June 9, 1931. J. S. STONE 1,808,867

DIRECTIONAL ANTENNA ARRAY

Filed Jan. 26, 1927 4 Sheets-Sheet 1

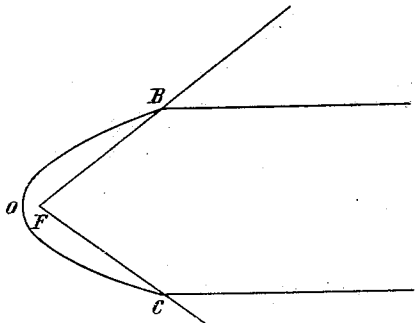

Fig. 1

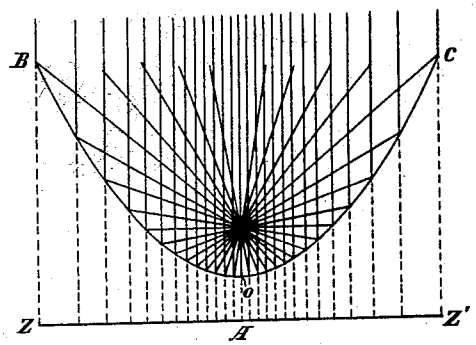

Fig. 2

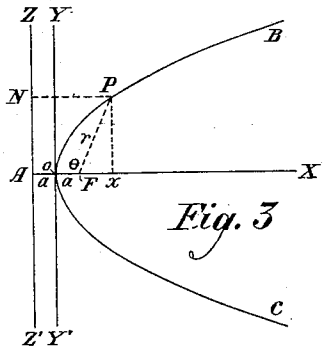

Fig. 3

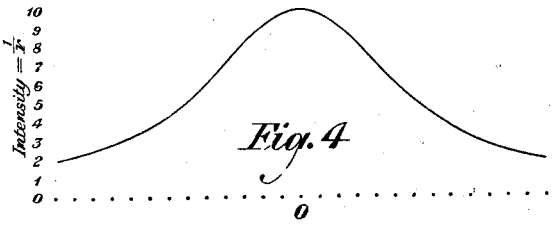

Fig. 4

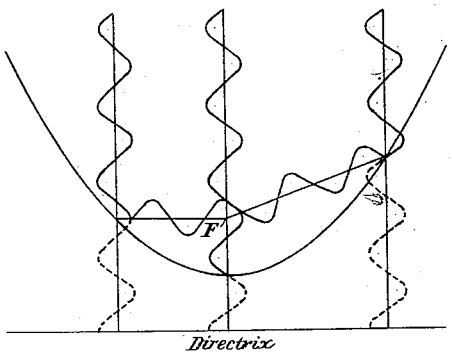

Fig. 5

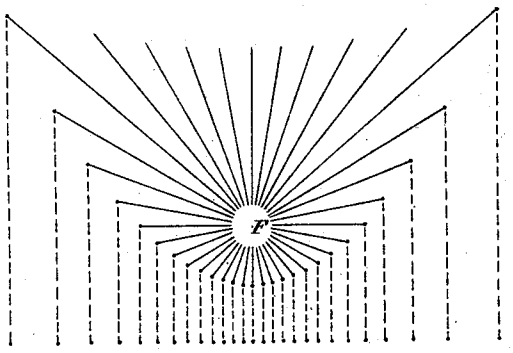

Fig. 6

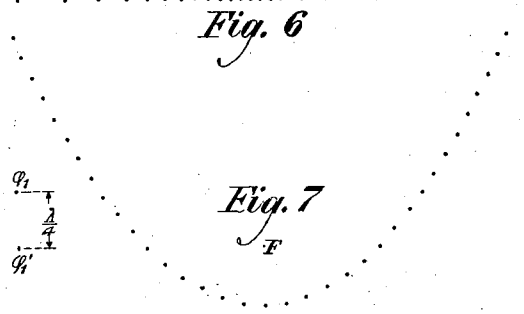

Fig. 7

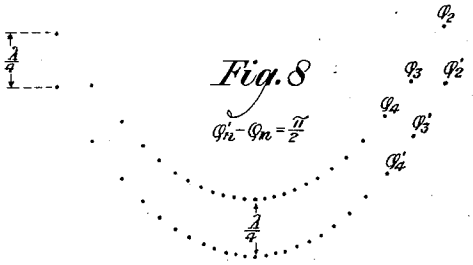

Fig. 8

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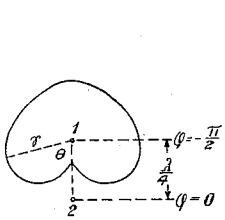

