

Oct. 13, 1942.

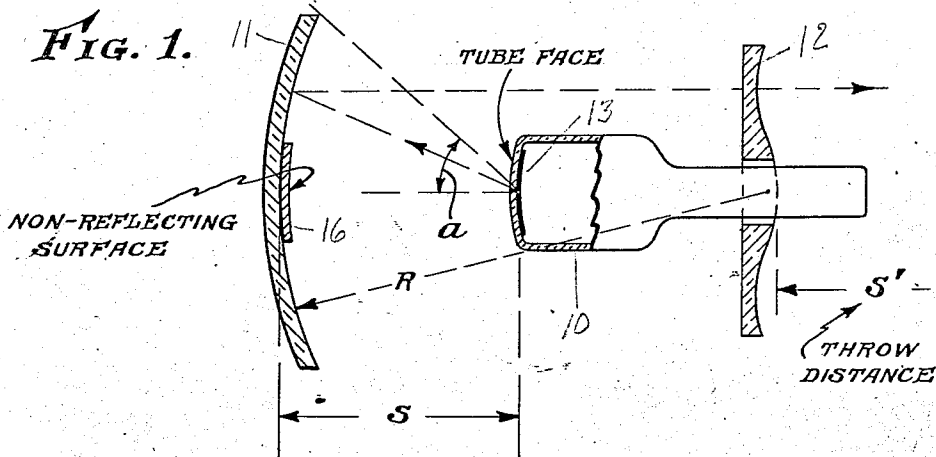
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2,298,808

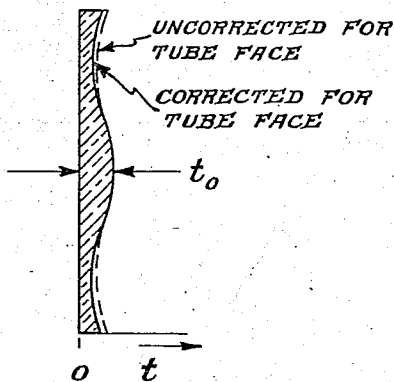
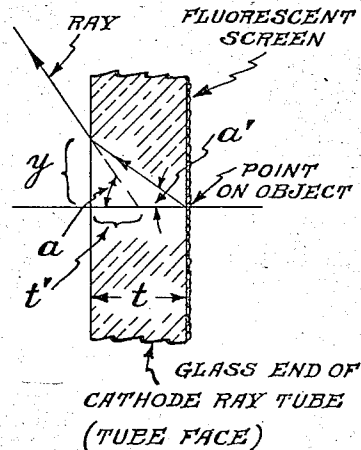
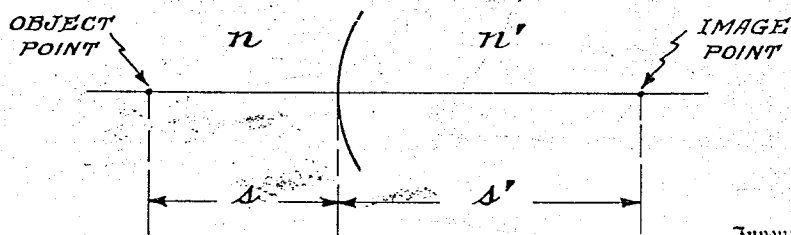
TELEVISION PROJECTION SYSTEM

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corrector plate corrects for error due to mirror and also to the tube wall (thickness)

FIG. 2.**FIG. 3.****FIG. 4.**

See Fig. 6, 21.3.

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Examiner

See British 559,265 Feb. 2, 1944

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FIG. 5.

