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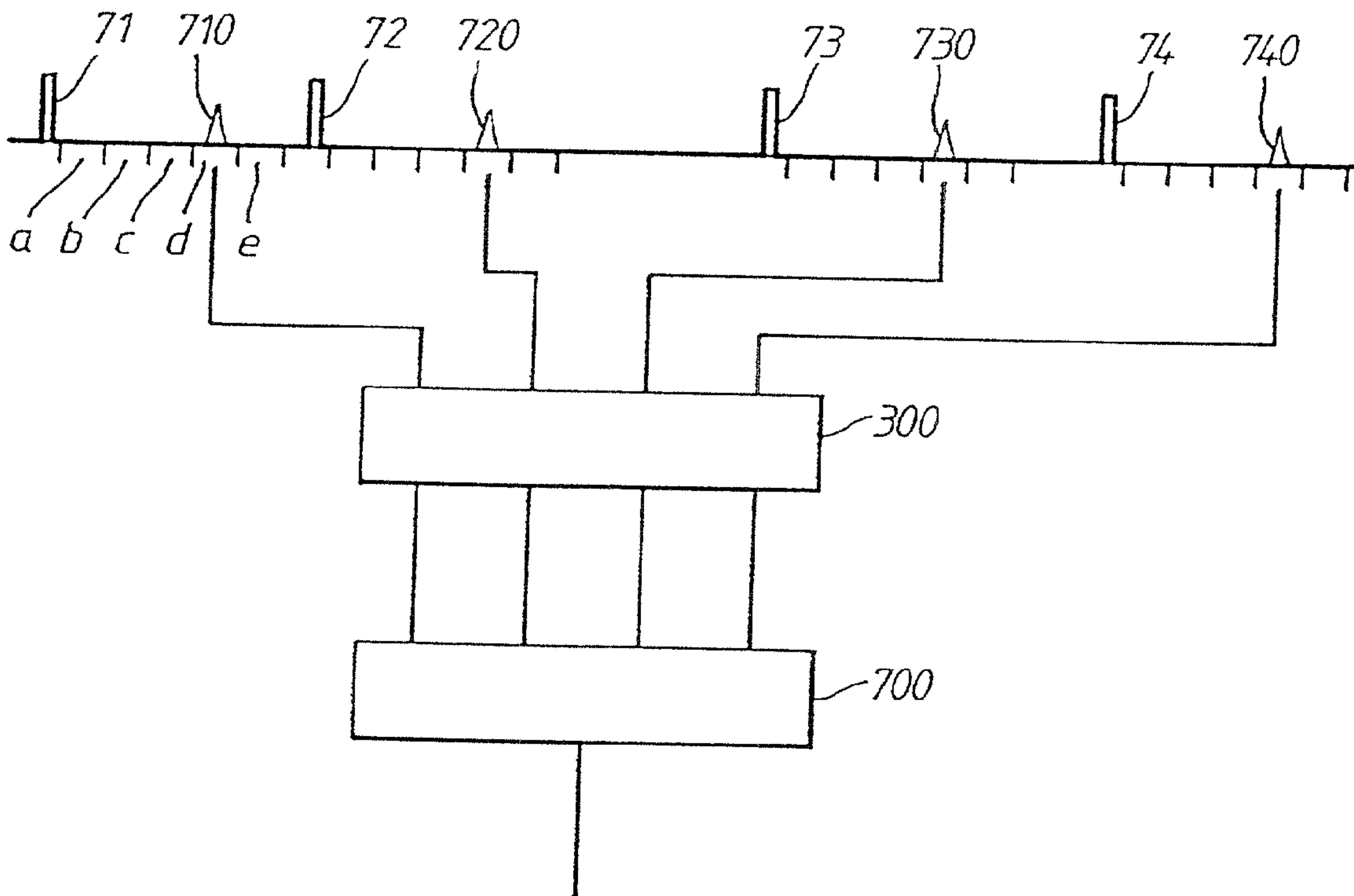
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(54) Title: SIGNAL PROCESSING APPARATUS



(57) Abrégé/Abstract:

The invention relates to an apparatus for and a method of processing signals. The apparatus includes circuitry for obtaining a first set of samples of a signal at a first set of non-uniformly-spaced sampling intervals; circuitry for reconstructing the signal from the first set of samples; and circuitry for generating, from the reconstructed signal, a second set of samples of the signal at a second set of sampling intervals different from the first. The invention can be used in a number of fields of technology, e.g. radar signal processing, to make use of information contained in the signal.

Abstract

The invention relates to an apparatus for and a method of processing signals. The apparatus includes circuitry for obtaining a first set of samples of a signal at a first set of non-uniformly-spaced sampling intervals; circuitry for reconstructing the signal from the first set of samples; and circuitry for generating, from the reconstructed signal, a second set of samples of the signal at a second set of sampling intervals different from the first. The invention can be used in a number of fields of technology, e.g. radar signal processing, to make use of information contained in the signal.

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SIGNAL PROCESSING APPARATUS

This invention relates to apparatus and for and a method of processing signals.

Signal sampling techniques are used in a number of fields of technology. A signal is sampled and the information contained in the samples is utilised to make use of information contained in the signal.

Sampling may take place in the time domain. For example, a continuous signal may be sampled at regular instants of time. As long as the sampling occurs at a rate greater than the Nyquist rate, all information about the original signal is retained.

Sampling may also take place in space.

Figure 1 shows a prior art antenna arrangement comprising an array of spaced-apart antenna elements 1-5. The elements are coupled via lines 6 to a signal processing arrangement 7 which is coupled to further circuitry via line 8. The elements are spaced at a constant pitch d . Signals are applied to, or obtained from, each of the antenna

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elements 1-5 via lines 6 such that a defined phase and/or amplitude relationship exists between the respective signals supplied to or obtained from, each element. In this way the directivity and direction of the antenna beam can be predetermined or adjusted. The signals received by each of the individual elements can be considered to comprise samples of a radiated signal taken at different points in space.

In general, sampling is performed at regular, equally spaced intervals. Sampling in the time domain is done at regular intervals of time under control of a clock. In an antenna array, such as a binomial array antenna, the individual elements are all spaced at a uniform pitch.

Uniform intervals are generally employed because the parameters of such systems are easier to determine on the basis of uniform intervals. As a consequence the properties of arrangements based on uniform intervals are well understood and arrangements based on uniform sampling intervals are almost universally employed.

While it has been suggested that, in theory, it would be possible to provide arrangements in which the sampling intervals are not uniform, it has also been acknowledged

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that such arrangements would be difficult to design. The provision of a non-uniform sampling interval would require the provision of a dedicated signal processing arrangement to process signals produced by such an arrangement. In general, a uniform sampling interval arrangement is perfectly satisfactory and so there has been no need to attempt to construct an arrangement having a non-uniform sampling interval because of the very great difficulties involved in designing such an arrangement.

However, in certain applications it would be advantageous to be able to use non-uniform sampling intervals. For example, in installing a phased array antenna it would be a great advantage to be able to site individual antenna elements according to site geography (e.g. to avoid roads or building etc) rather than having to space the elements apart by a constant pitch determined by electrical requirements. It would also be desirable in such a case to be able to use, for signal processing, conventional signal processing arrangements designed to operate on data obtained from an array of elements having a fixed pitch. While it might be possible to design a dedicated signal processing arrangement for use with a particular arrangement of non-uniformly distributed elements, this would obviously entail a great deal of extra

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expense and design complexity.

Similarly, in radar or sonar applications, it would be advantageous not to have to transmit pulses at a constant rate, yet be able to use conventional signal processing arrangements designed for use with pulses emitted at a constant rate.