Fig. 9

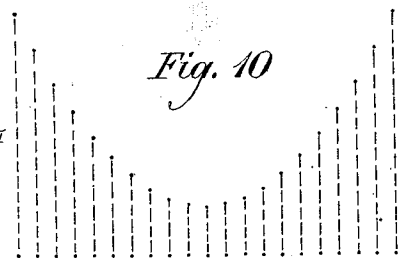

Fig. 10

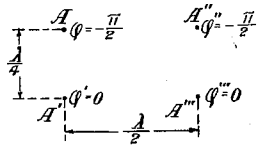

Fig. 13

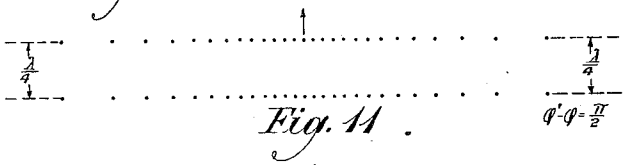

Fig. 11

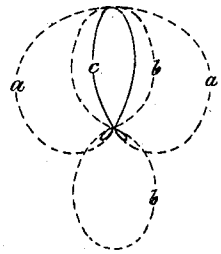

Fig. 14

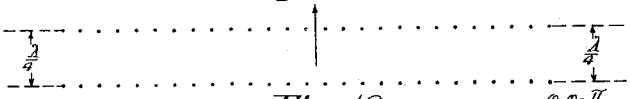

Fig. 12

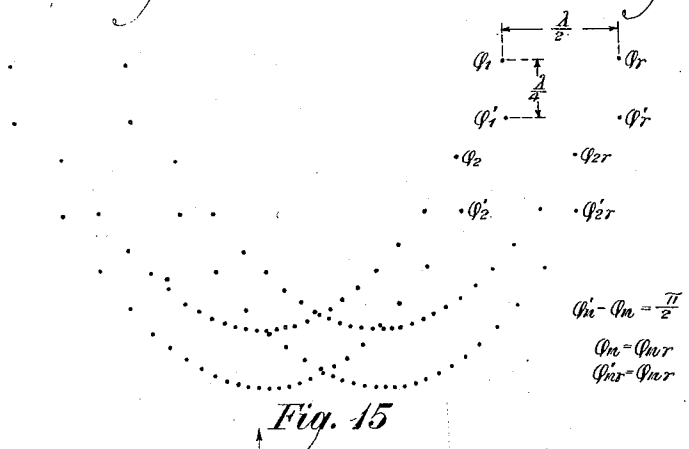

Fig. 15

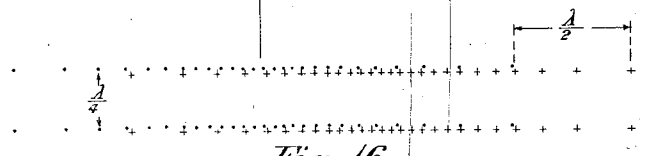

Fig. 16

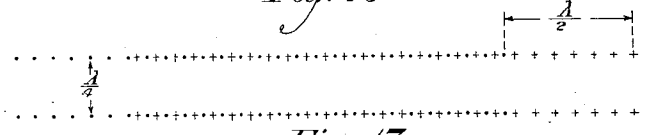

Fig. 17

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Fig. 18

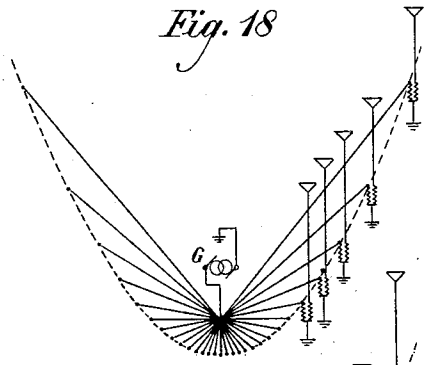


Fig. 19

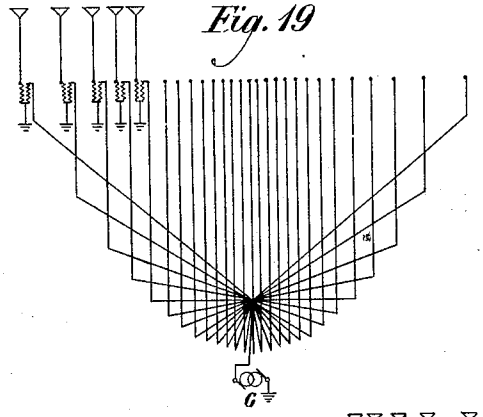


Fig. 20

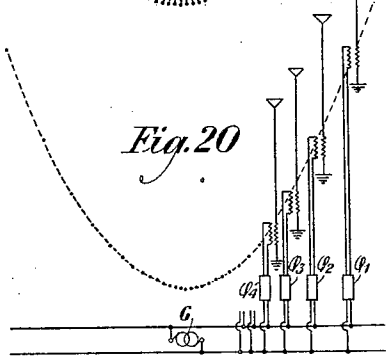


Fig. 21

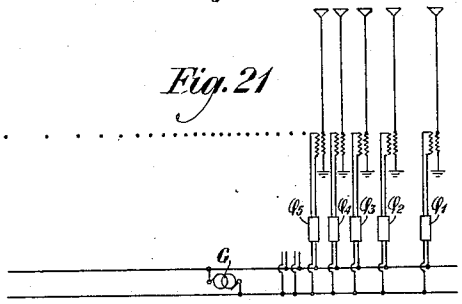


Fig. 22

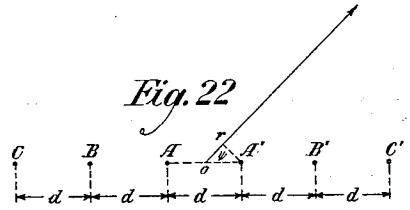
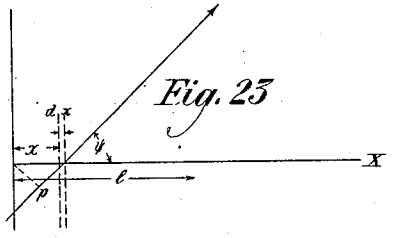


Fig. 23



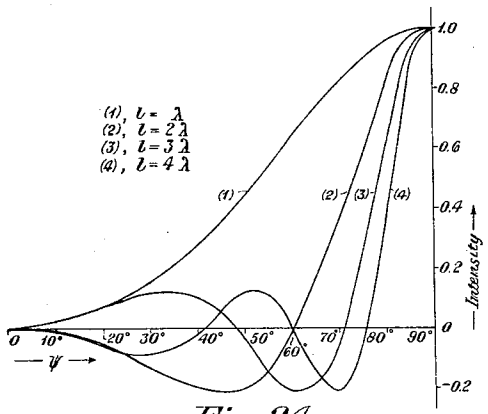


Fig. 24

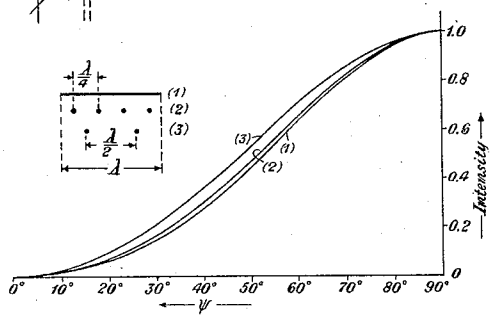


Fig. 25

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DIRECTIONAL ANTENNA ARRAY

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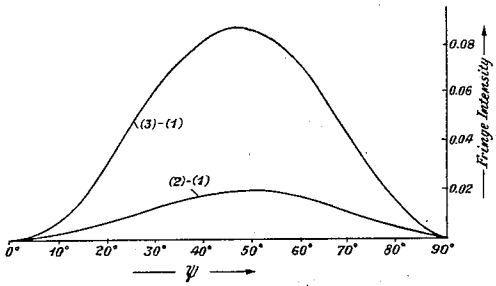

