2 is shown in Table III. Insertion loss as a function of Y_2 is shown in the graph of Figure 14.

Table III

[Resonator terminations a_0Z_k for line L_3 in Figure 7.]

Y_2 (normalized) $=\frac{1}{a_0}$

Y_2	Power in Matched Termination	Insertion Loss
		Per cent
.02	.0643e^2	74.2
.04	.0789e^2	68.4
.10	.1122e^2	55.0
.20	.155e^2	38.0
1.0	.231e^2	7.6
5.0	.248e^2	0.9
	Reference power with no stabilizer is .25e^2.	

A good operating region extends from above a_0=3 to about a_0=10. The resonance characteristic is not quite symmetrical about the resonance frequency, but the average loaded Q for Z_1 for a_0=10 is about .72 times the unloaded Q of the resonator out on line L_3. If the Q of the resonator is 5,000 to 10,000 initially, stabilization to fractions of 1 megacycle per second in 10,000 megacycles per second for reasonable variations in operating parameters is feasible. For a_0=10 the insertion loss is 55%. The average loaded Q for the "synthetic resonator" Z_1, for a_0=5, is about one times the unloaded Q of the initial resonator and the resonance characteristic again is not quite symmetrical about the resonance frequency. At a_0=5 the insertion loss is 38%.

The performance does not change rapidly for values of a_0 between about three and ten. For a value of a_0=3, the insertion loss is about 23%. The final choice of a_0 is a compromise. Operation is somewhat better for a_0=5 to a_0=10, but not tremendously better than for a_0=3. Therefore,

any value of a_0 between about 3 and 10 is satisfactory.

The case where

$$\beta L=\frac{2n-1}{2}\pi$$

should be considered. For the same various values of a_0, take as terminations for line L values of

$$\frac{1}{Y_1}$$

from Table I. For simplicity, consider the case where $n=1$. It is evident that the line transforms

$$\frac{1}{Y_1} \text{ to } Z_t$$

which is (normalized)

$$Z_t=Y_1 \qquad (120)$$

As before, corresponding values of Z_1, the mean value of the control impedance may be computed from Equation 118. The results are shown in Table IV. Comparison of phase angles for Y_2 at resonance and at frequencies ½Q above and below resonance, with the corresponding phase angles of Z_1, indicates that the rate of phase angle change with frequency is much slower for Z_1 than for Y_2. Also conditions are much worse than those in Table II where

$$L \text{ was } \frac{2n-1}{4}\pi$$

Table IV

[Resonator terminations a_0Z_k for line L_3 in Figure 7.]

$L=\frac{2n-1}{2}\pi$ $\beta L_3=n\pi$ Y_2 (normalized) $=\frac{1}{a_0}$

Y_2 (normalized)	Z_t (normalized)	Z_1
.02	.0743+i. 1240	$.270Z_k$
.02 (1+i)	.0847+i. 1365	$(.257-i.1475)Z_k$
.02 (1−i)	.0640+i. 0958	$(.206+i.111)Z_k$
.04	.0955+i. 1085	$.222Z_k$
.04 (1+i)	.1170+i. 148	$(.278-i.181)Z_k$
.04 (1−i)	.0792+i. 0848	$(.1655+i.0818)Z_k$
.10	.1520+i. 0809	$.195Z_k$
.10 (1+i)	.1506+i. 165	$(.320-i.116)Z_k$
.10 (1−i)	.1214+i. 0127	$(.1175+i.389)Z_k$
.20	.239 +i. 0793	$.272Z_k$
.20 (1+i)	.298 +i. 214	$(.439-i.122)Z_k$
.20 (1−i)	.232 −i. 0735	$(.184+i.00625)Z_k$
1.0	.564 −i. 0803	$.576Z_k$
1.0 (1+i)	.538 +i. 565	$(.438+i.0376)Z_k$
1.0 (1−i)	.785 −i. 2155	$(.828+i.0619)Z_k$
5.0	.868 −i. 164	$.895Z_k$
5.0 (1+i)	.937 −i. 1098	$(.874+i.0245)Z_k$
5.0 (1−i)	.909 −i. 323	$(1.14+i.088)Z_k$

Equations (33) which lead to the results of Equations 56 and 57 indicate that a line $\beta L=n\pi$ will not give good results. This prediction may be checked by numerical calculations. For $\beta L=(2n-1)\pi$ and the special case where $n=1$.

$$Z_t=\frac{1}{Y_1} \qquad (121)$$

which is the normalized value of Zt. The results are shown in Table V. As predicted by the general analysis, the rate of phase angle change of Z_1 with frequency departures above and below the resonant frequency by ½Q is less than the rate of phase angle change of Y_2, and much poorer than the results shown in Table II. Accordingly, the line βL should be

$$\frac{2n-1}{4}\pi$$

in length. As indicated heretofore, n should be greater than unity to utilize the greater phase sensitivity of a longer line. Also, it is advisable to have the wave guide operate fairly near cut off so that the rate of change of wavelength in the guide is greater than the rate of change of frequency or of free space wavelength.

