



(51) International Patent Classification:
G01S 5/14 (2006.01)

(21) International Application Number:
PCT/EP2009/054855

(22) International Filing Date:
22 April 2009 (22.04.2009)

(25) Filing Language: English

(26) Publication Language: English

(30) Priority Data:
08007781.1 22 April 2008 (22.04.2008) EP

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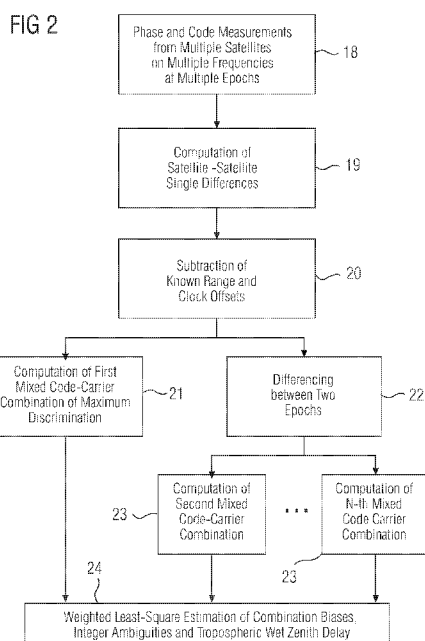
(81) Designated States (unless otherwise indicated, for every
kind of national protection available): AE, AG, AL, AM,
AO, AT, AU, AZ, BA, BB, BG, BH, BR, BW, BY, BZ,
CA, CH, CN, CO, CR, CU, CZ, DE, DK, DM, DO, DZ,
EC, EE, EG, ES, FI, GB, GD, GE, GH, GM, GT, HN,
HR, HU, ID, IL, IN, IS, JP, KE, KG, KM, KN, KP, KR,
KZ, LA, LC, LK, LR, LS, LT, LU, LY, MA, MD, ME,
MG, MK, MN, MW, MX, MY, MZ, NA, NG, NI, NO,
NZ, OM, PG, PH, PL, PT, RO, RS, RU, SC, SD, SE, SG,
SK, SL, SM, ST, SV, SY, TJ, TM, TN, TR, TT, TZ, UA,
UG, US, UZ, VC, VN, ZA, ZM, ZW.

(84) Designated States (unless otherwise indicated, for every
kind of regional protection available): ARIPO (BW, GH,
GM, KE, LS, MW, MZ, NA, SD, SL, SZ, TZ, UG, ZM,
ZW), Eurasian (AM, AZ, BY, KG, KZ, MD, RU, TJ,
TM), European (AT, BE, BG, CH, CY, CZ, DE, DK, EE,
ES, FI, FR, GB, GR, HR, HU, IE, IS, IT, LT, LU, LV,
MC, MK, MT, NL, NO, PL, PT, RO, SE, SI, SK, TR),
OAPI (BF, BJ, CF, CG, CI, CM, GA, GN, GQ, GW, ML,
MR, NE, SN, TD, TG).

Published:

— without international search report and to be republished
upon receipt of that report (Rule 48.2(g))

(54) Title: METHOD FOR A GLOBAL SATELLITE NAVIGATION SYSTEM



(57) Abstract: A method for estimating satellite-satellite single difference biases is described. The method uses an ionosphere-free mixed code-carrier combination of maximum ambiguities discrimination defined at the ration between wavelength and noise standard deviation. The accuracy of the biases estimation is further improved by an additional ionosphere-free mixed code-carrier combination of time-difference measurement that is uncorrelated with the first combination. Finally, an alternative method is based on a combination of carrier signals in a common frequency band which allows estimating the biases individually.

Description:

Method for a Global Satellite Navigation System

5 The invention relates to a method for a global satellite navigation system with at least two carriers, from which a linear combination is formed by a reference station for estimating a satellite-satellite single difference phase and code bias, wherein the estimated bias is provided to a mobile receiver, which computes the same linear combination, subtracts the bias from the linear combination and determines integer phase ambiguities.

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The invention further relates to a reference station, a mobile navigation device and a software product.

Such a method is known from GABOR, M. and NEREM, S., Satellite-satellite single
15 difference phase calibration as applied to ambiguity resolution, Navigation, Vol. 49, Nr. 4, pp. 223-242, 2002 [2]. The known method uses the ionosphere-free Melbourne-Wübbena (= MW) combination [1]. In a reference station a satellite-satellite single difference (= SD) phase and code bias is estimated, wherein the estimated bias is provided to a mobile receiver, which computes the same linear combination, subtracts the bias from the linear
20 combination and determines integer phase ambiguities.

The MW combination is widely used for precise point positioning (= PPP) to determine the widelane ambiguities. The MW combination is a geometry-free L1/L2 code-carrier combination that removes the tropospheric delay, the clock offset and further non-
25 dispersive error sources. The MW combination and an additional ionosphere-free L1/L2 phase combination are used in particular in [2], [3] and [4] to determine L1 phase bias estimates.

However, there exist a variety of disadvantages of this known method: First, the L1 and L2
30 phase biases can not be separated from the L1 and L2 code biases, i.e. the L1 phase bias estimate also includes weighted L1 and L2 code biases. Secondly, the Melbourne-Wübbena combination refers to a geometry-free, ionosphere-free linear combination which eliminates the range and can not be used for positioning. In principle, geometry-preserving,

ionosphere-free combinations can be found where the L1 and L2 bias of the known method are applicable, but these combinations are narrowlane combinations with a wavelength of at most 10.7 cm. This rather low wavelength prevents any reliable ambiguity resolution at the mobile receiver.

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Proceeding from this related art, the present invention seeks to provide improved methods for estimating biases.

This object is achieved by a method having the features of the independent claims.

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Advantageous embodiments and refinements are specified in claims dependent thereon.

In a first embodiment of the method, the satellite-satellite single difference bias of a single ionosphere-free, geometry-preserving combination of the code and phase signals of at least two carriers is determined by using a maximum combination discrimination requirement or a minimum noise requirement for the linear combination. Using such a combination enables the reference station to estimate biases that are directly applicable for positioning.

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An ionosphere-free mixed code-carrier bias, integer phase ambiguities and tropospheric wet zenith delays are advantageously obtained by the reference station using a least-square estimation that minimizes the errors in the estimation process.

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The combination discrimination requirement can be defined as the ratio of the wavelength of the combination and the weighted sum formed by the bias of the combination and the standard deviation of the combination. By maximizing such a combination discrimination requirement the combination is chosen such that the combination error and the bias of the combination is as small as possible with respect to the combination wavelength resulting in small relative errors in the determination of the combination biases.

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In a simplified method, the combination discrimination requirement is defined as the ratio of the wavelength of the combination and the standard deviation of the mixed code-carrier combination noise. Such a combination discrimination requirement still results in a robust determination of the biases of the linear combination.

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For allowing the receiver to judge on the reliability of the bias, the reference station is transmitting the bias together with the standard deviation associated with the bias of the combination to the mobile receiver.

- 5 The estimation of the biases can be further improved, if an additional geometry preserving, ionosphere-free combination of time differences of measurements or time differences of geometry-preserving ionosphere-free combinations of measurements are used for determining the biases, wherein the weighting coefficients of the additional combination fulfills the requirement that the ionosphere-free, geometry-preserving combination of the code and
10 phase signals of at least two carriers are uncorrelated.

- In an alternative embodiment of the method, the individual biases are determined for the phase signal and the code signal of at least three carriers, wherein at least two carriers belong to a common frequency band and have the same bias. The biases are transmitted to
15 the mobile receiver which uses the biases for determining integer phase ambiguities. Using this alternative embodiment of the method results in separates estimates of the ionospheric delay and the individual biases.

- For the Galileo global satellite navigation system the individual biases of the L1 phase
20 signal, the E5 signal, the L1 code signal and the E5 code signal are determined by dividing the Galileo E5 band into at least two sub-bands whose carrier signals have the same frequency and therefore the same bias.

- The Galileo E5 band can further be split into the sub-bands E5a and E5b and E5c, wherein
25 the later one corresponds to the central lobe between E5a and E5b allowing an estimation of an individual biases of the four carrier signals.

- The estimation of the individual biases is advantageously performed in a subspace of the range domain, by applying orthogonal projections on the measured phase signals that
30 transform the measurements into a subspace to the range domain that extends not to ionospheric and tropospheric errors. Thus, the ionospheric and tropospheric errors do not affect the estimation of the individual biases.

As in the previous embodiment of the method, the reference station is transmitting the biases together with the standard deviation associated with the biases for allowing the mobile receiver to judge on the accuracy of the biases.

- 5 It is also possible to perform both embodiments of the method and to use the former embodiment of the method to verify the individual biases resulting from the later embodiment of the method.

- 10 If the individual biases resulting from the later method are stated invalid, the biases resulting from the former method can be used instead of the individual biases resulting from the later method.

- 15 In both embodiments of the methods, the mobile receiver advantageously determines its integer ambiguities by a sequential bootstrapping and integer decorrelation after removal of the bias estimates. Such an approach has a higher success rate than the direct determination of the integer ambiguities.

- 20 Further advantages and properties of the present invention are disclosed in the following description, in which exemplary embodiments of the present invention are explained in detail based on the drawings:

- Figure 1 shows a global satellite navigation system and a receiver for the global satellite navigation system;
- 25 Figure 2 shows a flow chart of a method using an ionosphere-free code-carrier linear combination and an additional, uncorrelated code-carrier combination of time-differenced measurements for a bias estimation at a single reference station;
- 30 Figure 3 shows an estimation of satellite-satellite SD biases of the ionosphere-free code-carrier linear combination ($\lambda = 3.215m$) at a single reference station;

- Figure 4 demonstrates the benefit of an additional, uncorrelated L1-E5 ionosphere-free code-carrier combination of time-differenced measurements for bias estimation at a single reference station;
- 5 Figure 5 shows a SD bias estimation of the ionosphere-free code-carrier combination of maximum discrimination ($\lambda = 3.215m$): Reduced batch interval to 5 min with aiding by a time-differenced, ionosphere-free code-carrier combination of minimum noise;
- 10 Figure 6 shows single difference phase bias estimation with uncombined phase and code measurements on L1, E5a, E5b and E5c at a single reference station (Rates of iono./ tropo. estimation: $R_I = 1/10s$, $R_T = 1/120s$).
- 15 Figure 7 shows the probability of wrong fixing of SD integer ambiguities of the ionosphere-free code-carrier linear combination ($\lambda = 3.215m$) for known biases and single epoch measurements;
- Figure 8 demonstrates the impact of residual biases on the success rate of sequential integer ambiguity resolution: single epoch fixing of ionosphere-free code-carrier linear combination ($\lambda = 3.215m$) with integer decorrelation of float ambiguities;
- 20 Figure 9 illustrates multi-epoch sequential ambiguity fixing of ionosphere-free code-carrier linear combination ($\lambda = 3.215m$) with integer decorrelation: Residual biases of 2 cm for code-carrier combination and 10 cm for code-only comb;
- 25 Figure 10 shows the single epoch positioning accuracy with SD measurements by 120s ionosphere-free carrier smoothing of the ionosphere-free code-carrier combination with $\lambda = 3.215m$, aided by a code-only combination; and
- 30

Figure 11 demonstrates the impact of residual biases from Figure 3 on single epoch positioning with SD measurement by 120s ionosphere-free carrier smoothing of the ionosphere-free code-carrier combination with $\lambda = 3.215m$, aided by a code-only combination;

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In the following, various embodiments are described in detail:

1. Introduction

10 Figure 1 shows a global satellite navigation system 1, which comprises satellites 2 orbiting around the earth and emitting navigation signals 3 modulated on a number of carrier signals 4. The navigation signals 4 are received by a mobile navigation device 5 via an antenna 6. The antenna 6 is connected to a band pass filter and low noise amplifier 7, in which the received navigation signal 3 is sampled and amplified. In a subsequent down
15 converter 8, that is connected to the band pass filter and low noise amplifier 7 and to a reference oscillator 9, the received navigation signal 3 is converted to lower frequencies using the oscillating signal from the reference oscillator 9. The down-converted navigation signal is passing a band pass and sampling unit 10, in which the analog navigation signal 4 is sampled. The sampled navigation signal 3 is then passed to a tracking unit 11, where the
20 navigation signals 3, in particular the phases of the carrier signals 4 and the delay of code signals contained in the navigation signal 4, are tracked. The tracking unit 11 is followed by a bias subtraction unit 12, in which phase and code biases are subtracted from the phases of the carrier signals 4 and from the code signal. A subsequent position estimation unit 13 determines the actual position of the navigation device 5 based on phase signal
25 obtained by processing the carrier signal 4 and based on the codes signals. The results of the position estimation can finally be displayed on a monitoring device 14.

It should be noted that the position of the navigation device 5 is generally determined with respect to a reference station 15 that receives the satellite signals 3 by an antenna 16. A
30 base line 17 is the distance between the navigation device 5 and the reference station 15. The reference station 15 can also be used to determine various disturbances since the position of the reference station 15 is known. The parameter of the disturbances, such as ionospheric delay, tropospheric delay, code biases and phase biases can be transmitted

from the reference station 15 to the mobile navigation device 5 allowing PPP for the mobile navigation device 5, for instance by transmitting the phase bias and code biases from the reference station 15 to the mobile navigation device 5 so that the phase biases can be subtracted from the phases of the carrier signals and the code biases can be subtracted from the code signal.

It should be noted that the biases transmitted from the reference station 15 are subtracted from the the same linear combination of the phase signals and code signals that have been used in the reference station 15 for determining the biases of the phase signals and code signals.