Fig. 26

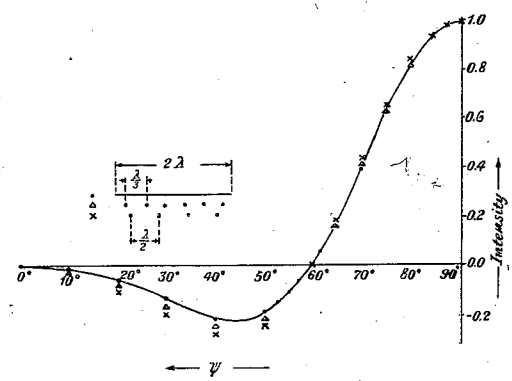

Fig. 27

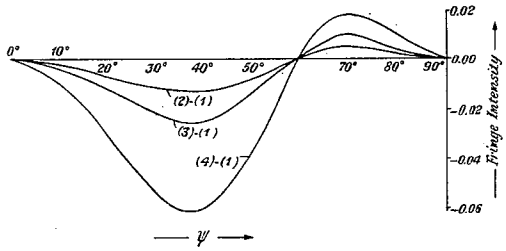

Fig. 28

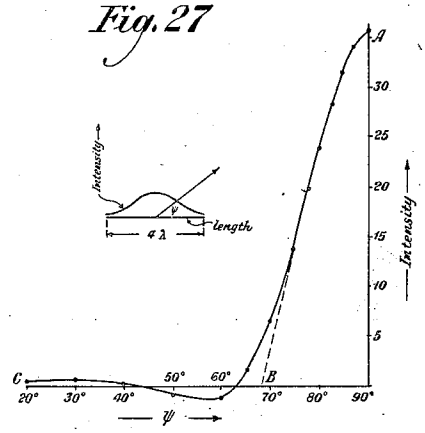

Fig. 29

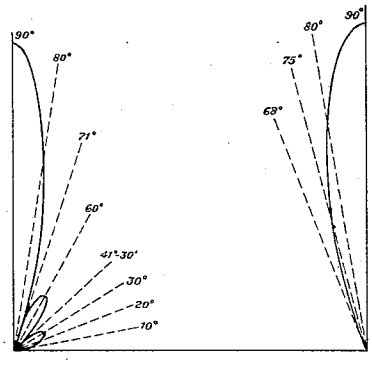

Fig. 30

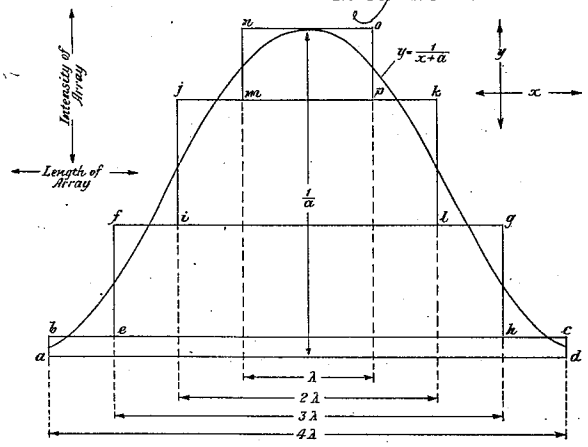

Fig. 31

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Patented June 9, 1931

1,808,867

UNITED STATES PATENT OFFICE

JOHN STONE STONE, OF SAN DIEGO, CALIFORNIA, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK

DIRECTIONAL ANTENNA ARRAY

Application filed January 26, 1927. Serial No. 163,774.

This invention relates to directive antenna arrays.

The earliest means employed to direct electromagnetic waves in an approximately parallel beam was the actual cylindrical parabolic reflector. A cylindrical parabolic reflector of, for example, sheet metal, having the oscillator substantially coincident with its focal line, directs rays to a conjugate cylindrical sheet metal parabolic reflector, in the focal line of which may be placed what is known as the resonator. Since the employment of this very early means of directional transmission, a number of modifications of this means have been made for securing further directive radio communication.

In considering the use of parabolic reflectors in radio or high frequency electrical transmission, the temptation is very strong to make inappropriate and illegitimate use of the analogue with the parabolic mirror in its use for producing a parallel beam of light. The analogue of the parabolic mirror is safe enough if it is remembered that the light must be monochromatic, i. e., light of a single wave length, and that the dimensions of the mirror must at most be but a few wave lengths in width. When these limitations are borne in mind, much of the utility of the light analogue as a means of clarification vanishes. In order to better understand this invention, the maximum legitimate use of the analogy with the light problem will be made in order to better understand the electromagnetic problem of short wave or high frequency transmission.

While this invention will be pointed out with particularity in the appended claims, the invention itself, both as to its objects and features, will be better understood from the detailed description hereinafter given, when read in connection with the accompanying drawings, in which Figures 2, 6 to 8, 10 to 12, and 15 to 21, inclusive, comprise various parabolic and parabolic image arrays. Fig. 13 represents a couplet of pairs of conductors. Figs. 1, 3 to 5, 9, 14, 24 to 31, inclusive, represent curves characteristic of the invention employed as aids to an analysis and understanding of the invention. Figs. 22 and 23 represent uniform linear arrays.

Referring to Fig. 1 of the drawings, there is shown a cross-section of a cylindrical parabolic mirror designated by the reference characters BOC. The reference character F designates the section of an incandescent filament whose axis lies in the focal line of the mirror. This figure shows that the light emanating from the organization consists of a parallel beam of light reflected from the mirror, and another and divergent beam of primary light emanating directly from the filament which is at the focal line of the mirror. This figure also shows that the primary light from the filament which escapes from the mirror is wasted, besides being detrimental so far as the directive effects of the system are concerned, and clearly suggests that an improvement could be effected in the organization if this primary light could be substantially suppressed without at the same time suppressing, or otherwise interfering with, the parallel beam of light reflected from the parabolic mirror. This can only be done in the imagination, at least, by dispensing with the incandescent filament and making the surface of the reflector at every point self-luminous in just the phase and degree of illumination to which the filament formerly illuminated it.

The degree to which the filament at the focal line of the mirror illuminates the different parts of a parabolic surface is illustrated in Fig. 2. In this figure, radial lines extend from the focus to the surface of the parabola at equal intervals of 10 degrees, and the spaces between them therefore represent substantially equal pencils of light emanating from the filament. The parabolic mirror is, for convenience, assumed to be a perfect reflector, and pencils of light are all reflected parallel to the axis of the parabola corresponding to those pencils of light originating at the filament and striking the surface of the parabola radially.

