OR 2,447,061

Aug. 17, 1948. C. W. FRANKLIN 2,447,061

PIEZOELECTRIC SYSTEM

Filed July 18, 1945 2 Sheets-Sheet 1

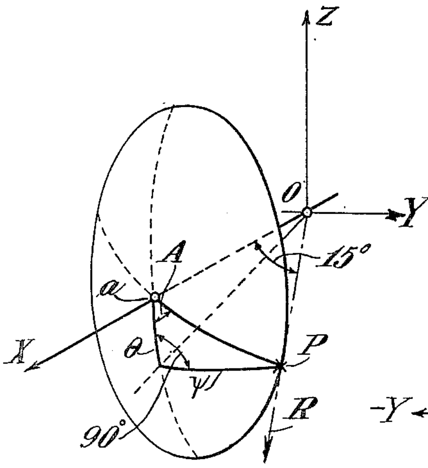

Fig.1

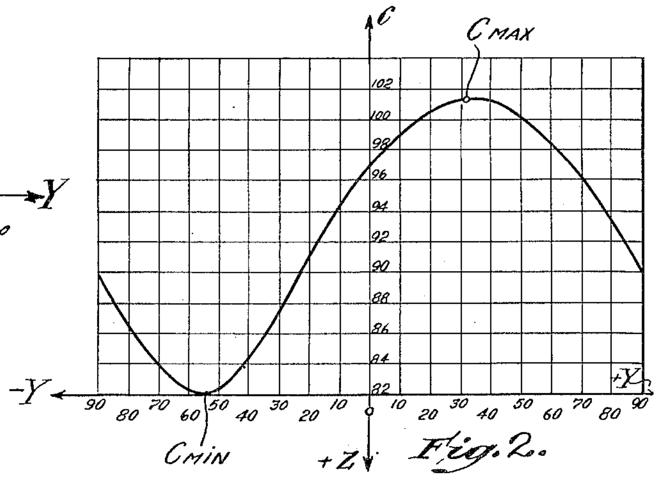

Fig.2.