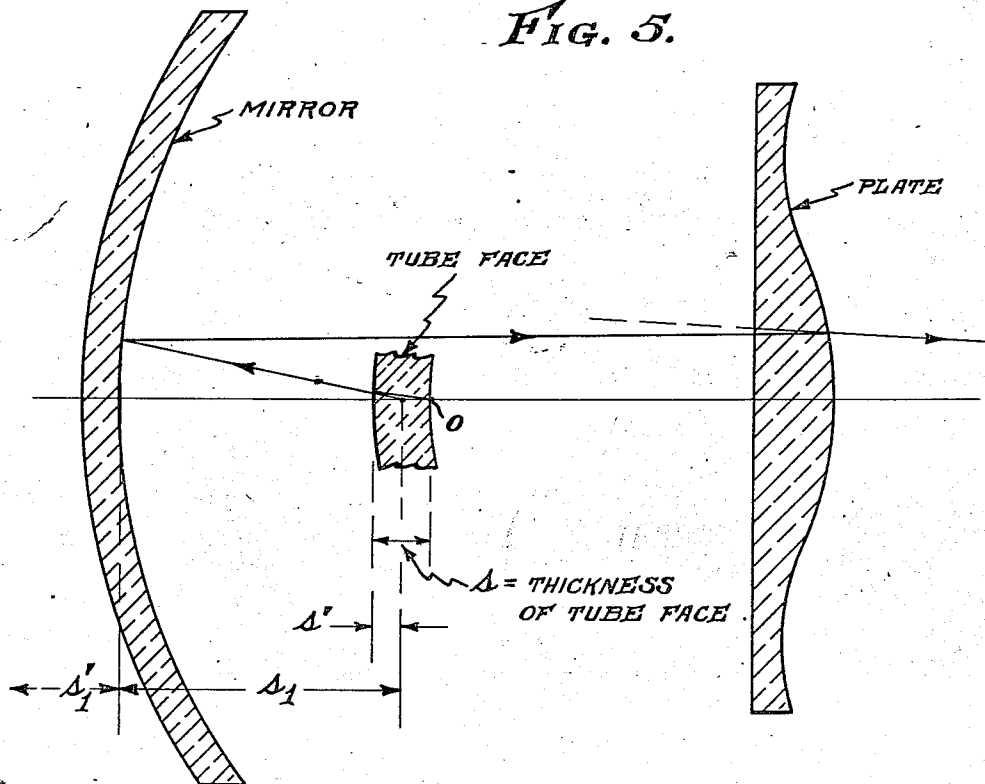
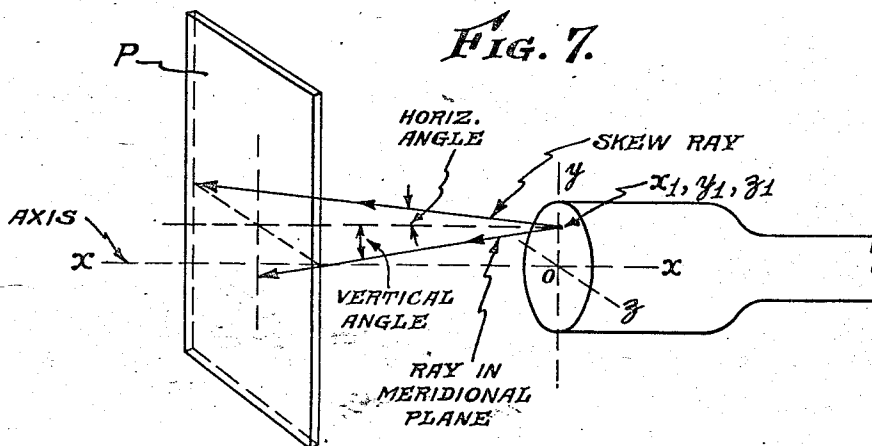


FIG. 7.