Table V

[Resonator termination a_0Z_k for line L_3 in Figure 7.]

$\beta L=n\pi$ $\beta L_3=n\pi$ Y_2 (normalized) $=\frac{1}{a_0}$

Y_2 (normalized)	Z_t (normalized)	Z_1
.02	3. 55 −i5. 93	$13.46Z_k$
.02 (1+i)	3. 29 − i. 531	$(11.7-i3.67)Z_k$
.02 (1−i)	4. 83 −i7. 23	$(13.05+i9.32)Z_k$
.04	4. 57 −i5. 20	$10.5Z_k$
.04 (1+i)	3. 29 −i4. 17	$(7.70-i4.98)Z_k$
.04 (1−i)	5. 89 −i6. 31	$(11.65+i5.69)Z_k$
.10	5. 24 −i2. 79	$6.70Z_k$
.10 (1+i)	3. 03 −i3. 33	$(4.91-i1.875)Z_k$
.10 (1−i)	8. 14 + i. 853	$(5.38+i1.586)Z_k$
.20	3. 77 −i1. 253	$4.19Z_k$
.20 (1+i)	2. 23 −i1. 60	$(2.83-i.823)Z_k$
.20 (1−i)	3. 92 +i1. 24	$(3.00+i.086)Z_k$
1.0	1. 735+ i. 247	$1.835Z_k$
1.0 (1+i)	.883− i. 928	$(.760+i.0988)Z_k$
1.0 (1−i)	1. 186+ i. 325	$(1.245-i.1315)Z_k$
5.0	1. 112+ i. 210	$1.155Z_k$
5.0 (1+i)	1. 055+ i. 123	$(1.070+i.0427)Z_k$
5.0 (1−i)	.976+ i. 3475	$(1.031+i.0555)Z_k$

Coupling of modulator to oscillator

Any of the usual types of coupling to the oscillator may be used. In the case where the oscillator is a magnetron a particularly useful type of coupling is shown in Figure 15. A coupling loop, not shown, is placed in and tightly coupled to the magnetron cavity or cavities and connected to a concentric line. The outer conductor **5** of the concentric line is terminated in the lower wall **7** of the wave guide as shown. The inner conductor **9** extends up in to the guide a length L_a. As before, the width of the guide is a and the height is b. The coupling loop in the magnetron cavity, length of concentric line and antenna of length L_a will, in general, present an impedance which can be represented by a complex number of the form

$$Z_{int}=R_{int}+ix_{int} \qquad (122)$$

By proper choice of distance h of the reactive stub **10**, the reactance x_{int} may be tuned out so that Z_{int} becomes

$$Z_{int}=R_{int} \qquad (123)$$

The greatest change in operating frequency of the magnetron cavity results when the current in the coupled concentric line **59** goes through its maximum value. This condition should occur if the radiation resistance of the antenna L_a goes through zero. This condition results, in turn, if the guide impedance in the plane A—A goes through zero.

If the antenna radiation resistance does become zero, or approximately zero, the power output of the magnetron will be very adversely affected. Therefore the radiation resistance chosen must be a compromise between a very low value which would give remarkable modulation control, and a somewhat higher value which will still provide acceptable control.

Inspection of Table II, especially in the recommended region $a_0=3$ to 5, indicates that this second condition may be met. The problem may be considered from either the point of view of the antenna L_a and the effect thereon of the impedance Z_1 coupled to it by the guide, or from the point of view of the impedance Z_1 and the effect thereon of the antenna coupled to it. In

either case, by proper choice of h in Figure 15 the reactance x_{int} must be cancelled out so that only R_{int} remains. Also, in either case the coupling factor k_a of the antenna L_a to the guide will appear as follows:

$$k_a=2\left(\frac{L_a}{b}\right)^2\sin^2\frac{\pi d}{a} \tag{124}$$

where a, b, and d are indicated in Figure 15.

If standard X-band wave guide, $L_a=\lambda/4$ at the operating frequency 9210 megacycles per second, and $d/a=1/2$, are employed, then

$$k_a=2(.795)^2\sin^2\pi/2=1.265 \tag{125}$$

In the first instance, the antenna of resistance R_{int} has connected to it the external impedance Z_1, but the coupling factor k_a enters to make the equivalent external impedance $k_aZ_1=1.265Z_1$. In the second instance, the impedance Z_1 has connected to it the resistance R_{int} but again the coupling factor k_a enters to make the equivalent resistance

$$\frac{R_{int}}{k_a}=\frac{R_{int}}{1.265}=.79\,R_{int} \tag{126}$$

The question of possible improvement in Q or of phase-frequency sensitivity by adding $n\pi$ lengths to the wave guides or transforming guide sections frequently arises in designing circuits.