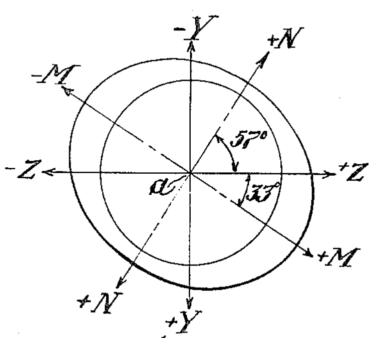

Fig.3

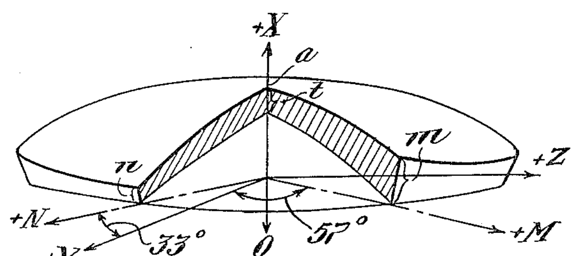

Fig.4

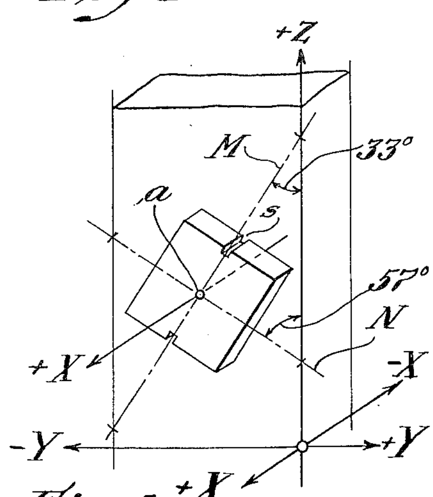

Fig.5

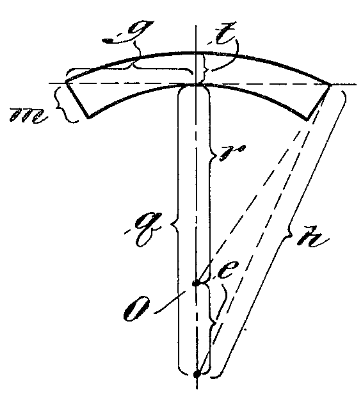

Fig.6

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PIEZOELECTRIC SYSTEM

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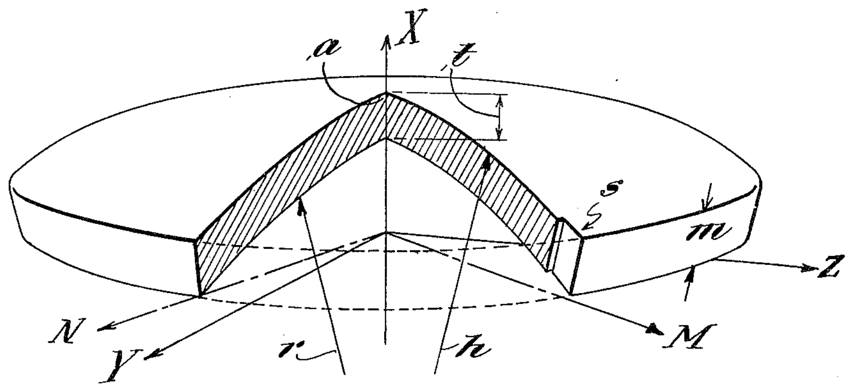

Fig. 7

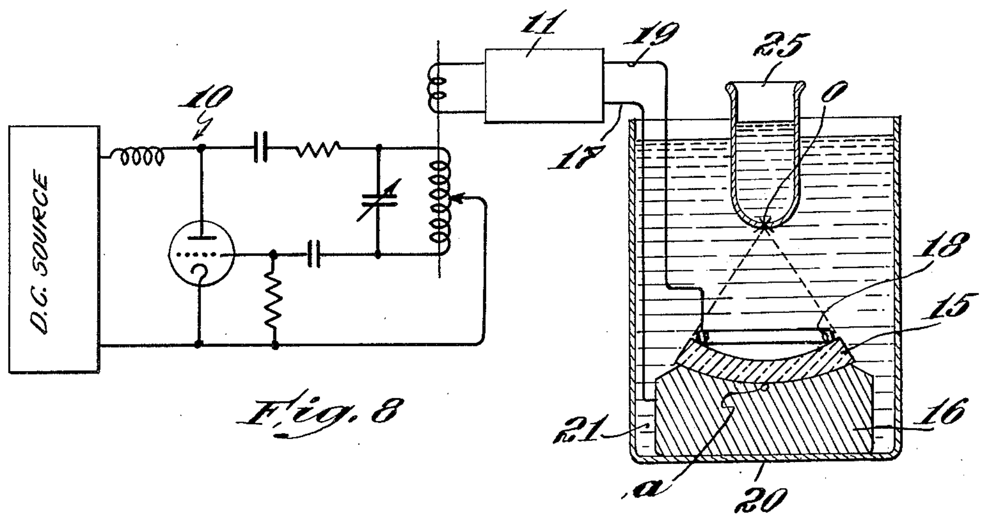

Fig. 8

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2,447,061

PIEZOELECTRIC SYSTEM

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11 Claims. (Cl. 171—327)

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The present invention relates to piezo electric systems, and more particularly to supersonic systems incorporating a piezo electric crystal body which is peculiarly cut for greater effectiveness in such systems.

The use of the piezo electric effect in oscillators for generating high frequency sound waves in fluids, either gaseous or liquid, of an appropriately selected and shaped body of piezo electric material serving as a wave emitter upon being subjected to an oscillating electric field is well known. Such sound waves have various practical uses. such as in the physical field for heating, signaling, sounding or degassing purposes; in the chemical field for purposes of emulsification and acceleration of reaction; and in the biological field for sterilizing. Most of these uses necessarily call for an energy output of maximum intensity, whereas the crystal body, as generator of such oscillation, inherently introduces limits in this respect due to its size as well as its dielectric and mechanical properties. Most of the above mentioned uses also call for an optimum distance of effectively maintained intensity, which requirement involves minimum spread of the beam angle of the supersonic vibration, in order to concentrate or "focus" it at a desired point or other predetermined locus of application.