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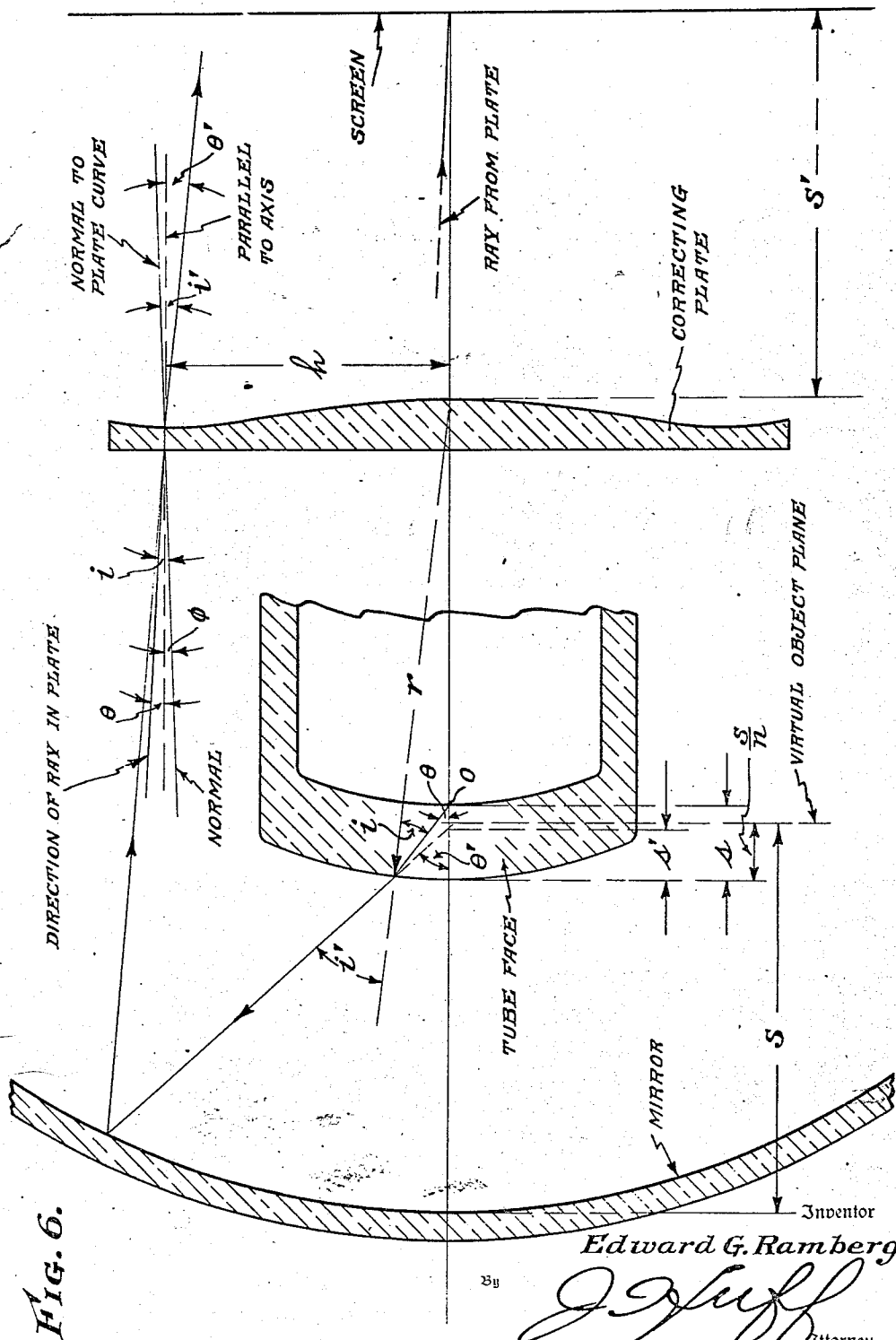
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FIG. 8.