Certain types of signal processing arrangement can only give a meaningful output if the correct number of signals are presented to it. If one of the signals is not present then no output can be obtained until such time as the correct number of signals are received. Thus if a single signal of a time-sequence of samples is missing, or if an antenna element of an array becomes faulty, then it is not possible to make use of the information contained in the remaining samples of data or the signal produced by the remaining elements. It would be advantageous to be able to provide an output based on such information as is available.

The present invention provides signal processing arrangement which allows signal processing arrangements intended for use with uniform sampling intervals to be used in conjunction with non-uniform sampling intervals.

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In accordance with the invention, signal processing apparatus comprises: means for obtaining a first set of samples of a signal at a first set of non-uniformly-spaced sampling intervals; means for reconstructing the signal from the first set of samples; and means for generating, from the reconstructed signal, a second set of samples of the signal at a second set of sampling intervals different from the first.

The second set of sampling intervals may comprise equally-spaced intervals.

This allows subsequent signal processing to be carried out with conventional signal processing apparatus designed to accept samples taken at regularly-spaced intervals.

The number of samples in the first set may be the same as the number of samples in the second set.

The samples may comprise samples taken at different instants of time. For example, a continuous signal may be sampled periodically, or respective echos from a train of pulses may be sampled in consecutive time frames in a fixed time relationship to the time of transmission of the transmitted pulses.

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The samples may comprise samples taken at different locations in space. For example the individual elements of an antenna array can be considered to sample a radiated signal at different points in space. These samples are then suitably processed to produce the desired antenna response.

In a further aspect of the invention, radar apparatus comprises means to transmit a sequence of non-uniformly spaced pulses; means to received a sequence of signals comprising reflections of the "sequence of pulses, the sequence of signals comprising said first set of samples; and means to convert the first set of samples into a second set of samples, said second set of samples representing the sequence of signals which would have been produced in response to uniformly-spaced transmitted pulses.

In this way the radar apparatus is not restricted to the use of pulses transmitted at equally-spaced intervals. Pulses having an arbitrary transmission interval can be employed, yet the information conveyed by the reflected pulses can be processed using conventional data processing apparatus which requires the transmission of regularly-spaced pulses.

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In another aspect of the invention, antenna apparatus comprises a plurality of non-uniformly spaced elements, means for coupling received signals received by individual ones of said elements to signal processing apparatus to produce a further set of signals, the further set of signals corresponding to signals which would have been produced had the elements been spaced uniformly.

In this way it is possible to receive signals employing an antenna array being a non-uniform antenna element spacing, in conjunction with a conventional signal processing arrangement designed for use with an antenna array having uniformly spaced elements.

In another aspect of the invention signal processing apparatus comprises; means for obtaining a first set of samples of a signal at a first set of uniformly-spaced sampling intervals; means for reconstructing the signal from the first set of samples; and means for generating, from the reconstructed signal, a second set of samples at a second set of non-uniformly-spaced sampling intervals.

In this way signals with a uniform-spaced sampling interval are converted to corresponding signals which would have been produced had sampling taken place using

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non-uniformly-spaced sampling intervals.

According to yet another aspect of the invention, a method of signal processing comprises the steps of: sampling a signal at a first plurality of non-uniform sampling intervals to obtain a first plurality of samples; using said first plurality of samples to reconstruct the signal; and sampling the reconstructed signal at a second plurality of uniform sampling intervals to produce a second plurality of samples.

The number of samples of the first plurality may be the same as the number of samples of the second plurality of samples.

According to yet another aspect of the invention, a method of signal processing comprise the steps of: sampling a signal at a first plurality of uniform sampling intervals to obtain a first plurality of samples; using said first plurality of samples to reconstruct the signal; and sampling the reconstructed signal at a second plurality of non-uniform sampling intervals to produce a second plurality of samples.

Exemplary embodiments of the invention will now be

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described by way of non-limiting example only with reference to the drawings in which;

Figure 1 shows a prior art antenna array;

Figure 2 shows an antenna array in accordance with the invention;

Figure 3 shows a known antenna array;

Figure 4 shows an antenna array for use in conjunction with the invention;

Figure 5 shows how the arrangement of Figure 4 behaves when used in conjunction with the invention;

Figure 6 shows waveforms produced by a conventional doppler radar system; and

Figure 7 shows waveforms of radar pulses produced by an arrangement in accordance with the invention.

Where a site is free from obstruction, a regularly-spaced antenna array can be employed. However, in the arrangement shown in Figure 2 the sites for the elements

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corresponding to 2, 3 and 4 of Figure 1 (shown in phantom in Figure 2) are obstructed by a stream 20, house 21 and marshy ground 22. Therefore the corresponding elements 12, 13 and 14 are sited at convenient locations having regard to the site conditions and are no longer arranged at a uniform spacing. A signal processing arrangement 30 accepts signals from elements 11-15 on lines 24. The signal processing apparatus 30 is provided with information as to the actual spacing of the elements 11-15 and the spacing of the corresponding uniformly-spaced array which it is to simulate. The signal processing apparatus 30 produces output signals on lines 60 corresponding to the signals which would have been produced had the antenna elements 11-15 been spaced uniformly. The signals on line 60 are then processed by a conventional fast fourier transform processor 7 to give an output signal 8. The signal processing arrangement 30 can be considered to take the original set of samples, reconstruct the signal from those samples and re-sample the reconstructed signal at a uniform sampling rate to produce the second set of output signals.

A second embodiment of the invention will now be described with reference to Figures 3, 4 and 5. Now as is known to those skilled in the art, the resolution of an antenna array is, inter alia, a function of its effective

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aperture, i.e. the distance between its extreme elements.

Consider an antenna array shown in Figure 3. The array comprises n equally-spaced elements $1', 2', \dots, n'$ spaced apart by a distance d' . It has an effective aperture A_1 . To obtain a higher resolution it would be necessary to increase the distance A_1 by providing more elements. Because of the weighting assigned to each element, it is a feature of such an array that elements contribute progressively less signal to the arrangement the further they are from the centre of the array. Thus while the extreme elements are necessary to obtain increased resolution, they contribute relatively little to the gain of the array, and the extra expense incurred in obtaining a higher resolution does not produce a corresponding increase in sensitivity.

In Figure 4, two additional elements 40, 41 have been added to the array of Figure 3. Elements 40, 41 are spaced apart from the end elements $1', n'$ of the original array by a much greater distance than the pitch d' of the original array. The effective aperture of the array now becomes distance A_2 . Thus it can be seen that the effective aperture of the array has been more than doubled without doubling the number of elements. The signals from the array

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are processed by signal processing apparatus 30 to produce a plurality of output signals 60 corresponding to signals which would have been produced by a uniform array comprising $n+2$ elements $1''$, $2''$, ... n'' , $(n+1)''$, $(n+2)''$ having an aperture A_2 . This effective antenna array as seen by signal processing circuit 3 is shown in Figure 5. Thus it can be seen that the effective aperture of an array can be dramatically increased by the addition of only two additional elements, and that the original signal processing arrangement 7 can be retained. The signal processing circuit 30 in accordance with the invention provides an interface between the antenna array having a non-uniform element spacing and a circuit 7 which is only capable of processing signals from an antenna array having a uniform element spacing.

A further embodiment of the invention will be described with reference to Figures 6 and 7.