It has been proposed to obtain better results in the above respects by using crystal shells with concentrically spherical surfaces instead of the customary parallel plane surfaces; compare, for example, the article of L. W. Labaw in "The Journal of the Acoustical Society of America," volume 16, No. 4, April 1945, pages 237 to 245. Such crystal shells actually do produce a vibratory beam which is thrown toward a focus of fairly strong amplitude instead of being projected with the spread of five to seven degrees that is produced by conventional plane crystals. However, these previously proposed curved crystals do not afford a consistent improvement in efficiency, in the above-mentioned respects, over that of equivalent plane crystals. This failure becomes increasingly apparent when the radius of curvature of the shell is decreased while its cross measurement is maintained constant.

It is the main object of the present invention to provide a piezo electric oscillator system which includes a crystal cut to secure maximum efficiency of translation of electrical into mechanical energy and in its application to supersonic systems, to provide optimum sound wave output due to large oscillation amplitude and energy concentration for all radii and all curvatures. Other

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objects are to provide a crystal of the above-mentioned desirable qualities, derived from a given body of piezo electric material, and to provide a method of cutting such a crystal according to the requirements of the invention.

In one of its aspects the invention employs, in an oscillator device of the above-mentioned type, a pulsating element which is cut from a piezo electric crystal and which has sections with boundaries that are curved in a fixed geometric relation to the characteristic axes of the crystal from which the element is cut, the dimensions which control the frequency at which each elementary portion of the crystal vibrates in a given electric field being varied through these sections as a function of the relation of the boundaries of the sections to the characteristic axes, in such a way that the pulse frequency in an electric field is substantially uniform throughout the element.

In another aspect, the invention contemplates the use, in supersonic systems, of piezo electric elements the thickness of which varies in such a manner that the pulsating frequencies in a given oscillatory field are the same for each elementary portion of the crystal, the surfaces of the crystal having peculiarly varying inclinations to the characteristic axes of the crystal, such as to provide a region of maximum vibratory amplitude at a given spatial relation to the crystal.

In still another aspect, a feature of the invention concerns a cup-shaped crystal shell, one of whose surfaces is substantially spherical whereas the other surface, preferably the outer surface, is bi-symmetrically curved so as to provide at two opposite points of the rim of the crystal regions of minimum thickness, and therebetween two opposite regions of maximum thickness, these regions being particularly oriented with respect to the characteristic axes of the crystal so as to provide the above-mentioned uniformity of frequency throughout the wave-emitting crystal surface, regardless of the relation of that surface to the directional properties of the peizo electric material from which the shell is cut.

Another feature of the invention is a preferred way of cutting a blank, oriented in accordance with the teaching of the invention, from a raw crystal, so as to facilitate the shaping of the crystal in its final form; and still another feature is a method of manufacture, which reduces delicate hand work in shaping the final crystal to a minimum.

These and other objects, aspects and features will be apparent from the following description of an embodiment illustrating the novel characteristics of the invention. This description refers to drawings in which

Fig. 1 is a diagram representing the relation of the shape and structure of a piezo electric shell according to the invention to the axes of the crystal from which it is cut;

Fig. 2 graphically represents the modulus of elasticity as a function of the angle in the ZY plane of the crystal;

Fig. 3 is a plan view of a crystal shell cut according to the invention;

Fig. 4 is an isometric view of a shell according to Fig. 3 with a quadrant removed on line **4—4—4** of Fig. 3;

Fig. 5 is a diagram illustrating the manner in which the blank for a shell according to Figs. 3 and 4 is cut from a quartz crystal;

Fig. 6 is a diagrammatic cross section through a crystal according to the invention, at one manufacturing stage;

Fig. 7 is an isometric view similar to Fig. 4, of the crystal at that stage; and

Fig. 8 is a schematic representation of supersonic apparatus incorporating a shell according to the invention.