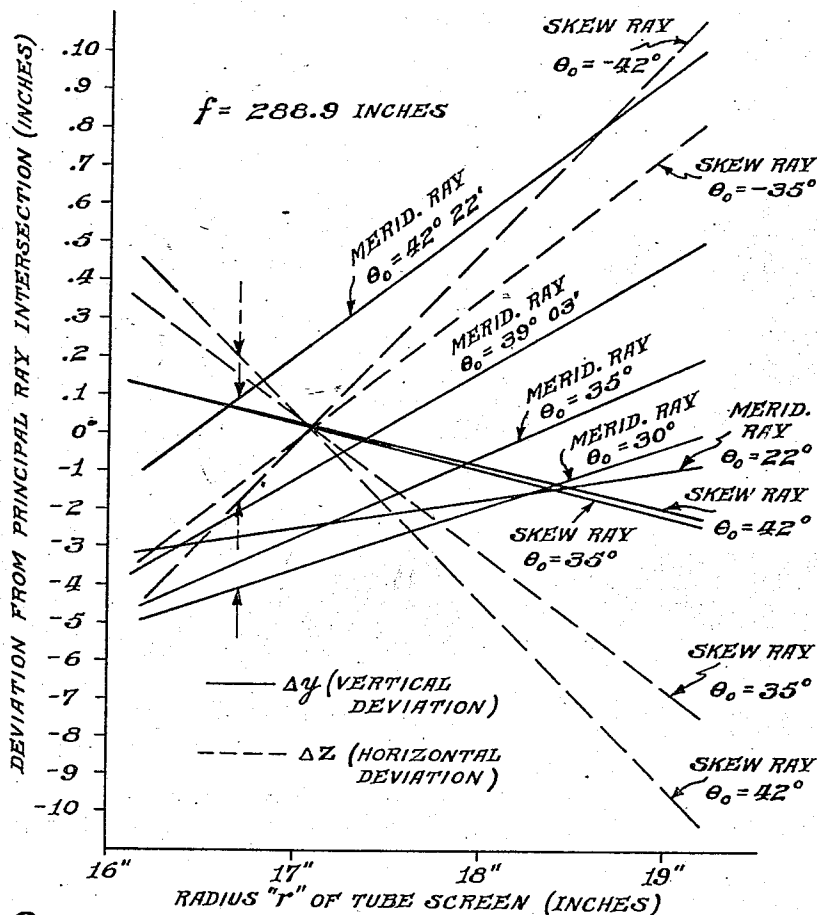


FIG. 9.

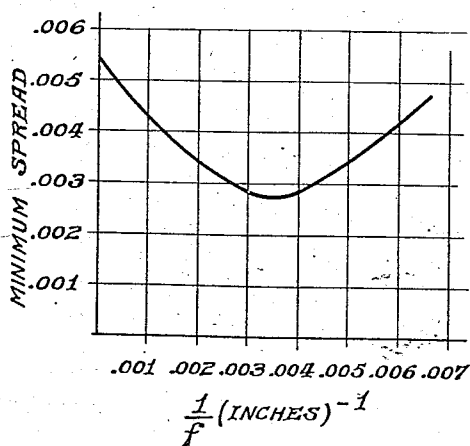
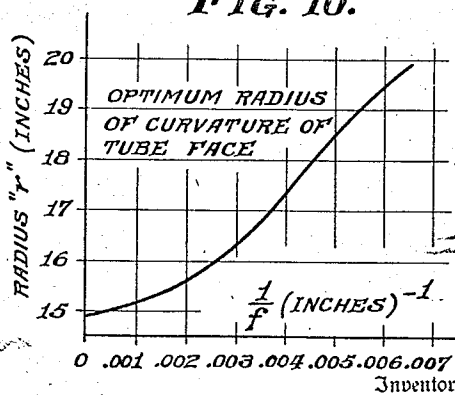


FIG. 10.



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2,298,808

TELEVISION PROJECTION SYSTEM

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Application April 26, 1941, Serial No. 390,459

7 Claims. (Cl. 178-7.5)

My invention relates to systems for projecting television pictures or the like and particularly to projection systems in which the optical system is of the type comprising a spherical mirror and a spherical-aberration correcting plate.

As explained in application Serial No. 248,569, filed December 30, 1938, in the name of Daniel O. Landis, it has been found that a large television picture can be projected with sufficient illumination and with good definition by employing a specially designed optical system of the above-described type. The present invention is an improvement on the system disclosed by the aforesaid application.

It is an object of the present invention to provide an optical projection system of the above-described type in which there is improved definition in the projected picture.

When the first television projector made in accordance with the Landis invention was completed and tested by projecting a picture from the end of a cathode ray tube, it was found that, while the projected image was much brighter than that obtained by any previously employed optical systems, the detail was less than had been expected for the known degree of accuracy of the system.

I have discovered that the end of the cathode ray tube envelope, that is, the tube face, introduces an error in addition to the spherical aberration error of the mirror which noticeably impairs the definition of the projected picture unless an additional correction is made. In other words, while an image of good definition may be projected by fully correcting only for the spherical aberration of the mirror, the quality of the image will be improved and will be of a very high order if the additional correction for the tube face thickness is made.

In accordance with my invention, this error is corrected by shaping or figuring the correcting plate of the optical system to correct not only for spherical aberration of the mirror but also for the error introduced by the thickness of the end of the cathode ray tube.

The invention will be better understood from the following description taken in connection with the accompanying drawings in which

Figure 1 is a side view of a television projection system embodying my invention,

Figure 2 is a sectional view of the correcting plate of the optical system indicating the difference, qualitatively, in the plate when figured to correct for spherical aberration of the mirror

only and when figured in accordance with the present invention,

Figure 3 is a diagram which is referred to in describing the error introduced by the thickness of the tube face,

Figure 4 is a diagram which is referred to with reference to formulas employed in tracing a paraxial ray,

Figures 5 and 6 are diagrams, not to scale, which are referred to in explaining how a paraxial ray and a ray at a large angle to the axis, respectively, are traced through the system,

Figure 7 illustrates the types of off-axis rays which are traced through the system in order to select the preferred values of tube face radius and of focal length for the center of the correcting plate, and

Figures 8, 9 and 10 are curves which are calculated and utilized for selecting the above-mentioned preferred values of radius and focal length.

