

Jan. 17, 1950

S. L. GOLDSBOROUGH
DISTANCE-TYPE RELAYS WITH LIMITED
DIRECTIONAL DISCRIMINATION

2,495,166

Filed Dec. 29, 1945

2 Sheets-Sheet 1

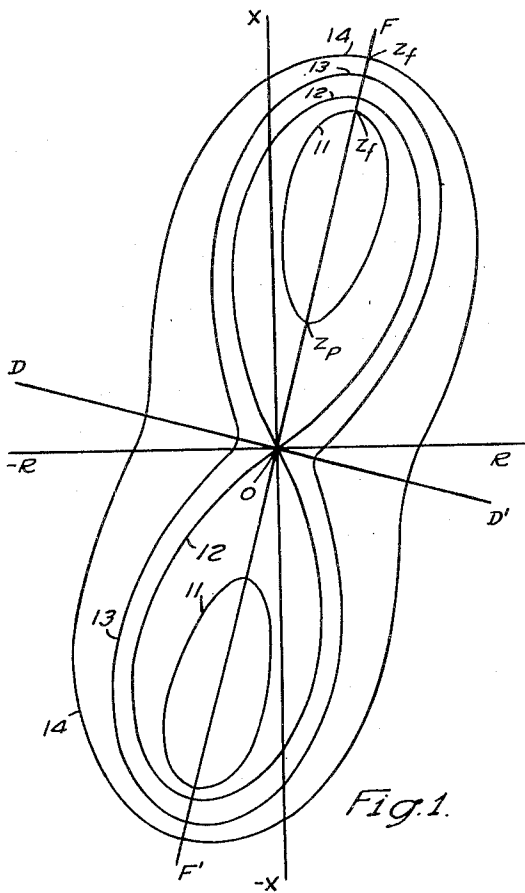


Fig. 1.

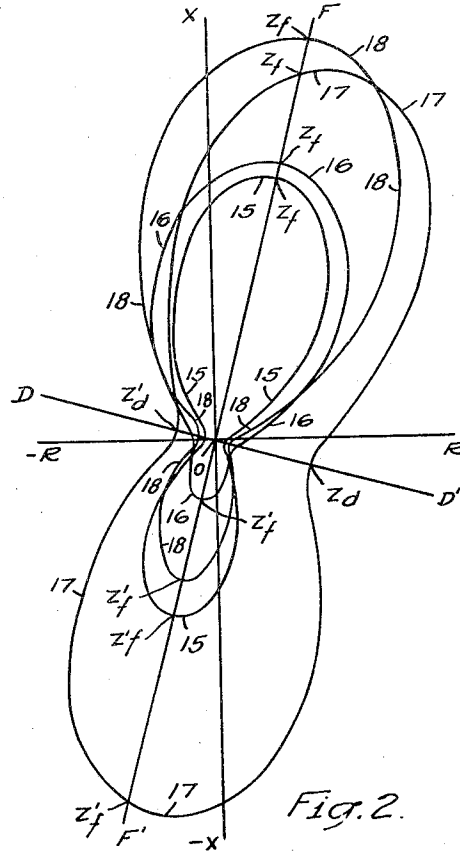


Fig. 2.

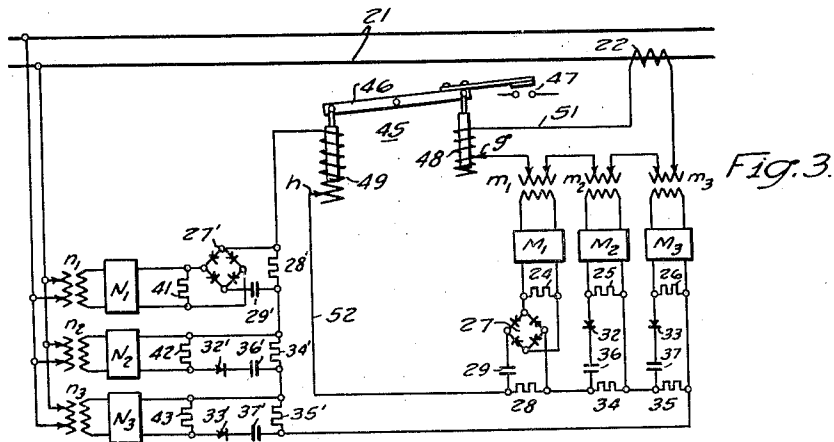


Fig. 3.

WITNESSES:

E. A. M. [Signature]

INVENTOR

Shirley L. Goldsborough.

BY O. B. Buchanan

ATTORNEY

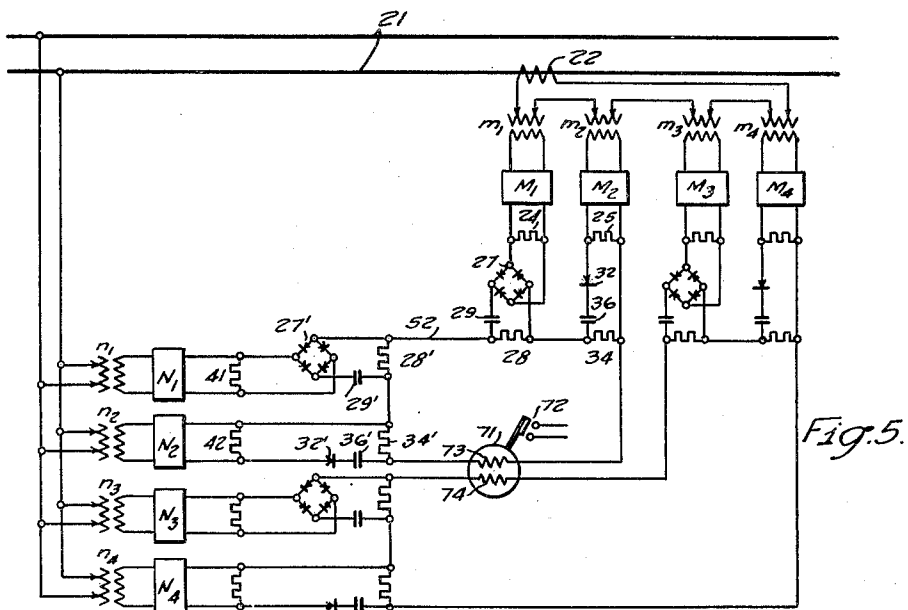
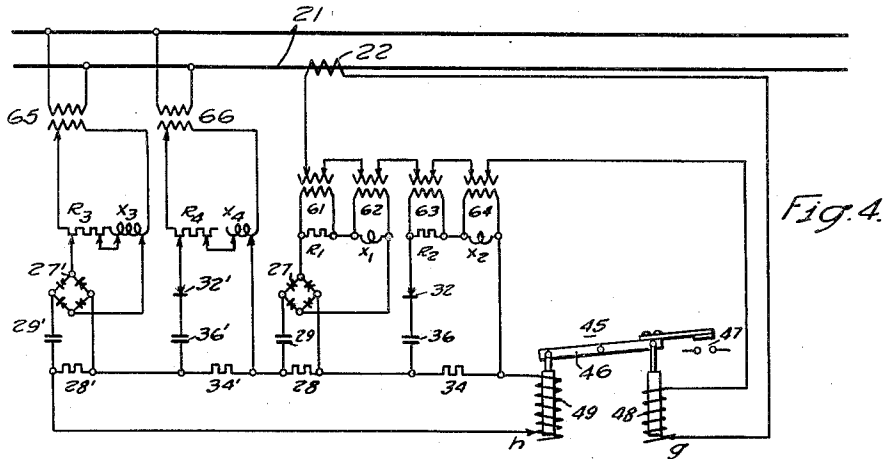
Jan. 17, 1950

S. L. GOLDSBOROUGH
DISTANCE-TYPE RELAYS WITH LIMITED
DIRECTIONAL DISCRIMINATION

2,495,166

Filed Dec. 29, 1945

2 Sheets-Sheet 2



WITNESSES:

E. A. M. Blakey.

INVENTOR

Shirley L. Goldsborough.

BY O. B. Buchanan
ATTORNEY

UNITED STATES PATENT OFFICE

2,495,166

DISTANCE-TYPE RELAY WITH LIMITED
DIRECTIONAL DISCRIMINATION

Shirley L. Goldsborough, Basking Ridge, N. J., assignor to Westinghouse Electric Corporation, East Pittsburgh, Pa., a corporation of Pennsylvania

Application December 29, 1945, Serial No. 638,355

20 Claims. (Cl. 175-294)

1

My present invention relates to distance-type relays for protecting alternating-current transmission-lines against faults.

An object of my invention is to produce a distance-relay with a limited directional discrimination, having a long "reach," in both directions, along the fault-ohms line or axis, and having a short "reach" for currents 90° out of phase with the fault-currents, or along the directional axis.

More generally stated, it is an object of my invention to provide a relay having a response-curve which is elongated along the fault-axis, and constricted along the directional axis, in the region of the origin.

In many relay-applications, as in carrier-current high-speed relaying, it is very desirable to have a third-zone distance-element which can "see" faults in either direction along the fault-axis, but having a limited response to load-ohms and to synchronizing-ohms. In the third-zone impedance-relay (Z_3) as has been common prior to my present invention, the center of the impedance-circle has been displaced from the origin, with the result that it has been necessary to utilize a separate Z_3 element to "look" in the reverse direction to start carrier. By my present invention, I provide an impedance-element in which, when the impedance-circle center is displaced from the origin, it is displaced in exactly opposite directions at the same time.