By way of example the computation, manufacture and use, according to the invention, of an X-cut quartz shell of 30° opening and with an inner spherical surface of 4 cm. radius will be described, but it is understood that the invention is limited neither to the axes relation, the particular curvature of the vibration-emitting surface, the material, nor the width of opening of the oscillating element. For example, the shell may be shaped with particular relation to the A-C cut, it may have the shape of a cylinder for producing a line instead of a point of focus, or piezoelectric material other than quartz, such as Rochelle salt might be used, and the shell will have smaller or larger openings and radii in accordance with the purpose to which this pulsating element is to be put.

The first step in manufacturing a crystal element according to the invention is the determination of the distribution of the modulus of elasticity over the crystal shell, as dependent on the orientation of the shell apex to the characteristic axes of the crystal.

In the present embodiment, as above indicated, the relation of the shell shape as defined by its apex is the one indicated in Fig. 1. In this figure X, Y and Z are the principal axes (right handed) of the quartz crystal. a is the apex of the shell, the inner surface of which is centered at O, and each peripheral point P of which lies on a line R through O. It will be evident that in this instance the wave propagation will proceed along lines through O as focus, the outermost of which lines are the above-mentioned lines R.

The computation of the modulus of elasticity along the shell rim follows in its initial stages the classical procedure as, for example, set forth by W. P. Mason in "The Bell System Technical Journal," volume XXII, No. 2, July 1943, pages 178 to 223, with particular reference also to the work of R. Bechmann mentioned in footnote 12 of this article.

This procedure is based on the fact that any anisotropic body has for a transmission frequency in any selected direction three speeds of propagation whose values depend upon the direction of the exciting disturbance and upon the relation of the selected direction to the characteristic axes of the crystal.

In accordance with this method of computation, the following equation is set up:

$$\begin{vmatrix} \lambda_{11}-C & \lambda_{12} & \lambda_{13} \\ \lambda_{12} & \lambda_{22}-C & \lambda_{23} \\ \lambda_{13} & \lambda_{23} & \lambda_{33}-C \end{vmatrix} = 0 \qquad (1)$$

where

$$\begin{aligned} \lambda_{11} &= C_{11}l^2 + C_{66}m^2 + C_{55}n^2 + 2C_{56}mn \\ \lambda_{22} &= C_{66}l^2 + C_{11}m^2 + C_{55}n^2 - 2C_{56}mn \\ \lambda_{33} &= C_{55}l^2 + C_{55}m^2 + C_{33}n^2 \\ \lambda_{12} &= 2C_{56}nl + (C_{12}+C_{66})lm \\ \lambda_{13} &= (C_{13}+C_{55})nl + 2C_{56}lm \\ \lambda_{23} &= C_{56}l^2 - C_{56}m^2 + (C_{13}+C_{55})mn \end{aligned} \qquad (1a)$$

and

$$\begin{aligned} C_{11} &= 85.1\times 10^{10} \text{ dynes/cm.}^2 \\ C_{66} &= 39.1\times 10^{10} \text{ dynes/cm.}^2 \\ C_{55} &= 57.1\times 10^{10} \text{ dynes/cm.}^2 \\ C_{56} &= 16.8\times 10^{10} \text{ dynes/cm.}^2 \\ C_{12}+C_{66} &= 46.05\times 10^{10} \text{ dynes/cm.}^2 \\ C_{13}+C_{55} &= 71.2\times 10^{10} \text{ dynes/cm.}^2 \\ C_{33} &= 105.3\times 10^{10} \text{ dynes/cm.}^2 \end{aligned} \qquad (1b)$$

and, with ψ and θ positive in counterclockwise direction,

$$\begin{aligned} l &= \cos\psi \cos\theta \\ m &= \sin\psi \\ n &= -\cos\psi \sin\theta \end{aligned} \qquad (1c)$$

In these equations, C is a constant or multiplicator which furnishes in the manner to be shown below the modulus values in direction R around the circumference of the shell under consideration; c_{ij} are the regular moduli of elasticity in the direction of axes X, Y, Z; λ_{ij} the transformed moduli corresponding to a disturbance in an arbitrarily selected propagation direction of specified relation to X, Y, Z, here in directions R; l, m, n the direction cosine values of the propagation direction defined by angles ψ and θ which determine the location of R relatively to the XZ and XY planes, as indicated in Fig. 1.