As is known to those skilled in the art, doppler radar systems utilise trains of pulses transmitted at regularly-spaced intervals. The receiver looks for echos arriving in respective range windows a-e, each range window being defined in terms of a predetermined range of elapsed times after the transmission of a pulse. Such an

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arrangement is shown diagrammatically in Figure 6, in which transmitted pulses 61 produce echos 62 from a target. As the target is at a constant range these echos 62 always lie within the same range window d. Signal processing circuitry 700 compares the phases of the echos 62 arriving in the same range windows of successive pulses and uses the change of phase between successive pulses to determine the rate of acceleration of the target which gave rise to the echo. The signal processing required for this is extremely complex and requires the use of pulses transmitted at regularly-spaced intervals. Unfortunately the transmission of regularly-spaced pulses means that it is relatively easy to detect the transmission of such pulses. It is therefore relatively easy for a target to detect that it is being observed and for the target to transmit signals synchronised to the pulses which will give rise to false echos in the receiver. Further, as the signal processor requires a continuous pulse train, any missing or corrupt pulses will severely upset the operation of the arrangement and give rise to unusable data.

Figure 7 shows an arrangement of pulses transmitted in an arrangement in accordance with the invention. Pulses 71-74 are transmitted at unequally-spaced intervals. The echos 710-740 are received in range windows a-e which are

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defined at respective constant ranges of times relative to their associated transmitted pulses but which occur at correspondingly unequally-spaced times relative to their adjacent corresponding range windows. The echo signals received in successive range windows are processed by processor 300 to extract the phase information so as to obtain a signal representing the rate of change of velocity of the target. The signal is then re-sampled so as to produce a plurality of equally-spaced signals corresponding to those echos which would have been produced had the transmitted pulses 71-74 been pulses 61 transmitted at uniformly spaced time intervals. These equally-spaced signals can then be processed by a conventional signal processing arrangement 700.

A number of modifications are possible within the scope of the invention. For example, it is possible to compensate for the loss of one a sample channel. A conventional fast fourier transform circuit requires all inputs to be present before it can give a meaningful result. By employing the invention, if one of the antenna elements of Figure 3 for example had to be taken out of commission or was otherwise unusable, the circuit 30 could be reprogrammed to calculate the intermediate signal on the basis of such information as was received by the remaining antenna

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elements. It would then convert this information into the correct number of samples required by the subsequent processors 30. While some inaccuracy would result from the absence of this information from the missing antenna, the information would still be meaningful.

Similarly, instead of calculating the signals which would have been produced had the antenna elements been uniformly distributed across aperture A2, the arrangement of Figure 4 could calculate the signals which would have been produced by an array having the 'same pitch d' as Figure 3 but an aperture of A2. While this might produce some degradation of performance in other respects, the resolution would be equivalent to an antenna of aperture A2. It would also be possible to selectively switch out the end elements 40, 41 and revert to operation using only the original n elements.

In the arrangement of Figure 7, the circuit could be modified to detect that interference has been received and to ignore information collected while interference was present. It would then calculate the acceleration on the basis of such samples as it has received. The signal processor would then produce the correct number of samples at the times required by the subsequent processing

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circuitry. Again, accuracy would be reduced but there would be no break in the production of meaningful information.

Further, although the operation of signal processing has been described as a sequence of operations in which a signal is reconstructed from samples taken at non-uniform intervals and the reconstructed signal re-sampled at uniform intervals, the re-sampled signals may be obtained from the original samples without actually reconstructing the signal as an intermediate step.

A mathematical analysis relevant to the operation of the invention will now be given.

Shannon's sampling theorem (Proc. IRE Vol. 37 pp 10-21, Jan 1949) demonstrated that a lowpass bandlimited signal was "completely determined" by its ordinate sample values taken at equispaced time intervals. A bandlimited signal is said to be "completely determined" if (and only if) the signal may be exactly reconstructed from these samples.

Proposition 1

An N^{th} order sampling scheme (denoted $sa_N(t/T_c)$), where

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T_c is the cyclic interval) is the addition of N equispaced sampling schemes of spacing T_c , each displaced from $t=0$ by an amount U_k , where $k=0,1,\dots,N-1$. That is

$$sa_N(t/T_c) = \sum_{k=0}^{N-1} \left[\sum_n \delta(t - nT_c - \psi_k) \right]. \quad (1)$$

Proposition 2

An alternative mathematical expression for N^{th} order sampling schemes to that given in proposition 1 is

$$sa_N(t/T_c) = \left[\sum_{k=0}^{N-1} \delta(t - \psi_k) \right] * \sum_n \delta(t - nT_c) \quad (2)$$

where the asterix signifies continuous-time convolution.

Proposition 3

A periodic time function $g_p(t)$ with periodic interval $N\tau$ may be written in the form:

$$g_p(t) = g'_p(t) * \sum_n \delta(t - nN\tau) \quad (3)$$

where $g'_p(t) = g_p(t)$ for $t \in [0, N\tau)$ and $g'_p(t) = 0$ for $t \notin [0, N\tau)$.

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If the periodic time interval $N\tau$ of a periodic time function is the same as the cyclic period of an N^{th} order sampling scheme T_c , then the N^{th} order sampled periodic time function $g_p sa_N(t/T_c)$ may be expressed as

$$g_p(t) \times sa_N(t/T_c) = \left[\sum_{k=0}^{N-1} g_p(\psi_k) \cdot \delta(t-\psi_k) \right] * \sum_n \delta(t-nN\tau) \quad (4)$$

It can be seen that the finite set of non-equispaced samples contains all the information about that signal; that is to say, additional samples over and above those contained within the periodic interval will reveal no further information about the signal. Therefore the reconstruction formula of a sampling theorem to cater for this case will ideally be based on only the finite set of samples contained by the square brackets [-] in (4).

Theorem 1

If the Fourier transform $G(f)$ of a complex-valued periodic time function $g_p(t)$ (periodic over time interval $N\tau$) contains no frequency components above and including the N^{th} harmonic, then $g_p(t)$ is completely determined by the

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finite sequence of complex-valued time samples

$$\{g'_p(\psi_k)\}_{k=0}^{N-1};$$

taken at arbitrary but distinct time locations

$$\{\psi_k\}_{k=0}^{N-1}$$

within the periodic time interval $[0, N\tau)$. The reconstruction is

$$g_p(t) = \left[\sum_{k=0}^{N-1} g'_p(\psi_k) \cdot e^{-\frac{j\pi(N-1)\psi_k}{N\tau}} \right] \cdot \left[\prod_{\substack{v=0 \\ v \neq k}}^{N-1} \frac{\sin((\pi/N\tau) \cdot (t-\psi_v))}{\sin((\pi/N\tau) \cdot (\psi_k-\psi_v))} \right] \cdot e^{\frac{j\pi(N-1)t}{N\tau}} \quad (5)$$

Alternatively, if $g_p(t)$ is in the form $a_p(t) + jb_p(t)$, then the real and imaginary components of $g_p(t)$ may be reconstructed independently as follows:

$$a_p(t) = \sum_{k=0}^{N-1} \left[a'_p(\psi_k) \cdot \cos\left(\frac{(\pi(N-1)) \cdot (t-\psi_k)}{N\tau}\right) - b'_p(\psi_k) \cdot \sin\left(\frac{(\pi(N-1)) \cdot (t-\psi_k)}{N\tau}\right) \right] \times \left[\prod_{\substack{v=0 \\ v \neq k}}^{N-1} \frac{\sin((\pi/N\tau) \cdot (t-\psi_v))}{\sin((\pi/N\tau) \cdot (\psi_k-\psi_v))} \right] \quad (6)$$

and

$$b_p(t) = \sum_{k=0}^{N-1} \left[a'_p(\psi_k) \cdot \sin\left(\frac{(\pi(N-1)) \cdot (t-\psi_k)}{N\tau}\right) + b'_p(\psi_k) \cdot \cos\left(\frac{(\pi(N-1)) \cdot (t-\psi_k)}{N\tau}\right) \right] \times \left[\prod_{\substack{v=0 \\ v \neq k}}^{N-1} \frac{\sin((\pi/N\tau) \cdot (t-\psi_v))}{\sin((\pi/N\tau) \cdot (\psi_k-\psi_v))} \right] \quad (7)$$

Note that the real and imaginary samples of $g_p(\psi_k)$ are

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both required in order to permit reconstruction of either $a_p(t)$ or $b_p(t)$.