In a waveguide having cross-sectional dimensions a and b, it is assumed that

$$a>\frac{\lambda_0}{2}\text{ and }b<\frac{\lambda_0}{2}$$

where λ_0 is the free space wavelength. If the E vector is initially in a plane parallel to the cross-sectional, or xy, plane only one wave of the form

$$E_x=Ae^{\gamma z}\sin\frac{n\pi}{a}y \tag{127}$$

will be propagated. The initial electric vector intensity is A, and γ is the propagation constant. No other modes will be propagated. If other modes were propagated the same analysis would hold provided the integrations are changed to accommodate the new E and H distributions. For 3 cm. waves, the standard 1'' x ½'' wave guide fulfills the conditions

$$a>\frac{\lambda_0}{2},\ b<\frac{\lambda_0}{2}$$

The length of resonator for $Z=\lambda/2$ is $L=\lambda/2\cong 2b$. Also $a\cong 2b$, thus the end area A_1 is

$$A_1=a^2 \tag{128}$$

and the area of the sides carrying current is

$$A_2=2aL\cong 2a^2 \tag{129}$$

Assuming a uniform substance for the walls, it follows that the respective end and side resistances are

$$R_1=R_0 \tag{130}$$

$$R_2=2R_0 \tag{131}$$

Considering the ends first, the current distribution in the y direction is sinusoidal, and the average current along y is

$$I_{av(xy)}=\frac{I_{max}}{\pi}\int_0^\pi\sin y\,dy=\frac{2I_{max}}{\pi} \tag{132}$$

The average current in a unit area of the sides in the Zy plane will be

$$I_{av(Zy)}=\frac{I_{max}}{\pi}\int_0^\pi\sin y\,dy\cdot\frac{I}{\pi}\int_0^\pi\sin Z\,dZ=\frac{4}{\pi^2}I_{max} \tag{133}$$

It follows, therefore, that the energy losses in

the ends and sides, from Equations 130, 131, 132, and 133 are as follows:

The energy loss in the ends is

$$J_1=\frac{4}{\pi^2}I_{max}^2R_0 \tag{134}$$

and the energy loss in the sides is

$$J_2=\frac{16}{\pi^4}I_{max}^2(2R_0) \tag{135}$$

so that the ratio of the energy loss in the ends to that in the sides is

$$\frac{J_1}{J_2}=\frac{4I_{max}^2}{\pi^2}\cdot\frac{\pi^4}{16I_{max}^2}\cong 1 \tag{136}$$

If the Q of a resonator is defined as

$$Q=\frac{\text{Energy stored in resonator}}{\text{Energy loss in resonator}}$$

then the Q of a standard half wave resonator is

$$Q_{(\lambda/2)}=\frac{J_3}{J_4} \tag{137}$$

where J_3 is the energy stored, and J_4 is the energy loss, in the resonator.

For $L=\lambda/2$, J_4 will be J_1+J_2 so that from Equation 137

$$Q_{(\lambda/2)}=\frac{J_3}{2J_1} \tag{138}$$

For a resonator

$$\frac{n\pi}{2}$$

long the Q becomes

$$Q_{\left(\frac{n\lambda}{2}\right)}=\frac{(n-1)J_3'}{nJ_1'}+\frac{1}{n}\frac{J_3'}{2J_1'} \tag{139}$$

where the primes place the energy on a per $\lambda/2$ section basis. Then

$$Q_{\left(\frac{n\lambda}{2}\right)}=\frac{J_3}{2nJ_1}(2n-1) \tag{140}$$

For the limiting case $n\to\infty$

$$Q_{\left(\frac{\infty\lambda}{2}\right)}=\frac{J_3}{J_1} \tag{141}$$

For the shortest resonator $n=1$, and

$$Q_{\lambda/2}=\frac{J_3}{2J_1}$$

so that the improvement in going from $n=1$ to $n=\infty$ is

$$\frac{Q_\infty}{Q_{\lambda/2}}=2 \tag{142}$$

The analysis heretofore has assumed a uniform wall material. Since the ends involve a soldering or welding operation, it probably is more accurate to assume the end wall resistance is about equal to the side wall resistance. Thus, the end wall energy loss is J_5 where

$$J_5=\frac{4I_{max}^2}{\pi^2}R_0 \tag{143}$$

and the side wall energy loss is

$$J_6=\frac{16}{\pi^4}I_{max}^2R_0 \tag{144}$$

The ratio of energy loss in end walls to energy loss in side walls is

$$\frac{J_5}{J_6}=\frac{4I_{max}^2}{\pi^2}\cdot\frac{\pi^4}{16I_{max}^2}\cong 2 \tag{145}$$

Therefore,

$$Q_{(\lambda/2)}=\frac{2J_7}{3J_5} \tag{146}$$

where J_7 is the energy stored in the length $\lambda/2$. For

$$L=\frac{n\lambda}{2}$$

$$Q_{\left(\frac{n\lambda}{2}\right)}=\frac{2J_7'}{3nJ_5'}(3n-2) \qquad (147)$$

where the primes again denote a per $\lambda/2$ section basis.

For the limiting case $n\to\infty$

$$Q_{(\infty)}=\frac{2J_7}{J_5} \qquad (148)$$

so that the improvement in Q in going from $n=1$ to $n=\infty$ is

$$\frac{Q_{(\infty)}}{Q_{(\lambda/2)}}=\frac{2J_7}{J_5}\frac{3J_5}{2J_7}=3 \qquad (149)$$

Equations 142 and 149 show that the Q of a resonator does not increase linearly with the number of half wavelengths of length for an assumed reasonable loss distribution.