In the following, two new approaches are proposed for PPP with Galileo: The first one is based on an ionosphere-free mixed code-carrier combination with a wavelength of 3.215 m and a noise level of 3.76 cm [5]. It is a geometry-preserving linear combination so that the bias estimates from the reference station 15 are directly applicable for positioning performed in the mobile navigation device 5. The large wavelength significantly increases the reliability of ambiguity resolution while estimating the position of the navigation device 5. Advantageously, a sequential bootstrapping with an integer decorrelation transformation [6] can be used for estimating the position of the mobile navigation device 5.

Figure 2 shows a flow chart of the method as performed in the reference station 15. As in the mobile navigation device 5, the navigation signal 4 is processed by a band pass filter and low noise amplifier, a down converter connected to a reference oscillator, a band pass and sampling unit and a tracking unit. These functional units are used to perform a phase and code measurement 18 from multiple satellites 2, on multiple frequencies at multiple epochs. Since the position of the reference station 15 is known, the position estimation unit 13 is replaced by a bias estimation unit that performs a computation 19 of satellite-satellite single differences of the measurements and further performs a subtraction 20 of the known range and clock offsets. The subtraction 20 is followed on the one hand by a computation 21 of a first mixed code-carrier combination of maximum discrimination or minimal noise and on the other hand by differencing 22 measurements between two epochs and a computation 23 of at least one further mixed code-carrier combination. The first

mixed code-carrier combination resulting from the computation 21 and the at least one further combination based on time differenced measurement of different epochs are finally used for a weighted least-square estimation 24 of the combination biases, integer ambiguities and tropospheric wet zenith delay.

5

The second method uses phase and code measurements on four frequencies without linear combinations. The estimation of independent phase and code biases on each frequency is not feasible as at least one phase bias can not be separated from the ionospheric delay.

However, the Galileo E5 band can be split into the E5a, E5b and E5c signal with the latter one corresponding to the central lobe between E5a and E5b. These three signals are modulated onto a single carrier which motivates the assumption of a common bias. In this case, the L1 and E5 code and phase biases can be determined separately.

15 2. Design of ionosphere-free mixed code-carrier combinations

In the following the design of ionosphere-free mixed code carrier combinations is described.

20 The received code signal $\rho_{u,m}^k(t_i)$ and carrier phase signal $\phi_{u,m}^k(t_i)$ at receiver u from satellite k on frequency m at epoch i is modeled as

$$\begin{aligned}
 \rho_{u,m}^k(t_i) = & r_u^k(t_i) + \delta r_u^k(t_i) + c(\delta \tau_u(t_i) - \delta \tau^k) \\
 & + q_{1m}^2 I_u^k(t_i) + T_u^k(t_i) \\
 & + b_{\rho_{u,m}} + b_{\rho_m}^k + \varepsilon_{\rho_{u,m}}^k(t_i)
 \end{aligned} \tag{1}$$

25

$$\begin{aligned}
 \lambda_m \phi_{u,m}^k(t_i) = & r_u^k(t_i) + \delta r_u^k(t_i) + c(\delta \tau_u(t_i) - \delta \tau^k) \\
 & - q_{1m}^2 I_u^k(t_i) + T_u^k(t_i) + \lambda_m N_{u,m}^k \\
 & + \lambda_m b_{\phi_{u,m}} + \lambda_m b_{\phi_m}^k + \varepsilon_{\phi_{u,m}}^k(t_i)
 \end{aligned} \tag{2}$$

30

with the receiver-satellite range $r_u^k(t_i)$, the projected orbital error $\delta r_u^k(t_i)$, the receiver / satellite clock errors $\{c\delta\tau_u, c\delta\tau^k\}$, the ionospheric delay I_u^k on L1, the ratio of frequencies $q_{ij} = f_i/f_j$, the tropospheric delay T_u^k , the integer ambiguity $N_{u,m}^k$, the receiver code/ phase bias $b_{\rho_{u,m}}, b_{\phi_{u,m}}$, the satellite code/ phase bias $b_{\rho_m}^k, b_{\phi_m}^k$ and code/ carrier phase noise $\varepsilon_{\rho_{u,m}}^k(t_i), \varepsilon_{\phi_{u,m}}^k(t_i)$ including multipath.

The phase weights α_m and code weights β_m of a geometry-preserving (= GP) code-carrier linear combination $\sum_{m=1}^M \alpha_m \lambda_m \varphi_m + \sum_{m=1}^M \beta_m \rho_m$ of the received code and phase signals at M frequencies are constraint by

$$\sum_{m=1}^M \alpha_m + \sum_{m=1}^M \beta_m = 1. \quad (3)$$

The ionospheric delay of first order is eliminated if

$$\sum_{m=1}^M \alpha_m q_{1m}^2 - \sum_{m=1}^M \beta_m q_{1m}^2 = 0. \quad (4)$$

Thus, a linear combination of the received code and phase signals is called ionosphere-free (= IF) if the condition of Equation (4) is fulfilled.

The linear combination is periodic with wavelength λ if the phase weights can be written as

$$\alpha_m = \frac{j_m \lambda}{\lambda_m} \quad (5)$$

where j_m denotes an integer. Correspondingly, a combination of phase signals is called integer nature preserving (= NP) if the condition according to Equation (5) is met.

The variance of the linear code-carrier combination is given by

$$\sigma_n^2 = \sum_{m=1}^M \alpha_m^2 \sigma_{\phi_m}^2 + \beta_m^2 \sigma_{\rho_m}^2 \quad (6)$$

and the combination discrimination is defined as $D = \lambda / \sigma_n$. For a three frequency mixed code-carrier combination with given integer coefficients j_m , there exist four degrees of freedom (three for β_m and one for λ) of which two are required to fulfill (3) and (4). The remaining two might be used to minimize the noise variance or to maximize the combination discrimination.

In the first case, the additional constraints are given by

$$\frac{\partial \sigma_n^2(\lambda, \beta_1)}{\partial \lambda} \stackrel{!}{=} 0 \quad \text{and} \quad \frac{\partial \sigma_n^2(\lambda, \beta_1)}{\partial \beta_1} \stackrel{!}{=} 0$$

$$\frac{\partial^2 \sigma_n^2(\lambda, \beta_1)}{\partial \lambda^2} \cdot \frac{\partial^2 \sigma_n^2(\lambda, \beta_1)}{\partial \beta_1^2} - \left(\frac{\partial}{\partial \lambda} \frac{\partial}{\partial \beta_1} \sigma_n^2(\lambda, \beta_1) \right)^2 \stackrel{!}{>} 0. \quad (7)$$

The standard deviations σ_{ρ_m} are obtained from the Cramer Rao bound given by

$$\Gamma_m = \frac{c^2}{\frac{C}{N_0} \cdot \frac{\int (2\pi f)^2 |S_m(f)|^2 df}{\int |S_m(f)|^2 df}}, \quad (8)$$

with the speed of light c , the carrier to noise power ratio $\frac{C}{N_0}$, and the power spectral

density $S_m(f)$ that has been derived by Betz in [15] for binary offset carrier (= BOC) modulated signals. Table 1 shows the Cramer Rao bounds of the wideband Galileo signals.

In the second case, the wavelength and code weights of the combination with maximum ambiguity discrimination D are given by the non-linear optimization

$$\max_{\substack{\alpha_1, \dots, \alpha_M \\ \beta_1, \dots, \beta_M}} D = \frac{\lambda(\alpha_1, \dots, \alpha_M, \beta_1, \dots, \beta_M)}{2\sigma_{n_1}(\alpha_1, \dots, \alpha_M, \beta_1, \dots, \beta_M)}. \quad (9)$$

The code weight β_2 is obtained from (3) as

5

$$\beta_2 = 1 - \sum_{m=1}^M \alpha_m - \beta_1 - \sum_{m=3}^M \beta_m, \quad (10)$$

and the code weight β_1 is computed with the ionosphere-free constraint of (4), i.e.

$$\begin{aligned} \beta_1 &= \sum_{m=1}^M \alpha_m q_{1m}^2 - \sum_{m=2}^M \beta_m q_{1m}^2 \\ &= \sum_{m=1}^M \alpha_m q_{1m}^2 - \left(1 - \sum_{m=1}^M \alpha_m - \beta_1 - \sum_{m=3}^M \beta_m\right) q_{12}^2 - \sum_{m=3}^M \beta_m q_{1m}^2. \end{aligned} \quad (11)$$

Replacing α_m by (5), using

$$\lambda = \tilde{\lambda}_0 \cdot w_\phi \quad \text{with} \quad \tilde{\lambda}_0 = \frac{1}{\sum_{m=1}^M \frac{j_m}{\lambda_m}} \quad \text{and} \quad w_\phi = 1 - \sum_{m=1}^M \beta_m, \quad (12)$$

and solving for β_1 yields

$$\beta_1 = s_1 + s_2 w_\phi + \sum_{m=3}^M s_m \beta_m \quad (13)$$

with

$$s_1 = -\frac{q_{12}^2}{1 - q_{12}^2} \quad (14)$$

$$s_2 = \frac{\tilde{\lambda}}{1 - q_{12}^2} \cdot \sum_{m=1}^M \frac{j_m}{\lambda_m} \cdot (q_{12}^2 + q_{1m}^2) \quad (15)$$

$$s_m = \frac{q_{12}^2 - q_{1m}^2}{1 - q_{12}^2} \quad \forall m \in \{3, \dots, M\} \quad (16)$$

20

Equation (13) is used to rewrite (10) as

$$\beta_2 = 1 - s_1 + (-1 - s_2)w_\phi - \sum_{m=3}^M (1 + s_m)\beta_m, \quad (17)$$

5

which allows us to express D as a function of w_ϕ and β_m , $m \geq 3$:

$$D = \frac{\tilde{\lambda}}{2} \cdot w_\phi \cdot \left(\tilde{\eta}^2 w_\phi^2 + \left(s_1 + s_2 w_\phi + \sum_{m=3}^M s_m \beta_m \right)^2 \sigma_{\rho_1}^2 \right. \\ \left. + \left(1 - s_1 + (-1 - s_2)w_\phi - \sum_{m=3}^M (1 + s_m)\beta_m \right)^2 \sigma_{\rho_2}^2 + \sum_{m=3}^M \beta_m^2 \sigma_{\rho_m}^2 \right)^{-1/2} \quad (18)$$

10

with $\tilde{\eta}^2 = \tilde{\lambda}^2 \cdot \sum_{m=1}^M \frac{j_m^2}{\lambda_m^2} \sigma_{\phi_m}^2$. The maximum discrimination is given by

$$\frac{\partial D}{\partial w_\phi} = 0 \quad (19)$$

and

$$\frac{\partial D}{\partial \beta_m} = 0 \quad \forall m \in \{3, \dots, M\}. \quad (20)$$

15

Equation (20) is equivalent to

$$s_m \sigma_{\rho_1}^2 \left(s_1 + s_2 w_\phi + \sum_{l=3}^M s_l \beta_l \right) - (1 + s_m) \sigma_{\rho_2}^2 \left(1 - s_1 - (1 + s_2)w_\phi - \sum_{l=3}^M (1 + s_l)\beta_l \right) + \beta_m \sigma_{\rho_m}^2 = 0, \quad (21)$$

20

which can be written also in matrix-vector notation as

$$\mathbf{A} \cdot [\beta_3, \dots, \beta_M]^T + \mathbf{b} \cdot w_\phi + \mathbf{c} = \mathbf{0}, \quad (22)$$

with

$$\begin{aligned}
\mathbf{A}_{m,l} &= s_l s_m \sigma_{\rho_1}^2 + (1+s_l)(1+s_m) \sigma_{\rho_2}^2 + \sigma_{\rho_m}^2 \delta(m-l) \\
\mathbf{b}_m &= s_2 s_m \sigma_{\rho_1}^2 + (1+s_m)(1+s_2) \sigma_{\rho_2}^2 \\
\mathbf{c}_m &= s_1 s_m \sigma_{\rho_1}^2 - (1+s_m)(1-s_1) \sigma_{\rho_2}^2,
\end{aligned} \tag{23}$$

5 and $\delta(m-l)$ being 1 for $m=l$ and 0 otherwise. Solving (22) for β_m yields

$$[\beta_3, \dots, \beta_M]^T = -\mathbf{A}^{-1}(\mathbf{c} + \mathbf{b} \cdot w_\phi). \tag{24}$$

Constraint (19) is written in full terms as

$$\begin{aligned}
&\left(s_1 + s_2 w_\phi + \sum_{m=3}^M s_m \beta_m \right) \left(s_1 + \sum_{m=3}^M s_m \beta_m \right) \sigma_{\rho_1}^2 + \left(1 - s_1 - (1+s_2) w_\phi - \sum_{m=3}^M (1+s_m) \beta_m \right) \\
&\cdot \left(1 - s_1 - \sum_{m=3}^M (1+s_m) \beta_m \right) \sigma_{\rho_2}^2 + \sum_{m=3}^M \beta_m^2 \sigma_{\rho_m}^2 = 0.
\end{aligned} \tag{25}$$