Fig. 1 which is approximately to scale for spacings and for curvature of mirror and tube face, shows a television projection system comprising a cathode ray projection tube 10 and a spherical mirror 11 and correcting plate 12 for projecting, upon a projection screen, the cathode ray tube image or object which, in the example illustrated, is on a coating of fluorescent material 13 on the inner surface of the end of the tube envelope. This end of the tube has a certain thickness of glass through which the object rays from the screen 13 must pass. The cathode ray tube 10 may be of any suitable type and, therefore, need not be described in detail.

The optical system comprising the mirror 11 and the correcting plate 12 is of the same type as that described and claimed in the above-mentioned Landis application. The spherical mirror 11 by itself would project an image having a large amount of spherical aberration. The aberration is removed by the correcting plate 12 which is properly shaped or figured to give an image of high quality on the projection screen. The shape of the particular correcting plate illustrated is such that it moves the normal focus of the edge rays from the mirror 11 away from the mirror while it moves the normal focus of the paraxial rays from the mirror 11 towards the mirror, the focus for all rays being moved to the plane of the projection screen. As stated in the above-identified Landis application, the use of the spherical mirror and the correcting plate for correcting spherical aberration is based upon the principle of the well known Schmidt

camera. It will be understood that, just as in the case of the Schmidt camera, the correcting plate need not have this particular shape. For example, the entire plate may be convex.

In accordance with my invention, correction for the thickness of the end of the tube envelope 10 is made by so figuring the correcting plate 12 that it is thinner at the edge than it would be if no such correction were made (assuming the same thickness at the center in both cases). This is illustrated in Fig. 2 where the correcting plate 12, as figured in accordance with my invention, is represented in solid lines, while the outline of the correcting plate as would be figured without correcting for the tube face thickness is indicated in dotted lines.

The fact that a substantial part of the total error in a system of the above-described type is introduced by the end of the cathode ray tube was discovered from the following considerations. For the particular system described in this application (which is one having a 30-inch mirror) it is assumed that the maximum permissible ratio Δ of the diameter of the consequent circle of diffusion in the image to the diameter of the image is 0.002. It is assumed that the axial position of the tube is adjusted for best focus of the paraxial rays and that the correcting plate has been figured to make the spherical aberration zero for zero thickness of the tube face. Since it is assumed that an object without an intervening thick tube face is projected with perfect definition, the value Δ also represents the maximum permissible ratio of the diameter of the consequent circle of diffusion in the virtual image of the object produced by the tube face considered as a lens to the diameter of the object.

Fig. 3 shows a light ray traced from a point of the object which is on the axis, through the glass comprising the end of the cathode ray tube, and towards the spherical mirror (not shown). If n is the index of refraction of the glass and t is the thickness of the glass,

$$\sin \alpha = n \sin \alpha'$$

$$y = t \tan \alpha'$$

$$t' = \frac{y}{\tan \alpha} = \frac{t \tan \alpha'}{\tan \alpha}$$

$$\tan \alpha' = \frac{\sin \alpha'}{\sqrt{1 - \sin^2 \alpha'}} = \frac{\sin \alpha}{\sqrt{n^2 - \sin^2 \alpha}}$$

$$t' = \frac{t \cos \alpha}{\sqrt{n^2 - \sin^2 \alpha}}$$

$$\Delta t' = \frac{t}{n} - t' = t \left(\frac{1}{n} - \frac{\cos \alpha}{\sqrt{n^2 - \sin^2 \alpha}} \right)$$

Let p be the radius of the circle of diffusion due to spherical aberration on virtual image of the object having a diameter d . Then

$$p = \Delta t' \tan \alpha$$

$$\Delta = \frac{2 \Delta t' \tan \alpha}{d} \text{ or}$$

$$\Delta = \frac{2t}{d} \left(\frac{\tan \alpha}{n} - \frac{\sin \alpha}{\sqrt{n^2 - \sin^2 \alpha}} \right)$$

Solving for t for the case where

$$\Delta = 0.002$$

$$d = 7 \text{ inches}$$

$$n = 1.474$$

$$\alpha = \text{aperture angle} = 42^\circ 45'$$

then

$$t = 0.053 \text{ inch}$$

Since the projection tube of the size assumed should have an envelope about three times this thick at the large end or face to withstand the atmospheric pressure, it is now apparent that the circle of diffusion in the image will be too large unless an additional correction is provided in the optical system.