This analysis holds only when end wall resistance and side wall resistances are assumed to approach 0.

Equations 140 and 147 indicate that the rate of improvement obtained by adding half wave sections to a resonator beyond 3 or 4 becomes negligible. The same equations, however, indicate that if the ratio of end wall losses to side wall losses is high or higher than that assumed, it may be desirable to add half wave sections beyond this 3 to 4 value.

Thus it is seen from the foregoing considerations that variation of the reactive term in Equation 118 in response to signal intelligence, may, if properly transformed and coupled to the generator as explained heretofore, provide extremely efficient and substantially linear frequency modulation of the oscillator carrier frequency for reasonable frequency modulation swings as compared to the mean frequency. Various devices for accomplishing such variations of said reactive term, and hence effective reactance variation of the control resonator, will be explained in detail hereinafter.

A first practical basic circuit configuration of the invention, adapted to waveguide construction, is illustrated in Figures 16 and 17.

An ultra-high frequency generator comprising a conventional magnetron 11, of which the magnetic structure is omitted for the sake of simplicity, is coupled to a transmission waveguide system 13 which is coupled to a load, not shown. The output coupling line 12 of the magnetron 11 includes a coupling loop, not shown, which is closely coupled to one or more of the magnetron resonant anode cavities. The end of the magnetron output line 12, remote from the magnetron proper, is terminated in a short antenna 15 which extends into the waveguide transmission 13. Reactance in the magnetron output line 12 is cancelled by means of a reactive stub 17 opening into the waveguide transmission line 13 adjacent the magnetron antenna 15, whereby the magnetron output is made substantially resistive at the plane A—A of the antenna 15 in the waveguide system. The load is matched approximately to the waveguide transmission system 13 by means of a first conventional tuning screw 19 extending through the waveguide wall adjacent the load connection.

The frequency modulation network includes a closed waveguide reactive high Q stub 21 extending normally from and opening into the transmission line 13 at a point on the transverse plane B—B intermediate the load and the generator. The reactance of the control stub 21 is adjustable by means of a second tuning screw 23 projecting therein. The degree of coupling from the control stub 21 to the transmission line 13 is determined by a coupling aperture device 25 in the control stub waveguide adjacent the point where it opens into the transmission waveguide 13. The plane B—B of the control stub 21 is selected to be some multiple of one-half wavelength plus or minus ⅛ wavelength from the plane A—A of the magnetron antenna. Thus, by means of a third tuning screw 27 extending into the transmission waveguide 13 intermediate the planes B—B and A—A, the effective transformation of the impedance of the control stub 21 provided by the impedance transformer 14, (comprising the portion of the waveguide system 13 intermediate the planes A—A and B—B), may be adjusted at the plane A—A which is effectively the resistive termination of the magnetron antenna. The reactive stub 21 and second tuning screw 23 may be considered to be a resonator only as seen from the coupling aperture 25. At the plane A—A of the magnetron antenna, the combination of the stub and tuning screw appear as an impedance.

A fourth tuning screw 29 extending into the transmission waveguide 13 on the opposite side of the plane A—A from the plane B—B of the control stub 21 provides means whereby some predetermined or mean value of the transformed impedance of the control stub may be resonated in the plane A—A at the output frequency of the generator 11.

The characteristics of the control stub 21 and the second tuning screw 23 coupled thereto are selected to have relatively high reactance with as high a "Q" value as conventional construction will permit. Thus, by means of the impedance transformation provided by the transformer 14 between the waveguide transverse planes B—B and A—A, the resultant reactance at the plane A—A due to the control stub 21 will be substantially lower than the reactance of the control stub itself, and will have a substantially higher "Q" value. The transformed reactance at the plane A—A thus may be resonated by means of the fourth tuning screw 29 effectively to provide an extremely high "Q" resonator at the plane A—A which will be effectively connected in shunt with the resonant cavity anode of the magnetron 11 through the magnetron coupling line which includes the antenna 15.

The characteristics of waveguide T junctions of the type connecting the control impedance 21 to the waveguide transformer 14 are well known and are discussed in detail in the literature. Similarly, the reactive effects of tuning stubs coupled into waveguide transmission systems are well known, as is the specific structure of such reactive devices. Detailed construction and theoretical consideration of appropriate tuning screws is included in the copending application of Vernon D. Landon, Serial No. 508,229, filed October 29, 1943, now Patent Number 2,427,107, issued September 9, 1947, which is assigned to the same assignee as the instant application.

Since the carrier frequency generated by the magnetron 11 is controlled by the reactive characteristics of the control stub 21, the magnetron may be efficiently and substantially linearly frequency-modulated by varying the reactance of

the control stub 21 in response to signal intelligence. For example, a conventional microphone 33, responsive to sound waves, may actuate any known device (indicated by the block 35 identified as a "reactance modulator"), to vary the reactance of the control stub 21. Various devices for this purpose are described hereinafter by reference to Figs. 18 to 25 of the drawings.