Even aside from carrier-current relay-applications in which the lack of the ability of a third-zone impedance-element to "see" in both directions is a disadvantage, a displaced-center relay in which the circle-center displacement is greater than the circle-radius has a further disadvantage by reason of the fact that the circle does not include the origin of the coordinate axes of line-resistance R and line-reactance X , so that the relay will not respond to close-in faults, of low ohms. A modification of my invention makes it possible to give the response-curve of the relay a bulge or protuberance which encircles the origin.

A still further refinement of my improved relay makes it possible to distort the response-curve in such direction as to enable it the better to respond to faults having a considerable fault-resistance.

A generally stated object of my invention is to

2

provide a distance-type relay utilizing rectified current and voltage-responsive quantities to produce one or both of the relay-fluxes.

In the accompanying drawing, Figures 1 and 2 are curve-diagrams showing some of the possibilities of my relay-adjustments, and Figs. 3, 4 and 5 are diagrammatic views of circuits and apparatus embodying my invention in three different forms of embodiment.

I shall first explain the mathematical principles of the invention, and then the specific illustrative forms of embodiment thereof.

Let us assume three derived current-responsive alternating voltages having instantaneous values,

$$i_1 = m_1 \sqrt{2} I \sin (wt - \theta + M_1) \quad (1)$$

$$i_2 = m_2 \sqrt{2} I \sin (wt - \theta + M_2) \quad (2)$$

and

$$i_3 = m_3 \sqrt{2} I \sin (wt - \theta + M_3) \quad (3)$$

where I is the R. M. S. value of the line-current, and θ is the power-factor angle of the line, counting θ positive for lagging power-factors, and counting M_1 , M_2 and M_3 positive when they make their derived voltages i_1 , i_2 and i_3 lead the line-current $\sqrt{2} I \sin (wt - \theta)$; m_1 , m_2 , m_3 , I and w being scalar quantities greater than zero.

Now let us assume three derived voltage-responsive alternating voltages having instantaneous values,

$$v_1 = -n_1 \sqrt{2} E \sin (wt + N_1) \quad (4)$$

$$v_2 = -n_2 \sqrt{2} E \sin (wt + N_2) \quad (5)$$

and

$$v_3 = -n_3 \sqrt{2} E \sin (wt + N_3) \quad (6)$$

where E is the R. M. S. value of the line-voltage, counting N_1 , N_2 and N_3 positive when they make their derived voltages v_1 , v_2 and v_3 lead the line-voltage $-\sqrt{2} E \sin wt$; n_1 , n_2 , n_3 and E being scalar quantities greater than zero.

Now let us assume that each of the voltages i_1 and v_1 is perfectly rectified by a separate double-wave rectifier; that each of the voltages i_2 , i_3 , v_2 and v_3 is perfectly rectified by a separate single-wave rectifier; that the direct-current component of each of the six rectified voltages is eliminated; and that the un-smoothed-out alternating-current components of the six rectified volt-

ages are added, to produce a restraint-voltage v_r for a differential relay. The harmonic analyses given in Jolley's Alternating Current Rectification, New York, Wiley, 1926, pages 32-33, show that the instantaneous value of the restraint-voltage is

$$v_r = + \frac{4n_1\sqrt{2}E}{\pi} \left[\frac{\cos 2(wt+N_1)}{1 \times 3} + \frac{\cos 4(wt+N_1)}{3 \times 5} + \dots + \frac{\cos 2k(wt+N_1)}{(2k-1)(2k+1)} + \dots \right] +$$

$$- \frac{4m_1\sqrt{2}I}{\pi} \left[\frac{\cos 2(wt-\theta+M_1)}{1 \times 3} + \dots + \frac{\cos 2k(wt-\theta+M_1)}{(2k-1)(2k+1)} + \dots \right] +$$

$$- \frac{1}{2}n_2\sqrt{2}E \sin (wt+N_2) + \frac{2n_2\sqrt{2}E}{\pi} \left[\sum \frac{\cos 2k(wt+N_2)}{(2k-1)(2k+1)} \right] +$$

$$+ \frac{1}{2}m_2\sqrt{2}I \sin (wt-\theta+M_2) - \frac{2m_2\sqrt{2}I}{\pi} \left[\sum \frac{\cos 2k(wt-\theta+M_2)}{(2k-1)(2k+1)} \right] +$$

$$- \frac{1}{2}n_3\sqrt{2}E \sin (wt+N_3) + \frac{2n_3\sqrt{2}E}{\pi} \left[\sum \frac{\cos 2k(wt+N_3)}{(2k-1)(2k+1)} \right] -$$

$$+ \frac{1}{2}m_3\sqrt{2}I \sin (wt-\theta+M_3) - \frac{2m_3\sqrt{2}I}{\pi} \left[\sum \frac{\cos 2k(wt-\theta+M_3)}{(2k-1)(2k+1)} \right] \quad (7)$$

where k is the k th term of an infinite series of terms which are added.

When this restraint-voltage v_r is applied to the restraint-coil of the differential relay, let us assume that the restraint-coil flux h_r leads the impressed voltage v_r by an angle T , and has a magnitude which is h times the magnitude of the impressed voltage v_r ; h being a scalar quantity greater than zero, and the value of the angle T making no difference in the operation of the relay. The instantaneous value of the restraint-coil flux or field-strength is

$$h_r = + \frac{4h\sqrt{2}}{\pi} \left[\sum \frac{n_1E \cos 2k(wt+N_1+T) - m_1I \cos 2k(wt-\theta+M_1+T)}{(2k-1)(2k+1)} \right] +$$

$$+ \frac{2h\sqrt{2}}{\pi} \left[\sum \frac{n_2E \cos 2k(wt+N_2+T) - m_2I \cos 2k(wt-\theta+M_2+T)}{(2k-1)(2k+1)} \right] +$$

$$+ \frac{2h\sqrt{2}}{\pi} \left[\sum \frac{n_3E \cos 2k(wt+N_3+T) - m_3I \cos 2k(wt-\theta+M_3+T)}{(2k-1)(2k+1)} \right] +$$

$$+ \frac{1}{2}h\sqrt{2}[-n_2E \sin (wt+N_2+T) + m_2I \sin (wt-\theta+M_2+T)] +$$

$$+ \frac{1}{2}h\sqrt{2}[-n_3E \sin (wt+N_3+T) + m_3I \sin (wt-\theta+M_3+T)] \quad (8)$$

The square of this instantaneous restraint-flux h_r is a measure of the restraint-force. As we are interested only in the average value H_r^2 of the instantaneous restraint-force h_r^2 , our calculations are much simplified, because we can drop all terms containing $\cos (yt+Y)$ or $\sin (yt+Y)$, because

Remembering also that

$$\cos^2 (yt+Y) = \frac{1}{2} + \frac{1}{2} \cos (yt+Y)$$

$$\sin^2 (yt+Y) = \frac{1}{2} - \frac{1}{2} \cos (yt+Y)$$

$$\cos x \cos y = \frac{1}{2} \cos (x-y) + \frac{1}{2} \cos (x+y)$$

$$\sin x \sin y = \frac{1}{2} \cos (x-y) - \frac{1}{2} \cos (x+y)$$

$$\sin x \cos y = \frac{1}{2} \sin (x-y) + \frac{1}{2} \sin (x+y) \quad (11)$$

and putting

$$S_1 = M_1 - N_1$$

$$S_2 = M_2 - N_2$$

$$S_3 = M_3 - N_3$$

(12)

we find, from Equation 8, that the average value of the restraint-force is

$$H_r^2 = + \frac{16h^2}{\pi^2} \left\{ (n_1^2E^2 + m_1^2I^2) \left[\sum \frac{1}{(4k^2-1)^2} \right] - 2m_1n_1EI \left[\sum \frac{\cos 2k(\theta-S_1)}{(4k^2-1)^2} \right] \right\} +$$

$$+ \frac{4h^2}{\pi^2} \left\{ (n_2^2E^2 + m_2^2I^2) \left[\sum \frac{1}{(4k^2-1)^2} \right] - 2m_1n_1EI \left[\sum \frac{\cos 2k(\theta-S_2)}{(4k^2-1)^2} \right] \right\} +$$

$$+ \frac{4h^2}{\pi^2} \left\{ (n_3^2E^2 + m_3^2I^2) \left[\sum \frac{1}{(4k^2-1)^2} \right] - 2m_3n_3EI \left[\sum \frac{\cos 2k(\theta-S_3)}{(4k^2-1)^2} \right] \right\} +$$

$$+ \frac{h^2}{4} [(n_2^2E^2 + m_2^2I^2) - 2m_2n_2EI \cos (\theta-S_2)] +$$

$$+ \frac{h^2}{4} [(n_3^2E^2 + m_3^2I^2) - 2m_3n_3EI \cos (\theta-S_3)] +$$

$$+ \frac{16h^2}{\pi^2} \left\{ n_1n_2E^2 \left[\sum \frac{\cos 2k(n_1-N_2)}{(4k^2-1)^2} \right] + m_1m_2I^2 \left[\sum \frac{\cos 2k(M_1-M_2)}{(4k^2-1)^2} \right] \right\} +$$