Proof

Since $g_p(t)$ is periodic over time interval $N\tau$, the spectrum of $g_p(t)$ is a discrete line spectrum consisting of N harmonics of spectral spacing $1/N\tau$. That is,

$$G(f) = \sum_{n=0}^{N-1} G(n/N\tau) \cdot \delta(f - n/N\tau) \quad (8)$$

Consequently, $g_p(t)$ may be expressed as

$$g_p(t) = G_0 + G_1 \cdot e^{j2\pi t/N\tau} + G_2 \cdot e^{j2\pi 2t/N\tau} + \dots + G_{(N-1)} \cdot e^{j2\pi(N-1)t/N\tau}$$

$$\text{or} \quad g_p(t) = \sum_{n=0}^{N-1} G_n \cdot e^{j2\pi n t/N\tau} = \sum_{n=0}^{N-1} G_n \cdot \left[e^{j2\pi t/N\tau} \right]^n \quad (9)$$

i.e. $g_p(t)$ is a polynomial of degree $(N-1)$.

Suppose we were to evaluate equation (9) at N arbitrary (but different) values of time

$$\{\psi_k\}_{k=0}^{N-1}$$

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within the periodic time interval $[0, N\tau)$ such that we obtained the discrete time sequence

$$\{g'_p(\psi_k)\}_{k=0}^{N-1}$$

that is

$$\{g'_p(\psi_k)\}_{k=0}^{N-1} = \left\{ \sum_{n=0}^{N-1} G_n \cdot \left[e^{j(2\pi/N\tau)\psi_k n} \right] \right\}_{k=0}^{N-1} \quad (10)$$

From equations (9) and (10), let the contents of the square brackets $[-]$ be denoted as

$$x = e^{j(2\pi/N\tau)t} \quad \text{and} \quad \{x_k\}_{k=0}^{N-1} = \left\{ e^{j(2\pi/N\tau)\psi_k} \right\}_{k=0}^{N-1} \quad \text{respectively.}$$

Using Lagrange's interpolation formula,

$$g_p(t) = \sum_{k=0}^{N-1} g'_p(\psi_k) \prod_{\substack{v=0 \\ v \neq k}}^{N-1} \left[\frac{(x - x_v)}{(x_k - x_v)} \right]. \quad (11)$$

'x' and 'x $_{\alpha}$ ' (where $\alpha = k$ or v) may be substituted -in (11)- to obtain

$$g_p(t) = \sum_{k=0}^{N-1} g'_p(\psi_k) \prod_{\substack{v=0 \\ v \neq k}}^{N-1} \left[\frac{(e^{j(2\pi/N\tau)t} - e^{j(2\pi/N\tau)\psi_v})}{(e^{j(2\pi/N\tau)\psi_k} - e^{j(2\pi/N\tau)\psi_v})} \right] \quad (12)$$

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The contents of the square brackets in [-] in equation (12) can be written in the form:

$$\left[\frac{e^{j(2\pi/N\tau)t} e^{j(2\pi/N\tau)\psi_v} e^{-j\pi t/N\tau} e^{-j(\pi/N\tau)\psi_v}}{e^{j(2\pi/N\tau)\psi_k} e^{j(2\pi/N\tau)\psi_v} e^{-j(\pi/N\tau)\psi_k} e^{-j(\pi/N\tau)\psi_v}} \right] e^{-j(\pi/N\tau)\psi_k} e^{j\pi t/N\tau}$$

Hence

$$\frac{\sin((\pi/N\tau) \cdot (t - \psi_k)) \cdot e^{j(\pi/N\tau) \cdot (t - \psi_k)}}{\sin((\pi/N\tau) \cdot (\psi_k - \psi_v))}$$

Substituting back into equation (12) results in:

$$g_p(t) = \sum_{k=0}^{N-1} g'_p(\psi_k) \cdot \left[\prod_{\substack{v=0 \\ v \neq k}}^{N-1} \frac{\sin((\pi/N\tau) \cdot (t - \psi_v)) \cdot e^{j(\pi/N\tau) \cdot (t - \psi_k)}}{\sin((\pi/N\tau) \cdot (\psi_k - \psi_v))} \right]$$

or

$$g_p(t) = \left[\sum_{k=0}^{N-1} g'_p(\psi_k) \cdot e^{-\frac{j\pi(N-1)\psi_k}{N\tau}} \right] \cdot \left[\prod_{\substack{v=0 \\ v \neq k}}^{N-1} \frac{\sin((\pi/N\tau) \cdot (t - \psi_v))}{\sin((\pi/N\tau) \cdot (\psi_k - \psi_v))} \right] \cdot e^{\frac{j\pi(N-1)t}{N\tau}} \quad (5)$$

It can be seen that Theorem 1 can be used to create a 'non-equispaced to equispaced transformation' (NESTEST) process which 'transforms' non-equispaced samples into an equivalent equispaced set. From then on, conventional equispaced processing techniques may be applied to the

sampled-data set.

The NESTEST process for lowpass bandlimited periodic time functions (NESTESTt) simply amounts to evaluating equation (5) at the equispaced time locations m , where $m=\{0,1,2,\dots,N-1\}$. Thus, given the non-equispaced sample set

$$\{g'_p(\psi_k)\}_{k=0}^{N-1}$$

we can transform this set into an equivalent equispaced sample set

$$\{g'_p(m\tau)\}_{m=0}^{N-1}$$

by means of the following equation:

$$\{g'_p(m\tau)\}_{m=0}^{N-1} = \left[\sum_{k=0}^{N-1} g'_p(\psi_k) \cdot e^{-\frac{j\pi(N-1)\psi_k}{N\tau}} \cdot \prod_{\substack{v=0 \\ v \neq k}}^{N-1} \frac{\sin((\pi/N\tau) \cdot (m\tau - \psi_v))}{\sin((\pi/N\tau) \cdot (\psi_k - \psi_v))} \right] \cdot e^{\frac{j\pi(N-1)m}{N}} \quad (13)$$

Therefore, since the Discrete Fourier transform (DFT) of a finite set of equispaced lowpass bandlimited periodic time function samples can be obtained from the well-known equation:

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$$\text{DFT}[\{g'_p(m\tau)\}_{m=0}^{N-1}] = \{G(n/N\tau)\}_{n=0}^{N-1} = (1/N\tau) \cdot \sum_{m=0}^{N-1} g'_p(m\tau) \cdot e^{-j\frac{2\pi mn}{N}} \quad (14)$$

Substitution of equation (13) into (14), produces the DFT of an non-equispaced sample set

$$\{g'_p(\psi_k)\}_{k=0}^{N-1}$$

as

$$\{G(n/N\tau)\}_{n=0}^{N-1} = \left\{ (1/N\tau) \cdot \sum_{m=0}^{N-1} \left[\sum_{k=0}^{N-1} g'_p(\psi_k) \cdot e^{-j\frac{\pi(N-1)\psi_k}{N\tau}} \right] \cdot \left[\prod_{\substack{v=0 \\ v \neq k}}^{N-1} \frac{\sin((\pi/N\tau) \cdot (m\tau - \psi_v))}{\sin((\pi/N\tau) \cdot (\psi_k - \psi_v))} \right] \cdot e^{-j\frac{\pi m \cdot (2n - (N-1))}{N}} \right\}_{n=0}^{N-1} \quad (15)$$

For completeness, inverse cases will now be provided without proof, since their derivation follows the approach above in a straightforward manner.