Adjustment and operation

In operation, the magnetron 11 is initially adjusted by any conventional frequency adjusting means included therewith to provide the desired output carrier frequency. The first tuning screw 19 is adjusted to provide adjustment of the load coupling impedance to match approximately the load to the transmission waveguide 13 and to permit, in the absence of modulation signals, nearly symmetrical tuning on either side of resonance by means of the second tuning screw 23. The second tuning screw 23 in the control waveguide stub 21 is adjusted to tune the control impedance 21 to resonance, as viewed from the aperture 25, at the operating frequency, and in the absence of modulation signals, as may be determined partially by initial calculations. The fourth tuning screw 29 then is adjusted to resonate the unmodulated control impedance at the plane A—A of the magnetron antenna. Next, the third tuning screw 27 is adjusted to vary the unmodulated impedance transformation at the plane A—A of the magnetron antenna to obtain the highest practicable "Q" value for the control impedance in view of the resistive load thus placed upon the magnetron as explained in detail heretofore. The fourth tuning screw is again adjusted to resonate the unmodulated control impedance more accurately to the oscillator output frequency. If necessary, the first tuning screw 19 is again adjusted to match approximately the load to the waveguide transmission system 13 under modulated or unmodulated operating conditions.

It is assumed that the reactive stub 17 adjacent the magnetron antenna 15 has been precalculated or preset to provide the desired cancellation of reactance in the magnetron output line to the antenna 15. If necessary, an adjustable shorting plug may be included in the reactive stub 17 to provide adjustment of the reactance thereof. This feature is indicated by means of the arrow and piston 31.

Figure 18 shows an electrodynamic system for modulating the reactance of the control impedance 21 in response to signal intelligence. The microphone 33 is coupled through an amplifier 37 and an output transformer 39 to the voice coil 41 of a conventional electrodynamic loudspeaker mechanism which includes, for example, a permanent magnet field 43 having pole pieces 45 adjacent the voice coil 41.

The voice coil support 47 comprises a cylindrical member which is connected to a tuning piston 49 disposed within, and adjacent to, one end of the control impedance cavity 21. Thus, the piston 49 is provided with a reciprocating motion in the control impedance cavity 21 in response to signal intelligence derived from the microphone 33.

In the absence of applied modulating signals, the second tuning screw 23 is adjusted to resonate the control impedance cavity 21 to the oscillator carrier frequency as described heretofore. Thus, reciprocating motion of the piston 49 in response to modulating signals modulates the reactance of the control impedance cavity to frequency-modulate the output of the carrier frequency generator as described heretofore.

Referring to Figure 19, a second embodiment of the invention includes ionic means for modulating the reactance of the control impedance cavity 21. The microphone 33 is coupled through a relatively high gain amplifier 37' to a gaseous discharge tube 51 which is disposed entirely within, and adjacent to, one end of the control impedance cavity resonator 21. The gaseous discharge tube 51 includes, for example, a pair of flat electrodes 53, 55 which are closely disposed at one edge and are more widely separated at the opposite edge.

Thus modulating signals derived from the amplifier 37' provide an ionic discharge between the electrodes 53, 55. The discharge occurs first at the more closely separated portions of the electrodes, and as the modulating potentials increase, the discharge extends toward the more widely separated portions of the electrodes. Thus, variations in modulating potentials in response to signal intelligence effectively vary the length and the reactance of the control impedance cavity resonator 21, since the ionic discharge between the electrodes 53, 55 of the gaseous discharge tube 51 provides a conductive path substantially across one end of the cavity resonator element 21, and the position of this conductive path is varied in response to said signal intelligence.

As in the modification of the invention described by reference to Figure 18, the second tuning screw 23 is adjusted to resonate the cavity resonator element 21 to the carrier frequency, in the absence of modulating signals. Thus the modulating potentials effectively vary the impedance of the control resonator 21 in response to applied signal intelligence.

Figure 20 illustrates a third embodiment of the invention wherein the control impedance cavity resonator 21 is modulated in response to signal intelligence by means of a conventional electronic device which utilizes the variation in dielectric constant in a coaxial line which includes a magnetron reactance modulating device. Such reactance modulating devices are described and explained in an article in the Journal of Applied Physics, volume 12 (1941), at pages 856 to 858.

The reactance modulating device includes a section of coaxial line having an inner conductor 61 and a concentrically disposed outer conductor 63. The central portion of the inner conductor 61 comprises the thermionic cathode 65 of a magnetron type thermionic tube. The cylindrical anode 67 of the magnetron tube is separated from, and capacitively coupled to, the outer conductor 63 of the coaxial line through the glass envelope 69 of the magnetron tube. A tuning piston 71 is insulated from the inner conductor 61 of the coaxial line by means of a central insulating insert 73. The tuning piston 71 may be moved longitudinally along the coaxial line intermediate the conductors 61, 63 thereof to tune the line to the desired mean reactance.