$$- \frac{16h^2}{\pi^2} \left\{ m_2n_1EI \left[\sum \frac{\cos 2k(\theta-M_2+N_1)}{(4k^2-1)^2} \right] + m_1n_2EI \left[\sum \frac{\cos 2k(\theta-M_1+N_2)}{(4k^2-1)^2} \right] \right\} +$$

$$+ \frac{16h^2}{\pi^2} \left\{ n_1n_3E^2 \left[\sum \frac{\cos 2k(N_1-N_3)}{(4k^2-1)^2} \right] + m_1m_3I^2 \left[\sum \frac{\cos 2k(M_1-M_3)}{(4k^2-1)^2} \right] \right\} +$$

$$\begin{aligned}
& -\frac{16h^2}{\pi^2} \left\{ m_3 n_1 EI \left[\sum \frac{\cos 2k(\theta - M_3 + N_1)}{(4k^2 - 1)^2} \right] + m_1 n_3 EI \left[\sum \frac{\cos 2k(\theta - M_1 + N_3)}{(4k^2 - 1)^2} \right] \right\} + \\
& + \frac{8h^2}{\pi^2} \left\{ n_2 n_3 E^2 \left[\sum \frac{\cos 2k(N_2 - N_3)}{(4k^2 - 1)^2} \right] + m_2 m_3 I^2 \left[\sum \frac{\cos 2k(M_2 - M_3)}{(4k^2 - 1)^2} \right] \right\} + \\
& - \frac{8h^2}{\pi^2} \left\{ m_3 n_2 EI \left[\sum \frac{\cos 2k(\theta - M_3 + N_2)}{(4k^2 - 1)^2} \right] + m_2 n_3 EI \left[\sum \frac{\cos 2k(\theta - M_2 + N_3)}{(4k^2 - 1)^2} \right] \right\} + \\
& + \frac{h^2}{2} [n_2 n_3 E^2 \cos(N_2 - N_3) + m_2 m_3 I^2 \cos(M_2 - M_3)] + \\
& - \frac{h^2}{2} [m_2 n_3 EI \cos(\theta - M_2 + N_3) + m_3 n_2 EI \cos(\theta - M_3 + N_2)] + \\
& + \frac{4h^2}{3\pi} [n_1 n_2 E^2 \sin(N_1 - N_2) + m_1 m_2 I^2 \sin(M_1 - M_2)] + \\
& + \frac{4h^2}{3\pi} [m_2 n_1 EI \sin(M_2 - N_1 - \theta) + m_1 n_2 EI \sin(\theta - M_1 + N_2)] + \\
& + \frac{4h^2}{3\pi} [n_1 n_3 E^2 \sin(N_1 - N_3) + m_1 m_3 I^2 \sin(M_1 - M_3)] + \\
& + \frac{4h^2}{3\pi} [m_3 n_1 EI \sin(M_3 - N_1 - \theta) + m_1 n_3 EI \sin(\theta - M_1 + N_3)] \quad (13)
\end{aligned}$$

In Equation 13,

$$\left[\sum \frac{1}{(4k^2 - 1)^2} \right] = \frac{1}{9} + \frac{1}{225} + \frac{1}{1225} + \frac{1}{3969} + \dots = \left(\frac{\pi^2}{16} - \frac{1}{2} \right) = 0.1168503 \quad (14)$$

$$\frac{16}{\pi^2} = 1.62114 \quad (15)$$

$$\begin{aligned}
\left[\sum \frac{\cos 2kx}{(4k^2 - 1)^2} \right] = f(x) = & + \frac{\cos 2x}{9} + \frac{\cos 4x}{225} + \frac{\cos 6x}{1225} + \frac{\cos 8x}{3969} + \\
& + \frac{\cos 10x}{9801} + \frac{\cos 12x}{20449} + \frac{\cos 14x}{38025} + \frac{\cos 16x}{62500} + \\
& + \frac{\cos 18x}{104329} + \dots \quad (16)
\end{aligned}$$

The values of $f(x)$ and $[f(x)]^2$, for different values of the angle x , are shown in the table.

Table

x				$f(x)$	$[f(x)]^2$
Degrees	Degrees	Degrees	Degrees		
0	180	180	360	+ .11685	+ .01365
5	175	185	355	+ .1147	+ .01316
10	170	190	350	+ .1084	+ .01175
15	165	195	345	+ .0982	+ .00964
20	160	200	340	+ .0852	+ .00726
25	155	205	335	+ .0697	+ .00486
30	150	210	330	+ .0526	+ .00276
35	145	215	325	+ .0340	+ .00116
40	140	220	320	+ .0150	+ .000225
43.0	136.1	223.0	316.1	0	0
45	135	225	315	-.0042	-.000018
50	130	230	310	-.0229	+.00052
55	125	235	305	-.0407	+.00166
60	120	240	300	-.0572	+.00327
65	115	245	295	-.0718	+.005155
70	110	250	290	-.08425	+.00710
75	105	255	285	-.0941	+.008355
80	100	260	280	-.1014	+.01023
85	95	265	275	-.1058	+.01119
90	90	270	270	-.1073	+.01151

Rewriting Equation 13, we find the average restraint-force (divided by the constant, h^2) to be

$$\begin{aligned}
\frac{H_r^2}{h^2} = & + E^2 [0.18943 n_1^2 + 0.29736 (n_2^2 + n_3^2) + 0.5 n_2 n_3 \cos(N_2 - N_3)] + \\
& + 0.81057 E^2 [2 n_1 n_2 f(N_1 - N_2) + 2 n_1 n_3 f(N_1 - N_3) + n_2 n_3 f(N_2 - N_3)] + \\
& + 0.42441 E^2 [n_1 n_2 \sin(N_1 - N_2) + n_1 n_3 \sin(N_1 - N_3)] + \\
& + I^2 [0.18943 m_1^2 + 0.29736 (m_2^2 + m_3^2) + 0.5 m_2 m_3 \cos(M_2 - M_3)] + \\
& + 0.81057 I^2 [2 m_1 m_2 f(M_1 - M_2) + 2 m_1 m_3 f(M_1 - M_3) + m_2 m_3 f(M_2 - M_3)] + \\
& + 0.42441 I^2 [m_1 m_2 \sin(M_1 - M_2) + m_1 m_3 \sin(M_1 - M_3)] + \\
& - 0.81057 EI [4 m_1 n_1 f(\theta - S_1) + m_2 n_2 f(\theta - S_2) + m_3 n_3 f(\theta - S_3)] + \\
& - 0.5 EI [m_2 n_2 \cos(\theta - S_2) + m_3 n_3 \cos(\theta - S_3)] + \\
& - 0.81057 EI [2 m_1 n_2 f(\theta - M_1 + N_2) + 2 m_2 n_1 f(\theta - M_2 + N_1) + 2 m_1 n_3 f(\theta - M_1 + N_3)] + \\
& - 0.81057 EI [2 m_3 n_1 f(\theta - M_3 + N_1) + m_2 n_3 f(\theta - M_2 + N_3) + m_3 n_2 f(\theta - M_3 + N_2)] + \\
& - 0.42441 EI [m_2 n_1 \sin(\theta - M_2 + N_1) + m_3 n_1 \sin(\theta - M_3 + N_1)] + \\
& + 0.42441 EI [m_1 n_2 \sin(\theta - M_1 + N_2) + m_1 n_3 \sin(\theta - M_1 + N_3)] \quad (17)
\end{aligned}$$

Now let us assume that the differential relay has an operating-coil flux or field-strength having an instantaneous value

$$h_o = g \sqrt{2} I \sin(wt - \theta + U) \quad (18)$$

and having an average-square value or operating force (again divided by the constant, h^2) of

$$\frac{H_o^2}{h^2} = \frac{g^2 I^2}{h^2} \quad (19)$$

where g is a scalar quantity greater than zero, and U is any angle, which makes no difference in the operation.

The relay will operate when the restraining force is less than the operating force, or when

$$H_r^2 < H_o^2 \quad (20)$$

To determine the relay-operation in terms of the line-impedance Z , it is necessary to note that the numerical value of the line-impedance is

$$Z = \frac{E}{I} \quad (21)$$

This is a scalar quantity, and it is always positive, because E and I are both scalar quantities, both always positive. Z represents a length which is to be measured, from the pole or origin, on the radius vector which thus becomes a line representing an impedance-angle or power-factor angle. The radius vector or impedance-line is at an angle θ from the polar axis or the line representing line-resistance R .

Substituting from Equations 17, 19 and 21 in 10 the inequality (20), rearranging the terms, and

understanding that I am not limited, of course, to the particular settings chosen for illustration.

The simplest case is when I utilize only the two derived voltages i_1 and v_1 , which are double-wave rectified.