Replacing $[\beta_3, \dots, \beta_M]^T$ by (24) yields

$$\begin{aligned}
&\left(s_1 + s_2 w_\phi - \mathbf{t}^T \mathbf{A}^{-1}(\mathbf{c} + \mathbf{b} w_\phi) \right) \cdot \left(s_1 - \mathbf{t}^T \mathbf{A}^{-1}(\mathbf{c} + \mathbf{b} w_\phi) \right) \cdot \sigma_{\rho_1}^2 \\
&+ \left(1 - s_1 - (1+s_2) w_\phi + \mathbf{u}^T \mathbf{A}^{-1}(\mathbf{c} + \mathbf{b} w_\phi) \right) \cdot \left(1 - s_1 + \mathbf{u}^T \mathbf{A}^{-1}(\mathbf{c} + \mathbf{b} w_\phi) \right) \cdot \sigma_{\rho_2}^2 \\
&+ (\mathbf{c} + \mathbf{b} w_\phi)^T \mathbf{A}^{-T} \mathbf{\Sigma} \mathbf{A}^{-1}(\mathbf{c} + \mathbf{b} w_\phi) = 0,
\end{aligned} \tag{26}$$

20 with $\mathbf{t} = [s_3, \dots, s_M]^T$, $\mathbf{u} = \mathbf{s} + \mathbf{1}$, and the diagonal matrix $\mathbf{\Sigma}$ that is given by

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{\rho_3}^2 & \mathbf{0} & 0 \\ \mathbf{0} & \ddots & \mathbf{0} \\ 0 & \mathbf{0} & \sigma_{\rho_M}^2 \end{bmatrix}. \tag{27}$$

Equation (26) can be simplified to

$$r_1 + r_2 \cdot w_\phi = 0, \quad (28)$$

with

$$r_1 = (s_1 - \mathbf{t}^T \mathbf{A}^{-1} \mathbf{c})^2 \sigma_{\rho_1}^2 + (1 - s_1 + \mathbf{u}^T \mathbf{A}^{-1} \mathbf{c})^2 \sigma_{\rho_2}^2 + \mathbf{c}^T \mathbf{A}^{-T} \Sigma \mathbf{A}^{-1} \mathbf{c}, \quad (29)$$

5 and

$$\begin{aligned} r_2 = & ((s_1 - \mathbf{t}^T \mathbf{A}^{-1} \mathbf{c})(-\mathbf{t}^T \mathbf{A}^{-1} \mathbf{b}) + (s_2 - \mathbf{t}^T \mathbf{A}^{-1} \mathbf{b})(s_1 - \mathbf{t}^T \mathbf{A}^{-1} \mathbf{c})) \cdot \sigma_{\rho_1}^2 \\ & + ((1 - s_1 + \mathbf{u}^T \mathbf{A}^{-1} \mathbf{c})\mathbf{u}^T \mathbf{A}^{-1} \mathbf{b} - (1 + s_2 - \mathbf{u}^T \mathbf{A}^{-1} \mathbf{b})(1 - s_1 + \mathbf{u}^T \mathbf{A}^{-1} \mathbf{c})) \cdot \sigma_{\rho_2}^2 \\ & + (\mathbf{c}^T \mathbf{A}^{-T} \Sigma \mathbf{A}^{-1} \mathbf{b} + \mathbf{b}^T \mathbf{A}^{-T} \Sigma \mathbf{A}^{-1} \mathbf{c}). \end{aligned} \quad (30)$$

10 The optimum phase weighting $w_{\phi_{opt}} = -r_1/r_2$ is used in (24), (17) and (13) to obtain the code weights. Equation (12) provides the optimum wavelength for the computation of the phase weights with (5). Table 2 shows the weighting coefficients and properties of GP-IF-NP linear combinations of maximum ambiguity discrimination based on code and carrier phase measurements on up to five frequencies. The dual frequency E1-E5a combination is
 15 characterized by a noise level of 31.4 cm and a wavelength of 4.309 m which allows reliable ambiguity resolution within a few epochs. As only the E1 and E5a frequencies lie in aeronautical bands, this linear combination might be useful for aviation.

Linear combinations that comprise the wideband E5 and E6 code measurements benefit
 20 from a substantially lower noise level which turns into a larger ambiguity discrimination. It increases to 25.1 for the E1-E5 combination, to 39.2 for the E1-E5-E6 combination, and to 41.2 for the E1-E5a-E5b-E5-E6 combination. The large wavelength of these combinations makes them robust to the non-dispersive orbital errors and satellite clock offsets. The linear combination of measurements on 5 frequencies has the additional
 25 advantageous property of $|\beta_m| < 1$ and $|j_m| < 2$ for all m .

Table 3 shows the weighting coefficients and properties of the optimum widelane and narrowlane linear combinations for an increased noise level. The larger noise assumptions result in lower code weights and a slightly larger E6 phase weight for the four-frequency
 30 E1-E5a-E5b-E6 widelane combination. For all other frequency settings, the weighting

coefficients are the same as in Table 2. For narrowlane combinations, the use of additional frequencies has only a negligible impact on λ and σ_n . The ambiguity discrimination varies between 10.1 and 11.2 which is larger than in the case of the first three combinations of Table 3 but smaller than in the case of the remaining widelane combinations.

The previous maximization of $\frac{\lambda}{2\sigma_n}$ does not take the biases into account. The worst case combination bias is obtained from the upper bounds b_{ϕ_m} and b_{ρ_m} on the measurement biases as

$$b_n = \sum_{m=1}^M |\alpha_m| \cdot b_{\phi_m} + |\beta_m| \cdot b_{\rho_m}. \quad (31)$$

Table 4 shows triple frequency mixed code-carrier widelane combinations of maximum ambiguity discrimination $D = \frac{\lambda}{2\sigma_n}$ with an additional constraint on the combination bias.

Obviously, the bias constraint results in significantly lower weighting coefficients.

A more generalized ambiguity discrimination is suggested which is defined as the ratio between the wavelength λ and a weighted sum of the combination standard deviation σ_n and the combination bias b_n , i.e.

$$D = \frac{\lambda}{\kappa_1 \sigma_n + \kappa_2 b_n}, \quad (32)$$

with the weighting coefficients κ_1 and κ_2 . Table 5 shows the weighting coefficients and properties of GP-IF-NP mixed code-carrier combinations that maximize the discrimination of Equation (32) for $\kappa_1 = 1$ and $\kappa_2 = 1$. As the maximization of Equation (32) tends to large wavelength, a wavelength constraint has been introduced to limit the noise amplification.

Advantageously, a combination that maximizes Equation (32) is used for precise point positioning and bias estimation as it benefits from a large robustness over both biases and statistical errors.

5

3. PPP with satellite-satellite SD measurements and mixed code-carrier combinations

Precise point positioning requires precise phase bias estimation to maintain the integer nature of ambiguities. Satellite-satellite single difference measurements (= SD) are evaluated in [2] to eliminate the receiver bias and clock offset. The satellite-satellite SD phase biases are computed at a network of reference stations [3] and provided to the users. The received SD code and carrier phase are obtained from (1) as

10

$$\rho_{u,m}^{kl}(t_i) = r_u^{kl}(t_i) + q_{1m}^2 I_u^{kl}(t_i) + T_u^{kl}(t_i) + b_{\rho_m}^{kl} + \varepsilon_{\rho_{u,m}}^{kl}(t_i) \quad (33)$$

15

$$\lambda_m \phi_{u,m}^{kl}(t_i) = r_u^{kl}(t_i) - q_{1m}^2 I_u^{kl}(t_i) + T_u^{kl}(t_i) + \lambda_m N_{u,m}^{kl} + \lambda_m b_{\phi_m}^{kl} + \varepsilon_{\phi_{u,m}}^{kl}(t_i) \quad (34)$$

where the SD clock biases and the projected SD orbital errors have been mapped to the SD code/ phase biases.

20

A linear combination of these SD biases is computed at a single reference station and is applicable for users up to a distance of several 100 km away. These biases are updated every 10 min and do not require an extensive averaging over a large network of reference stations.

25

3.1 Bias estimation with Melbourne-Wübbena combination

The L1/ L2 phase bias calibration of [2],[3] and [4] uses two linear combinations:

30

In a first step, the ionosphere-free geometry-free Melbourne-Wübbena (MW) combination [1] is computed:

$$\begin{aligned}
\lambda_{WL} \phi_{MW}^{kl} &= \left(\frac{f_1}{f_1 - f_2} \lambda_1 \phi_1^{kl} - \frac{f_2}{f_1 - f_2} \lambda_2 \phi_2^{kl} \right) \\
&\quad - \left(\frac{f_1}{f_1 + f_2} \rho_1^{kl} + \frac{f_2}{f_1 + f_2} \rho_2^{kl} \right) \\
&= \lambda_{WL} (N_1^{kl} - N_2^{kl}) + \lambda_{WL} (b_{\phi_1}^{kl} - b_{\phi_2}^{kl}) \\
&\quad - \frac{f_1}{f_1 + f_2} b_{\rho_1}^{kl} - \frac{f_2}{f_1 + f_2} b_{\rho_2}^{kl} + \varepsilon_{MW}^{kl}
\end{aligned} \tag{35}$$

where the receiver and time indices have been omitted and λ_{WL} denotes the wavelength of the widelane combination.

10 In a second step, the L1/ L2 ionosphere-free (IF) carrier phase combination is determined, i.e.

$$\begin{aligned}
\lambda_{IF} \phi_{IF}^{kl} &= \frac{f_1^2}{f_1^2 - f_2^2} \lambda_1 \phi_1^{kl} - \frac{f_2^2}{f_1^2 - f_2^2} \lambda_2 \phi_2^{kl} \\
&= r^{kl} + T^{kl} + \frac{f_1^2}{f_1^2 - f_2^2} \lambda_1 (N_1^{kl} + b_{\phi_1}^{kl}) \\
&\quad - \frac{f_2^2}{f_1^2 - f_2^2} \lambda_2 (N_2^{kl} + b_{\phi_2}^{kl}) + \varepsilon_{IF}^{kl}
\end{aligned} \tag{36}$$

which includes the joint ambiguity/ bias term B_{IF}^{kl} that is defined as

$$B_{IF}^{kl} = \frac{f_1^2}{f_1^2 - f_2^2} \lambda_1 (N_1^{kl} + b_{\phi_1}^{kl}) - \frac{f_2^2}{f_1^2 - f_2^2} \lambda_2 (N_2^{kl} + b_{\phi_2}^{kl}) + \varepsilon_{IF}^{kl}. \tag{37}$$

20 Finally, the MW and IF ambiguity/ bias terms are combined by

$$\frac{f_1 + f_2}{c} B_{IF}^{kl} - \frac{f_2}{f_1 - f_2} \phi_{MW}^{kl} = N_1^{kl} + \tilde{b}_{\phi_1}^{kl} + \tilde{\varepsilon}^{kl} \tag{38}$$

with

$$\tilde{b}_{\phi_1}^{kl} = b_{\phi_1}^{kl} + \frac{f_2}{f_1 + f_2} \frac{b_{\rho_1}^{kl}}{\lambda_1} + \frac{f_2}{f_1 + f_2} \frac{b_{\rho_2}^{kl}}{\lambda_2}. \quad (39)$$

5

The bias estimate on L2 is obtained in a similar way by combining this bias and the MW bias, i.e.

$$\tilde{b}_{\phi_2}^{kl} = b_{\phi_2}^{kl} + \frac{f_1}{f_1 + f_2} \frac{b_{\rho_1}^{kl}}{\lambda_1} + \frac{f_1}{f_1 + f_2} \frac{b_{\rho_2}^{kl}}{\lambda_2}. \quad (40)$$

10

These biases are provided by the reference station 15 and enable the mobile navigation device 5 to determine unbiased L1 and L2 integer ambiguities.

However, there exist some critical aspects of this approach: First, the L1 and L2 phase
 15 biases can not be determined separately as (39) and (40) also include L1 and L2 code
 biases. Secondly, the geometry-preserving, ionosphere-free linear combination to which
 the bias estimates correspond is a narrowlane combination with a wavelength of only 10.7
 cm. Any linear combination of the bias estimates in (39) and (40) does not refer to a
 geometry-preserving, ionosphere-free linear combination with a larger wavelength than
 20 10.7 cm. Another critical point is the significant code noise in the narrowlane combination.
 All these aspects are the foundation for the design of new ionosphere-free mixed code-
 carrier combinations for bias estimation.

25 3.2 Bias estimation with mixed code-carrier combinations

The following scheme is suggested for bias estimation:

In a first step, satellite-satellite single difference measurements of code and phase signals are computed where the satellite 2 of highest elevation is typically chosen as a reference satellite.

- 5 In a second step, a geometry-preserving, ionosphere-free combination with maximum combination discrimination is then computed from the single difference measurements.

Alternatively, the geometry-preserving, ionosphere-free combination is computed in the first step and single differences are performed in a second step.

10

For the linear combination, a linear combination of Table 5 is advantageously be used due to its robustness with respect to both stochastic errors and biases. The bias robustness has been introduced as there exist biases like multipath that depend on the location of the mobile navigation device and can not be estimated at the reference station 15. If the maximum bias shall be bounded to a predefined value, a linear combination of Table 4 is advantageously used. If residual biases due to multipath are negligible, a linear combination of either Table 2 or 3 can be chosen. In the following simulations, the first combination of Table 2 has been chosen.

15

- 20 The single differences of this mixed code-carrier combination are written in matrix-vector notation as

$$\lambda\Delta\boldsymbol{\varphi} = [\lambda\Delta\boldsymbol{\varphi}^T(t_1) \dots \lambda\Delta\boldsymbol{\varphi}^T(t_{N_T})]^T \quad \text{with} \quad \lambda\Delta\boldsymbol{\varphi}(t_i) = [\lambda\varphi^{1K}(t_i) \dots \lambda\varphi^{(K-1)K}(t_i)]^T. \quad (41)$$

- 25 where N_T denotes the number of measurement times and K designates the reference satellite 2.