A strong magnetic field is generated in the space occupied by the magnetron tube elements by means of a winding 75 surrounding, and coaxial with that portion of the coaxial line. The magnetic winding 75 is energized, for example, by means of a battery 77 connected in series with the output of a speech amplifier 37 which is responsive to signals from the microphone 33.

Filament current for energizing the cathode 65 of the magnetron tube is derived from the sec-

ondary winding 79 of a conventional filament transformer 81, the primary winding 83 of which is connected to a source of alternating potential, not shown. A source of high voltage, not shown, is connected between the cathode and anode electrodes of the magnetron to operate the cathode at the required negative potential.

Thus a rotating space charge is maintained in that portion of the coaxial line 61, 63 which is occupied by the magnetron tube. The effect of the rotating space charge is to vary the effective dielectric constant in the coaxial line in the region occupied by the magnetron. Thus, since the dielectric constant may be varied from negative through zero to positive values by proper selection of the strength of the magnetic field and the applied voltage between the magnetron electrodes, the effective reactance of the coaxial line may be varied between relatively wide limits.

The coaxial line is coupled to the control impedance cavity resonator 21 by means of a short coupling line 85 having coupling loops 87 and 89 disposed within the coaxial line and control impedance cavity resonator, respectively.

The graph of Figure 24 illustrates the variation in tuning of the coaxial line device as a function of the strength of the magnetic field of the magnetron tube. Since the magnetic field strength is varied in response to modulating signals derived from the microphone amplifier 37, it is apparent that the reactance variations in the magnetron-modulated coaxial line will be reflected in the control cavity resonator 21 and will vary the effective reactance thereof as a function of said modulating signals.

Figure 21 illustrates a first modification of the third embodiment of the invention, previously described by reference to Figure 20, wherein the magnetic field produced by the winding 75 of the magnetron tube is maintained at a substantially constant value by means of the energizing battery 77 which is directly connected to the winding 75. In this modification of the invention, modulating signals derived from the microphone 33 and amplifier 37 are combined with and effectively vary the high potential applied between the magnetron cathode 65 and anode 67. For a predetermined magnetic field strength in the magnetron reactance modulating device, the effect of variation of the high potential applied between the cathode and anode electrodes of the magnetron is illustrated in the graph of Figure 23. Thus, by proper selection of the applied magnetron anode voltage, the effective reactance variations in the coaxial line device, and hence in the control impedance cavity resonator 21 coupled thereto, may be efficiently controlled in response to applied signal intelligence.

The graph of Figure 25 shows the effective change in tuning of the coaxial line reactance device in response to adjustment of the position of the tuning piston 71 disposed within the coaxial line device.

Figure 22 illustrates a second modification of the third embodiment of the invention, previously described by reference to Figure 20, wherein the reactance-modulating coaxial line and the control impedance cavity resonator are combined in one unit to provide a coaxial line control impedance element 21' which is directly coupled through a coupling loop 91 to the transformer section 14 of the waveguide transmission line 13. As in the device described by the reference to Figure 20, the reactance modulation is accomplished by variation of magnetic field strength in the magnetron tube in response to signal intelligence. It should be understood that the coaxial line control impedance 21' may be coupled in any other known manner to the waveguide transmission system 13. It also should be understood that the effective dielectric constant of the portion of the coaxial line control impedance 21 in the region occupied by the magnetron reactance tube may be varied by modulation of the anode potential thereof, as illustrated and described by reference to Figure 21.

In Figure 26, a modification of the basic circuit configuration, shown in Figs. 16 and 17, includes a cylindrical very high Q variable control reactance 21'' which includes a tuning piston 37, the longitudinal position of which may be adjusted by means of a micrometer screw 39. The control reactor 21'' is coupled to a T junction 41' which extends from the narrow side of the waveguide transmission system 13. A second T junction 43 extending from the opposite narrow side of the waveguide system 13 includes the third tuning screw 27 which is employed to vary the coupling of the control impedance 21'' to the impedance transformer 14. The first tuning screw 19 is operable through the transmission waveguide wall intermediate the plane B—B of the control impedance and a load, not shown. The magnetron generator has been omitted for the sake of simplicity. The reactance modulator 35 for the variable control reactor 21'' is shown in block form, and may comprise any of the devices described heretofore by reference to Figs. 18 to 25.

The operation and adjustment of the system is similar to the device described by reference to Figures 16 and 17. However, the cylindrical construction of the control impedance 21'' permits a higher "Q" to be obtained from this element than with rectangular construction, with greater phase sensitivity in response to modulating reactance variations.

The higher Q control impedance results usually in higher values of resonant resistance and higher reactances at the coupling aperture 25 opening into said control impedance at frequencies slightly off resonance. This would result in an unfavorable situation if used in the arrangement of Fig. 16 and Fig. 17 in which transmission and control waveguide branches 13 and 41 are substantially in series insofar as the impedance transformer 14 is concerned, since under these conditions the power delivered to the load would be small.

For this reason the T junctions of the type shown in Fig. 26, in which the transmission and control waveguide branches 13 and 41' (with the additional reactance of the tuning waveguide stub 43) are substantially in parallel, is used with a high Q control impedance 21'', since the high impedances are then in parallel with the useful load.