In this case,

$$m_2 = m_3 = n_2 = n_3 = 0 \quad (32)$$

Substituting from Equations 27, 28 and 32 in Equations 24, 25 and 26, we find, from Equation 23, that the balance-point of the relay is at any line-impedance which satisfies the equation

$$\begin{aligned} Z &= \frac{+32}{\pi^2} m_1 n_1 f(\theta - S_1) \pm \sqrt{\frac{1024}{\pi^4} m_1^2 n_1^2 [f(\theta - S_1)]^2 - \left(4 - \frac{32}{\pi^2}\right) \left[\frac{g^2}{h^2} + \left(1 - \frac{8}{\pi^2}\right) m_1^2\right] n_1^2} \\ &\quad \left(2 - \frac{16}{\pi^2}\right) n_1^2 \\ &= \frac{16}{\pi^2 - 8} \frac{m_1}{n_1} f(\theta - S_1) \pm \frac{m_1}{n_1} \sqrt{\left(\frac{256}{\pi^2 - 8}\right) 2[f(\theta - S_1)]^2 - 1 + \frac{\pi^2}{\pi^2 - 8} \frac{g^2}{h^2 m_1^2}} \\ &= 8.558 \frac{m_1}{n_1} f(\theta - S_1) \pm \frac{m_1}{n_1} \sqrt{73.24[f(\theta - S_1)]^2 - 1 + \frac{5.2790g^2}{h^2 m_1^2}} \\ &= 8.558 \frac{m_1}{n_1} f(\theta - S_1) \pm 8.558 \frac{m_1}{n_1} \sqrt{[f(\theta - S_1)]^2 - 0.01365 + 0.07208g^2/h^2 m_1^2} \end{aligned} \quad (33)$$

dividing through by I^2 , we find that the relay will respond when

$$aZ^2 + bZ + c < 0 \quad (22)$$

whence

$$Z < \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (23)$$

where

$$\begin{aligned} a &= +0.18943n_1^2 + 0.29736(n_2^2 + n_3^2) + 0.5n_2n_3 \cos(N_2 - N_3) + \\ &\quad + 0.81057[2n_1n_2f(N_1 - N_2) + 2n_1n_3f(N_1 - N_3) + n_2n_3f(N_2 - N_3)] + \\ &\quad + 0.42441[n_2n_3 \sin(N_1 - N_2) + n_1n_3 \sin(N_1 - N_3)] \end{aligned} \quad (24)$$

$$\begin{aligned} b &= -0.81057[4m_1n_1f(\theta - S_1) + m_2n_2f(\theta - S_2) + m_3n_3f(\theta - S_3)] + \\ &\quad - 0.5[m_2n_2 \cos(\theta - S_2) + m_3n_3 \cos(\theta - S_3)] + \\ &\quad - 1.62114[m_1n_2f(\theta - M_1 + N_2) + m_2n_1f(\theta - M_2 + N_1)] + \\ &\quad - 1.62114[m_1n_3f(\theta - M_1 + N_3) + m_3n_1f(\theta - M_3 + N_1)] + \\ &\quad - 0.81057[m_2n_3f(\theta - M_2 + N_3) + m_3n_2f(\theta - M_3 + N_2)] + \\ &\quad - 0.42441[m_2n_1 \sin(\theta - M_2 + N_1) + m_3n_1 \sin(\theta - M_3 + N_1)] + \\ &\quad + 0.42441[m_1n_2 \sin(\theta - M_1 + N_2) + m_1n_3 \sin(\theta - M_1 + N_3)] \end{aligned} \quad (25)$$

$$\begin{aligned} c &= -\frac{g^2}{h^2} + 0.18943m_1^2 + 0.29736(m_2^2 + m_3^2) + 0.5m_2m_3 \cos(M_2 - M_3) + \\ &\quad + 0.81057[2m_1m_2f(M_1 - M_2) + 2m_1m_3f(M_1 - M_3) + m_2m_3f(M_2 - M_3)] + \\ &\quad + 0.42441[m_1m_2 \sin(M_1 - M_2) + m_1m_3 \sin(M_1 - M_3)] \end{aligned} \quad (26)$$

$$0.18943 = \frac{16}{\pi^2} \left[\sum \frac{1}{(4k^2 - 1)^2} \right] = \frac{16}{\pi^2} \left(\frac{\pi^2}{16} - \frac{1}{2} \right) = 1 - \frac{8}{\pi^2} \quad (27)$$

$$0.81057 = \frac{8}{\pi^2} \quad (28)$$

$$0.29736 = \frac{1}{4} + \left(\frac{1}{4} - \frac{2}{\pi^2} \right) = \frac{1}{2} - \frac{2}{\pi^2} \quad (29)$$

$$0.42441 = \frac{4}{3\pi} \quad (30)$$

$$1.62114 = \frac{16}{\pi^2} \quad (31)$$

$f(x)$ is as defined in Equation 16, and Z must always be positive. Only positive values of Z are valid solutions of the inequality (23).

At the balance-point of the relay, the inequality-sign, $<$, in Equation 23 becomes an equality-sign, $=$. The locus of the line-impedance Z , at various power-factor angles θ , is usually plotted on rectangular coordinates representing the line-resistance R and the lagging-power-factor line-reactance X , yielding a separate response-curve for each relay-setting.

The performance of the relay represented by the Equation 23, and the effects of some of the variables in the relay-adjustment, will best be illustrated by several concrete examples, with the

lay, at four settings of the relay-constants of Equation 33, as follows:

For curve 11,

$$S_1 = (M_1 - N_1) = 75^\circ \text{ and } 5.279g^2/h^2 m_1^2 = 0.2 \quad (34)$$

For curve 12,

$$S_1 = (M_1 - N_1) = 75^\circ \text{ and } 5.279g^2/h^2 m_1^2 = 1 \quad (35)$$

For curve 13,

$$S_1 = (M_1 - N_1) = 75^\circ \text{ and } 5.279g^2/h^2 m_1^2 = 1.5 \quad (36)$$

For curve 14,

$$S_1 = (M_1 - N_1) = 75^\circ \text{ and } 5.279g^2/h^2 m_1^2 = 4 \quad (37)$$

The scale of ohms or distance has not been indicated in Fig. 1, because that scale may be made anything desired, by the proper choice of the value of m_1/n_1 .

Equation 33 expresses the line-impedances Z , at the balance-points of the relay, in polar coordinates, in terms of the angle $x = (\theta - S_1)$, using

the fault-impedance line OF as the polar axis. This line represents a lagging power-factor angle $\theta = S_1 = (M_1 - N_1)$, which has been chosen as 75° because that is a typical or illustrative value of the fault-impedance angle of a transmission line. The polar axis OF in Fig. 1 can be given any inclination by properly choosing the phase-shift angles M_1 and N_1 which are defined in Equations 1 and 4.

Since S_1 is a fixed angle, fixed by the selection of the relay-constants M_1 and N_1 Equation 33 also expresses the balance-point line-impedances Z , in polar coordinates, in terms of the lagging-power-factor angle θ of the line-current, using the pure-resistance impedance-line R as the polar axis. By thus plotting the characteristic response-curve in polar coordinates, and then adding a line representing the lagging-power-factor line-reactance X, we have a representation of the relay-characteristics in rectangular coordinates (R,X) as shown in Fig. 1.

The "reach" of the relay, or the distance to which it responds, is shown by the intercept of the response-curve on the fault-axis FOF'. It is found by putting $(\theta - S_1) = 0$ or 180° in Equation 33. It is

$$Z_f = \frac{m_1}{n_1} (1 + \sqrt{5.279g^2/h^2m_1^2}) = \frac{m_1}{n_1} \left(1 + 2.298 \frac{g}{hm_1} \right) \quad (38)$$

When $5.279g^2/h^2m_1^2 < 1$, or

$$\frac{g}{hm_1} < 0.4352$$

there will be a second intercept on each of the fault-lines OF and OF', corresponding to negative values of the radical in Equation 38, this second intercept being an inner intercept, closer to the origin, as shown by the equation

$$Z_o = \frac{m_1}{n_1} (1 - \sqrt{5.279g^2/h^2m_1^2}) = \frac{m_1}{n_1} \left(1 - 2.298 \frac{g}{hm_1} \right) = \frac{1}{n_1} \left(m_1 - 2.298 \frac{g}{h} \right) \quad (39)$$

The response-curve of the relay then consists of two separate loops, or ellipse-like figures, bisected by the line-fault axis FOF', as shown in curve 11 of Fig. 1. The ratio of the two fault-axis intercepts is

$$\frac{Z_o}{Z_f} = \frac{1 - 2.298g/hm_1}{1 + 2.298g/hm_1} \quad (40)$$

When $5.279g^2/h^2m_1^2 > 1$, or

$$\frac{g}{hm_1} > 0.4352$$

the response-curve of the relay has an hour-glass shape, with its length along the line-fault axis FOF', and its waist along the directional axis DOD' which passes through the origin at right angles to FOF'. Two typical response-curves of this class are shown at 13 and 14 in Fig. 1. The "reach" of the relay, or the intercept of its characteristic curve on the fault-axis FOF', is Z_f , as indicated in Equation 38. Its intercept on the directional axis DOD' is found by putting $(\theta - S_1) = 90^\circ$ or 270° in Equation 33. It is

$$Z_d = \frac{m_1}{n_1} (-0.9183 + \sqrt{-0.1568 + 5.279g^2/h^2m_1^2}) \\ = \text{approximately } \frac{m_1}{n_1} \left(-0.92 + 2.3 \frac{g}{hm_1} - 0.03 \frac{hm_1}{g} \right) \quad (41)$$

The ratio of these two intercepts is

$$\frac{Z_d}{Z_f} = \text{approximately } \frac{1 - 0.40hm_1/g - 0.015h^2m_1^2/g^2}{1 + 0.435hm_1/g} \quad (42)$$

It will be observed that the ratio g/hm_1 , in Equation 33, affects the shape of the response-curve, or the ratio of its intercepts Z_o/Z_f or Z_d/Z_f , as the case may be. The "reach" Z_f of the relay, or the distance to which it responds, measured in impedance-ohms, as shown in Equation 38, is directly responsive to the ratio of the relay-constants m_1 and n_1 , which are defined in Equations 1 and 4. It is also affected by the magnitudes of the current-response coefficient m_1 and the ratio of the relay-constants g and h , which determine the flux-strength, or ratio of coil-turns to magnetic reluctance, of the operating and restraining coils respectively, as defined in Equations 18 and 8. It is thus clear that the shape of the curve can be controlled by varying the current-response coefficient m_1 , the operating-coil coefficient g , or the restraint-coil coefficient h ; and after the shape has been determined, the size of the curve, or the scale of impedance-ohms to be applied to it, can be controlled by varying the voltage-response coefficient n_1 , if it is desired to retain the selected shape undisturbed; or the size-scale or distance-calibration could alternatively be controlled by varying the current-response coefficient m_1 , if a simultaneous change in the intercept Z_o or Z_d is unobjectionable.