Theorem 2

If the inverse Fourier transform $g(t)$ of a complex-valued periodic frequency function $G_p(f)$ (periodic over frequency interval $1/\tau$) contains no temporal components

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above and including the N^{th} harmonic, then $G_p(f)$ is completely determined by the finite sequence of complex-valued frequency samples

$$\{G'_p(\psi_n)\}_{n=0}^{N-1}$$

taken at arbitrary but distinct frequency locations

$$\{\psi_n\}_{n=0}^{N-1}$$

within the periodic frequency interval $[0, 1/\tau)$. The term 'harmonic' is taken from Cepstral terminology and is taken to mean 'harmonic in the time domain'. The reconstruction is

$$G_p(f) = \left[\sum_{n=0}^{N-1} G'_p(\psi_n) \cdot e^{j\pi(N-1)\tau\psi_n} \cdot \prod_{\substack{v=0 \\ v \neq n}}^{N-1} \frac{\sin \pi\tau(f - \psi_v)}{\sin \pi\tau(\psi_n - \psi_v)} \right] \cdot e^{-j\pi(N-1)\tau f} \quad (16)$$

Similarly, the 'non-equispaced to equispaced transformer' NESTESTf is

$$\{G'_p(m/N\tau)\}_{m=0}^{N-1} =$$

$$\left\{ \sum_{n=0}^{N-1} G'_p(\psi_n) \cdot e^{j\pi(N-1)\tau\psi_n} \cdot \prod_{\substack{v=0 \\ v \neq n}}^{N-1} \frac{\sin \pi\tau((m/N\tau) - \psi_v)}{\sin \pi\tau(\psi_n - \psi_v)} \right\} \cdot e^{-\frac{j\pi(N-1)m}{N}} \quad (17)$$

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and the inverse discrete Fourier transform (IDFT) for a non-equispaced frequency sample set is

$$\{g(k\tau)\}_{k=0}^{N-1} =$$

$$\left\{ \sum_{m=0}^{N-1} \left[\sum_{\substack{n=0 \\ n \neq m}}^{N-1} \frac{G'_n(\psi_n)}{P_n} \cdot e^{j\pi(N-1)\tau\psi_n} \cdot \prod_{\substack{v=0 \\ v \neq n}}^{N-1} \frac{\sin \pi\tau((m/N\tau) - \psi_v)}{\sin \pi\tau(\psi_n - \psi_v)} \right] \right\} \cdot e^{\frac{j\pi m \cdot (2k - (N-1))}{N}} \Bigg|_{k=0}^{N-1} \quad (18)$$

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CLAIMS

1. Signal processing apparatus comprising
means for obtaining a first set of samples of a signal
at a first set of non-uniformly-spaced sampling intervals;
means for reconstructing the signal from the first set
of samples; and
means for generating, from the reconstructed signal, a
second set of samples of the signal at a second set of
sampling intervals different from the first.
2. Signal processing apparatus as claimed in claim 1
in which the second set of sampling intervals comprises
equally-spaced intervals.
3. Signal processing apparatus as claimed in claim 1
or 2 in which the number of samples in the first set is the
same as the number of samples in the second set.
4. Signal processing apparatus as claimed in any one
of claims 1 to 3, in which the samples comprise samples
taken at different instants of time.
5. Signal processing apparatus as claimed in any one

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of claims 1 to 3 in which the samples comprise samples taken at different locations in space.

6. Radar apparatus comprising signal processing apparatus as claimed in any one of claims 1 to 4.

7. Radar apparatus as claimed in claim 6 comprising means to transmit a sequence of non-uniformly spaced pulses;

means to receive a sequence of signals comprising reflections of the sequence of pulses, the sequence of signals comprising said first set of samples, said second set of samples representing the sequence of signals which would have been produced in response to uniformly-spaced transmitted pulses.

8. Antenna apparatus comprising a plurality of non-uniformly spaced elements, signal processing apparatus according to any one of claims 1 to 3 or claim 5, and

means for coupling received signals received by individual ones of said elements to said signal processing apparatus, said received signals comprising said first set of signals.

9. Signal processing apparatus comprising;

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means for obtaining a first set of samples of a signal at a first set of uniformly-spaced sampling intervals;

means for reconstructing the signal from the first set of samples;

means for generating, from the reconstructed signal, a second set of samples at a second set of non-uniformly-spaced sampling intervals.

10. Signal processing apparatus as claimed in claim 9 in which the number of samples in the first set is the same as the number of samples in the second set.

11. Signal processing apparatus as claimed in claim 9 or 10, in which the samples represent values of signal at different instants of time.

12. Signal processing apparatus as claimed in claim 9 or 10, in which the samples comprise samples taken at different locations in space.

13. Antenna apparatus comprising a plurality of non-uniformly-spaced elements, signal processing apparatus according to any one of claims 9, 10 or 12, and means for coupling said second set of signals to said elements.

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14. A method of signal processing comprising the steps of

(a) sampling a signal at a first plurality of non-uniform sampling intervals to obtain a first plurality of samples;

(b) using said first plurality of samples to reconstruct the signal; and

(c) sampling the reconstructed signal at a second plurality of uniform sampling intervals to produce a second plurality of samples.

15. A method as claimed in claim 14 in which the number of samples of the first plurality of samples is the same as the number of samples of the second plurality of samples.

16. A method of signal processing comprising the steps of:

(a) sampling a signal at a first plurality of uniform sampling intervals to obtain a first plurality of samples;

(b) using said first plurality of samples to reconstruct the signal; and

(c) sampling the reconstructed signal at a second plurality of non-uniform sampling intervals to produce a second plurality of samples.

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17. A method as claimed in claim 16 in which the number of samples of the first plurality is the same as the number of samples of the second plurality.

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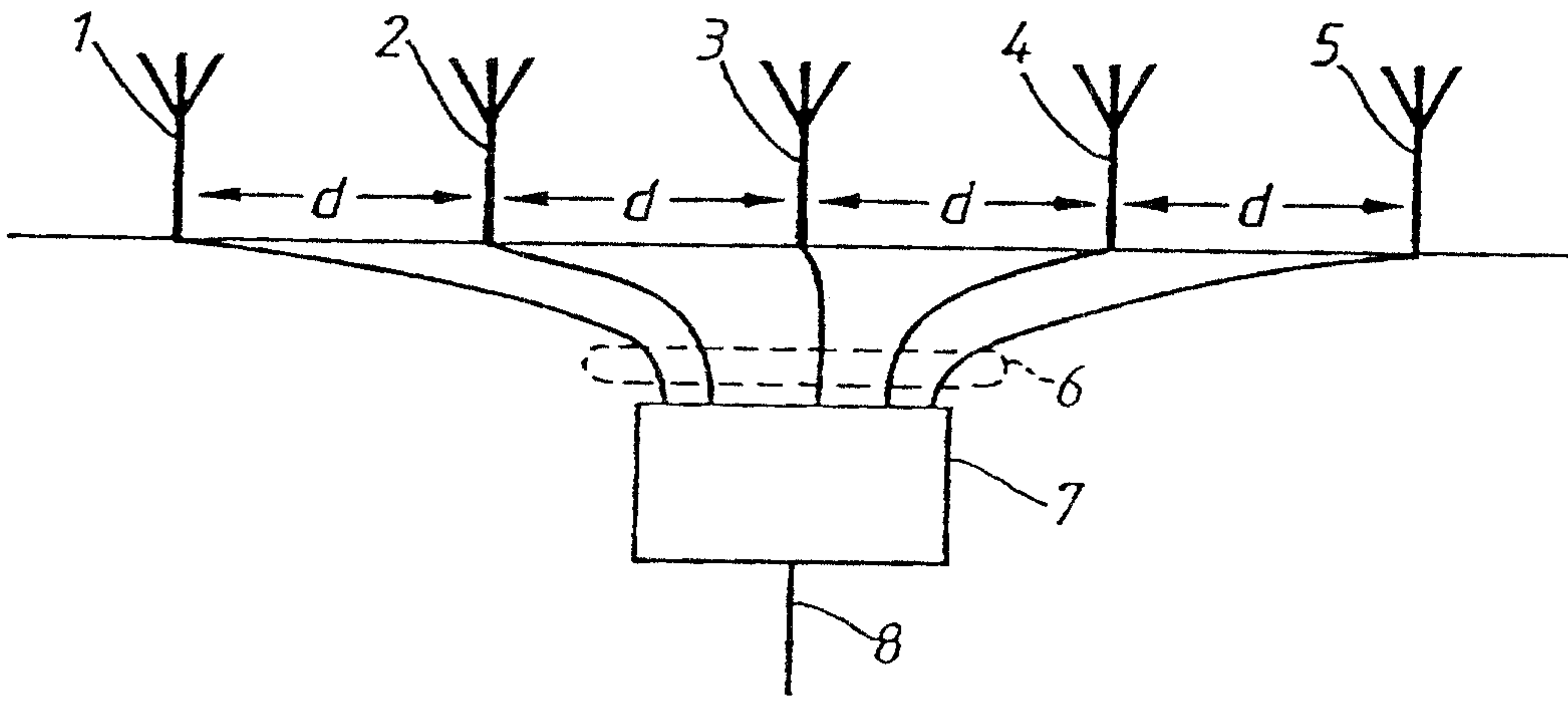