In a third step, the single difference ranges and the single difference satellite clock offsets are computed from the known ephemeris and the known position of the reference station

30

15. The sum of the single difference ranges and the single difference satellite clock offsets is denoted as Δr and subtracted from the single difference measurements. The obtained linear combinations are modeled as

$$\lambda \Delta \tilde{\boldsymbol{\varphi}} = \lambda \Delta \boldsymbol{\varphi} - \Delta \mathbf{r} = \mathbf{H}_1 (\Delta \mathbf{N} + \Delta \mathbf{b}) + \mathbf{H}_2 T_z + \Delta \boldsymbol{\varepsilon} \quad (42)$$

with

$$\begin{aligned} \mathbf{H}_1 &= \lambda \cdot \mathbf{1}^{N_T \times 1} \otimes \mathbf{I}^{(K-1) \times (K-1)} \\ \mathbf{H}_2 &= \left[\Delta \mathbf{m}_w^T(t_1) \dots \Delta \mathbf{m}_w^T(t_{N_T}) \right]^T \end{aligned} \quad (43)$$

and

$$\Delta \mathbf{m}_w(t_i) = \begin{bmatrix} m_w^1(t_i) - m_w^K(t_i) \\ \vdots \\ m_w^{K-1}(t_i) - m_w^K(t_i) \end{bmatrix} \quad (44)$$

where m_w denotes a mapping function for the tropospheric wet zenith delay. Typically, the Niell mapping function of [10] is chosen due to its independence from meteorological data. Then, a weighted least-square estimation of SD combination biases $\Delta \mathbf{b}$ and of the tropospheric wet zenith delay T_z is performed. As the integer valued ambiguities and real-valued biases can not be estimated individually, a common ambiguity/bias term $\Delta \mathbf{N} + \Delta \mathbf{b}$ is estimated and then split into an integer part $\Delta \mathbf{N}$ and a fractional part $\Delta \mathbf{b}$. The fractional part $\Delta \mathbf{b}$ is then broadcasted as a correction term. Any mobile navigation device can then use this bias for precise point positioning with the linear combinations.

In the following simulations, 1 Hz measurements are considered over 10 minutes. The covariance matrix of SDs with a common reference satellite 2 is characterized by $2\sigma_n^2$ for the diagonal elements and by σ_n^2 for the off-diagonal elements.

Figure 3 shows that a standard deviation between a few millimetres and 2.5 cm is achievable for the SD bias estimates of the ionosphere-free mixed code-carrier combination with $\lambda = 3.215m$.

The accuracy of bias estimation can be significantly improved by using an additional L1-E5 ionosphere-free code-carrier combination. Time-differenced satellite-satellite single difference measurements are used for this combination to avoid the introduction of

additional biases and ambiguities. The weighting coefficients are uniquely described by four constraints: Preservation of geometry, elimination of ionosphere, uncorrelated to first combination (of maximum discrimination) and minimum noise amplification. The L1/ E5 phase weights are obtained as $\tilde{\alpha}_1 = 1.7321$, $\tilde{\alpha}_2 = -0.8617$ and the respective code

5 coefficients are $\tilde{\beta}_1 = 0.0002$, $\tilde{\beta}_2 = 0.1294$. The noise level of this linear combination is characterized by a standard deviation of only 2.3 mm. Both linear combinations are written in matrix-vector notation as

$$\begin{bmatrix} \lambda \Delta \tilde{\phi}_1 \\ \lambda \Delta \tilde{\phi}_2 \end{bmatrix} = \begin{bmatrix} \lambda \Delta \phi_1 - \Delta \mathbf{r} \\ \lambda \Delta \phi_2 - \Delta \mathbf{r} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ \mathbf{0} & \mathbf{H}_3 \end{bmatrix} \cdot \begin{bmatrix} \Delta \mathbf{N} + \Delta \mathbf{b} \\ T_z \end{bmatrix} + \begin{bmatrix} \Delta \boldsymbol{\varepsilon}_1 \\ \Delta \boldsymbol{\varepsilon}_2 \end{bmatrix} \quad (45)$$

10 with

$$\mathbf{H}_3 = \left[\Delta \mathbf{m}_w^T(t_2) - \Delta \mathbf{m}_w^T(t_1), \dots, \Delta \mathbf{m}_w^T(t_{N_T}) - \Delta \mathbf{m}_w^T(t_{N_T} - t_{N_T-1}) \right]^T. \quad (46)$$

Figure 4 demonstrates the benefit of this second combination of time-differenced measurements for bias estimation. The uncertainty of critical SD bias estimates from the first code-carrier combination can be reduced by at least one order of magnitude. Each point refers to one SD for one epoch and the different colors represent different batch intervals.

The benefit of the second linear combination of time-differenced measurements motivates the reduction of the measurement batch interval to 5 min. Figure 5 demonstrates an achievable SD bias accuracy between 3 mm and 11 mm whereas the higher values refer to low elevation satellites. The respective SD biases can be estimated more accurately by another reference station with better visibility.

25

3.3 PPP with mixed code-carrier combinations

PPP is performed with mixed code-carrier phase combinations using the bias estimates from the reference station 15. The SD mixed code-carrier phase measurements are modeled at the mobile navigation device 5 similar to (45) as

30

$$\lambda \Delta \tilde{\boldsymbol{\phi}} = \lambda \Delta \boldsymbol{\phi} - \mathbf{H}_1 \Delta \mathbf{b} = \mathbf{H}_0 \mathbf{x} + \mathbf{H}_1 \Delta \mathbf{N} + \mathbf{H}_2 T_z + \boldsymbol{\varepsilon} \quad (47)$$

with the geometry matrix $\mathbf{H}_0 = [\mathbf{e}^{1K} \dots \mathbf{e}^{(K-1)K}]^T$, the user position $\mathbf{x}$ and the SD direction

5 vectors $\mathbf{e}^{kK} = \mathbf{e}^k - \mathbf{e}^K$.

The least-square estimation of $\mathbf{x}$, $\Delta \mathbf{N}$ and T_z is supported by a second L1-E5 code-only combination of minimum noise amplification. This geometry-preserving, ionosphere-free combination ($\beta_1 = 2.338, \beta_2 = -1.338$) has a noise level of $\sigma_n = 46.8 \text{ cm}$ and is

10 uncorrelated with the mixed code-carrier combination ($\lambda = 3.215 \text{ m}$). In the further analysis, the single difference operator Δ is omitted to simplify notation.

The integer ambiguity resolution is based on sequential bootstrapping [11] which achieves a higher success rate than direct rounding. The traditional bootstrapping fixes the

15 ambiguities in the following order: First, the integer ambiguity of the lowest variance is fixed. Then, this ambiguity is removed from the measurements, a new real-valued "float solution" of the position of the mobile navigation device 5, ambiguities and tropospheric zenith delay are computed, and the most reliable float ambiguity is fixed. This step is repeated until all ambiguities are fixed. The sequential fixing takes the correlation between

20 float ambiguity estimates into account and achieves a lower probability of wrong fixing than traditional rounding. Teunissen has expressed the integer ambiguity estimates in [6] as a function of the conditional variances, i.e.

$$\begin{aligned} \tilde{N}_{B,1} &= [\hat{N}_1] \\ \tilde{N}_{B,2} &= [\hat{N}_{2|1}] = \left[\hat{N}_2 - \sigma_{\hat{N}_2 \hat{N}_1} \sigma_{\hat{N}_1}^{-2} (\hat{N}_1 - \tilde{N}_{B,1}) \right] \end{aligned} \quad (48)$$

and

$$\tilde{N}_{B,k} = [\hat{N}_{k|k}] = \left[\hat{N}_k - \sum_{l=1}^{k-1} \sigma_{\hat{N}_k \hat{N}_{l|l}} \sigma_{\hat{N}_{l|l}}^{-2} (\hat{N}_{l|l} - \tilde{N}_{B,l}) \right] \quad (49)$$

with $[\cdot]$ denoting the rounding to the nearest integer and $\mathbf{k} = \{1, \dots, k-1\}$. The sequential ambiguity estimator can also be written in matrix-vector notation as

$$\begin{bmatrix} \hat{N}_1 - \tilde{N}_{B,1} \\ \hat{N}_2 - \tilde{N}_{B,2} \\ \hat{N}_3 - \tilde{N}_{B,3} \\ \vdots \end{bmatrix} = \mathbf{L} \cdot \begin{bmatrix} \hat{N}_1 - \tilde{N}_{B,1} \\ \hat{N}_{2|1} - \tilde{N}_{B,2} \\ \hat{N}_{3|1,2} - \tilde{N}_{B,3} \\ \vdots \end{bmatrix} \quad (50)$$

5 with $\mathbf{L}(i, j) = \sigma_{\hat{N}_i \hat{N}_{j|\mathbf{j}}} \sigma_{\hat{N}_{j|\mathbf{j}}}^{-2}$ for $i > j$, $\mathbf{L}(i, j) = 0$ for $i < j$ and $\mathbf{L}(i, i) = 1$. The conditional variances of $\hat{N}_{k|\mathbf{k}}$ are obtained from a triangular decomposition of the float ambiguity covariance matrix $\Sigma_{\hat{N}} = \mathbf{L} \mathbf{D} \mathbf{L}^T$ as $\sigma_{\hat{N}_{k|\mathbf{k}}}^2 = \mathbf{D}(k, k)$. The success rate of the bootstrapped estimator depends on the order of ambiguity fixings and is given in [12] as

$$10 \quad P_s = \prod_{k=1}^{K-1} \Phi \left(\frac{1 - 2b_{\hat{N}_{k|\mathbf{k}}}}{2\sigma_{\hat{N}_{k|\mathbf{k}}}} \right) + \Phi \left(\frac{1 + 2b_{\hat{N}_{k|\mathbf{k}}}}{2\sigma_{\hat{N}_{k|\mathbf{k}}}} \right) - 1 \quad (51)$$

with the conditional biases $b_{\hat{N}_{k|\mathbf{k}}}$ and

$$\Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}v^2\right) dv. \quad (52)$$

15

The sequential fixing can be applied to the float ambiguities in original order or to a permutation of the float ambiguities, e.g. sorted w.r.t. the variances. The success rate of the bootstrapped estimator is significantly increased if an integer decorrelation transformation $\mathbf{Z}$ [12] is applied to the float ambiguities before sequential fixing.

20

Figure 6 shows the probability of wrong fixing of SD ambiguities for different bootstrapping estimators and known biases. The integer decorrelation transformation reduces the error rate by up to ten orders of magnitude for some epochs. The high reliability of single epoch ambiguity fixing is caused by the large wavelength of
 25 $\lambda = 3.215m$ and the low code noise level of the mixed code-carrier combination.

The conditional float ambiguity biases $b_{\hat{N}_{k|k}}$ are related to the residual biases of the mixed code-carrier combination ($\mathbf{b}_1$) and of the code-only combination ($\mathbf{b}_2$) by

$$\begin{aligned} \begin{bmatrix} b_{\hat{N}_1} \\ b_{\hat{N}_{2|1}} \\ \vdots \end{bmatrix} &= \mathbf{L}^{-1} \mathbf{b}_{\hat{N}}, = \mathbf{L}^{-1} \mathbf{Z} \mathbf{b}_{\hat{N}} \\ &= \underbrace{\mathbf{L}^{-1} \mathbf{Z} \mathbf{P} (\mathbf{X}^T \mathbf{\Sigma}^{-1} \mathbf{X})^{-1} \mathbf{X}^T \mathbf{\Sigma}^{-1}}_{[\mathbf{M}_1, \mathbf{M}_2]} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} \end{aligned} \quad (53)$$

with

$$\begin{aligned} \mathbf{X} &= \begin{bmatrix} \mathbf{H}_0 & \mathbf{H}_1 & \mathbf{H}_2 \\ \mathbf{H}_0 & \mathbf{0} & \mathbf{H}_2 \end{bmatrix} \\ \mathbf{P} &= \begin{bmatrix} \mathbf{0}^{K-1 \times 3} & \mathbf{1}^{K-1 \times K-1} & \mathbf{0}^{K-1 \times 1} \end{bmatrix}. \end{aligned} \quad (54)$$

An upper bound for the k -th conditional ambiguity bias is given for $-b_{c,\max}^j < b_c^j < +b_{c,\max}^j$, $c \in \{1,2\}$ by

$$\begin{aligned} \max \left(b_{\hat{N}_{k|k}} \right) &\leq \sum_{j=1}^{K-1} |M_1(k, j)| \cdot b_{1,\max}^j \\ &+ \sum_{j=1}^{K-1} |M_2(k, j)| \cdot b_{2,\max}^j. \end{aligned} \quad (55)$$

Figure 7 shows the impact of (uncorrected) residual SD biases on the probability of wrong fixing. A residual bias of 2 cm for the mixed code-carrier combination and of 10 cm for the code-only combination reduces the success rate only slightly due to the large wavelength of the linear combination.

The success rate of instantaneous ambiguity resolution can be significantly increased by using measurements from multiple epochs. Position and tropospheric wet zenith delay are estimated once per epoch for PPP.

Figure 8 shows that the probability of wrong fixing can be reduced to less than 10^{-10} within 5 s. Consequently, CAT III requirements for aircraft landing can be fulfilled for ambiguity resolution of the mixed code-carrier combination.