Examination of the harmonic analysis of the full-wave-rectified restraint-voltage v_r , as given in Equation 7, and the resulting derivations, will show that the second harmonic term, containing the factor $\cos 2(\omega t + N_1)$, is the largest harmonic, and that this harmonic could be segregated and utilized alone—or, in general, the relative magnitudes of the different harmonics could be considerably distorted—without qualitatively changing the results to any material extent, merely changing the magnitudes of the various constants and the magnitudes of the effects which they have on the relay-characteristics.

The relay-characteristics can be changed by superimposing, on the full-wave-rectified component, one or more half-wave-rectified components of either the line-current, or the line-voltage, or both. Four illustrative cases will be given, and plotted in Fig. 2, with the understanding that these particular cases are only illustrative, as the inequality (23) is perfectly general, and applicable to all possible values of all of the constants (or variable factors of the relay), within the limitations which were indicated when the inequality (23) was first written.

In general, the effect of adding a half-wave-rectified component, by assigning finite values to m_2 , n_2 , m_3 and/or n_3 , is to make the response-curve of the relay unsymmetrical with respect to the directional axis D—D'. If m_2 and n_2 are equal to each other, and if the angle S_2 is equal to S_1 (or to $S_1 + 180^\circ$), the effect of adding the m_2 and n_2 components to the curves shown in Fig. 1 is to make the portion of the curve above (or below) the line D—D' larger, and the other portion smaller, without destroying the symmetry with respect to the fault-line axis F'OF. If m_3 and n_3 are equal to each other, and if the angle S_3 is equal to S_1 plus a small angle, the effect is to move the fault-axis bodily to the right, while slightly decreasing its inclination to the X-axis, and at the same time increasing the "reach" of the relay for internal faults.

If

$$m_3 = n_3 = 0, \text{ and } M_2 = M_1, \text{ and } N_2 = N_1 \quad (43)$$

then

$$a = n_1^2 \left(.18943 + .29736 \frac{n_2^2}{n_1^2} + .18943 \frac{n_2}{n_1} \right) \quad (44)$$

$$\frac{-b}{m_1 n_1} = .81057 \left(4 + \frac{m_2 n_2}{m_1 n_1} + 2 \frac{m_2}{m_1} + 2 \frac{n_2}{n_1} \right) f(\theta - S_1) + 0.5 \frac{m_2 n_2}{m_1 n_1} \cos(\theta - S_1) \quad (45)$$

$$c = -\frac{g^2}{h^2} + m_1^2 \left(.18943 + .29736 \frac{m_2^2}{m_1^2} + .18943 \frac{m_2}{m_1} \right) \quad (46)$$

$$K_1 Z = B + \sqrt{B_1^2 + K_2} \quad (47)$$

$$K_1 = \frac{.37886(1 + n_2/n_1) + .59472 n_2^2/n_1^2}{.81057(4 + m_2 n_2/m_1 n_1 + 2m_2/m_1 + 2n_2/n_1)} \frac{n_1}{m_1} \quad (48)$$

$$B_1 = f(\theta - S_1) + \frac{m_2 n_2}{1.62114(4m_1 n_1 + m_2 n_2 + 2m_2 n_1 + 2m_1 n_2)} \cos(\theta - S_1) \quad (49)$$

$$K_2 = a \text{ function of } g^2/h^2 m_1^2 \quad (50)$$

The internal-fault intercept, on the fault-axis OF, is found by putting $(\theta - S_1) = 0$. It is

$$K_1 Z_f = B_2 + \sqrt{B_2^2 + K_2} \quad (51)$$

where

$$B_2 = 0.11685 + \frac{m_2 n_2}{1.62114(4m_1 n_1 + m_2 n_2 + 2m_2 n_1 + 2m_1 n_2)} \quad (52)$$

The internal-fault intercept, on the fault-axis OF', is found by putting $(\theta - S_1) = 180^\circ$. It is

$$K_1 Z_f' = B_3 + \sqrt{B_3^2 + K_2} \quad (53)$$

where

$$B_3 = 0.11685 - \frac{m_2 n_2}{1.62114(4m_1 n_1 + m_2 n_2 + 2m_2 n_1 + 2m_1 n_2)} \quad (54)$$

The differential-axis intercept, on the axis DOD', is found by putting $(\theta - S_1) = 90^\circ$ or 270° . It is

$$K_1 Z_d = -.1073 + \sqrt{.01151 + K_2} \quad (55)$$

The value of K_2 , and hence $g^2/h^2 m_1^2$, determines the magnitude of the differential-axis intercept Z_d , which is one-half of the width of the waist of the hour-glass-shaped curve.

The relative magnitudes of the two fault-axis intercepts, Z_f and Z_f' , or, in other words, the "reaches" of the relay, for internal and external faults, respectively, may then be adjusted to any desired value by the proper choice of $m_2 n_2$, which determines the magnitude of the quantity which is added to 0.11685 to make B_2 , for the into-the-line-looking intercept Z_f , this same quantity being subtracted from 0.11685 to make B_3 , which enters into the expression for the out-of-the-protected-line-section-looking intercept Z_f' .

Two illustrative curves, 15 and 16, are plotted, in Fig. 2, for the values of Z shown in Equation 47, that is, for a relay in which the restraint-winding current is made up of a full-wave-rectified current-responsive component $m_1 I$, a full-wave-rectified voltage-responsive component $n_1 E$, a half-wave-rectified current-responsive component $m_2 I$, and a half-wave-rectified voltage-responsive component $n_2 E$, under the special conditions expressed in Equations 43.

Curve 15, in Fig. 2 is chosen to illustrate a case 75

in which it is desired for the relay to have a very small "reach" along the directional axis DOD', a long forward-looking "reach" along the fault-axis OF, corresponding to a long length of the protected line-section, and a shorter "reach" along the reverse-current fault-axis OF', corresponding to a shorter line-section, having a length about two-thirds of the length of the protected line-section. The constants for curve 15, in addition to the constants indicated in Equation 43 are

$$m_2 = 0.5 m_1, \text{ and } n_2 = 0.5 n_1 \quad (56)$$

Curve 16, in Fig. 2, is chosen to illustrate the manner in which my relay may be adjusted to have a response-curve which is elongated along the fault-axis OF, so as to be sensitively and selectively responding to currents having a power-factor approximating the impedance-angle, S_1 , of the line, while not responding to power-load currents, or out-of-step or switching surges, and at the same time the response-curve is elongated, or bulged, near the origin O, so that the curve encircles the origin, so as to make sure that the relay will respond to zero-impedance bus-faults at the relaying station. This is a very useful property, never before attained in a distance-responsive relay except with saturated current-transformers which have given special curve-distortions only when the line-current has been exceptionally high.

The relay-constants or adjustments, necessary to obtain a curve of the type shown at 16 in Fig. 2, are obtained by assigning a small value to B_2 , and solving Equation 54 for the value of m_2/m_1 , which was assumed to be equal to n_2/n_1 , for purposes of symmetry and ease of calculation. This determines the value of the coefficient of $\cos(\theta - S_1)$ in Equations 49, which is the term which changes from positive to negative when the curve circles around the origin in the reverse-current direction. Various small values of K_2 may then be tried, to determine the relative magnitudes of the three critical intercepts Z_f , Z_f' and Z_d , to determine the relay-constants which will produce the desired relay-characteristics. In curve 16, the constants are

$$\frac{m_2}{m_1} = \frac{n_2}{n_1} = 1.5, \text{ and } K_2 = .01 \quad (57)$$

in addition to the conditions indicated in Equations 43.

Curves 17 and 18, in Fig. 2, have been drawn to illustrate the manner in which a still further, very much wanted, distortion, or departure from a true circle, is obtained in the relay-characteristics. Here, the half-wave-rectified current- and voltage-responsive components, $m_3 I$ and $n_3 E$, have been utilized, with an angle, $S_3 = M_3 - N_3 = S_1 - 10^\circ$, for the sake of illustration in a concrete example. This shifts the curve in such manner as to enable the relay to respond to faults having considerable fault-resistance. In curve 17, this shifting-effect is made more pronounced than in curve 18, by assigning larger values to m_3/m_1 and n_3/n_1 . In both curves, the magnitudes of the m_2 and n_2 components have been adjusted to make the external-fault intercept Z_f' have any desired value relative to the internal-fault intercept Z_f . Thus, in curve 17, the m_2 and n_2 components are utilized to make Z_f' larger, which is done by reversing either the current-component m_2 or the voltage-component n_2 , so as to be opposite to the effects in curves 15 and 16. Then a value of

$$m_2/m_1 = n_2/n_1$$

was chosen, so as to make (for example) the Z_1 intercept equal to the Z_2 intercept. In curve 18, however, the m_2 and n_2 components are utilized to make Z_1 smaller, so as to provide a relay characteristic having a small bulge or protuberance encircling the origin O.