Taking for c_{ij} the values obtainable from standard works in this field for example the above-mentioned article by W. P. Mason, the following values for λ_{ij} in 10^{10} dynes/cm.^{2}, are obtained.

$$\begin{aligned} \lambda_{11} &= 85.1\cos^2\psi\cos^2\theta + 39.1\sin^2\psi + 57.1\cos^2\psi\sin^2\theta - 16.8\sin^2\psi\sin\theta \\ \lambda_{22} &= 39.1\cos^2\psi\cos^2\theta + 85.1\sin^2\psi + 57.1\cos^2\psi\sin^2\theta + 16.8\sin^2\psi\sin\theta \\ \lambda_{33} &= 57.1\cos^2\psi\cos^2\theta + 57.1\sin^2\psi + 105.3\cos^2\psi\sin^2\theta \\ \lambda_{12} &= -16.8\sin^2\theta\cos^2\psi + 23.025\sin^2\psi\cos\theta \\ \lambda_{13} &= -35.6\sin^2\theta\cos^2\psi + 16.8\sin^2\psi\cos\theta \\ \lambda_{23} &= 16.8\cos^2\psi\cos^2\theta - 16.8\sin^2\psi - 35.6\sin^2\psi\sin\theta \end{aligned} \qquad (2)$$

In order to compute these values, the trigonometric components thereof have to be expressed in terms of R, the directions of propagation at the shell rim. In the present example these directions are defined by one-half of the shell opening angle, namely 15°, and spherical angle A, subtended at the apex of the shell by the intersection of the shell with the XZ plane, and the intersection of the shell with a plane through axis X and that point P of the shell periphery which is under consideration; compare also Fig. 1.

In these terms, the above-mentioned trigonometric components of equation (2) are as follows:

$$\begin{aligned}
\sin^2\psi &= \sin^2 A \sin^2 15^\circ \\
\cos^2\psi \cos^2\theta &= \cos^2 15^\circ \\
\cos^2\psi \sin^2\theta &= \cos^2 A \sin^2 15^\circ \\
\sin^2\psi \sin\theta &= \sin A \sin^2 15^\circ \qquad (3) \\
\cos^2\psi \sin^2\theta &= \cos A \sin (30^\circ) \\
\sin^2\psi \cos\theta &= \sin A \sin (30^\circ)
\end{aligned}$$

Writing the determinant given above under (1) in the form

$$C^3-\Sigma\lambda_{kk}C^2+(\Sigma\lambda_{kk}\lambda_{jj}-\Sigma\lambda_{ij}{}^2)C-(\lambda_{11}\lambda_{22}\lambda_{33}+2\lambda_{12}\lambda_{13}\lambda_{23}-\Sigma\lambda_{pp}\lambda_{kj}{}^3)=0 \qquad (4)$$

the factors according to Equation 2 are obtained with the aid of Equation 3.

Using these factors, the cubic function (4), and its derivative

$$f'(C)=3C^2-2\Sigma\lambda_{kk}C+(\Sigma\lambda_{kk}\lambda_{jj}-\Sigma\lambda_{kj}{}^2) \qquad (5)$$

in Newton's method of approximate solution, consecutive approximations C_0, C_1, C_2, etc., are set up, which values rapidly converge to an accurate solution.

Having solved the cubic equation and thus obtained the C constants in direction R for a given value of A, the solution for a nearby value is readily obtained by using next, for the first value of C, the value corresponding to the previous value of A. In this manner, there can be no doubt that the variations of the same root are obtained and that all the values of C correspond to equivalent wave-fronts in the shell.