Fig. 1.

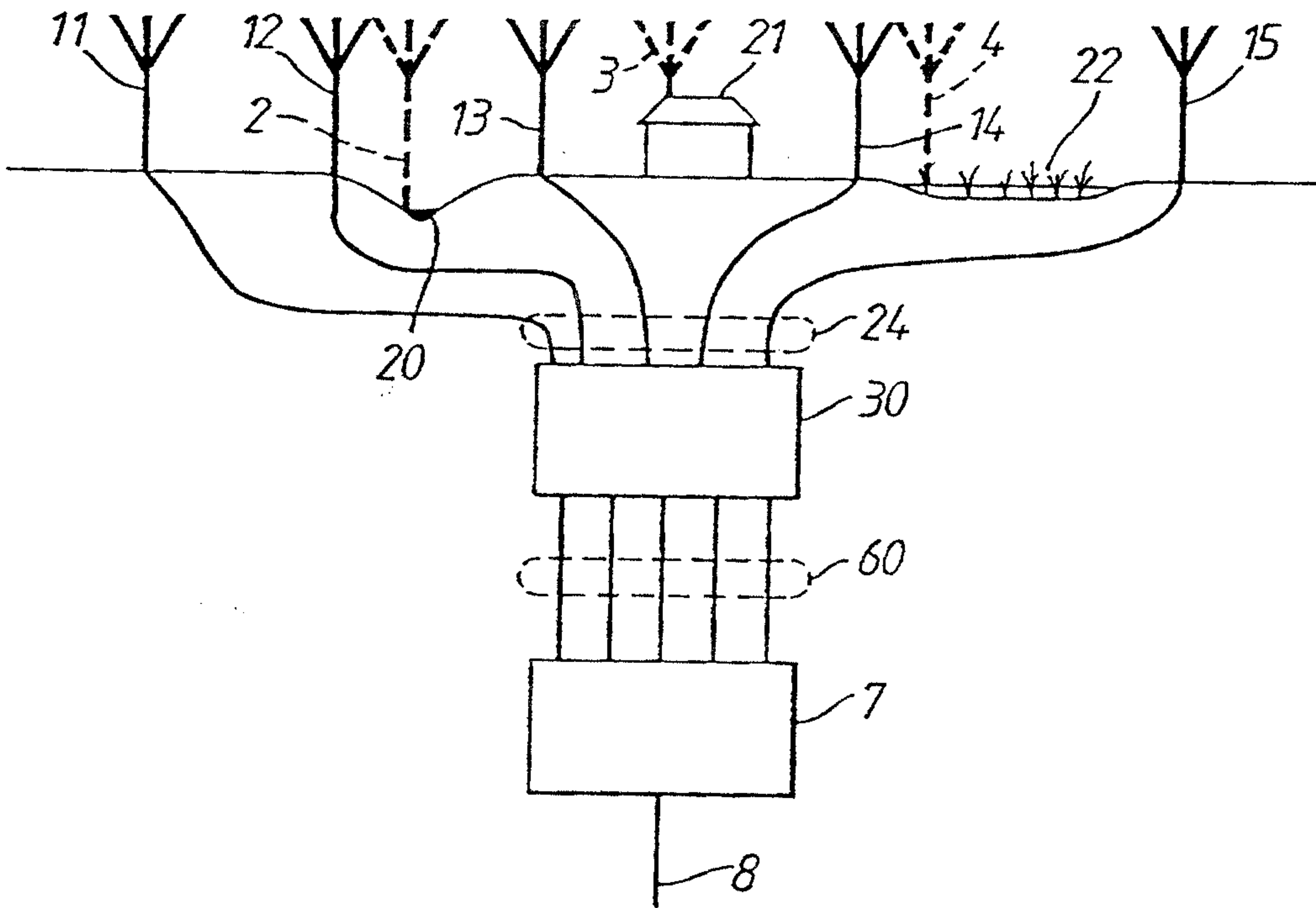


Fig. 2.

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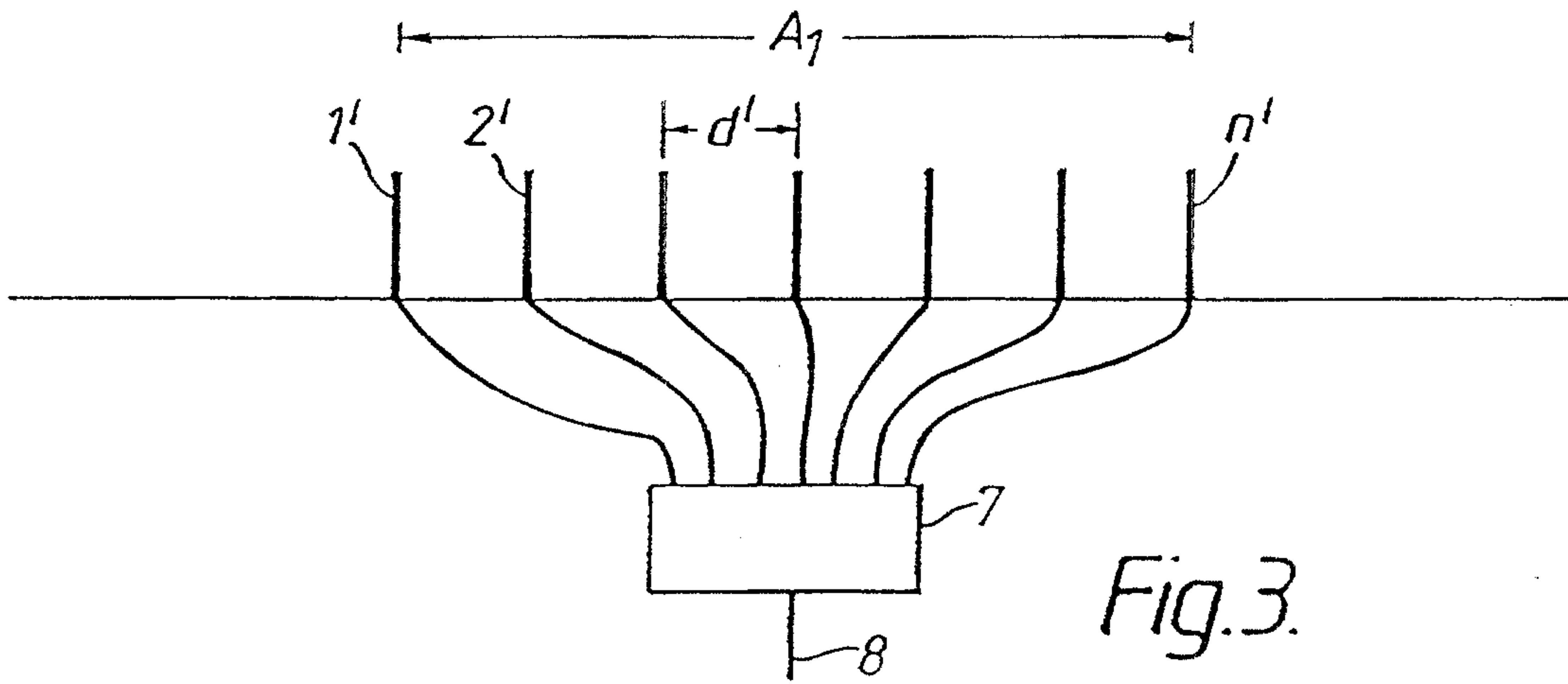