An inspection of Fig. 2 shows that the surface of the parabolic mirror is illuminated most intensely at and near the vertex, and that the intensity gradually diminishes with

a departure from that point outwardly along the curve. Similarly, this figure shows the varying intensity of the parallel reflected beam. The intensity of this beam is seen to be greatest at the center, and to wane gradually from that point to the periphery of the beam.

The geometry of the system is derived from Fig. 3 of the drawings showing another cross-sectional view of a parabolic surface. In a parabolic curve having the equation

$$y^2=4ax,\ x=r-a \text{ and } r=\frac{a}{\cos^2\frac{\theta}{2}},$$

r being the radial distance of any point on the parabola from the focus and θ being the angle between the radial line from the focus to the point on the parabola and the axis of the parabola. Clearly, the intensity of the light of the parallel beam reflected from the mirror is proportional to

$$\frac{d\theta}{dy},$$

and this intensity may be expressed mathematically as follows:

$$\frac{d\theta}{dy}=\frac{\cos^2\frac{\theta}{2}}{a}=\frac{1}{r} \quad (1)$$

The lag ϕ in the phase of the light which falls on the surface of the parabolic mirror behind that of the light emanating from the focus is given by the following expression:

$$\phi=-\frac{2\pi r}{\lambda}=-\frac{2\pi a}{\lambda\cos^2\frac{\theta}{2}} \quad (2)$$

In this expression λ represents the wave length. It will also be readily understood that the intensity of light falling upon any surface is given by

$$\frac{d\theta}{ds}=\frac{1}{\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}},$$

in which s is the length of arc and r is the radius vector. From this it follows that

$$\frac{d\theta}{ds}=\frac{\cos^3\frac{\theta}{2}}{a} \quad (3)$$

and that

$$s=\frac{y}{4a}\sqrt{y^2+4a^2}+a\log\left[\frac{1}{2a}(y+\sqrt{y^2+4a^2})\right] \quad (4)$$

Viewed from a great distance, the image of the filament at the focus appears to be a continuously illuminated surface lying in the plane of the directrix ZZ′ of the parabola BOC of Figs. 2 and 3. This plane, however, is not uniformly illuminated, the intensity of illumination being greatest at A (directly opposite the focus) and gradually diminishing with the departure from A toward the boundaries Z and Z′. Fig. 2 illustrates this variation of intensity qualitatively by the relative crowding together of the dotted vertical lines near the center of the directrix A, while the variation in intensity is illustrated quantitatively in Fig. 4 of the drawing.

Just as a parallel beam may, in the imagination, be reproduced without the intervention of the primary source at the focus by making the reflector, itself, independently luminous at every point in the degree and phase to which it would be illuminated at that point by the primary source at the focus, so this parallel reflected beam may also be reproduced without the intervention of either the source at the focus or the independently luminous sources upon the surface of the parabolic reflector, provided the plane of the image in the directrix is made independently luminous at every point to the degree illustrated by the curve of Fig. 4, and provided also the phase of the illumination at all points of the plane is the same. Equality of phase is essential because all points on the directrix are as distant from corresponding points on the parabolic surface as the focus is from those points.

In understanding the reflection of monochromatic light and of high frequency or short electromagnetic waves, the matter of the phase of the waves becomes very significant, but the determination of the phase in the case of the image of a source in an ideally perfect reflector is rather simple. In such a case, since a source and its image are at all times at equal distances from the reflecting surface, the phase of the image lags by the angle $k\pi$ behind that of the source, in which k is an odd integer. It is a fact that at any point on the perfectly reflecting surface, the motion of the resulting magnetic force due to the primary source and its image must be either zero or parallel to the surface.

Fig. 5 illustrates the operation of a parabolic reflector with a source of monochromatic light, or with an oscillator, preferably of constant frequency, at its focus. It is to be specially noted that the phase of the radiation from the image is constant throughout, its length being $k\pi$ behind that of the primary source at the focus of the reflector, k having the integral value of 3 in the case assumed for illustration.

Clearly, when the primary source is a linear oscillator concentric with the focal line of the reflector, there is no need of transverse conductivity in the mirror, so that the metallic sheet of the continuous parabolic reflector of the well-known type may be replaced by the parabolic grid arrangements of wires or conductors parallel to the focal line of the reflector, as illustrated in Figs. 6 and 7. In the parabolic grid of Fig. 7, the wires

or conductors are uniformly spaced, while in the parabolic grid of Fig. 6 they are so spaced that the density of their distribution in any area is proportional to the illumination which that area would receive from a source of light at the focus, or

$$\frac{d\theta}{ds},$$

given mathematically, as determined by Equation (3), given hereinabove. In passing from a continuous reflecting surface to one formed of a grid or array of parallel wires or conductors, caution must be exercised lest the distance between the sending wires of the grid or array be insufficiently small relative to the wave length. The requisite distance between the grid or array will be considered in detail hereinafter. It must be borne in mind that the classical laws of reflection of light from a mirror surface, upon which is based the ordinary theory of a parabolic mirror, depend upon the continuity of the surface of the mirror. This is so because the incident light falling on a given point of the ordinary mirror surface is in fact reflected in all possible directions, but all of the light reflected from this point, except that having an angle of reflection equal to the angle of incidence of the primary light, is substantially neutralized and rendered substantially ineffective by interference with light waves reflected from neighboring points on the surface of the mirror.

However, in the case of a surface formed of parallel wires or conductors, this interference can never be quite complete. Instead of complete neutralization, diffractive fringes are formed in addition to the main reflected beam, which, except for diffraction, follows the classical laws of reflection. As the wires or conductors are brought closer and closer, these diffraction fringes become less important, until there remains sensibly but a uniform reflected beam, as in the case of light reflected from a continuous reflecting surface. It should be remembered that the matter of radio differs widely from the matter of light in that the dimensions of the radio reflector correspond to those of a microscopically small light mirror in which the phenomena of diffraction are of the utmost importance. For this reason, the light analogue, in spite of its great simplicity and familiarity, will be used with utmost caution hereinafter as it is employed to aid in a better understanding of this invention.