Since the impedance transformation and resonating is otherwise similar to that described in Figures 16 and 17, the effective control impedance "Q" at the plane A—A is substantially improved, thereby providing greater modulation swing of the generator frequency in response to modulating reactance variations, as indicated heretofore.

Thus the invention disclosed and explained herein comprises several embodiments and modifications of a frequency modulator for carrier wave generators wherein a variable impedance

represented by a complex number is selected, is transformed to a new impedance having a higher Q and lower absolute value, and then is parallel-resonated to the carrier frequency by another reactance of opposite sign. The reactance of the variable control impedance is varied in response to signal intelligence to modulate efficiently the carrier frequency of the generator. Circuits having rapid rates of change of phase angle with frequency are employed, and critical optimum line lengths are determined to provide practical and efficient circuit configurations. Several devices are disclosed for varying the reactance of the variable control impedance either electromagnetically, ionically or electronically in response to signal intelligence.

It should be understood that several sections of the control impedance and impedance transformation elements of the circuit configurations described may be cascaded to provide a more efficient modulation system. However, circuit adjustment in such a cascaded waveguide system may become quite difficult, due to present waveguide measurement limitations and coupling technique.

I claim as my invention:

1. The method of employing a variable control impedance for modulating the frequency of a signal generator comprising the steps of transforming the impedance value of said variable impedance to a lower variable impedance value having a relatively higher Q, resonating a predetermined value of said transformed impedance value substantially to said generator frequency, and shunting said generator by said resonated transformed variable impedance value.

2. The method of employing a variable control impedance for modulating the frequency of a signal generator comprising the steps of transforming the impedance value of said variable impedance to a lower variable impedance value having a relatively higher Q, resonating a predetermined value of said transformed impedance value substantially to said generator frequency, shunting said generator by said resonated transformed variable impedance value, and cancelling normal coupling reactance between said generator and said resonated transformed impedance.

3. The method of utilizing a variable control impedance for modulating the frequency of a signal generator coupled to a load comprising the steps of cancelling normal coupling reactance from said generator, transforming the impedance value of said variable impedance to a lower variable impedance value having a relatively higher Q, resonating a predetermined value of said transformed variable impedance value substantially to said generator frequency, and shunting said reactance cancelled generator coupling by said resonated transformed variable impedance value.

4. The method of utilizing a variable control impedance for modulating the frequency of a signal generator coupled to a load comprising the steps of cancelling normal coupling reactance from said generator, transforming the impedance value of said variable impedance to a lower variable impedance value having a relatively higher Q, resonating a predetermined value of said transformed variable impedance value substantially to said generator frequency, shunting said reactance cancelled generator coupling by said resonated transformed impedance value, and coupling said load to said generator through said impedance transformation.

5. The method of utilizing a variable control impedance for modulating the frequency of a signal generator coupled to a load comprising the steps of cancelling normal coupling reactance from said generator, transforming the impedance value of said variable impedance to a lower variable impedance value having a relatively higher Q, resonating a predetermined value of said transformed variable impedance value substantially to said generator frequency, shunting said reactance cancelled generator coupling by said resonated transformed variable impedance value, coupling said load to said generator through said impedance transformation, and matching said load to said load coupling.

6. The method of providing a reactance modulated high-Q resonant network including a variable impedance element comprising the steps of transforming the impedance value of said element to a lower impedance value having a relatively higher Q value, resonating said transformed impedance value and reactance modulating said variable impedance element.

7. A high-Q resonant network including a high Q impedance element, means for reactance-modulating said element, means for transforming said modulated element to a lower impedance having a relatively higher Q value, and reactive means connected to said transforming means for resonating said transformed reactance-modulated impedance.

8. Apparatus for modulating the frequency of a carrier signal generator, including a high Q control impedance element, means for varying the impedance of said element in response to signal intelligence, means for effectively transforming said variable impedance to a lower variable impedance having a relatively higher Q value, reactive means for resonating a predetermined value of said transformed variable impedance substantially to said generator frequency, and means for connecting said resonated transformed variable impedance in shunt with said generator.

9. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining resonant element, including a high Q control impedance element, means for varying the impedance of said element in response to signal intelligence, means for effectively transforming said variable impedance to a lower variable impedance having a relatively higher Q value, reactive means for resonating a predetermined value of said transformed variable impedance substantially to said generator frequency, and means for connecting said resonated transformed variable impedance in shunt with said generator resonant element.

10. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining resonant element, including a load coupled to said generator, a high Q control impedance element, means for varying the impedance of said element in response to signal intelligence, means for effectively transforming said variable impedance to a lower variable impedance having a relatively higher Q value, reactive means for resonating a predetermined value of said transformed variable impedance substantially to said generator frequency, and means for connecting said resonated transformed variable impedance in shunt with said generator resonant element and said load.

11. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining resonant element, including a load, a

transmission line coupling said generator resonant element to said load, a high Q control impedance element coupled to said line, means for varying the impedance of said element in response to signal intelligence, an impedance transformer interposed in said line between said variable control impedance element and said generator for providing an effectively lower variable impedance and a relatively higher Q value, means for providing a resistive termination of said generator at a predetermined point on said line, and reactive means for resonating a predetermined value of said transformed variable control impedance to said generator frequency at said point on said line.

12. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining cavity resonator, including a load, a transmission waveguide coupling said generator cavity resonator to said load, a high Q control reactive cavity element coupled to said waveguide, means for varying the impedance of said element in response to signal intelligence, a section of said waveguide providing an impedance transformer interposed between said variable control reactive cavity element and said generator for providing an effectively lower variable impedance and a relatively higher Q value, means for providing a resistive termination of said generator at a predetermined point on said waveguide, and reactive means for resonating a predetermined value of said transformed variable control impedance to said generator frequency at said point on said waveguide.

13. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining cavity resonator, including a load, a transmission waveguide coupling said generator cavity resonator to said load, a second waveguide forming a T junction with said transmission waveguide and comprising a high Q control reactive cavity element coupled to said waveguide, means for varying the impedance of said element in response to signal intelligence, a section of said transmission waveguide providing an impedance transformer interposed between said variable control reactive cavity element and said generator for providing an effectively lower variable impedance and a relatively higher Q value, means for providing a resistive termination of said generator at a predetermined point on said transmission waveguide, and reactive means for resonating a predetermined value of said transformed variable control impedance to said generator frequency at said point on said transmission waveguide.

14. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining cavity resonator, including a load, a transmission waveguide coupling said generator cavity resonator to said load, a T junction in said waveguide, a high Q control reactive cavity element of adjustable volume coupled through said junction to said waveguide, means for varying the impedance of said element in response to signal intelligence, a section of said transmission waveguide providing an impedance transformer interposed between said variable control reactive cavity element and said generator for providing an effectively lower variable impedance and a relatively higher Q value, means for providing a resistive termination of said generator at a predetermined point on said transmission waveguide, and reactive means for resonating a predetermined value of said transformed variable control impedance to said generator frequency at said point on said transmission waveguide.

15. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining cavity resonator, including a load, a transmission waveguide coupling said generator cavity resonator to said load, a high Q control reactive cavity element coupled to said waveguide, electromechanical means disposed in said element for varying the impedance of said element in response to signal intelligence, a section of said waveguide providing an impedance transformer interposed between said variable control reactive cavity element and said generator for providing an effectively lower variable impedance and a relatively higher Q value, means for providing a resistive termination of said generator at a predetermined point on said waveguide, and reactive means for resonating a predetermined value of said transformed variable control impedance to said generator frequency at said point on said waveguide.

16. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining cavity resonator, including a load, a transmission waveguide coupling said generator cavity resonator to said load, a high Q control reactive cavity element coupled to said waveguide, ionizable means disposed in said element for varying the impedance of said element in response to signal intelligence, a section of said waveguide providing an impedance transformer interposed between said variable control reactive cavity element and said generator for providing an effectively lower variable impedance and a relatively higher Q value, means for providing a resistive termination of said generator at a predetermined point on said waveguide, and reactive means for resonating a predetermined value said transformed variable control impedance to said generator frequency at said point on said waveguide.

17. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining cavity resonator, including a load, a transmission waveguide coupling said generator cavity resonator to said load, a high Q control reactive cavity element coupled to said waveguide, variably reactive electronic means coupled to said reactive cavity element for varying the impedance of said element in response to signal intelligence, a section of said waveguide providing an impedance transformer interposed between said variable control reactive cavity element and said generator for providing an effectively lower variable impedance and a relatively higher Q value, means for providing a resistive termination of said generator at a predetermined point on said waveguide, and reactive means for resonating a predetermined value of said transformed variable control impedance to said generator frequency at said point on said waveguide.

18. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining cavity resonator, including a load, a transmission waveguide coupling said generator cavity resonator to said load, a high Q control reactive cavity element coupled to said waveguide, variably reactive electronic means for varying the impedance of said element in response to signal intelligence, a section of said waveguide providing an impedance transformer interposed between said variable control reactive cavity element and said generator for providing an effectively lower variable impedance and a relatively

higher Q value, means for providing a resistive termination of said generator at a predetermined point on said waveguide, and reactive means for resonating a predetermined value of said transformed variable control impedance to said generator frequency at said point on said waveguide.

19. Apparatus for modulating the frequency of a carrier signal generator having a frequency determining cavity resonator, including a load, a transmission waveguide coupling said generator cavity resonator to said load, a high Q control reactive cavity element including variably reactive electronic means disposed therein, means coupling said element to said waveguide, means for varying the reactance of said electronic means and hence said impedance of said element in response to signal intelligence, a section of said waveguide providing an impedance transformer interposed between said variable control reactive cavity element and said generator for providing an effectively lower variable impedance and a relatively higher Q value, means for providing a

resistive termination of said generator at a predetermined point on said waveguide, and reactive means for resonating a predetermined value of said transformed variable control impedance to said generator frequency at said point on said waveguide.

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