Fig. 3.

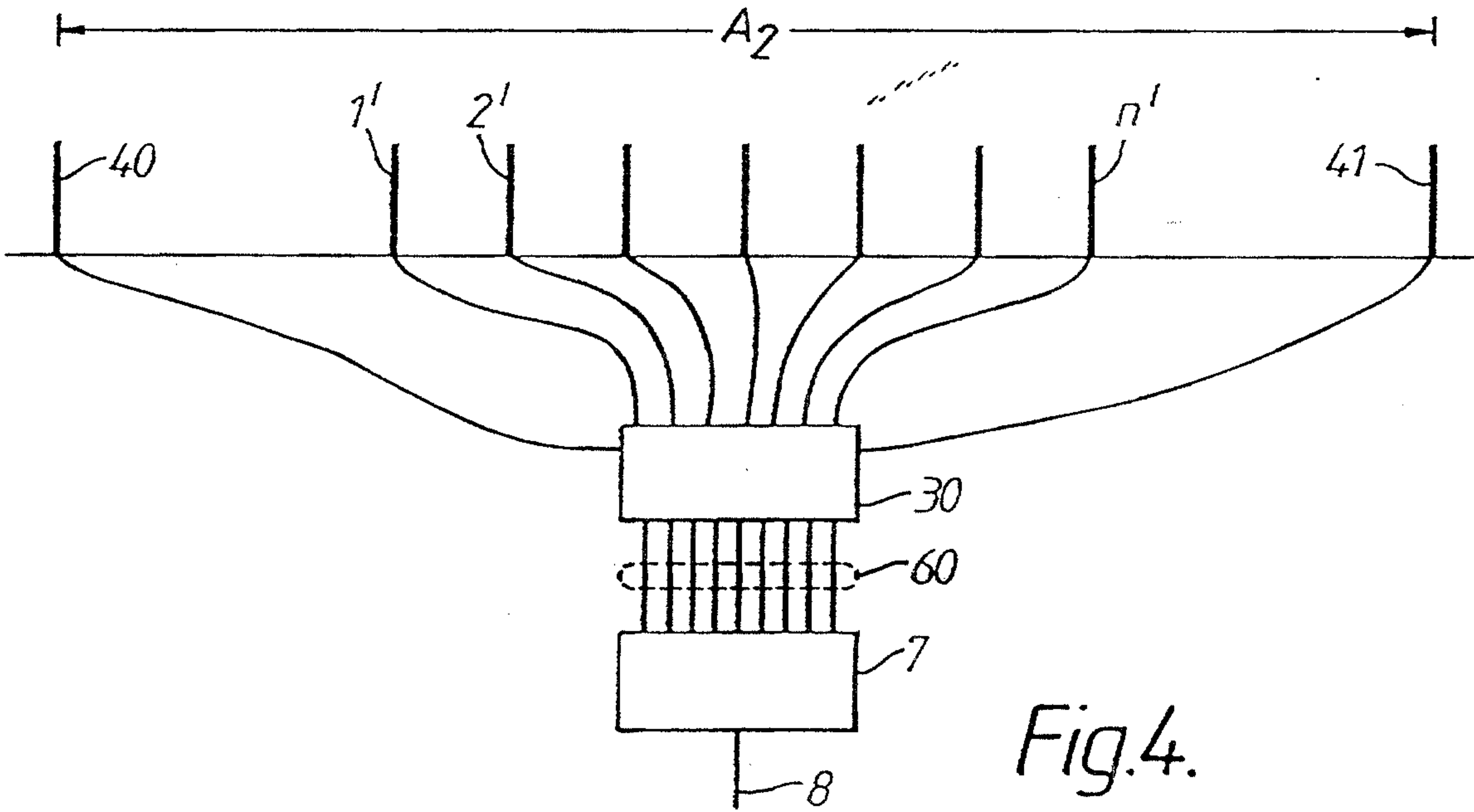


Fig. 4.

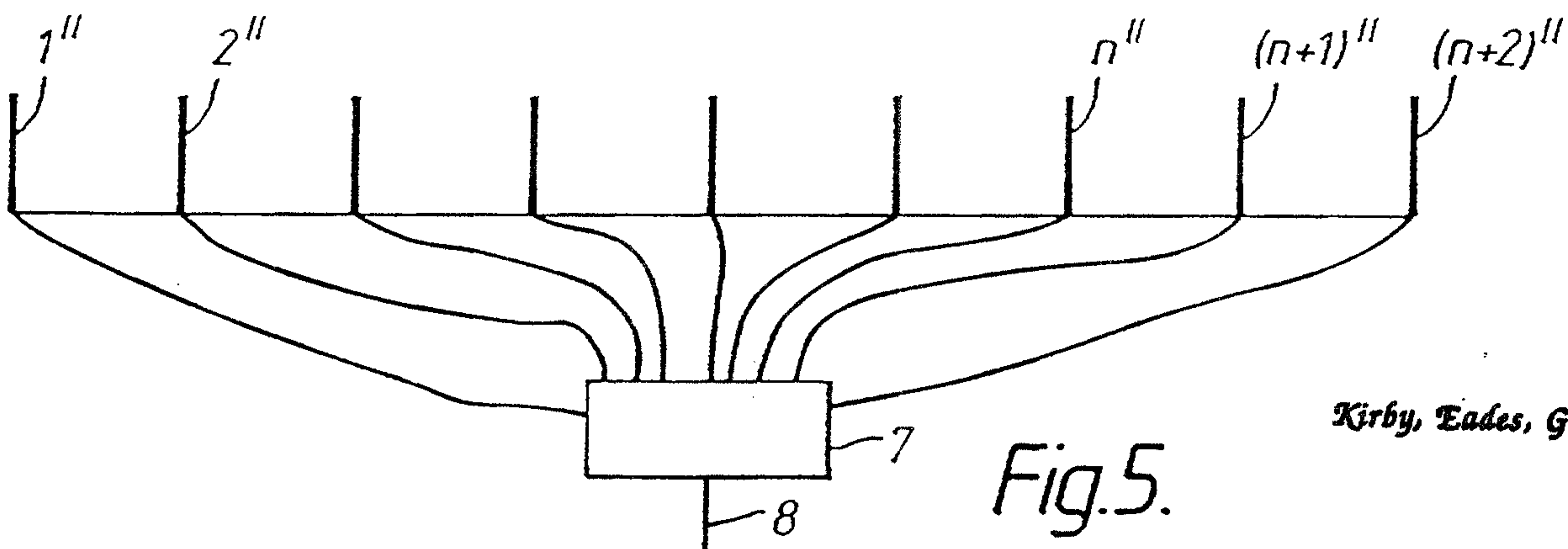


Fig. 5.