- 5 After ambiguity fixing, the mixed-code carrier and code-only combinations of SD measurements are corrected by ambiguities/ biases and the receiver position is estimated, i.e.

$$\hat{\mathbf{x}} = \mathbf{S}(\mathbf{X}^T \mathbf{\Sigma}^{-1} \mathbf{X})^{-1} \mathbf{X}^T \mathbf{\Sigma}^{-1} \begin{bmatrix} \lambda \tilde{\phi} \\ \tilde{\rho} \end{bmatrix} \quad (56)$$

10

with the selection matrix $\mathbf{S} = [\mathbf{1}^{3 \times 3}, \mathbf{0}^{3 \times (K-1)}, \mathbf{1}^{3 \times 1}]$. The positioning accuracy is improved if the ionosphere-free carrier smoothing of [13] is applied to the mixed code-carrier combination. The smoothed linear combination is written as

$$15 \quad \lambda \bar{\phi}(t_i) = \bar{\chi}(t_i) + \lambda_c \phi_c(t_i) \quad (57)$$

with $\chi(t_i) = \lambda \phi(t_i) - \lambda_c \phi_c(t_i)$. This geometry-free, ionosphere-free term is filtered by a low pass filter, i.e.

$$20 \quad \bar{\chi}(t_i) = \left(1 - \frac{1}{\tau}\right) \bar{\chi}(t_{i-1}) + \frac{1}{\tau} \chi(t_i). \quad (58)$$

The smoothing combination $\lambda_c \phi_c$ is geometry-preserving, ionosphere-free and of minimum noise. The variance of the smoothed mixed code-carrier combination is given by [5] as

25

$$\begin{aligned} \sigma_n^2 &= \sigma_c^2 + \frac{1}{2\tau_s - 1} \cdot (\sigma_n^2 + \sigma_c^2 - 2\sigma_{nc}) \\ &+ \frac{2}{\tau_s} \cdot (\sigma_{nc} - \sigma_c^2) \end{aligned} \quad (59)$$

with the smoothing time constant τ_s , the variance σ_c^2 of the smoothing combination, the variance σ_n^2 of the unsmoothed mixed code-carrier combination and the covariance σ_{nc} between these two combinations. The L1-E5 smoothing combination has a noise level of $\sigma_c = 2.7mm$ and allows a noise reduction of the mixed code-carrier combination from 3.76cm to 3.7mm for $\tau_s = 120s$.

Figure 9 shows the single epoch positioning accuracy with SD measurements by 120s ionosphere-free carrier smoothing of the ionosphere-free code-carrier combination with $\lambda = 3.215m$, aided by a code-only combination. Figure 9 further shows a standard deviation of a few millimetres for the horizontal position and of less than 5 cm for the vertical position in 90 % of the epochs. A high correlation between the vertical position error and tropospheric zenith delay is observed and degrades the vertical positioning accuracy to a few decimetres for a few epochs.

The bias estimation at the reference station leaves some residual biases whose impact on the position is shown in Figure 10. The mixed-code carrier SD residual biases have been chosen according to the standard deviations of Figure 3. The residual biases of the code-only SD combination are assumed to be 10 times larger. The vertical position bias is highly correlated with the tropospheric zenith delay bias and varies between 1 cm and 10 cm.

A comparison of Figure 9 and 10 shows that standard deviation and bias of the receiver position are characterized by a similar order of magnitude although the biases are slightly more critical. The lower standard deviation is achieved by the ionosphere-free carrier smoothing which does not affect the biases of the mixed code-carrier combination.

4. PPP with satellite-satellite SD measurements without linear combinations

The estimation of SD phase and code biases on each frequency is not feasible as at least one bias can not be distinguished from the ionospheric delay. However, the Galileo E5a and E5b signals are modulated onto the same carrier which motivates the assumption of a

common bias. In this case, the L1 and E5 SD code and phase biases can be determined separately. The accuracy can be increased if the E5c signal which corresponds to the central lobe between E5a and E5b is also taken into account.

- 5 First, the widelane ambiguities between E5a and E5b ($\lambda = 9.76m$) as well as between E5a and E5c ($\lambda = 19.52m$) are determined with the Melbourne-Wubben combination [1]. Thus, the three E5 ambiguities and phase biases are reduced to a single ambiguity and a single phase bias. The SD code and phase measurements on all four frequencies are modeled for a reference station 15 of known position as

$$\begin{aligned} \tilde{\mathbf{y}} = \mathbf{y} - \mathbf{A}_0 \Delta \mathbf{r} = & \mathbf{A}_1 \Delta \mathbf{b}_\phi + \mathbf{A}_2 \Delta \mathbf{b}_\rho + \mathbf{A}_3 \Delta \mathbf{I}_1 \\ & + \mathbf{A}_4 \frac{\partial}{\partial t} \Delta \mathbf{I}_1 + \mathbf{A}_5 T_z + \mathbf{A}_6 \frac{\partial}{\partial t} T_z + \boldsymbol{\varepsilon} \end{aligned} \quad (60)$$

with

$$\mathbf{A}_1 = \mathbf{1}^{N_T \times 1} \otimes \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \\ 0 & \lambda_3 \\ 0 & \lambda_4 \\ \mathbf{0}^{4 \times 1} & \mathbf{0}^{4 \times 1} \end{bmatrix} \otimes \mathbf{1}^{(K-1) \times (K-1)}$$

$$\mathbf{A}_2 = \mathbf{1}^{N_T \times 1} \otimes \begin{bmatrix} \mathbf{1}^{4 \times 1} & \mathbf{1}^{4 \times 1} \\ 1 & 0 \\ \mathbf{0}^{3 \times 1} & \mathbf{1}^{3 \times 1} \end{bmatrix} \otimes \mathbf{1}^{(K-1) \times (K-1)}$$

$$\mathbf{A}_3 = \mathbf{1}^{N_T \times 1} \otimes \mathbf{A}_I$$

$$= \mathbf{1}^{N_T \times 1} \otimes \begin{bmatrix} +q_{11}^2 \\ \vdots \\ +q_{14}^2 \\ -q_{11}^2 \\ \vdots \\ -q_{14}^2 \end{bmatrix} \otimes \mathbf{1}^{(K-1) \times (K-1)}$$

$$\mathbf{A}_s = \begin{bmatrix} \mathbf{1}^{8 \times 1} \otimes \Delta \mathbf{m}_w(t_1) \\ \vdots \\ \mathbf{1}^{8 \times 1} \otimes \Delta \mathbf{m}_w(t_N) \end{bmatrix} \quad (61)$$

Both ionospheric and tropospheric delays are assumed piecewise linear. The design matrices for ionospheric/ tropospheric gradients are given by

5

$$\mathbf{A}_4 = \begin{bmatrix} 1\delta t & 0 & 0 & \dots \\ 2\delta t & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \\ \delta t/r_I & 0 & 0 & \\ \delta t/r_I & 1\delta t & 0 & \dots \\ \delta t/r_I & 2\delta t & 0 & \dots \\ \vdots & \vdots & \vdots & \\ \delta t/r_I & \delta t/r_I & 0 & \\ \delta t/r_I & \delta t/r_I & 1\delta t & \\ \vdots & \vdots & \vdots & \end{bmatrix} \otimes \mathbf{A}_I$$

$$\mathbf{A}_6 = \begin{bmatrix} \mathbf{1}^{8 \times 1} \otimes \Delta \mathbf{m}_w(t_1) & & \\ & \ddots & \\ & & \mathbf{1}^{8 \times 1} \otimes \Delta \mathbf{m}_w(t_N) \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1\delta t & 0 & 0 & \dots \\ 2\delta t & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \\ \delta t/r_T & 0 & 0 & \\ \delta t/r_T & 1\delta t & 0 & \dots \\ \delta t/r_T & 2\delta t & 0 & \dots \\ \vdots & \vdots & \vdots & \\ \delta t/r_T & \delta t/r_T & 0 & \\ \delta t/r_T & \delta t/r_T & 1\delta t & \\ \vdots & \vdots & \vdots & \end{bmatrix} \quad (62)$$

10 where r_I and r_T denote the rate of ionospheric/ tropospheric gradient estimation and δt represents the time interval between two measurements. Both ionosphere and troposphere

are nuisance parameters for bias estimation and are eliminated by an orthogonal projection [14]. The phase bias estimates are obtained as

$$\Delta \hat{\mathbf{b}}_{\phi} = \left(\overline{\mathbf{A}}_1^T \boldsymbol{\Sigma}^{-1} \overline{\mathbf{A}}_1 \right)^{-1} \overline{\mathbf{A}}_1^T \boldsymbol{\Sigma}^{-1} \tilde{\mathbf{y}} \quad (63)$$

with

$$\overline{\mathbf{A}}_1 = \mathbf{P}_{\mathbf{A}_{2,\dots,6}}^{\perp} \quad \text{and} \quad \mathbf{A}_{2,\dots,6} = [\mathbf{A}_2, \dots, \mathbf{A}_6] \quad (64)$$

and the orthogonal projection

$$\mathbf{P}_{\mathbf{A}_{2,\dots,6}}^{\perp} = \mathbf{I} - \mathbf{A}_{2,\dots,6} \left(\mathbf{A}_{2,\dots,6}^T \boldsymbol{\Sigma}^{-1} \mathbf{A}_{2,\dots,6} \right)^{-1} \mathbf{A}_{2,\dots,6}^T \boldsymbol{\Sigma}^{-1}. \quad (65)$$

Moreover, the complexity can be further reduced if the sparse properties of $\mathbf{A}_1, \dots, \mathbf{A}_6$ and the block diagonal structure of the covariance matrix $\boldsymbol{\Sigma}$ are taken into account.

Figure 11 shows the achievable accuracy for SD phase bias estimation with uncombined phase and code measurements at a single reference station. Simulated 1 Hz measurements on L1, E5a, E5b and E5c have been generated for periods of 10 min. A phase noise of $\sigma_{\phi} = 1$ mm has been assumed and the code noise has been chosen according to the CRB for 45 dBHz, i.e. $\sigma_{\rho_1} = 11.80$ cm for E1 (MBOC(6,1,1/11), 20 MHz), $\sigma_{\rho_{\{2,3\}}} = 8.29$ cm for E5a/b (BPSK(10), 20 MHz) and $\sigma_{\rho_4} = 15.10$ cm for E5c (AltBOC(15,10), 10 MHz).

The ionospheric gradients are estimated once per 10 s and the gradient of the tropospheric wet zenith delay is updated every 120 s.

The standard deviation of the SD L1 phase bias estimates varies between 2 mm and 7 mm with the exception of two epochs where one SD shows a standard deviation of 2 cm. This outlier is explained by the low elevation angle (5°) of the respective satellite and the low number of visible satellites. This SD phase bias can be determined more accurately at a different reference station.

The standard deviation of SD phase bias estimates in Figure 11 is one to two orders of magnitude lower than the weighted code bias ($b_{\rho_{\{1,2\}}} = 10\text{cm}$) in the L1 phase bias estimate of (38). The uncombined phase biases might also be beneficial for single frequency receivers which can not form the MW combination.

5

5 Advantages

Precise point positioning with satellite-satellite single difference measurements requires
10 precise estimates of the phase and code biases for ambiguity resolution.

A method for satellite-satellite single difference bias estimation has been described for precise point positioning. It uses an ionosphere-free mixed code-carrier combination of maximum ambiguity discrimination defined as ratio between wavelength and noise
15 standard deviation. The L1-E5 linear combination of maximum discrimination is characterized by a wavelength of 3.215 m, a low noise level of 3.76 cm, and an E1 code multipath suppression by 23.5 dB. The wavelength of 3.215 m of the L1-E5 linear combination of maximum discrimination is four time larger than the wavelength of the L1-E5 Melbourne-Wübbena combination. In contrast to the Melbourne-Wübbena
20 combination, the proposed mixed code-carrier combination is a geometry-preserving linear combination so that the bias estimates are directly applicable at the mobile receiver 5. The single difference biases of the discrimination maximizing linear combination are determined at a single reference station 15 with an accuracy between a few millimeters and 1 cm within 5 min. As these biases refer to a geometry-preserving linear combination, they
25 are directly applicable at the mobile navigation device 5.

The accuracy of bias estimation is further improved by an additional ionosphere-free L1-E5 mixed code-carrier combination of time-differenced measurements that is uncorrelated with the discrimination maximizing combination. The additional combination uses time-
30 differenced measurements to avoid the introduction of additional ambiguities and biases.

Moreover, the SD phase and code biases on L1 and E5 can also be determined separately by subdividing the Galileo E5 band into the E5a, E5b and E5c band whereas the latter one corresponds to the central lobe between E5a and E5b.

5 The methods described above can also be implemented in software products that contain program code for performing the methods. The software product can in particular be stored on a computer readable storage medium or data carrier that can also be an electrical signal of a data network.

10 Generally, it should be noted that linear operations on phase or code signals can be performed in any order. For instance, the subtraction 20 of the known ranges and clock offsets in Figure 2 can also be performed before the computation 19 of satellite-satellite single differences. In a corresponding way, the sequence of any other linear operations can be inverted.

15

It should further be noted that, throughout the description and claims of this specification, the singular encompasses the plural unless the context otherwise requires. In particular, where the indefinite article is used, the specification is to be understood as contemplating plurality as well as singularity, unless the context requires otherwise.

20

Features, integers, characteristics, compounds or groups described in conjunction with a particular aspect, embodiment or example of the invention are further to be understood to be applicable to any other aspect, embodiment or example described herein unless incompatible therewith.