In this invention, it is proposed to employ the parabolic grids or arrays of Figs. 6 and 7 without any source of radiation at the focus. In the organization of Fig. 6, each wire or conductor is made the seat of an alternating current of the same frequency and amplitude as any other wire or conductor, but the phase of the current in each wire or conductor is determined from equation (2), given hereinabove. In the organization of Fig. 7, the wires or conductors are made the seats of alternating currents of the very same frequency, but both the amplitudes and the phases of these currents vary from wire to wire. The phase of the current is determined from Equation (2), given herein above, and the amplitude

$$\frac{d\theta}{ds}$$

is determined from Equation (3), also given hereinabove.

In any parabolic array, it is of the utmost importance to remember that the currents do not have the same initial phase, and, moreover, that the currents do not have different frequencies. When an improper relationship is maintained between the various wires or conductors, it is fatal to the production of a directive system, especially one of the highly efficient directive systems contemplated in this invention.

In the ordinary parabolic mirror, which is employed in a well-known manner to secure a parallel beam of light, none of the light from the primary source at the focus passes through the reflector to produce an illumination behind the reflector; yet, if the primary source of light at the focus be suppressed and the parallel beam be secured by making the mirror, itself, incandescent throughout its surface, a large portion of the total light developed is radiated backwardly through the convex surface of the mirror. Similarly, in radio, or rather in high frequency or short wave electromagnetic energy, a beam of radiation may be secured by setting up appropriate currents in the wires or conductors of the parabolic grid or array. However, half of the energy is radiated backwardly from the convex side of the array, as well as forwardly. The arrangement disclosed in Fig. 8 is intended to obviate this difficulty. In the organization of this figure, a second and identical parabolic array is shown, for the purpose of illustration, situated at a predetermined distance behind the first parabolic array. The distance between these parabolic arrays is one-quarter of a wave length,

$$\frac{\lambda}{4}.$$

Currents identical as to amplitude and frequency with those in the first array are set up in the second array, but the phases of the currents in the second array lead the corresponding currents in the first array by a predetermined phase angle, one-quarter of a cycle in the case illustrated,

$$\frac{\pi}{2}.$$

In this arrangement, any pair of correspond-

ing conductors in the two arrays constitutes a uni-directional couplet of the type having a directional characteristic which is a cardioid, shown in Fig. 9 of the drawings. In this latter figure, the reference characters 1 and 2 designate a corresponding pair of conductors in the first and second parabolic arrays, respectively, and the cardioid is the polar curve representing its directional transmitting and receiving properties.

In the production of a parallel beam of light by means of a cylindrical parabolic mirror, it has been pointed out in connection with Fig. 2, which has an incandescent filament coaxial with the line of the focus, that the image of the luminous source appears to be a continuously illuminated plane which is coincident with the plane of the directrix of the cylindrical parabola. It has also been shown that in the absence of the incandescent filament and of the parabolic mirror, it is possible to develop the same parallel beam of light by making the plane of the image in the directrix independently luminous throughout, provided the real or actual luminosity is at every point equal to the apparent luminosity as an image of the incandescent filament at the focus.

It is similarly possible to secure the same approximately parallel electromagnetic beam of radiation from a linear array of sources, such as is shown at the lower part of Fig. 6, as from the parabolic array of the figure, provided the currents throughout the linear array are all of equal magnitude and of the same phase. The magnitude of the current common to all of these conductors is the same as the amplitude of the currents common to the conductors of the parabolic array, in order that the two beams may have the same, or substantially the same, absolute intensities at all points. Moreover, the same approximately parallel beam of radiation as that just considered may be secured by the employment of a horizontal array of oscillators or other sources shown at the lower part of Fig. 10, provided the currents in these oscillators are all of the same phase, and provided the amplitudes of these currents are proportional to the ordinates of the curve of Fig. 4. In general, whatever may be the distribution of the wires or conductors in any linear array, or in any parabolic array, the distribution of the current amplitudes must be such as to provide a distribution of intensity in the beam, such as is shown in Fig. 2 of the drawings, and more explicitly given in Fig. 4. In other words, the amplitude of a current in any wire of a parabolic array must be proportional to

$$\frac{1}{r}$$

if the wires are located at equal intervals on the arc, but the amplitudes are all equal if the angular spacing is uniform. The amplitude of the current in any wire of a parabolic image array is determined from the fact that it is the same as the amplitude of the current in the wire of the parabolic array of which it is a projection on the directrix.

Radiation will take place backwardly as well as forwardly in the case of the image or linear arrays of a single layer just as in the case of the parabolic arrays of a single layer, with the result that there will be a loss of some of the advantages to be gained by making the system directive. However, this may be overcome by employing the arrangements illustrated in Figs. 11 and 12 of the drawings. The direction of the radiated beam in each of these cases is indicated by the arrow. A separation between the pairs of arrays is predetermined and taken each of one-quarter of a wave length, and consequently the applied currents in the front layer lag, by a quarter of a cycle behind those in the rear layer, in order that these currents may be in phase and their effects cumulative in the desired direction.

Although the cardioid directive characteristic of the couplet shown in Fig. 9 permits no radiation, or substantially negligible radiation, backwardly, i. e., in the direction $\theta=0$, it nevertheless permits very considerable radiation in almost every other direction, particularly forwardly, i. e., in the direction $\theta=180°$. Yet, when greater efficiency of transmission and reception and greater directive exclusiveness are desired than are provided by the arrangements shown in Figs. 8, 11 and 12, in each of which a couplet of the type shown in Fig. 9 is employed, it is desirable to use two couplets which are so relatively spaced and phased as to restrict the radiation to an even narrower range in the general direction of the parallel beam which is intended to be transmitted or received.

A preferred form of the array produced by the combination of two couplets of the type shown in Fig. 9 is illustrated in Fig. 13. The reference characters A and A′ and A″ and A‴ designate two pairs of conductors, the conductors of each pair being separated by a predetermined distance, such as one-quarter of a wave length

$$\frac{\lambda}{4}.$$

In this arrangement, the transmission takes place in the direction from A′ and A‴ to A and A″, and the phase of the currents impressed on the conductors A and A″ lags by the angle

$$\frac{\pi}{2}$$

behind the phase of the currents impressed on the conductors A′ and A‴.