Curve 17 is plotted for the following constants:

$$\frac{m_2}{m_1} = \frac{n_2}{n_1} = 0.564, M_2 = M_1 + 180^\circ, N_2 = N_1 \quad (58)$$

$$\frac{m_3}{m_1} = \frac{n_3}{n_1} = 0.5, M_3 = M_1 - 10^\circ, N_3 = N_1 \quad (59)$$

$$a = 0.7276n_1^2 \quad (60)$$

$$\begin{aligned} \frac{-b}{m_1 n_1} = & 6.2679 f(\theta - S_1) + 1.2418 f(\theta - S_1 + 10^\circ) - 0.0218 \sin(\theta - S_1 + 5^\circ) + \\ & + 0.037 \cos(\theta - S_1 + 5^\circ) - 0.034 \cos(\theta - S_1) - 0.4787 \sin(\theta - S_1) \end{aligned} \quad (61)$$

$$c = -\frac{g^2}{h^2} + 0.4758m_1^2 \quad (62)$$

$$0.2285 \frac{n_1}{m_1} Z = B_1 + \sqrt{B_1^2 - 0.0723 + 0.15203g^2/h^2 m_1^2} \quad (63)$$

$$\begin{aligned} B_1 = & f(\theta - S_1) + 0.19501 f(\theta - S_1 + 10^\circ) - 0.00342 \sin(\theta - S_1 + 5^\circ) + \\ & + 0.00581 \cos(\theta - S_1 + 5^\circ) + 0.00535 \cos(\theta - S_1) - 0.07518 \sin(\theta - S_1) \end{aligned} \quad (64)$$

Curve 18 is plotted for the following constants:

$$\frac{m_2}{m_1} = \frac{n_2}{n_1} = 1, M_2 = M_1, N_2 = N_1 \quad (65)$$

$$\frac{m_3}{m_1} = \frac{n_3}{n_1} = 0.2, M_3 = M_1 - 10^\circ, N_3 = N_1 \quad (66)$$

$$a = 0.84495n_1^2 \quad (67)$$

$$\begin{aligned} \frac{-b}{m_1 n_1} = & 7.7815 f(\theta - S_1) + 0.5188 f(\theta - S_1 + 10^\circ) - 0.00349 \sin(\theta - S_1 + 5^\circ) + \\ & + 0.0148 \cos(\theta - S_1 + 5^\circ) + 0.52 \cos(\theta - S_1) \end{aligned} \quad (68)$$

$$c = -\frac{g^2}{h^2} + 0.87745m_1^2 \quad (69)$$

$$0.2171 \frac{n_1}{m_1} Z = B_2 + \sqrt{B_2^2 - 0.6441 + 0.7340g^2/h^2 m_1^2} \quad (70)$$

$$\begin{aligned} B_2 = & f(\theta - S_1) + 0.06667 f(\theta - S_1 + 10^\circ) - 0.00045 \sin(\theta - S_1 + 5^\circ) + \\ & + 0.0019 \cos(\theta - S_1 + 5^\circ) + 0.0668 \cos(\theta - S_1) \end{aligned} \quad (71)$$

All of the foregoing curves are intended to be merely illustrative or typical of some of the many types of response-curves which are obtainable with the general inequality (23). Any positive values may be assigned to any of the constants in this inequality (23), and all positive values of Z are valid.

One of many combinations of apparatus for accomplishing the functions hereinabove discussed, is shown in Fig. 3. The protected alternating-current line-section is indicated at 21. The line-current, for relaying purposes, is derived by a line-current transformer 22. The current-response coefficients are introduced by three adjustable line-current-energized auxiliary transformers m_1 , m_2 and m_3 , and the phase-angle is shifted by any desired amount by adjustable phase-shifters M_1 , M_2 and M_3 , respectively. The three derived current-responsive alternating-current voltages i_1 , i_2 and i_3 are derived by passing the derived currents through resistors or other impedances 24, 25 and 26, respectively, which may, or may not, be adjustable. The voltage i_1 which appears across the resistor 24 is rectified by any suitable full-wave rectifier-means, such as a rectifier-bridge 27, which may comprise tube-rectifiers, contact rectifiers, mechanical rectifiers, or any other rectifying-means. The output of the full-wave rectifier 27 is supplied to a rectifier-output resistor 28, through a large capacitor 29 or other means for blocking or excluding the direct-

current component of the rectified wave-form, from the resistor 28. The voltages i_2 and i_3 , which appear across the resistors 25 and 26, respectively, are separately rectified by half-wave rectifiers which are symbolically indicated at 32 and 33, respectively, and the rectified outputs of these half-wave rectifiers are supplied to resistors 34 and 35, through blocking-means 36 and 37, respectively, for removing the respective direct-current components.

The three derived voltage-responsive alternating-current voltages v_1 , v_2 and v_3 are applied to the resistors or impedances 41, 42 and 43, through

adjustable line-energized potential-transformers n_1 , n_2 and n_3 , and adjustable phase-shifters N_1 , N_2 and N_3 , respectively, as shown in Fig. 3, the full-wave and half-wave rectifiers 27', 32', and 33', the blocking-capacitors 29', 35', 37', and the rectifier-output resistors 28', 34' and 35', corresponding to the similarly numbered equipment

which has already been described for the current-responsive voltages.

The protective relay is illustrated, in Fig. 3, as a differential relay 45, which is shown as comprising a balanced beam 46, carrying make-contacts 47 and an operating-coil mechanism 48 on its front end, and having a restraint-coil mechanism 49 on its back end. The operating and restraining coils 48 and 49 have tap-changing means g and h , respectively, which is intended to be symbolically representative of any flux-strength adjusting-means. The operating-coil 48 is energized from the line-current transformer 22 through a circuit 51 which serially includes the primary windings of the auxiliary current-transformers m_1 , m_2 , m_3 . The restraint-coil 49 is energized from the vectorial sum of the voltages appearing across the several output-resistors 28', 34', 35', 35, 34 and 28, through an energizing-circuit 52 which serially includes said output-resistors.

The relay 45 of Fig. 3 has an operating-characteristic as expressed in inequality (23), some of the important features of which are shown in the eight curves of Figs. 1 and 2, which have already been discussed. It may be noted, in general, that the effect of separately rectifying both the current-responsive-component and the voltage-responsive component on the back end of the relay (or on the front end, in relays which have both current and voltage on the front end), is to

displace the center of the well-known response-circle, equally in both directions from the origin, if the adjustments are symmetrical, or, in general, to elongate the response-curve along the fault-axis. Thus, if only voltage were utilized on the back end of the relay, the response-curve of the relay would be the well-known circle, with its center at the origin. Heretofore, it has been possible to displace the center, in one direction, from the origin, by vectorially adding an alternating-current current-component to the alternating-current voltage-component on the back end of the relay. By rectifying both the current and voltage components, before combining them on the back end of the relay, the impedance-circle center is, in effect, displaced in opposite directions at the same time. The rectification renders the relay insensitive to 180° reversals in current, but leaves it responsive to 90° shifts, particularly with the full-wave rectification obtained by the m_1 and n_1 components alone. It is to be noted that un-smoothed-out rectification is utilized, and that the direct-current components of the rectified waves are substantially eliminated.

By adding an unrectified fundamental sine-wave component of each of the current and voltage-waves, to the full-wave-rectified components, it is possible to change the duo-directional symmetry of the full-wave-rectified response, while retaining the effect of a response-curve which is elongated along the fault-ohms axis and constricted along the directional axis, near the origin. One of the means for introducing such an unrectified fundamental current or voltage wave-form is to utilize a single-wave-rectified component, m_2 and n_2 , or m_3 and n_3 , or both. Thus, in Equation 7, it is seen that the half-wave-rectified components, having the coefficients m_2 , n_2 , m_3 and n_3 , have terms including the sine of a time-function of the type $(\omega t + Y)$, which is an unrectified component. I desire my derivations and my illustrated apparatus to be symbolic of any means for introducing such unrectified components onto the full-wave-rectified components which are expressed, in Equation 7, by the infinite series

$$\left[\sum \frac{\cos 2k(\omega t + Y)}{4k^2 - 1} \right]$$

Since an alternating current having a magnitude $m_1 I$ and a phase-displacement $(-\theta + M_1)$ with respect to another wave, has an in-phase component, $m_1 I \cos(-\theta + M_1)$, and a quadrature-phase component, $j m_1 I \sin(-\theta + M_1)$, the phase-angle $(-\theta + M_1)$ and the magnitude-coefficient m_1 may be, in effect, adjusted indirectly, by separately controlling or adjusting the in-phase coefficient $m_1 \cos(-\theta + M_1)$ and the out-of-phase coefficient $m_1 \sin(-\theta + M_1)$. Also, since the two half-wave-rectified components m_2 and m_3 , or n_2 and n_3 , are treated in the same way by the half-wave rectifiers, the two alternating currents having the coefficients m_2 and m_3 could be combined before rectification, and then rectified with a single half-wave rectifier, and the same applies to the voltages having the coefficients n_2 and n_3 .