One specific optical system designed in accordance with my invention will now be described by way of example. The method of designing this system will also be given, and, since the method applies to a system of any size, it will be clear to those skilled in the art how to apply the invention to optical systems having different size mirrors, different projection throws, etc.

In this particular system, which is illustrated in Figure 1, a front surface mirror 30 inches in diameter is employed. A correcting plate diameter of three-fourths the mirror diameter was selected, since this results in even light distribution over the projection screen for the case where the central portion of the mirror is masked to improve contrast as described and claimed in my copending application Serial No. 367,817, filed November 29, 1940, and entitled "Reflective projection system." The correcting plate is crown glass with an index of refraction equal to 1.532.

The radius of curvature and, hence, the focal length of the mirror are determined by deciding what maximum aperture angle α is permissible from the point of view of aberration in the system. The angle which was adopted is $42^\circ 45'$, corresponding to an f -number of 0.737. The corresponding radius of curvature of the mirror is $R = 32.4$ inches.

The size of the cathode ray tube fluorescent screen is such as to give, at least approximately, maximum brilliance of the projected image. A tube screen diameter d of 7 inches was selected as being satisfactory.

The radius of curvature r of the tube screen should be made approximately equal to, or slightly greater than, one-half the radius of curvature of the mirror in order that the image will be sharp over the entire surface of a flat projection screen. A curvature r of 16.7 inches was selected by a procedure which will be explained hereinafter. The type of glass for the tube face is Pyrex, with a mean index of refraction equal to 1.474. The thickness of the end of the tube or tube face is $\frac{1}{8}$ inch.

The object and throw distances S and S' , respectively, may now be determined by the approximate formulas:

$$\frac{S}{R} = \frac{\frac{f}{R} \left(1 + \frac{1}{m} \right) - 1}{\frac{2f}{R} - 1}; \quad \frac{S'}{R} = \frac{(m-1)\frac{f}{R}}{\frac{2f}{R} - 1}$$

where R is radius of mirror, m is the magnification, and f is the focal length of the correcting plate at its center or apex. In these formulas, such parts in the path of a ray that are glass (i. e., the thickness of the cathode ray tube face and the correcting plate) are treated as equivalent to paths in air which are shorter in the inverse proportion to the index of refraction.

A magnification m of about 25 is desired. After assuming this value and later recalculating the optical system, the value $m = 25.809$ was obtained.

Now, if a value for f is selected, the object and throw distances S and S' , respectively, can be found. In accordance with considerations explained below, the value $f = 8.918$, $R = 288.9$ inches is selected.

Then

$$\begin{aligned} S/R &= 0.4704 + .5298/m = .491 \\ S &= 15.91 \text{ inches} \\ S'/R &= 0.5298 (m-1) = 13.14. \\ S' &= 425.74 \text{ inches} \end{aligned}$$

It may be noted that in a first calculation of the optical system, the value of S' thus obtained would be an approximate value and would be used as a check on other calculations. As explained hereinafter, the exact value of S' is obtained by means of a more exact magnification formula.

With respect to the selection of the focal length f of the correcting plate, while an image which is perfectly sharp at the center can be obtained by the proper shaping of the correcting plate for any value of the central refracting power or focal length f of the correcting plate, the quality of the image off the axis is dependent on the choice of the value of f . In order to obtain the optimal value of f , numerous off-axis rays should be calculated as will be explained hereinafter. Thus, by trial and error, the best value for f can be determined. This value of f is normally used in the preceding formulas in a preliminary calculation for obtaining values for the object distance S and the throw distance S' . The exact value of f ($f=8.918R$), which corresponds to the object distance thus determined and to the given magnification, is obtained by tracing a paraxial ray from the center of the tube screen through the system to the center of the image by the use of certain exact formulas given below. Thus, there is found the exact focal length which will bring the rays to focus at the desired throw distance. This ray tracing is done, as explained below, by means of the following well known formulas for paraxial rays, in which s is the object distance and s' is the image distance as illustrated in Fig. 4:

Refraction at a spherical surface of radius r separating media of indices of refraction n and n' :

$$n'/s' = n/s + (n' - n)/r$$

Refraction at a plane surface:

$$s' = (n'/n)s$$

Reflection at a mirror of radius R :

$$1/s' = 1/s + 2/R$$

Referring to Fig. 5. (not to scale), a paraxial ray will now be traced by the formulas I through the system beginning with the center of the object at o . In order to illustrate the ray path, the ray in this figure is shown at a larger angle to the axis than a paraxial ray. First tracing the ray through the glass face of the tube and into air, from the formula $n'/s' = n/s + (n' - n)/r$, we can find s' , r in this instance being the curvature of the outside surface of the tube face.

We find that $s' = -1.277$ inch.

The ray is next reflected at the mirror. For this reflection the object distance (s_1) equals the distance from the tube face to the mirror plus s' .

From the formula

$1/s'_1 = 1/s_1 + 2/R$, the subscript 1 referring to the mirror surface, we find that $s'_1 = -873.9$ (is negative since a paraxial ray diverges slightly after leaving the mirror).

The ray is next refracted at the plane surface of the correcting plate, which will be indicated by the subscript 2.

For this refraction, the object distance (s_2) equals s'_1 plus the distance from the mirror face

to the plane surface of the correcting plate or $s_2 = -873.9 - 31.91 = -905.8$

Then

$$s'_2 = (n'/n) s_2 = -1387.7$$

The ray is next refracted at the curved surface of the correcting plate indicated by subscript 3. The radius of curvature of the correcting plate at its center may now be found by the formula

$$n'/s'_3 = n/s_3 + (n' - n)/r$$

Here

s_3 = the originally selected thickness t_0 of the correcting plate at its center plus s'_2

or

$$-t_0 - 1387.7 = -1388.5$$

$$s'_3 = \frac{M}{n} \cdot \frac{s}{s'} \cdot \frac{s_1}{s'_1} \cdot \frac{s_2}{s'_2} \cdot s_3$$

where M is the magnification,

s'_3 = the exact throw distance $S' = 425.74$ inches

Solving for r ,

$$r = -153.7 \text{ inches}$$

As for any plano-convex lens, the relation between the radius of curvature and the focal length is:

$$f = -r/n - 1 = -288.9 \text{ inches}$$

We have now found the exact focal length f of the correcting plate at its apex (and also the exact throw distance S'). The value f will be used in the process of obtaining the coefficients in the equation for the correcting plate curve.

The equation for the correcting plate curve is:

$$t = t_0 + ah^2 + bh^4 + ch^6 + dh^8 \dots$$

where

t_0 is the thickness of the correcting plate at the axis and t is the thickness of the plate at the radial distance h from the axis. For the apertures here considered, it is satisfactory to break off the series beyond the term with h^8 .

The slope of the corresponding plate curve is then given by

$$dt/dh = 2ah + 4bh^3 + 6ch^5 + 8dh^7$$

The coefficients may be determined as follows:

The first coefficient, $2a$, is the reciprocal of the central radius of curvature of the correcting plate, or $2a = 1/r$. This can be shown as follows:

The reciprocal radius of curvature r of any curve, such as t , regarded as a function of the independent variable h [that is, of the curve $t = t(h)$] is, according to a well known law of analytic geometry, given by

$$\frac{1}{r} = \frac{\frac{d^2 t}{dh^2}}{\left[1 + \left(\frac{dt}{dh}\right)^2\right]^{3/2}}$$

In particular, at the point $h=0$, $dt/dh=0$, as is shown by substituting $h=0$ in the above equation for dt/dh . Hence, at this point, that is, at the vertex of the correcting plate curve,

$$\frac{1}{r} = \frac{d^2 t}{dh^2}$$

A differentiation of the equation for dt/dh with respect to h yields for $h=0$

$$\frac{d^2 t}{dh^2} = 2a$$

Therefore

$$1/r = 2a$$

Previously it has been mentioned that

$$f = \frac{-r}{n-1} \text{ or } \frac{1}{f} = \frac{-1}{r(n-1)}$$

where n is the index of refraction of the correcting plate and f is its axial focal length. It follows that

$$2a = \frac{-1}{f(n-1)}$$

Since the exact value of f has been found by calculating or tracing a paraxial ray as described above, we obtain the first coefficient as follows:

$$2a = \frac{-1}{(n-1)f} = \frac{-1}{(1.532-1) 288.9} \\ = -6.50568(10)^{-3}$$

As t_0 is fixed to begin with, only three further coefficients, b , c and d , need be determined to fully establish the shape of the correcting plate.