The polar curve (*a*) of Fig. 14 exhibits the directive characteristics of one of these couplets, such as A and A′. However, A and A″ and A′ and A‴ represent two pairs of conductors separated by a distance of, for example, one half wave length

$$\frac{\lambda}{2},$$

and the currents in each pair of conductors are of the same phase. The polar curve (*b*) of Fig. 14 exhibits the directive characteristics of the couplet such as A and A″.

A couplet comprising two conductors, such as A and A′, may be regarded as a single conductor situated midway between A and A′ having a mean phase of, for example

$$-\frac{\pi}{4},$$

and having the directive characteristic (*a*) of Fig. 14. Yet, Fig. 13 comprises an array which may be regarded as a pair of such equivalent conductors. The directive characteristics of such a pair of equivalent conductors may be obtained from the product of the characteristics of the two types of couplets of which it is composed, and these directive characteristics are shown graphically by curve (*c*) of Fig. 14. It will be understood that when a parabolic array or a parabolic image array is composed of directive couplets instead of simple radiating conductors, it becomes possible to use arrays having lateral widths less than those which would otherwise be possible, without at the same time suffering too great a diffractive spreading of the beam.

Fig. 15 illustrates a parabolic array based on the type shown in Fig. 13. Such an array may be employed to produce a narrow beam in which there will be no radiation, or substantially no radiation, from the convex side of the array. Fig. 16 illustrates a parabolic image array based on the type shown in Fig. 13 for the production of a narrow beam in one direction only. This arrangement is a modification of the linear or image arrays of Figs. 6 and 11. Fig. 17 illustrates another parabolic image array based on the type shown in Fig. 13, for the production of a parallel beam in a single direction. This arrangement is a modification of the linear arrays of Figs. 11 and 12. From the foregoing disclosure, it will become apparent that one skilled in the art may properly construct a parabolic array of a parabolic image array that will produce an approximately parallel beam of radiation. The number of conductors or wires and their relative proximity will be considered in some detail hereinafter.

The specific means by which currents in the radiated conductors are given their essential relative amplitudes and phases is in each case determined by different conditions. Figs. 18 and 19 illustrate how the length of the supply mains may be employed to secure the requisite relative phase variation between the currents in the radiating conductors of a parabolic array and a parabolic image array, respectively, the phases of the currents in Fig. 18 being different from one another, and the phases of the currents in Fig. 19 being all alike. Figs. 20 and 21 of the drawing illustrate the use of, for example, what may be called lumped phase shifters in the supply mains to bring about the requisite phase variation. Throughout Figs. 18 to 21, inclusive, the reference character G designates a common energizing source, and the equal radiating conductors are made to symbolize the equality of current amplitudes in these conductors. In Figs. 20 and 21, a number of boxes are shown to which the reference characters $\phi 1$, $\phi 2$, $\phi 3$, etc. are attached, these boxes enclosing the lumped phase shifters mentioned hereinabove. Yet it will be understood that it is within the scope of this invention to employ any well-known phase shifting means instead thereof.

In Figs. 18 to 21, inclusive, the supply systems are shown to consist of a number of ground return circuits. In practice, these ground return circuits may preferably be replaced by metallic circuits to eliminate the large end effects due to grounding. Otherwise the velocity of phase propagation in the various conductors will not be the same throughout their lengths from source to sink; nor will the phase be the same at corresponding points of the various conductors.

The parabolic and linear or image arrays developed herein have in each instance been described in connection with the development of a beam of radiation, but it is clear that since the problems of directive transmission and of directive reception are conjugate problems, these arrays developed explicitly for transmission are implicitly also directive receivers. The common source G of alternating current in the radiating conductors of the transmitting arrays is connected to each of these conductors through a suitable phase shifter, as has been mentioned hereinabove. Be this phase shifter linear, as in Figs. 18 and 19, or lumped, as in Figs. 20 and 21, the common receiving device of the receiving array may nevertheless be connected to each receiving wire or conductor of this array through some equally suitable phase shifter. These phase shifters may preferably be identical, wire for wire, with those of the corresponding transmitter. In other words, it is sufficient to replace the generator G by a suitable receiver or demodulator to convert the arrangement from the directive radiating system to the corresponding directive receiving system.

Precautions must be exercised in developing and constructing the various arrays in view of the effects of diffraction due to the relative smallness of the length of the arrays when measured in wave lengths, and, moreover, due to the building up of arrays of discrete oscillators separated by some distance comparable with the wave length. These matters will be considered hereinafter.

Fig. 22 of the drawings shows a plurality of radiators or oscillators equi-distant from one another, through which flow currents of equal magnitude and of the same phase. Let A and A′ be right sections through the equatorial plane of two equal oscillators separated by a distance d. Let these oscillators support equal oscillations, each of which may be given by the following expression:

$$I \cos \omega t \quad (5)$$

in which ω is the periodicity. Consider lines drawn from a very distant point of observation in the equatorial plane making an angle ψ with the line joining A and A′; then the effect at the point of observation due to the current in oscillator A′ will be proportional to the following expression:

$$\cos\left(\omega t+\frac{\pi d}{\lambda}\cos\psi-\phi\right) \quad (6)$$

in which λ is the wave length and ϕ is the arbitrary angular retardation which a wave of periodicity ω experiences in passing from the central point 0 of Fig. 22 to the very distant point of observation, or $\phi=2\pi l/\lambda$, where l is the distance from the 0 to the point of observation, while the effect due to the current in the oscillator A will be correspondingly proportional to the following expression:

$$\cos\left(\omega t-\frac{\pi d}{\lambda}\cos\psi-\phi\right) \quad (7)$$

It is clear that the joint effect at the point of observation will be proportional to the following expression:

$$\cos\left(\frac{\pi d}{\lambda}\cos\psi\right)\cos(\omega t-\phi) \quad (8)$$

Another pair of oscillators B and B′ identical with oscillators A and A′ and supporting the same oscillations given by expression (5) are separated by the distance $3d$. The effect of this pair of oscillators at the distant point of observation will be correspondingly proportional to the following expression:

$$\cos\left(\frac{3\pi d}{\lambda}\cos\psi\right)\cos(\omega t-\phi) \quad (9)$$

Another and third pair of oscillators C and C′ identical with the first and second pairs and supporting identical oscillations are separated by a distance $5d$. The effect of this pair of oscillators at the distant point of observation will be correspondingly proportional to the following expression:

$$\cos\left(\frac{5\pi d}{\lambda}\cos\psi\right)\cos(\omega t-\phi) \quad (10)$$