25

Table 1: Cramer Rao Bounds for $C/N_0 = 45dB/Hz$

	Signal	BW [MHz]	Γ [cm]
E1	MBOC	20	11.14
E5	AltBOC(15,10)	51	1.95
E5a	BPSK(10)	20	7.83
E5b	BPSK(10)	20	7.83
E6	BOC(10,5)	40	2.41

5 Table 2: GP-IF-NP mixed code-carrier widelane combinations of max. discrimination for $\sigma_\phi = 1mm$, $\sigma_{\rho_m} = \Gamma_m$

	E1	E5a	E5b	E5	E6	λ	σ_n	D
j_m	1.0000			-1.0000		3.285m	6.5cm	25.1
α_m	17.2629			-13.0593				
β_m	-0.0552			-3.1484				
j_m	1.0000	-1.0000				4.309m	31.4cm	6.9
α_m	22.6467	-16.9115						
β_m	-1.0227	-3.7125						
j_m	1.0000	4.0000	-5.0000			3.531m	13.3cm	13.3
α_m	18.5565	55.4284	-71.0930					
β_m	-0.2342	-0.8502	-0.8075					
j_m	1.0000			1.0000	-2.0000	4.019m	5.1cm	39.2
α_m	21.1223			15.9789	-34.2894			
β_m	-0.0200			-1.1422	-0.6495			
j_m	1.0000	1.0000	0.0000		-2.0000	4.469m	6.3cm	35.3
α_m	23.4845	17.5371	0.0000		-38.1242			
β_m	-0.0468	-0.1700	-0.1615		-1.5191			
j_m	1.0000	1.0000	0.0000	0.0000	-2.0000	3.9387m	4.8cm	41.0
α_m	20.6978	15.4562	0.0000	0.0000	-33.6004			
β_m	-0.0159	-0.0578	-0.0549	-0.9084	-0.5166			

Table 3: GP-IF-NP mixed code-carrier widelane and narrowlane combinations of maximum ambiguity discrimination for $\sigma_\phi = 2mm$, $\sigma_{\rho_m} = 3 \cdot \Gamma_m$

	E1	E5a	E5b	E5	E6	λ	σ_n	D
j_m	1.0000			-1.0000				
α_m	17.2629			-13.0593		3.285m	19.0cm	8.6
β_m	-0.0552			-3.1484				
j_m	1.0000	-1.0000						
α_m	22.6467	-16.9115				4.309m	93.8cm	2.3
β_m	-1.0227	-3.7125						
j_m	1.0000	4.0000	-5.0000					
α_m	18.5565	55.4284	-71.0930			3.531m	34.0cm	5.2
β_m	-0.2342	-0.8502	-0.8075					
j_m	1.0000			1.0000	-2.0000			
α_m	21.1223			15.9789	-34.2894	4.019m	11.9cm	16.9
β_m	-0.0200			-1.1422	-0.6495			
j_m	1.0000	1.0000	1.0000		-3.0000			
α_m	22.5147	16.8130	17.2516		-54.8249	4.284m	13.7cm	15.6
β_m	-0.0186	-0.0676	-0.0642		-0.6040			
j_m	1.0000	1.0000	0.0000	0.0000	-2.0000			
α_m	20.6978	15.4562	0.0000	0.0000	-33.6004	3.9387m	10.9cm	18.1
β_m	-0.0159	-0.0578	-0.0549	-0.9084	-0.5166			

	E1	E5a	E5b	E5	E6	λ	σ_n	D
j_m	4.0000			-3.0000		10.87cm	5.3mm	10.3
α_m	2.2853			-1.2966				
β_m	0.0002			0.0111				
j_m	4.0000	-3.0000				10.88cm	5.4mm	10.1
α_m	2.2870	-1.2809						
β_m	-0.0013	-0.0048						
j_m	4.0000	-2.0000	-1.0000			10.87cm	5.0mm	10.8
α_m	2.2853	-0.8533	-0.4378					
β_m	0.0007	0.0026	0.0025					
j_m	4.0000			-3.0000	0.0000	10.87cm	5.3mm	10.3
α_m	2.2841			-1.2959	0.0000			
β_m	0.0001			0.0075	0.0042			
j_m	4.0000	-2.0000	-1.0000		0.0000	10.87cm	5.0mm	10.9
α_m	2.2844	-0.8529	-0.4376		0.0000			
β_m	0.0002	0.0006	0.0005		0.0049			
j_m	4.0000	-1.0000	-1.0000	-1.0000	0.0000	10.87cm	4.8mm	11.2
α_m	2.2845	-0.4265	-0.4376	-0.4321	0.0000			
β_m	0.0002	0.0004	0.0004	0.0068	0.0039			

Table 4: GP-IF-NP mixed code-carrier widelane combinations of maximum ambiguity

discrimination $D = \frac{\lambda}{2\sigma_n}$ for $\sigma_\varphi = 1 \text{ mm}$, $\sigma_{\rho_m} = \Gamma_m$ with constraint worst-case combination

biases for $b_{\varphi_m} = 1 \text{ cm}$ and $b_{\rho_m} = 5 \text{ cm}$ on all frequencies

5

	E1	E5a	E5b	λ	σ_n	b_n	D
j_m	0.0000	1.0000	-1.0000	0.986m	26.9cm	25.0cm	1.84
α_m	0.0000	3.8699	-3.9708				
β_m	2.2890	-0.7957	0.0000				
j_m	0.0000	1.0000	-1.0000	1.475m	27.4cm	30.0cm	2.69
α_m	0.0000	-5.7874	5.9384				
β_m	2.2519	-1.4029	0.0000				
j_m	0.0000	1.0000	-1.0000	2.320m	26.6cm	35.0cm	4.35
α_m	0.0000	9.1037	-9.3412				
β_m	2.2743	-1.0368	0.0000				
j_m	1.0000	-1.0000	0.0000	2.726m	22.2cm	40.0cm	6.15
α_m	14.3260	-10.6980	0.0000				
β_m	0.1836	-2.8116	0.0000				

Table 5: GP-IF-NP mixed code-carrier widelane combinations of maximum

discrimination $D = \frac{\lambda}{\sigma_n + b_n}$ for $\sigma_\varphi = 1 \text{ mm}$, $\sigma_{\rho_m} = \Gamma_m$ for $b_{\varphi_m} = 1 \text{ cm}$ and $b_{\rho_m} = 5 \text{ cm}$ on all frequencies

5

	E1	E5a	E5b	λ	σ_n	b_n	D
j_m	1.0000	0.0000	-1.0000	0.80m	21.7cm	24.3cm	1.74
α_m	4.2040	0.0000	-3.2213				
β_m	1.6978	-1.3107	-0.3699				
j_m	1.0000	0.0000	-1.0000	1.00m	20.6cm	26.0cm	2.15
α_m	5.2550	0.0000	-4.0266				
β_m	1.5554	-1.3366	-0.4472				
j_m	1.0000	0.0000	-1.0000	2.00m	16.3cm	34.3cm	3.96
α_m	10.5101	0.0000	-8.0532				
β_m	0.8414	-1.4805	-0.8178				
j_m	1.0000	0.0000	-1.0000	3.00m	16.0cm	42.5cm	5.13
α_m	15.7651	0.0000	-12.0798				
β_m	0.1229	-1.6604	-1.1478				
j_m	0.0000	1.0000	-1.0000	4.00m	26.6cm	48.1cm	5.36
α_m	0.0000	15.6969	-16.1063				
β_m	2.3306	-0.5115	-0.4097				
j_m	0.0000	1.0000	-1.0000	5.00m	26.5cm	55.5cm	6.10
α_m	0.0000	19.6211	-20.1329				
β_m	2.3310	-0.4579	-0.3613				

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Claims:

1. A method for a global satellite navigation system with at least two carriers (4), from which a linear combination is formed combined by a reference station (15) for estimating a
5 satellite-satellite single difference phase and code bias, wherein the estimated bias is provided to a mobile receiver (5), enabling the mobile receiver to compute the same linear combination, subtracts the bias from the linear combination and determines integer phase ambiguities,
c h a r a c t e r i z e d i n t h a t
10 the satellite-satellite single difference bias of a single ionosphere-free, geometry-preserving combination of the code and phase signals of at least two carriers (4) is determined by using a maximum combination discrimination requirement or a minimum noise requirement for the linear combination.
- 15 2. The method according to Claim 1,
wherein an ionosphere-free mixed code-carrier bias, integer phase ambiguities and tropospheric wet zenith delays are obtained by the reference station (15) using a least-square estimation.
- 20 3. The method according to Claim 1 or 2,
wherein the combination discrimination requirement is defined as the ratio of the wavelength of the combination and the weighted sum formed by the bias of the combination and by the standard deviation of the combination.
- 25 4. The method according to Claim 3,
wherein the combination discrimination requirement is defined as the ratio of the wavelength of the combination and standard deviation of the mixed code-carrier combination noise.
- 30 5. The method according to any one of Claims 1 to 4,
wherein the reference station (15) is transmitting the bias together with the standard deviation associated with the bias.
- 35 6. The method according to any one of Claims 1 to 5,
wherein an additional geometry-preserving, ionosphere-free combination of time differences of measurements or time differences of geometry-preserving, ionosphere-free combinations of measurements are used for determining the biases, wherein the weighting

coefficients of the additional combination fulfill the requirement that the additional combination and the combination according to any one of Claim 1 to 5 are uncorrelated.

7. A method for a global satellite navigation system with a plurality of carriers used
5 by a reference station (15) for estimating a satellite-satellite single difference phase and code bias, wherein the estimated bias is provided to a mobile receiver (5), enabling the mobile receiver (5) to use the bias for determining integer phase ambiguities,
c h a r a c t e r i z e d i n t h a t
individual biases are determined for the phase signal and the code signal of at least three
10 carriers (4), wherein at least two carriers (4) belong to a common frequency band and have the same bias, and that the biases are transmitted to the mobile receiver (5) which uses the biases for determining integer phase ambiguities.

8. The method according to Claim 7,
15 wherein the individual biases of the L1 phase signal, the E5 phase signal, the L1 code signal and the E5 code signal are determined by dividing the Galileo E5 band into at least two subbands.

9. The method according to Claim 8,
20 wherein the Galileo E5 band is split into the subbands E5a, E5b and E5c, the latter one corresponding to the central lobe between E5a and E5b.

10. The method according to any one of Claims 7 to 9,
25 wherein an orthogonal projection is used for the bias estimation to eliminate both ionospheric and tropospheric errors.

11. The method according to any one of Claims 7 to 10,
wherein the reference station (15) is transmitting the biases together with the standard
30 deviations associated with the biases.

12. A method for a global satellite navigation system with a plurality of carriers (4)
used by a reference station (15) for estimating a satellite-satellite single difference phase
and code bias, wherein the estimated bias is provided to a mobile receiver (5), enabling the
mobile receiver (5) to use the bias for determining a integer phase ambiguity,
35 c h a r a c t e r i z e d i n t h a t
the methods according to any one of Claims 1 to 6 and the method according to any one of
Claims 6 to 11 are both performed and that the bias resulting from the method according to

any one of Claims 1 to 6 is used by the reference station (15) for verifying the individual biases resulting from the method according to any one of Claims 7 to 11.

13. The method according to Claim 12,

5 wherein the bias resulting from the method according to any one of Claims 1 to 6 is used if the individual biases resulting from the method according to any one of Claims 7 to 11 are stated invalid.

14. A reference station for a global satellite navigation system adapted for estimating
10 phase and code bias

c h a r a c t e r i z e d i n t h a t

the reference station (15) is arranged for executing a method according to any one of the Claims 1 to 13.

15. A method for a global satellite navigation system with a plurality of carriers, in which a mobile navigation device (5) receives a bias from a reference station (15) and uses the bias for determining integer phase ambiguities,

c h a r a c t e r i z e d i n t h a t

15 the mobile navigation device (5) uses the same linear combination as used by the referenced station (15) while executing a method according to anyone of the Claims 1 to 6, and
20 that mobile navigation device (5) subtract the biases received by the reference station (15) from the linear combination and determines phase ambiguities.

16. A mobile navigation device for a global satellite navigation adapted for using phase
25 and code bias

c h a r a c t e r i z e d i n t h a t

the device is arranged for executing a method according to Claims 15.

17. A software product for navigation purposes,

30 c h a r a c t e r i z e d i n t h a t

the software product comprises program code for implementing any one of the methods according to claim 1 to 13 and 15.