These principles are illustrated in the alternative form of construction shown in Fig. 4, wherein the six alternating-current components $m_1 I \angle (-\theta + M_1)$, $m_2 I \angle (-\theta + M_2)$, $m_3 I \angle (-\theta + M_3)$, $n_1 E \angle N_1$, $n_2 E \angle N_2$ and $n_3 E \angle N_3$, as previously described and illustrated in Fig. 3, have been

condensed into four, namely $(R_1 I + j X_1 I)$, $(R_2 I + j X_2 I)$, $(R_3 E + j X_3 E)$ and $(R_4 E + j X_4 E)$, where

$$R_1 = m_1 \cos(-\theta + M_1) \quad (72)$$

$$R_2 = m_2 \cos(-\theta + M_2) + m_3 \cos(-\theta + M_3) \quad (73)$$

$$R_3 = n_1 \cos N_1 \quad (74)$$

$$R_4 = n_2 \cos N_2 + n_3 \cos N_3 \quad (75)$$

$$X_1 = m_1 \sin(-\theta + M_1) \quad (76)$$

$$X_2 = m_2 \sin(-\theta + M_2) + m_3 \sin(-\theta + M_3) \quad (77)$$

$$X_3 = n_1 \sin N_1 \quad (78)$$

$$X_4 = n_2 \sin N_2 + n_3 \sin N_3 \quad (79)$$

Thus, in Fig. 4, the line-current transformer 22 energizes four adjustable transformers 61, 62, 63, 64, which in turn energize the resistance R_1 , the reactance X_1 , the resistance R_2 , and the reactance X_2 , respectively. The sum of the resistance-drop $R_1 I$ and the reactance-drop $j X_1 I$ is utilized, in lieu of the drop across the resistor 24 in Fig. 3, to produce the full-wave-rectified A. C. component appearing across the output-resistor 28, as in Fig. 3. The drops $R_2 I + j X_2 I$ are utilized, in lieu of the drop across the resistor 25 in Fig. 3, to produce the half-wave-rectified A. C. component appearing across the output-resistor 34, as in Fig. 3.

In Fig. 4 also, an adjustable potential-transformer 65 is utilized to energize the serially connected tapped resistance R_3 and tapped reactance X_3 , and an adjustable potential-transformer 66 is utilized to energize the serially connected tapped resistance R_4 and tapped reactance X_4 . The drops $(R_3 E + j X_3 E)$ and $(R_4 E + j X_4 E)$ energize the output-resistors 28' and 34' in a manner which will be obvious from the drawing and the preceding explanations. Although the adjustments are different, it is obviously possible, by the proper choice of the eight variables, R_1 , X_1 , R_2 , X_2 , R_3 , X_3 , R_4 and X_4 , in Fig. 4, to obtain the same kinds and sizes of response-curves, as with the apparatus which was exemplified in Fig. 3, and which was mathematically analyzed hereinabove.

In the field of distance-type relays with unrectified currents and voltages, it is known that the same types of responses can be obtained by multiplying two quantities $(AD + BI)$ and $(\dot{C}E + \dot{D}I)$ in a torque-type or product-responsive relay, as by opposing $D^2 I^2$ with the square of $(\dot{A}E + \dot{B}I)$ in a differential relay, where the superposed dots indicate complex numbers. The same thing is true in my use of rectified quantities to change the relay-characteristics as previously described. The product of the sum of the infinite series representing the rectified quantities $\dot{A}E$ and $\dot{B}I$, multiplied by the sum of the infinite series representing the rectified quantities $\dot{C}E$ and $\dot{D}I$, is not different, in kind, from the square of either sum. In either event, the result involves the function $f(x)$ which is shown in the table, and various functions such as $\sin x$ and $\cos x$, very much in the manner shown in Equation 17, but with different coefficients and angles. In a product-type relay, the balance-point is found by putting the product or torque equal to zero. Without carrying out the precise mathematical calculations, which follow from the principles already enunciated, I wish it to be understood that my general principle of utilizing rectified current-responsive and voltage-responsive quantities is applicable also to product-type relays as well as to differential relays.

Thus, in Fig. 5, I illustrate a wattmetric or product-type relay 71, having make-contacts 72, and two coils 73 and 74 for producing the fluxes

which are to be multiplied. In general, such a relay develops a torque equal to the product of the two fluxes (or quantities), times a function of the angle between them, and this observation applies separately to each term of the product, when (as here) the product is the resultant of the algebraic sum of a plurality of product-terms.

In Fig. 5, one coil 73 is energized in response to a full-wave-rectified current-responsive voltage, $m_1 I \angle (-\theta + M_1)$, and a full-wave-rectified voltage-responsive voltage, $n_1 E \angle N_1$, either with or without the addition of one or more pairs of half-wave-rectified quantities $m_2 I \angle (-\theta + M_2)$ and $n_2 E \angle N_2$. The other coil 74 is energized in the same way, with the subscripts changed to 3 and 4, instead of 1 and 2, the different subscripts indicating, in general, different magnitudes. The apparatus illustrated in Fig. 5 utilizes amplitude-changers and phase-shifters, as already fully described in Fig. 1, and no further description of Fig. 5 is believed to be necessary, corresponding numerals being utilized in so far as applicable, the numerals being applied mainly to the energizing circuits for only the coil 73, the other coil, 74, being energized by identical apparatus, as shown, except for different adjustments of the constants.

I claim as my invention:

1. A distance-responsive alternating-current relay, comprising a relay-means having at least two magnetizing-means for producing at least two separate relay-fluxes and having flux-responsive force-producing means for developing, from said fluxes, a response to predetermined impedance values, in combination with line-energized energizing-circuit means for energizing the respective magnetizing-means of the relay, the energizing-circuit means which energizes the magnetizing-means for producing at least a predetermined one of said relay-fluxes being a composite energizing-circuit means including at least two line-energized circuit-means for carrying at least two derived alternating-current relaying-quantities, at least one of said quantities being responsive to a line-current, at least another one of said quantities being responsive to a line-voltage, said composite energizing-circuit means also including separate transformation-means for deriving, from each of said relaying-quantities, an alternating voltage which is substantially free of a direct-current component and which includes at least a double-frequency harmonic of the relaying-quantity, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial combination of the voltages which are thus derived by the several transformation-means.

2. A distance-responsive differential alternating-current relay having a magnetizing-means for producing an operating flux, a flux-responsive force-producing means for producing an operating force responsive to the operating flux, a magnetizing-means for producing a restraining flux, and a flux-responsive force-producing means for producing a restraining force responsive to the restraining flux, in combination with line-energized energizing-circuit means for energizing the respective magnetizing-means of the relay, the energizing-circuit means which energizes the magnetizing-means for producing at least said restraining flux being a composite energizing-circuit means including at least two line-energized circuit-means for carrying at least two de-

derived alternating-current relaying-quantities, at least one of said quantities being responsive to a line-current, at least another one of said quantities being responsive to a line-voltage, said composite energizing-circuit means also including separate transformation-means for deriving, from each of said relaying-quantities, an alternating voltage which is substantially free of a direct-current component and which includes at least a double-frequency harmonic of the relaying-quantity, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial combination of the voltages which are thus derived by the several transformation-means.

3. A distance-responsive differential alternating-current relay having a magnetizing-means for producing an operating flux, a flux-responsive force-producing means for producing an operating force responsive to the operating flux, a magnetizing-means for producing a restraining flux, and a flux-responsive force-producing means for producing a restraining force responsive to the restraining flux, in combination with a line-energized energizing-circuit means for energizing the magnetizing-means for producing said operating flux being exclusively responsive to a line-derived current, and a composite line-energized energizing-circuit means for energizing the magnetizing-means for producing said restraining flux including at least two line-energized circuit-means for carrying at least two derived alternating-current relaying-quantities, at least one of said quantities being responsive to a line-current, at least another one of said quantities being responsive to a line-voltage, said composite energizing-circuit means also including separate transformation-means for deriving, from each of said relaying-quantities, an alternating voltage which is substantially free of a direct-current component and which includes at least a double-frequency harmonic of the relaying-quantity, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial combination of the voltages which are thus derived by the several transformation-means.

4. A distance-type product-responsive alternating-current relay having magnetizing-means for producing two diverse cooperating alternating magnetic fluxes, and flux-responsive force-producing means for so utilizing said fluxes as to produce a torque in response to the product of said fluxes, multiplied by a function of the angle between the two cooperating fluxes, in combination with two diverse line-energized energizing-circuit means for energizing the respective magnetizing-means, characterized by each of said line-energized energizing-circuit means including line-energized circuit-means for carrying at least two derived alternating-current relaying-quantities, at least one of said quantities being responsive to a line-current, at least another one of said quantities being responsive to a line-voltage, each of said line-energized energizing-circuit means also including separate transformation-means for deriving, from each of its relaying-quantities, an alternating voltage which is substantially free of a direct-current component and which includes at least a double-frequency harmonic of the relaying-quantity, each energizing-circuit means and its

associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to the vectorial combination of the voltages which are thus derived by the several transformation-means.

5. The invention as defined in claim 1, characterized by the vectorial combination of voltages comprising a line-frequency fundamental-wave component of a current-responsive relaying-quantity and a line-frequency fundamental-wave component of a voltage-responsive relaying quantity.