It will be obvious that if there be a linear array of n such equal pairs of equal oscillators supporting equal oscillations, and in which each oscillator is a distance d from its adjacent oscillator, the amplitude of the effect at the point of observation will be proportional to the following expression:

$$\sum_{m=1}^{m=2n-1}\cos\left(\frac{m\pi d}{\lambda}\cos\psi\right) \quad (11)$$

If the amplitude of the current in each oscillator is made inversely proportional to the number of pairs of oscillators, this latter expression may be replaced by the following:

$$\sum_{m=1}^{m=2n-1}\frac{1}{n}\cos\left(\frac{m\pi d}{\lambda}\cos\psi\right) \quad (12)$$

in which m is odd; and the maximum amplitude, i. e., the amplitude at

$$\psi=\frac{\pi}{2},$$

will be unity. Expression (12) is convenient for comparing the directive characteristics of arrays. It is to be noted, however, that the array of Fig. 22 is not a parabolic image array, but is an array which differs from the parabolic image array in that the intensity or current amplitude is uniform throughout. Such an array will be referred to hereinafter as a uniform amplitude array. A study of this array will clearly bring out that the diffractive effect is dependent upon and is due to the length of the array and to the relation of the interval d between oscillators to the wave length λ.

If the number of equal oscillators in an array of finite length l be infinite and the amplitude of oscillations common thereto be infinitesimal, the array represented in Fig. 23 becomes in effect a continuous conducting sheet supporting a uniform oscillatory current sheet in which the current per unit length of the sheet is given by the following equation:

$$i=A\cos\omega t \quad (13)$$

If dh is the effect at the distant point of observation in the equatorial plane due to the element dx of the current sheet, then the

total effect at the point of observation will be given by the following expression:

$$h = A\int_0^l \cos\left(\omega t - \frac{2\pi l}{\lambda}\cos\psi\right)dx \quad (14)$$

Equation (14), after integration, becomes

$$|h| = A\lambda\frac{\sin\left(\frac{\pi l}{\lambda}\cos\psi\right)}{\pi\cos\psi} \quad (15)$$

For the sake of simplicity, it may be assumed that

$$\frac{A}{\pi} = 1.$$

Thereupon, the field strength may be determined from the following expression:

$$h = \lambda\frac{\sin\left(\frac{\pi l}{\lambda}\cos\psi\right)}{\cos\psi} \quad (16)$$

But when

$$\psi = \frac{\pi}{2}, h = \pi l$$

and therefore

$$\frac{h}{\pi l} = \frac{\sin\left(\frac{\pi l}{\lambda}\cos\psi\right)}{\frac{\pi l}{\lambda}\cos\psi} \quad (17)$$

represents the directional characteristic. The array of Fig. 23 is in effect the limiting case of the array of Fig. 22 which is reached when d becomes infinitesimal and when n becomes infinite. Such an array will be referred to hereinafter as a continuous uniform amplitude array.

Fig. 24 shows in contrast the cartesian directive characteristics of continuous uniform amplitude arrays of different lengths, such, for example, as λ to 4λ, inclusive. This figure shows, among other things, that the beam is very much broadened by diffraction in the case of an array whose length is equal to the wave length. The beam becomes progressively narrower as the length of the array is increased, but fringes appear and multiply, however, as the length of this array is increased.

Fig. 25 shows the cartesian characteristics of two uniform amplitude arrays as contrasted with a continuous uniform amplitude array. The length common to these arrays is one wave length. In this diagram, curve (1) corresponds to the continuous array, while curve (2) corresponds to an array of four oscillators, and curve (3) corresponds to an array of two oscillators.

Added or greater diffraction in the cases of two and four oscillators is clearly shown, but the absolute magnitude of this added or greater diffraction is still more clearly shown in Fig. 26, in which the ordinates of the two curves are respectively the distances between the ordinates of curves (3) and (1) of Fig. 25, and the distances between the ordinates of curves (2) and (1) of the same figure. It is to be specially noted that the effect of the separation between the oscillators of the array is to enhance the diffraction which results in the case of the continuous array. The greater the separation, the greater the magnification of the diffraction.

Fig. 27 shows the cartesian characteristics of two uniform amplitude arrays as contrasted with a continuous uniform amplitude array. A length common to the three arrays has been chosen merely for the purpose of illustration, to be two wave lengths, 2λ. In this figure, the continuous line curve corresponds to the continuous array, while the curves (not drawn) indicated respectively by the triangles and crosses correspond to arrays of six and four oscillators, respectively. The added diffraction in the case of the arrays composed of discrete oscillators is again apparent, as is also its increase with the separation between the oscillators. The absolute magnitude of this effect is made much clearer in Fig. 28 of the drawing. In this figure, curve (4)—(1) is the enhanced diffraction in the case of four oscillators. Curve (3)—(1) is the enhanced diffraction in the case of an array of six oscillators, and (2)—(1) is the enhanced diffraction in the case of an array of eight oscillators. Curves (4)—(1) and (2)—(1) of Fig. 28 should be compared with curves (3)—(1) and (2)—(1) of Fig. 26, respectively. Since in these corresponding curves the separation between the oscillators, expressed in wave lengths, is the same, it appears that the longer the array, expressed in wave lengths, the larger the separation between the oscillators, expressed in wave lengths, may be, though this permissible increase is not directly proportional to the increase in the length of the array. It is clear that the enhancement of the diffraction is already practically negligible, when the separation between the oscillators is as small as a quarter of the wave length, and that a greater separation may often be tolerated. However, the degree of diffraction may readily be determined from the information given herein, though the computation be somewhat tedious.

The directive characteristics of the continuous parabolic array and the continuous parabolic image array do not lend themselves as easily to direct analytical solutions as does the continuous constant amplitude array.

By approximation, the characteristics of the continuous parabolic array and the continuous parabolic image array are given herein in Figs. 29 and 30.