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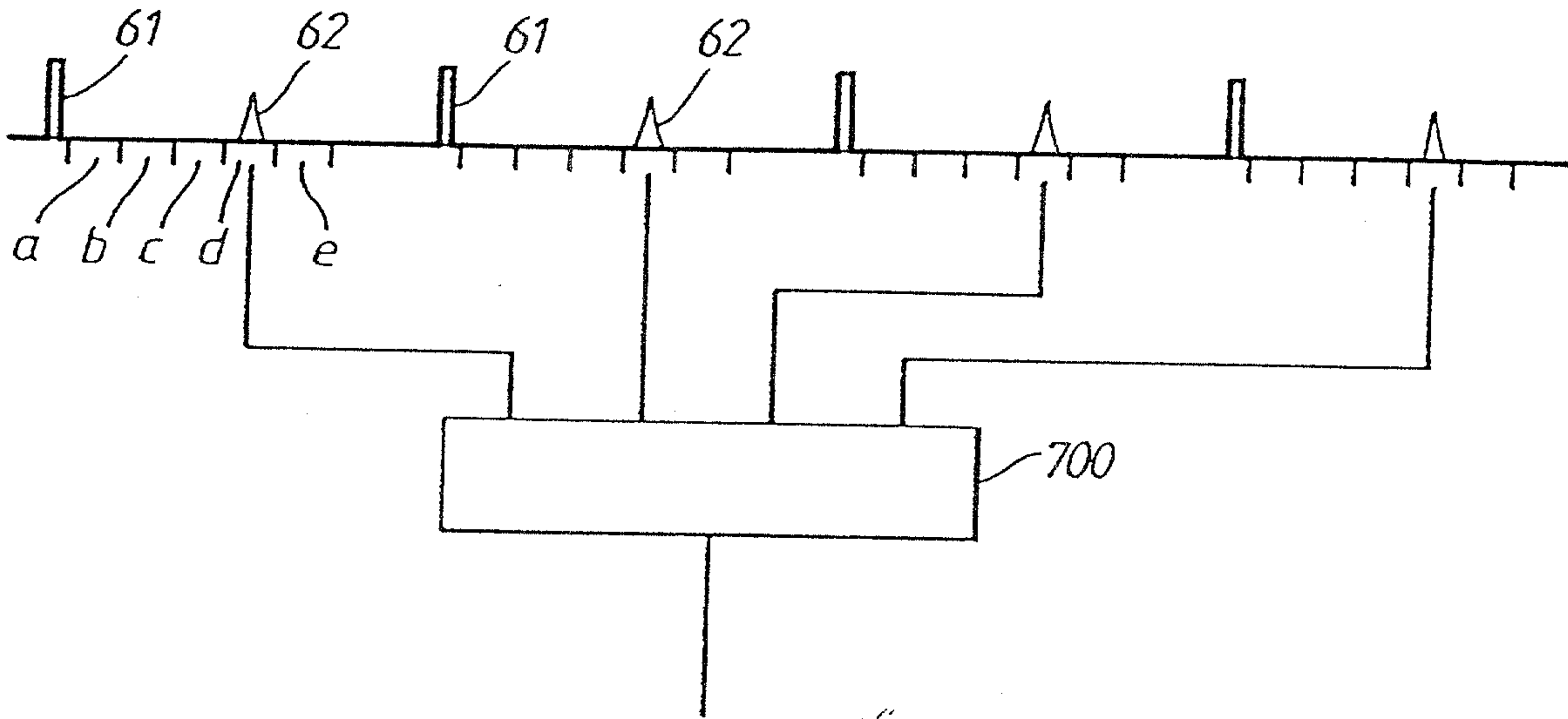


Fig. 6.

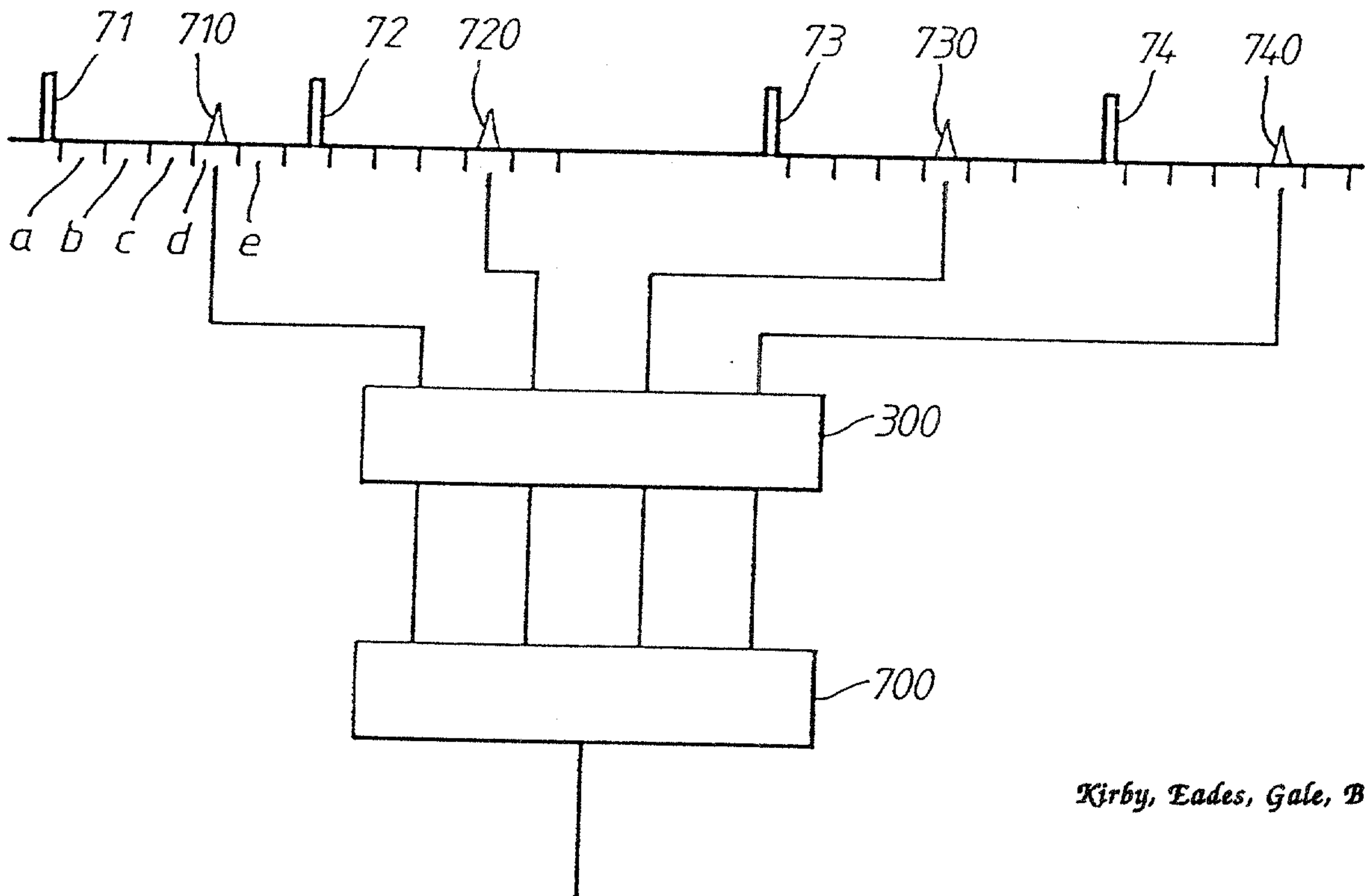


Fig. 7.

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