In this manner these values of the C factors, as varying around the circumference of the 30° X-cut shell, are obtained. For the present purpose, these values are plotted in the form of a curve, given in Fig. 2, which is self-explanatory.

It will be observed that the C value repeats itself for $A\pm180^\circ$, which characteristic is inherent in the nature of the cubic equation from which these values are derived. It will further be observed that, for the example in question, the C value and hence the modulus of elasticity has double symmetry about the perpendicular lines for which A lies 57° from $+Z$ in the direction of $-Y$ and for which A lies 33° from $+Z$ toward $+Y$.

With the distribution of the modulus of elasticity over the shell rim being known, the thickness of the latter can now be computed as follows:

The frequency constant in kilocycles per second per mm. thickness is given by the formula

$$10^3\times\tfrac{1}{2}\sqrt{\frac{C}{d}} \qquad (6)$$

wherein C is the above obtained factor, and d the density of quartz, namely 2.654 g./cc. For the apex of the shell, $C=c_{11}=85.1$, and hence the frequency constant is 2831 at that point. The frequency constant divided by the thickness t at the point in question, which was assumed to be 2 mm. gives the natural frequency as 1416 kc./sec. at the apex.

Taking now from the curve according to Fig. 2, the value of C minimum, we obtain for that point a frequency constant of 2779. Calculating from formula (6) the thickness necessary to provide at that point the same natural frequency which prevails at the apex, namely 1416 kc./sec., it is found that the crystal must at this point have the minimum thickness of $n=1.964$ mm.

For the maximum thickness, corresponding to the maximum C value of 101.4, we obtain a frequency constant of 3090 and a corresponding maximum dimension of $m=2.183$ mm., corresponding to the frequency of 1416 kc./sec.

It will now be evident that the moduli of elasticity can similarly be computed for every point of the shell, for example by assuming, instead of the maximum spread of 15° on each side of the apex, intermediate angles, for example of 5 and 10°, and computing for these angles curves similar to that shown in Fig. 2. The shell thickness for any point can then be computed as above indicated for the minimum and maximum thicknesses. A crystal cut and dimensioned according to the above computation is indicated in Figs. 3 and 4. These figures indicate the apex, maximum and minimum thicknesses, and the co-relation of these values to the characteristic axes of the shell.

The manner of manufacturing a crystal having the characteristics indicated in Figs. 3 and 4 will now be described.

The original blank is cut from the quartz crystal as indicated in Fig. 5, which shows the maximum and minimum A angles measured in the YZ plane, as derived from the chart of Fig. 2. The axes of symmetry of the blank are indicated in Fig. 5 at M and N. After having cut the blank as indicated in Fig. 5, the lens grinder marks the XM plane with deep saw marks on both XN faces as indicated at s of Fig. 5. The apex is marked on the blank, and the lens grinder is given the radii of curvature of the blank in its second or envelope stage, obtained as follows.

The inner surface is spherical in its final shape, with a radius of curvature, as above pointed out, of 4 cm., and the maximum edge thickness is $m=2.183$ mm. Referring now to Fig. 6 where these data are indicated, the radius of the spherical envelope of the final outer surface is obtained from the relations:

$$q^2+g^2=h^2$$

where

$$\begin{aligned}
q &= (r+m)\cos 15^\circ+e \\
g &= (r+m)\sin 15^\circ \\
h &= r+t+e \\
r &= 4\text{ cm.},\ m=0.2183\text{ cm.},\ t=0.2\text{ cm.};
\end{aligned}$$

as

$$h=4.815\text{ cm.}$$

The first step in grinding of the shell will therefore be the production of a lens 2.025 mm. thick at the apex, the extra thickness in excess of 2 mm. being allowed for possible grinding corrections, and slightly in excess of 2.183 mm. at the edge, by using an inner matrix of 4 cm. and outer matrix of 4.8147 cm. radius. The above mentioned saw cuts will, after this operation, be reduced to edge nicks and might even be altogether lost, in which case the Y and Z directions can be recovered according to conventional methods from the knowledge of the X axis, defined and preserved by the apex. Fig. 7, which corresponds to Fig. 4, shows the blank at this stage.