In Fig. 31, an approximately continuous parabolic image array of continuously uni-

form amplitude arrays is built up. In this figure, the continuous curve represents the amplitude variation in a continuous parabolic image array having a predetermined length of, for example, 4λ. The approximate parabolic image array *a b e f i j m n o p k l g h c d* consists of the sum of the continuous uniform amplitude arrays *abcd*, *efgh*, *ijkl*, and *mnop*, which have, for example, the relative amplitudes of 0.3, 1.7, 1.9 and 1.1, respectively, and which have lengths 4λ to λ, respectively.

The fringes exhibited by the curve of Fig. 29 are due to the echelon character of the array of Fig. 31. The directional characteristic of the corresponding uniform parabolic image array is given by the curve ABC having a discontinuity at B. It is this discontinuity which prevents a simple analytical solution for the continuous parabolic image array.

Fig. 30 contrasts the polar directional characteristics of a continuous uniform amplitude array of, for example, three wave lengths, 3λ with that of the continuous parabolic image array of, for example, four wave lengths, 4λ. Though the principal beam of the shorter uniform amplitude array is a somewhat more nearly parallel beam than the beam of the parabolic image array, nevertheless the secondary beams or fringes of the uniform amplitude array militate somewhat against its usefulness. The parabolic image array and a corresponding parabolic array have the extraordinary property of radiating no energy, or negligible energy, beyond a certain critical angle, which in the array whose length is, for example, four wave lengths, is approximately 68°. In other words, the broading of the beam by diffraction is, in this case, limited to 22° on either side of the normal. It is due to the absence of diffraction fringes that parabolic arrays and parabolic image arrays become of considerable importance.

Though the method adopted of exhibiting the effects of diffraction, which is based upon computations of these effects in the case of uniform amplitude arrays, is undoubtedly the one best adapted to the purpose, nevertheless, in determining the characteristic of a given parabolic array, or its corresponding parabolic image array, when these are of the type illustrated in Fig. 10, it is best to use the specific series for these arrays, which is as follows:

$$\sum_{m=1}^{m=2n-1} \sqrt{\frac{1}{a+\frac{\left(\frac{md}{2}\right)^2}{4a}}} \cos\left(\frac{m\pi d}{\lambda}\cos\psi\right) \quad (18)$$

If, on the other hand, the array be of the parabolic or image type, illustrated in Fig. 6, it is best to use the specific series for this type, which is as follows:

$$\sum_{m=1}^{m=2n-1} \cos\left(\frac{4\pi}{\lambda}a\tan\frac{m\theta}{4}\cos\psi\right) \quad (19)$$

These last two series are easily deduced by the same principles and in the same manner as Equation (12). When some other law of spacing the oscillators in a parabolic or image array is used, the corresponding series can be easily determined by the principles and methods already disclosed, if the equation of the parabola is borne in mind and the steps given in connection with Equations (5)–(12) be followed.

This invention has been described in some detail so as to aid in better understanding the complex phenomena of this invention. It will be understood, however, that while this invention has been described in certain particular embodiments, merely for the purpose of illustration, the general principles of this invention may be applied to other and widely varied organizations without departing from the spirit of the invention and the scope of the appended claims.

What is claimed is:

1. A directive antenna system comprising a plurality of pairs of parabolic image arrays, the arrays of each pair being separated from each other by a definite distance, said pairs of arrays being placed side by side and at a predetermined distance from each other.

2. A directive antenna system comprising a plurality of pairs of linear arrays, each linear array consisting of a plurality of radiators, the radiators of each array being separated by unequal and predetermined distances, the linear arrays of each pair being a definite distance from each other, said pairs of arrays being situated side by side and at a predetermined distance apart.

3. A directive antenna array for beam transmission and reception, comprising a linear array of parallel conductors which are unequally spaced, all of which are energized with currents of equal frequencies, of equal amplitudes and of the same phase relationship.

4. A directive antenna system comprising a linear array of conductors which are placed side by side at unequal and predetermined distances from one another, and means for energizing said conductors with currents of the same frequencies, of the same amplitudes and of the same phase relationship.

5. A directive antenna system comprising a linear array of conductors which are placed side by side unequally spaced though at predetermined distances from each other, and means for energizing said conductors with currents of equal amplitudes, of the same frequencies and in a definite phase relationship, said array effecting a beam of energy equiv-

alent to the beam of a cylindrical parabolic surface having a conductor coincident with its focal line receiving current of a definite amplitude.

6. A directive antenna system comprising a pair of linear arrays, each linear array consisting of a plurality of conductors which are separated by unequal and predetermined distances, said linear arrays being similar and being separated by a definite distance, and means for energizing the conductors of each linear array with currents of equal amplitudes, of the same frequency and of the same phase, the currents impressed upon the conductors of one of the linear arrays differing in phase from the currents impressed upon the other linear arrays by a definite angle.

7. A directive antenna system comprising a pair of linear arrays, the conductors of each array being unequally spaced though separated by predetermined distances, and means for impressing currents upon both of said linear arrays so that the phase of the currents impressed upon one of these arrays lags behind the phase of the current impressed upon the other of these arrays by an angle dependent upon the distance between these arrays.

8. A directive antenna system comprising a pair of identical linear arrays, the conductors of each linear array being unequally spaced and at predetermined distances from each other, one linear array being positioned directly behind the other.

9. The method of securing uni-directional transmission with a pair of linear antenna arrays each composed of a plurality of conductors unequally spaced and at predetermined distances from each other, which consists in exciting the conductors in both of the arrays with currents of the same frequency and of equal amplitudes, so that their cumulative directive effect shall approximate the directive effect of equivalent parabolic arrays.

10. The method of securing uni-directional transmission or reception with a pair of linear antenna arrays, which consists in exciting the conductors in both of the linear antenna arrays with currents of equal amplitude and of the same frequency, and adjusting the phases of the currents in the conductors in one of the linear antenna arrays to lag behind the phases of the currents in the conductors in the other of the linear antenna arrays by an angle dependent upon the distance between said linear antenna arrays.

11. A directive antenna array comprising a plurality of radiators in parallel relationship, which are spaced from each other in accordance with the projections upon the directrix of a cylindrical parabola of a corresponding plurality of parallel lines located on its surface equi-angularly displaced with respect to each other about the focal line.

12. A directive antenna array comprising a plurality of radiators placed side by side unequally spaced symmetrically about the center line, and means for energizing said radiators with equal currents which are in synchronism.

In testimony whereof, I have signed my name to this specification this 19th day of January 1927.

JOHN STONE STONE.