FIG 1

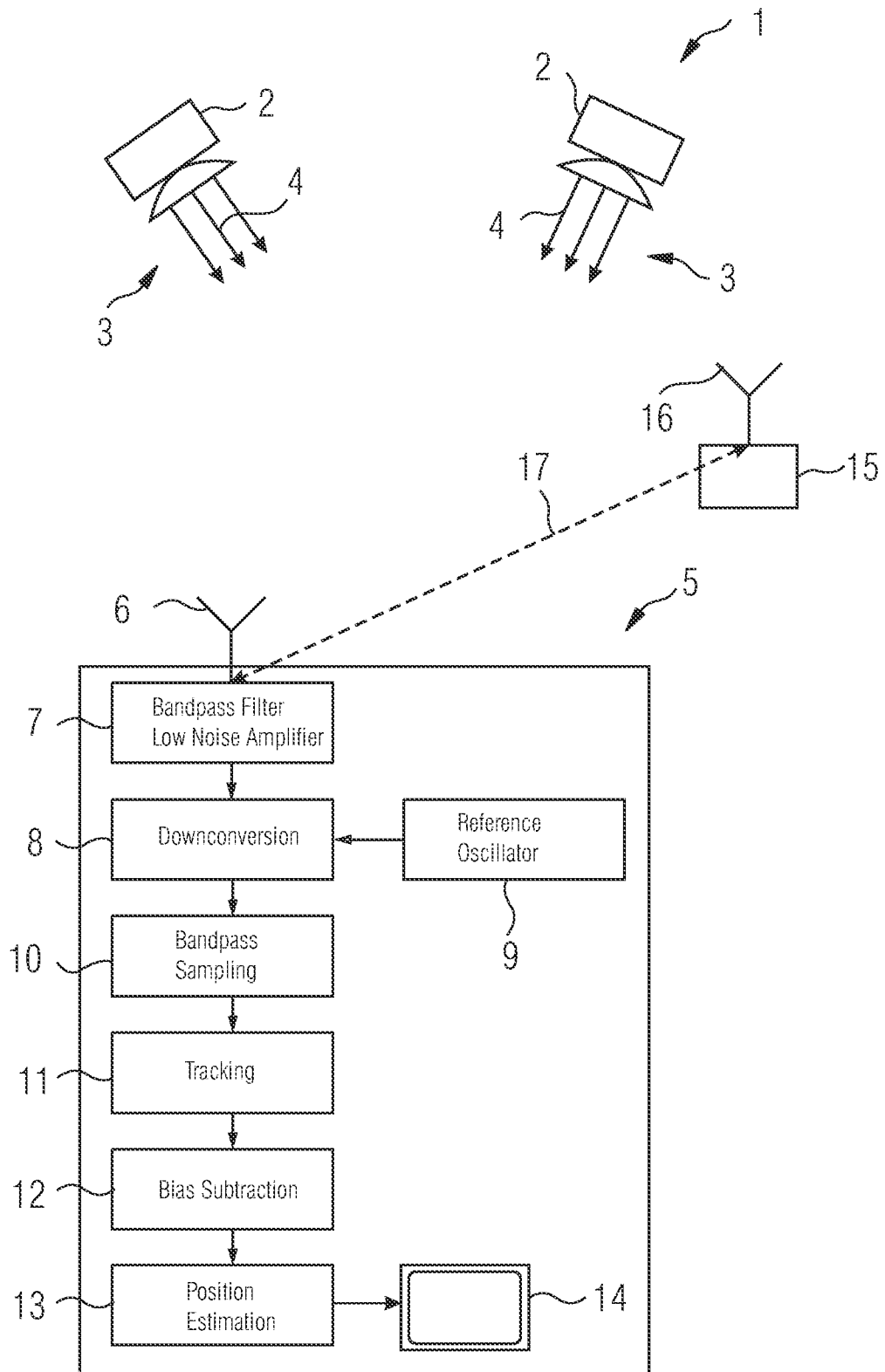


FIG 2

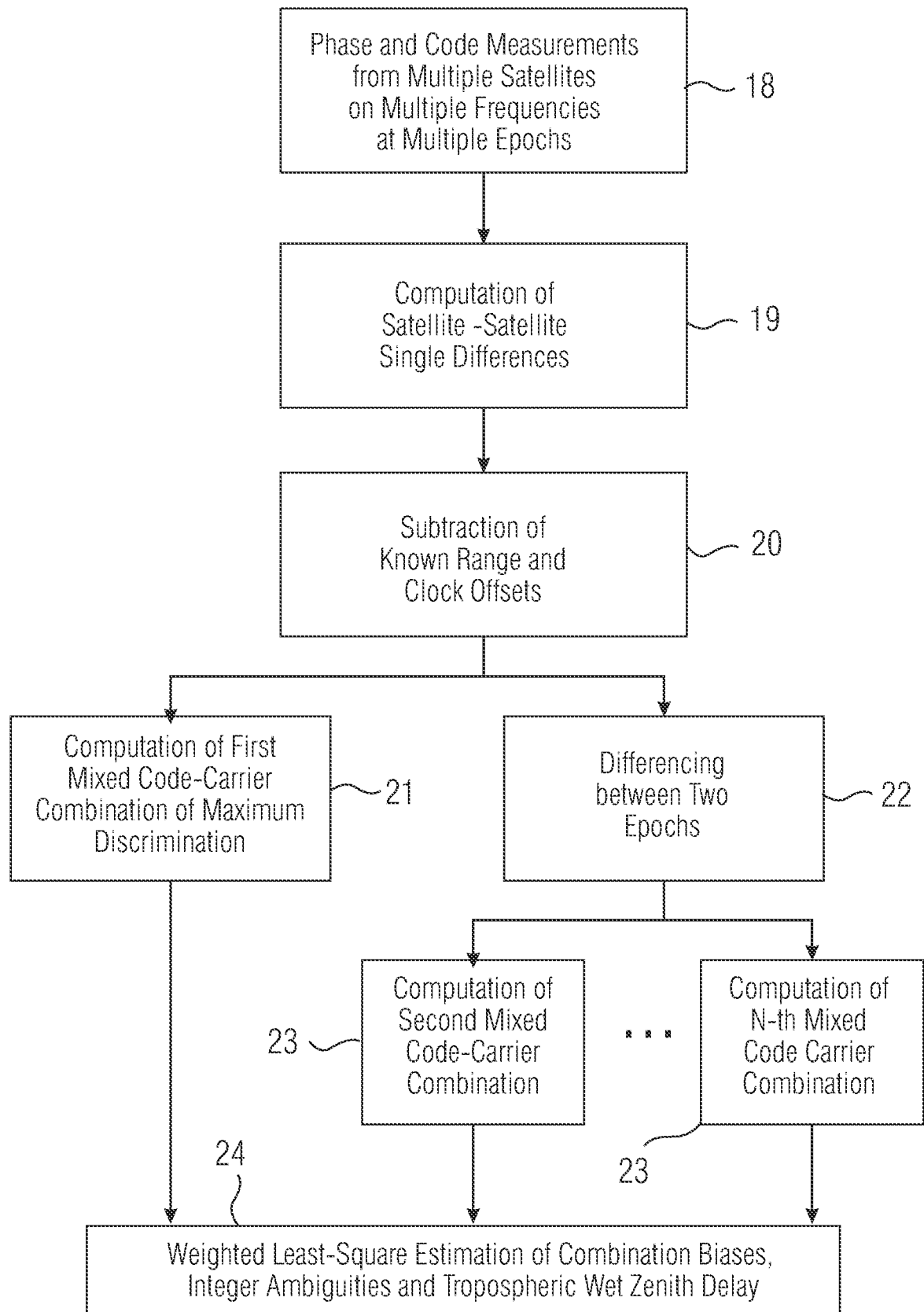


FIG 3

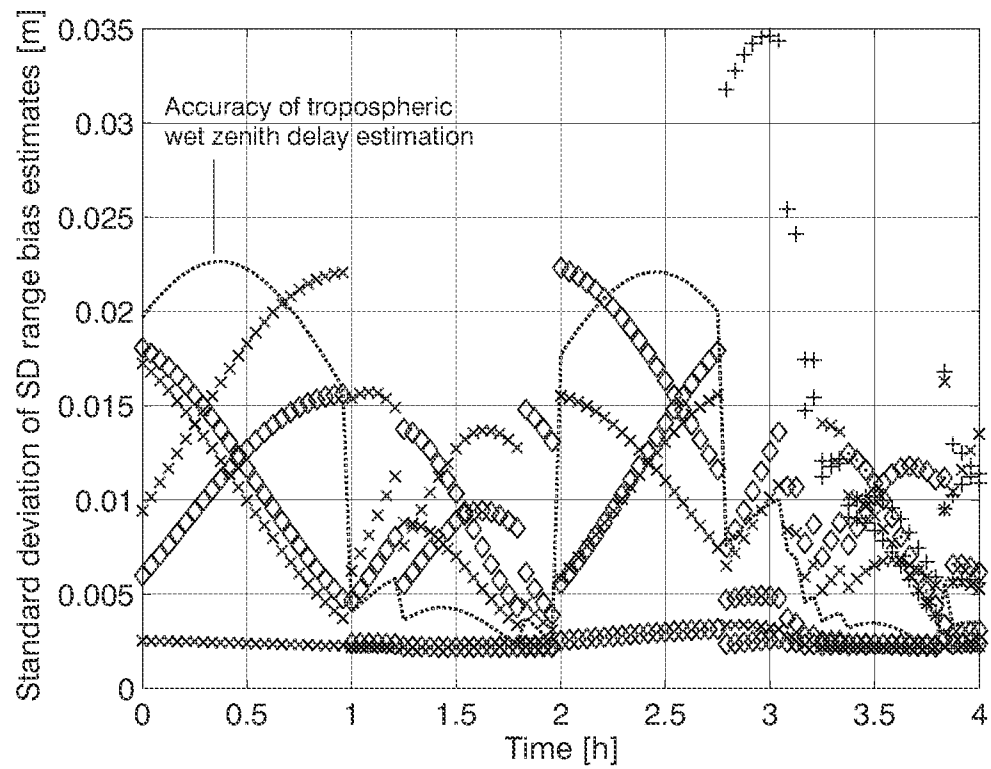


FIG 4

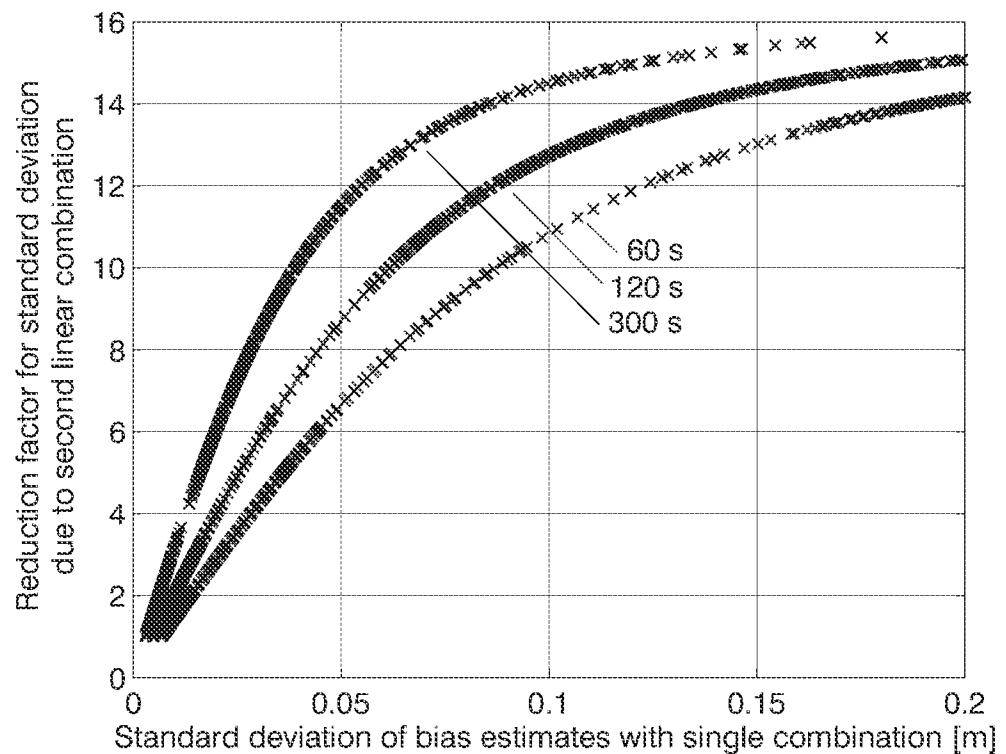


FIG 5

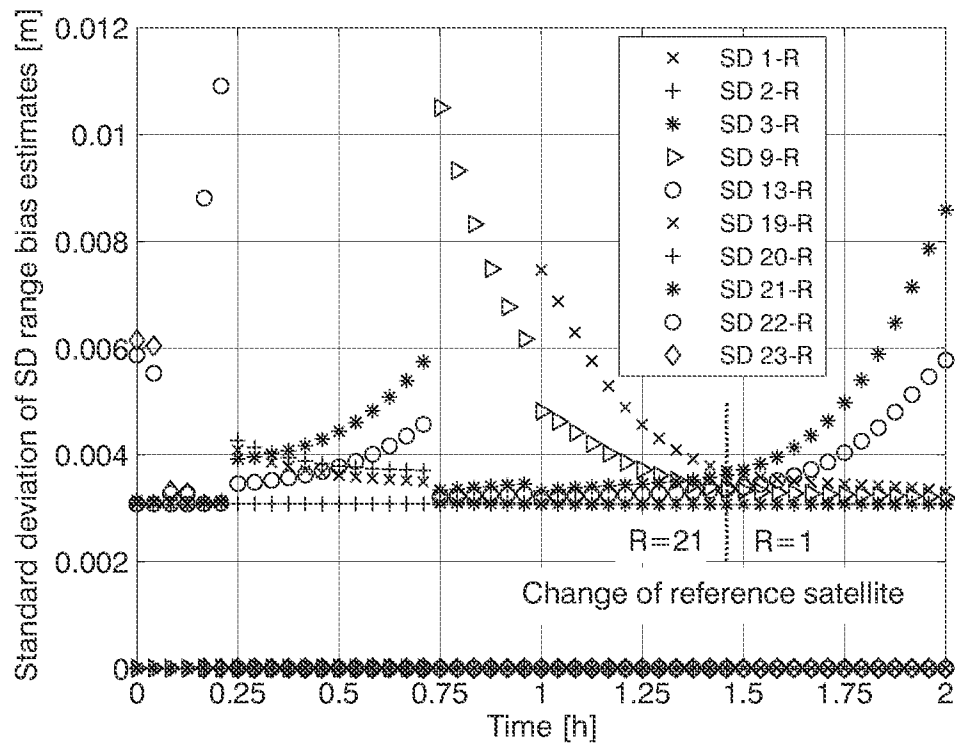


FIG 6