The crystal should now be checked for twinning before proceeding to the more laborious and expensive final stage, which consists of flattening the outer surface in the direction of the minima. After verification that the shell is untwinned, a mark is applied running from the apex to the ends of the grooves s, thus indicating the maxima axis M. The minima direction N perpendicular to the maxima axis M is also marked, conveniently in a different color.

The lens grinder now cuts the edge of the blank so that it is normal to the inner surface and which is then thinned from the outside along the N axis until the dimension $n=1.964$ cm. is reached at the end of the N line, on the edge of the shell, shading this dimension over the entire shell to figures furnished him, until the shape indicated in Figs. 3 and 4 is reached.

Fig. 8 indicates supersonic apparatus incorporating a shell according to Figs. 3 and 4. In Fig. 8, numeral **10** indicates a conventional oscillator circuit, for example a stabilized, amplitude limited Hartley oscillator feeding into an amplifier **11**. The shell **15** may be supported on a lead block holder **16** connected to one output terminal **17** of the amplifier. Near the rim of the crystal **15** rests a copper ring **18**, connected to the other terminal **19**. The entire vibrating unit may be immersed in a tank **20** containing insulating fluid **21**. When the oscillating field is applied to electrodes **16** and **18**, the shell vibrates through its entire body uniformly at the selected frequency, the vibration energy being concentrated at focus O. The specimen to be treated, indicated in Fig. 8 by a test tube **25**, is held in the focus by appropriate means depending upon the particular use to which the system is put.

It should be understood that the present disclosure is for the purpose of illustration only and that this invention includes all modifications and equivalents which fall within the scope of the appended claims.

I claim:

1. In an oscillator device of the type described a pulsating element cut from piezo electric material with sections having curved boundaries in fixed geometric relation to the characteristic axes of said material, the frequency controlling dimension of the element varying through said sections with varying relation of said boundaries to said axes according to the mathematical function which defines the condition that said element furnishes in an oscillating electric field a pulse frequency which is substantially uniform throughout the element.

2. In an oscillator device of the type described a pulsating element cut from piezo electric material with sections having curved boundaries in fixed geometric relation to the characteristic axes of said material, one boundary being curved to propagate mechanical vibrations substantially in predetermined directions and the second boundary being curved to provide frequency controlling dimensions of the element which vary through said sections with varying relation of said boundaries to said axes according to the mathematical function which defines the condition that said element furnishes in a given oscillating electric field a pulse frequency which is substantially uniform throughout the element.

3. In an oscillator device of the type described a pulsating element cut from piezo electric material with sections having curved boundaries in fixed geometric relation to the characteristic axes of said material, one boundary being curved to propagate mechanical vibrations substantially in predetermined directions and the second boundary being curved to define with said first boundary thicknesses of said sections which vary through said element with varying direction and modulus of elasticity as dependent upon direction according to the mathematical function which defines the condition that said element furnishes in a given oscillating electric field frequency responses in said directions which are substantially uniform over said propagating boundary.

4. In an oscillator device of the type described a shell shaped pulsating element cut from piezo electric material with sections having curved boundaries in fixed geometric relation to the characteristic axes of said material, the concave boundary being spherical and the convex boundary being curved to provide frequency controlling dimension of the element which vary through said sections with varying relation of said boundaries to said axes according to the mathematical function which defines the condition that said element furnishes in a given oscillating electric field a pulse frequency which is substantially uniform throughout the element.

5. A cup-shaped shell of piezo electric material for generating mechanical vibration while pulsating in an electric field, having one surface which is substantially spherical whereas the other surface is curved to define with said first surface two points of minimum thickness at two regions of the rim of said shell and two points of thickness at two regions between said minimum thickness regions, the thicknesses of said shell gradually varying between the apex of said shell and said points, and between said points, and said points being oriented regarding the characteristic axes of said material and the direction of said field to provide pulsation at substantially uniform frequency of said shell while in said field.