6. The invention as defined in claim 2, characterized by the vectorial combination of voltages comprising a line-frequency fundamental-wave component of a current-responsive relaying-quantity and a line-frequency fundamental-wave component of a voltage-responsive relaying-quantity.

7. The invention as defined in claim 3, characterized by the vectorial combination of voltages comprising a line-frequency fundamental-wave component of a current-responsive relaying-quantity and a line-frequency fundamental-wave component of a voltage-responsive relaying-quantity.

8. The invention as defined in claim 4, characterized by the vectorial combination of voltages comprising a line-frequency fundamental-wave component of a current-responsive relaying-quantity and a line-frequency fundamental-wave component of a voltage-responsive relaying-quantity.

9. A distance-responsive alternating-current relay, comprising a relay-means having at least two magnetizing-means for producing at least two separate relay-fluxes and having flux-responsive force-producing means for developing, from said fluxes, a response to predetermined impedance values, in combination with line-energized energizing-circuit means for energizing the respective magnetizing-means of the relay, the energizing-circuit means which energizes the magnetizing-means for producing at least a predetermined one of said relay-fluxes being a composite energizing-circuit means including at least two line-energized circuit-means for carrying at least two derived alternating-current relaying-quantities, at least one of said quantities being responsive to a line-current, at least another one of said quantities being responsive to a line-voltage, said composite energizing-circuit means also including separate rectifying-means for separately rectifying said quantities, and means for deriving, from each of the rectified quantities, an alternating-current component or components including substantially none of the direct-current component, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial combination of the derived alternating-current components of the rectified quantities.

10. The invention as defined in claim 9, characterized by each of said separate rectifying-means being a full-wave rectifying-means.

11. A distance-responsive alternating-current relay, comprising a relay-means having at least two magnetizing-means for producing at least two separate relay-fluxes and having flux-responsive force-producing means for developing, from said fluxes, a response to predetermined impedance values, in combination with line-energized energizing-circuit means for energizing

the respective magnetizing-means of the relay, the energizing-circuit means which energizes the magnetizing-means for producing at least a predetermined one of said relay-fluxes being a composite energizing-circuit means including at least two line-energized circuit-means for carrying a plurality of diverse current-responsive alternating-current quantities and line-energized circuit-means for carrying a plurality of diverse voltage-responsive alternating-current quantities, said composite energizing-circuit means also including separate full-wave rectifying-means for separately rectifying one of said current-responsive quantities and one of said voltage-responsive quantities, separate half-wave rectifying-means for separately rectifying one or more of said current-responsive quantities and one or more of said voltage-responsive quantities, and means for deriving, from each of the rectified quantities, an alternating-current component or components including substantially none of the direct-current component, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial combination of the derived alternating-current components of the rectified quantities.

12. A distance-responsive alternating-current relay, comprising a relay-means having at least two magnetizing-means for producing at least two separate relay-fluxes and having flux-responsive force-producing means for developing, from said fluxes, a response to predetermined impedance values, in combination with line-energized energizing-circuit means for energizing the respective magnetizing-means of the relay, the energizing-circuit means which energizes the magnetizing-means for producing at least a predetermined one of said relay-fluxes being a composite energizing-circuit means including at least two line-energized circuit-means for carrying at least two derived alternating-current relaying-quantities, at least one of said quantities being responsive to a line-current, at least another one of said quantities being responsive to a line-voltage, said composite energizing-circuit means also including separate rectifying-means for separately rectifying said quantities, and means for substantially removing the direct-current component of the wave-form from each of the rectified quantities, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial combination of the remainders of the several rectified quantities.

13. The invention as defined in claim 12, characterized by each of said separate rectifying-means being a full-wave rectifying-means.

14. A distance-responsive alternating-current relay, comprising a relay-means having at least two magnetizing-means for producing at least two separate relay-fluxes and having flux-responsive force-producing means for developing, from said fluxes, a response to predetermined impedance values, in combination with line-energized energizing-circuit means for energizing the respective magnetizing-means of the relay, the energizing-circuit means which energizes the magnetizing-means for producing at least a predetermined one of said relay-fluxes being a composite energizing-circuit means including at least two line-energized circuit-means for carrying a plurality of diverse current-responsive alternat-

ing-current quantities and line-energized circuit-means for carrying a plurality of diverse voltage-responsive alternating-current quantities, said composite energizing-circuit means also including separate full-wave rectifying-means for separately rectifying one of said current-responsive quantities and one of said voltage-responsive quantities, separate half-wave rectifying-means for separately rectifying one or more of said current-responsive quantities and one or more of said voltage-responsive quantities, and means for substantially removing the direct-current component of the wave-form from each of the rectified quantities, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial combination of the remainders of the several rectified quantities.

15. A distance-responsive differential alternating-current relay having a magnetizing-means for producing an operating flux, a flux-responsive force-producing means for producing an operating force responsive to the operating flux, a magnetizing-means for producing a restraining flux, and a flux-responsive force-producing means for producing a restraining force responsive to the restraining flux, in combination with a line-energized energizing-circuit means for energizing the magnetizing-means for producing said operating flux being exclusively responsive to a line-derived current, and a composite line-energized energizing-circuit means for energizing the magnetizing-means for producing said restraining flux including at least two line-energized circuit-means for carrying at least two derived alternating-current relaying-quantities, at least one of said quantities being responsive to a line-current, at least another one of said quantities being responsive to a line-voltage, said composite energizing-circuit means also including separate rectifying-means for separately rectifying said quantities, and means for deriving, from each of the rectified quantities, an alternating-current component or components including substantially none of the direct-current component, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial combination of the derived alternating-current components of the rectified quantities.

16. The invention as defined in claim 15, characterized by each of said separate rectifying-means being a full-wave rectifying-means.

17. A distance-responsive differential alternating-current relay having a magnetizing-means for producing an operating flux, a flux-responsive force-producing means for producing an operating force responsive to the operating flux, a magnetizing-means for producing a restraining flux, and a flux-responsive force-producing means for producing a restraining force responsive to the restraining flux, in combination with a line-energized energizing-circuit means for energizing the magnetizing-means for producing said operating flux being exclusively responsive to a line-derived current, and a composite line-energized energizing-circuit means for energizing the magnetizing-means for producing said restraining flux including at least two line-energized circuit-means for carrying a plurality of diverse current-responsive alternating-current quantities and line-energized circuit-means for carrying a plurality of diverse voltage-responsive alternating-

current quantities, said composite energizing-circuit means also including separate full-wave rectifying-means for separately rectifying one of said current-responsive quantities and one of said voltage-responsive quantities, separate half-wave rectifying-means for separately rectifying one or more of said current-responsive quantities and one or more of said voltage-responsive quantities, and means for deriving, from each of the rectified quantities, an alternating-current component or components including substantially none of the direct-current component, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial combination of the derived alternating-current components of the rectified quantities.

18. A distance-responsive differential alternating-current relay having a magnetizing-means for producing an operating flux, a flux-responsive force-producing means for producing an operating force responsive to the operating flux, a magnetizing-means for producing a restraining flux, and a flux-responsive force-producing means for producing a restraining force responsive to the restraining flux, in combination with a line-energized energizing-circuit means for energizing the magnetizing-means for producing said operating flux being exclusively responsive to a line-derived current, and a composite line-energized energizing-circuit means for energizing the magnetizing-means for producing said restraining flux including at least two line-energized circuit-means for carrying at least two derived alternating-current relaying-quantities, at least one of said quantities being responsive to a line-current, at least another one of said quantities being responsive to a line-voltage, said composite energizing-circuit means also including separate rectifying-means for separately rectifying said quantities, and means for substantially removing the direct-current component of the wave-form from each of the rectified quantities, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial combination of the remainders of the several rectified quantities.

19. The invention as defined in claim 18, characterized by each of said separate rectifying-means being a full-wave rectifying-means.

20. A distance-responsive differential alternating-current relay having a magnetizing means for producing an operating flux, a flux-responsive force-producing means for producing an operating force responsive to the operating flux, a magnetizing-means for producing a restraining flux, and a flux-responsive force-producing means for producing a restraining force responsive to the restraining flux, in combination with a line-energized energizing-circuit means for energizing the magnetizing-means for producing said operating flux being exclusively responsive to a line-derived current, and a composite line-energized energizing-circuit means for energizing the magnetizing-means for producing said restraining flux including at least two line-energized circuit-means for carrying a plurality of diverse current-responsive alternating-current quantities and line-energized circuit-means for carrying a plurality of diverse voltage-responsive alternating-current quantities, said composite energizing-circuit means also including separate

full-wave rectifying-means for separately rectifying one of said current-responsive quantities and one of said voltage-responsive quantities, separate half-wave rectifying-means for separately rectifying one or more of said current-responsive quantities and one or more of said voltage-responsive quantities, and means for substantially removing the direct-current component of the wave-form from each of the rectified quantities, said composite energizing-circuit means and its associated magnetizing-means including, in their combined structures, means for causing the resultant flux to be responsive to a vectorial com-

bination of the remainders of the several rectified quantities.

SHIRLEY L. GOLDSBOROUGH.

REFERENCES CITED

The following references are of record in the file of this patent:

UNITED STATES PATENTS

10	Number	Name	Date
	2,393,983	Goldsborough	Feb. 5, 1946
	2,404,955	Goldsborough	July 30, 1946