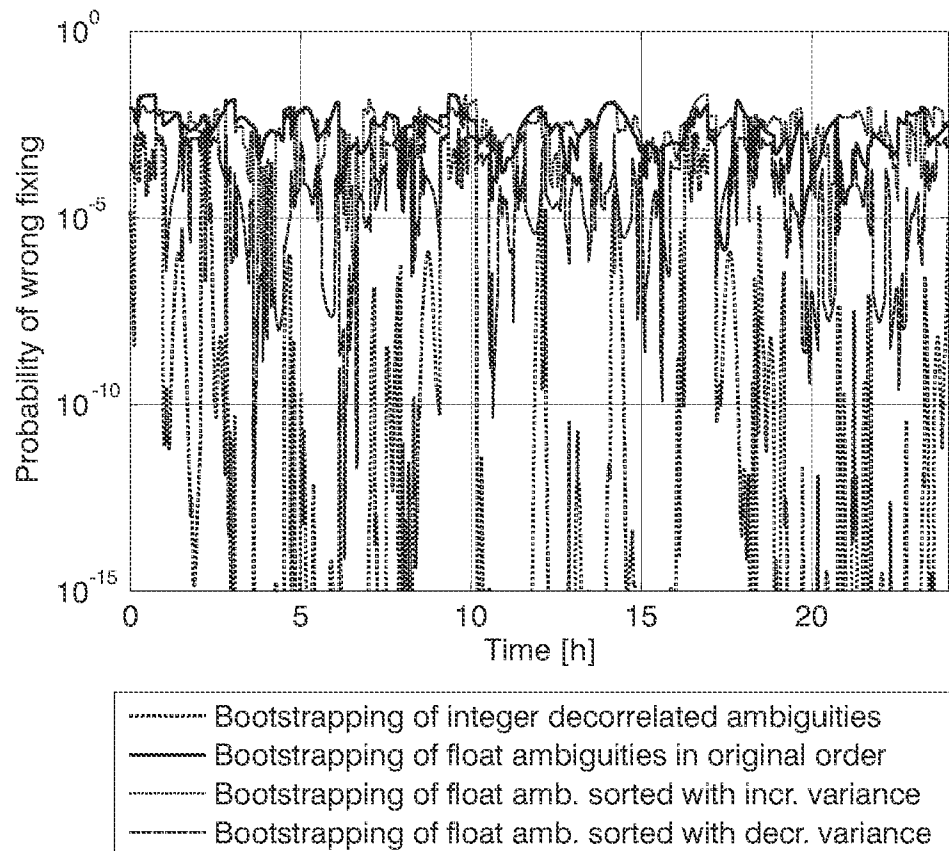


FIG 7

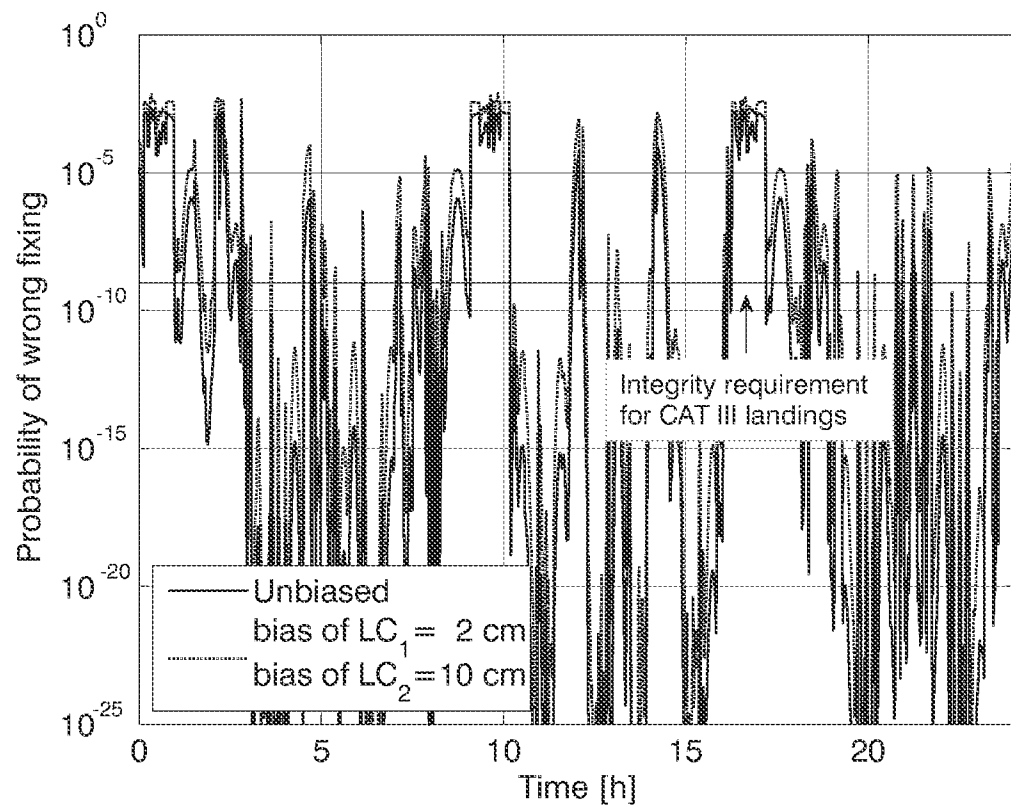
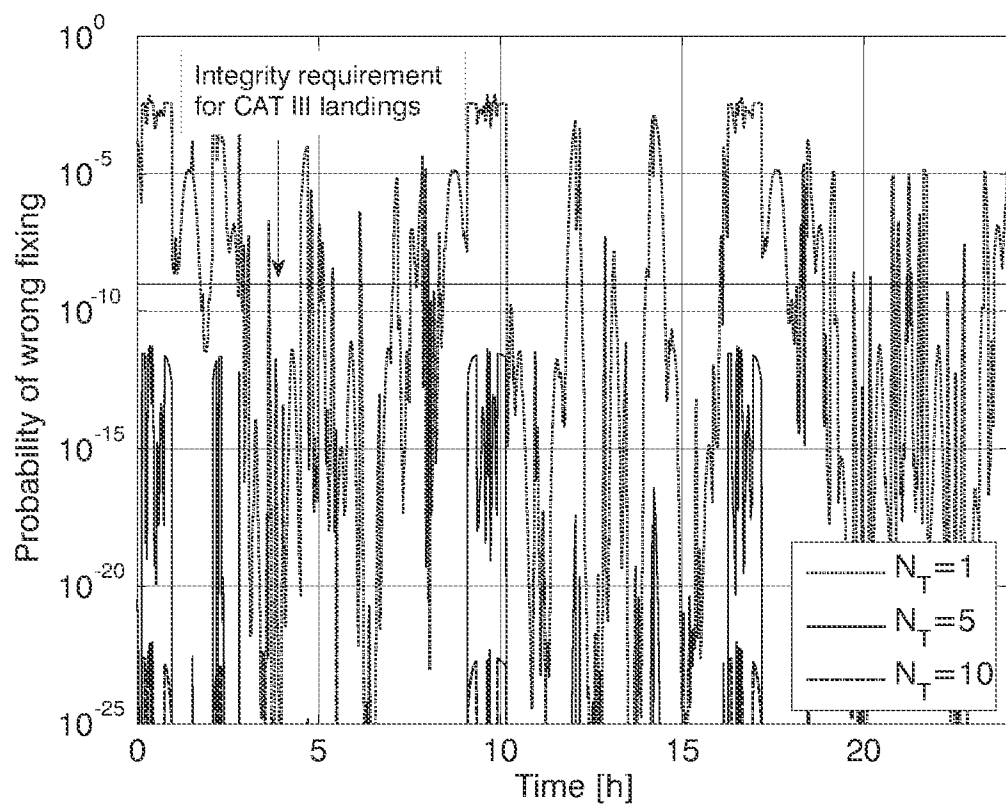


FIG 8



6/7

FIG 9

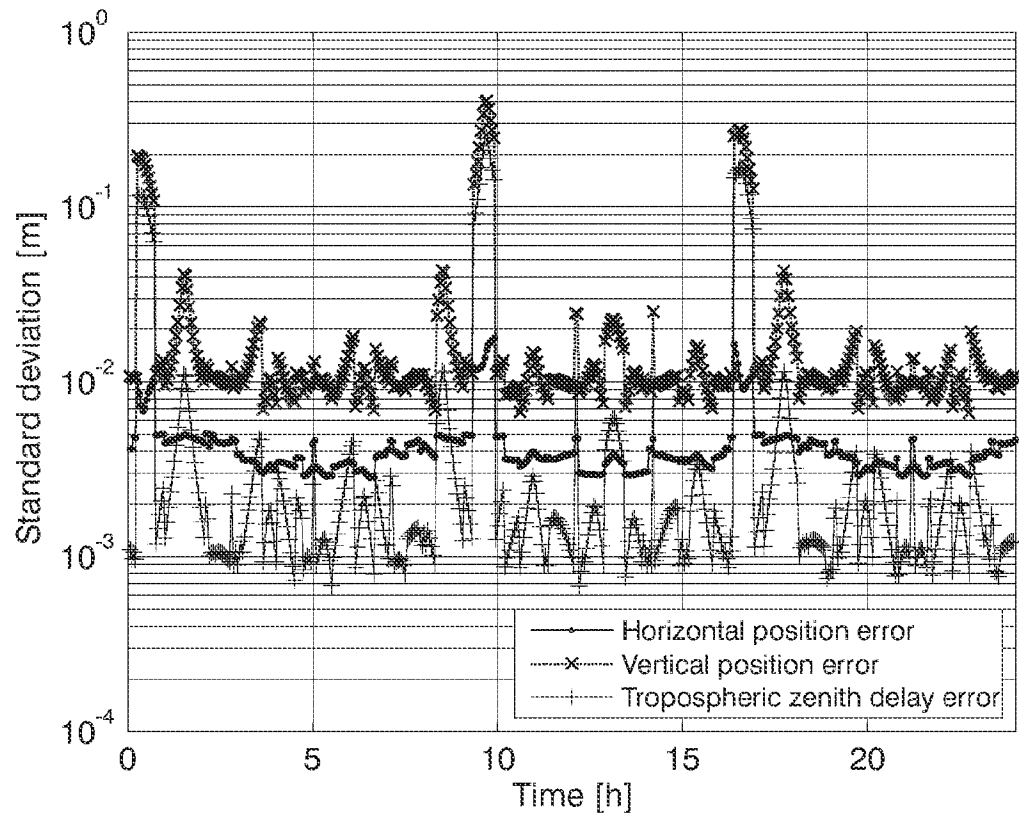


FIG 10

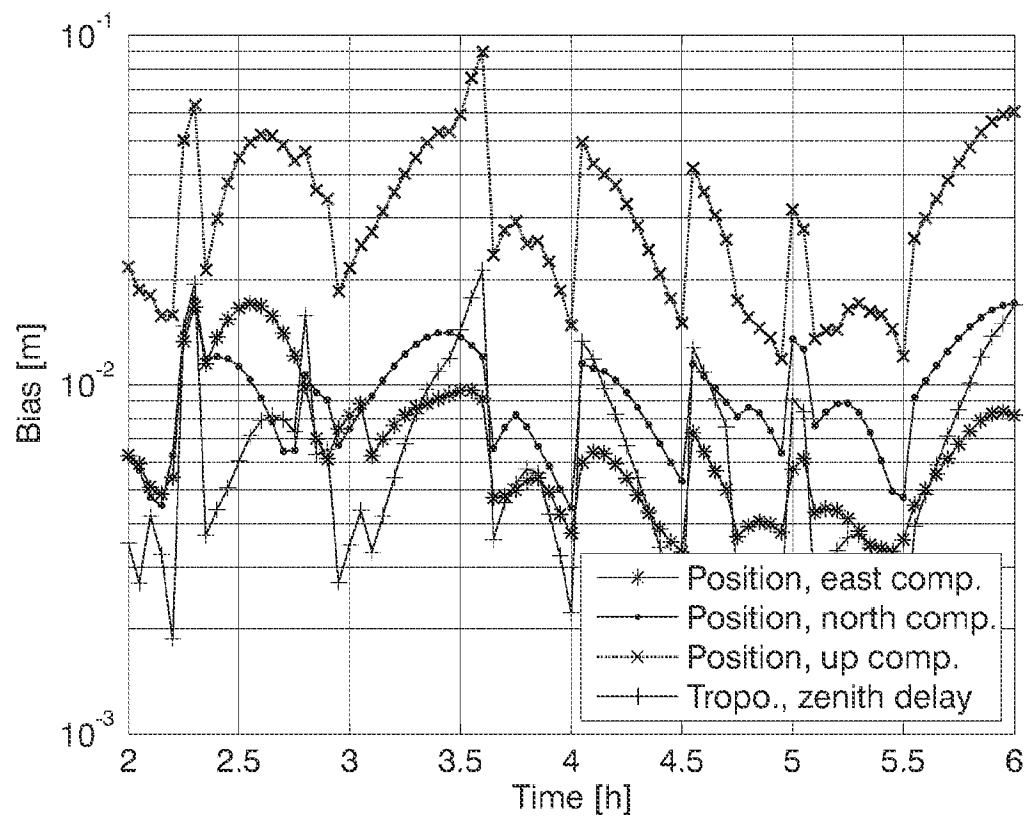


FIG 11