6. A cup-shaped shell of piezo electric material for generating mechanical vibration while pulsating in an electric field, having an inner substantially spherical surface whereas the outer surface is curved with two points of minimum thickness at two opposite regions of the rim of said shell and two points of maximum thickness at two opposite regions between said minimum regions, the thicknesses of said shell gradually varying between the apex of said shell and said points, and between said points, and said points being oriented regarding the characteristic axes of said material and the direction of said field to provide pulsation at substantially uniform frequency of said shell while in said field.

7. A cup-shaped shell of piezo electric material for generating mechanical vibration while pulsating in an electric field, having an apex region of predetermined thickness, one surface which is substantially spherical whereas the other surface is curved towards two peripheral regions in which the shell is thinner than in said apex region and two peripheral regions in which the shell is thicker than in said apex region and located between said thinner regions, the thicknesses of said shell gradually varying between said apex region and said peripheral region and increasing gradually from minimum to maximum thickness along said periphery, and said regions being oriented regarding the characteristic axes of said material and the direction of said field to provide pulsation at substantially uniform frequency of said shell while in said field.

8. A cup-shaped shell of piezo electric material for generating mechanical vibration while pulsating in an electric field, having one surface which is substantially spherical whereas the other surface is bisymmetrically curved with two points of minimum thickness at two opposite regions of the rim of said shell and two points of maximum thickness at two opposite regions between said minimum thickness regions, the thicknesses of said shell gradually varying between the apex of

said shell and said points and increasing gradually from minimum to maximum thickness along said rim and said points being oriented regarding the characteristic axes of said material and the direction of said field to provide pulsation at substantially uniform frequency of said shell while in said field.

9. A cup-shaped shell of piezo electric material for generating mechanical vibration while pulsating in an electric field, having an apex region of given thickness, an inner surface which is substantially spherical whereas the outer surface is bisymmetrically curved with two points of minimum thickness smaller than said apex thickness at two opposite regions of the rim of said shell and two points of maximum thickness greater than the apex thickness at two opposite regions between said minimum regions, the thicknesses of said shell gradually varying between said apex region and said points and increasing gradually from minimum to maximum thickness along said rim and said points being oriented regarding the characteristic axes of said crystal material and the direction of said field to provide pulsation at substantially uniform frequency of said shell while in said field.

10. In the art of manufacturing a curved body of piezo electric material having an apex of given thickness and geometric relation to a characteristic axis of said material and thickness dimensions varying from said apex towards a region which is thicker, and a region which is thinner than said apex, said regions defining lines of predetermined inclination relatively to two characteristic axes of said material: the method which comprises cutting from said material a block having a face which is substantially perpendicular to said apex axis and a face which is substantially parallel to one of said lines, grinding inner and outer surfaces of said body to envelop said apex and said thicker region portion, and thinning said body from said apex and said thicker region towards said thinner regions while preserving said apex axis, until said shell conforms to said dimensions.

11. In the art of shaping a shell of piezo electric material having an apex of given thickness and geometric relation to a characteristic axis of said material and shell thickness dimensions which vary from said apex towards two opposite rim regions which are thicker and two rim regions oppositely therebetween which are thinner than said apex, said opposite regions defining lines substantially within the plane of two other characteristic axes of said material and in predetermined angular relation to said axes: the method which comprises cutting from said material a substantially parallelepipedal block two of whose faces are substantially parallel to said plane whereas pairs of oppositely located ones of said other faces are substantially parallel to respective ones of said lines; marking each face of one of said pairs to indicate the direction of one of said lines; spherically grinding inner and outer shell surfaces to envelop said apex and said thicker regions; and guided by said marking thinning said shell from said apex and said thicker regions towards said thinner regions, until said shell conforms to said dimensions.

CONSTANCE W. FRANKLIN.

REFERENCES CITED

The following references are of record in the file of this patent:

FOREIGN PATENTS

Number	Country	Date
654,673	Germany	Dec. 24, 1937