The procedure for finding the coefficients b , c and d is as follows: From five to seven rays leaving the center point of the object at a number of more or less evenly spaced angles to the axis are calculated through the system by the aid of the well known trigonometric formulas given below with reference to Fig. 6 (not to scale). These formulas are for rays in a meridional plane, that is, in a plane containing the axis of symmetry.

Refraction at surface of radius r :

$$\sin i = \frac{(s-r)}{r} \sin \theta$$

$$\sin i' = (n/n') \sin i$$

$$\theta' = \theta + i - i'$$

$$s' = \frac{r \sin i'}{\sin \theta'} + r$$

Refraction at plane surface:

$$\sin \theta' = (n/n') \sin \theta$$

$$s' = s \tan \theta / \tan \theta'$$

Reflection at mirror:

$$\sin i = \frac{(s-r)}{r} \sin \theta$$

$$i' = -i$$

$$\theta' = \theta + 2i$$

$$s' = \frac{r \sin i'}{\sin \theta'} + r$$

In the above Formulas II, the object and image distances are represented by s and s' , respectively. The angle θ is the angle of incidence of the ray with respect to the axis, angle θ' is the angle the refracted or reflected ray makes with respect to the axis, i is the angle of incidence of the ray at the surface under consideration, and the angle i' is the angle of refraction or reflection at the surface in question.

Each of the several rays is traced through the system from the central point o of the object in steps similar to those described for a paraxial ray.

First considering the refraction caused by the tube face, the image distance s' and the angle θ' that the refracted ray makes with the axis are found by the formulas for refraction at a surface of radius r . The point at which the ray strikes the mirror is now found.

By the formulas for reflection at the mirror, the point at which the ray strikes the plane surface of the correcting plate is found.

The ray is next traced to the plane tangent to

the vertex of the correcting plate by means of the formulas for refraction at a plane surface.

Having traced a number of rays by the Formulas II from the axial point of the object through the system up to the plane tangent to the vertex of the correcting plate curve, the height of incidence h and angle of incidence θ , with respect to the axis, are obtained for each. It is then possible to find that slope of the plate surface which will just bend the incident ray so that it will strike the axis at its intersection with the image plane. This slope is found as follows:

The throw distance S' being known the angle θ' of the refracted ray with the axis is given by

$$\tan \theta' = h/S'$$

An application of Snell's law to the case in question then yields for the angle of incidence i , at the curved surface of the plate:

$$\tan i = \frac{\sin (\theta' - \theta)}{n - \cos (\theta' - \theta)}$$

and the slope of the curve is given, finally, by

$$dt/dh = \tan \phi = \tan (\theta = i)$$

Table I shows the results for h and dt/dh for the rays calculated for the system being described.

Table I

Initial angle (in g's)	Angle of emergence from tube	h	dt/dh (from $\tan \phi$)
4°30'	6°37'	1.900585	-.0120150
8°	11°48'	3.371326	-.0199484
13°	19°17'	5.449183	-.0267316
18°	26°58'	7.485631	-.0246847
22°	33°21'	9.075275	-.0133551
25°	38°19'	10.24731	.0031083
27°30'	42°38'	11.19369	.0237557

Three of the slopes dt/dh thus calculated from $\tan \phi$ and the corresponding values of h are then substituted in turn in the equation

$$dt/dh = 2ah + 4bh^3 + 6ch^5 + 8dh^7$$

Thus the values of h and dt/dh substituted in the equation are those for the rays leaving the center point of the object and making 13°, 22° and 27°30' to the axis. Solving the resulting three simultaneous equations by determinants, the following values were found:

$$4b = 5.08994 \cdot 10^{-5}; 6c = 8.72839 \cdot 10^{-8};$$

$$8d = 4.47325 \cdot 10^{-10}$$

The depths of the curve as a function of h for this system is shown in Table II below:

Table II

$$t - t_0 = -3.25284 \cdot 10^{-3} h^2 + 1.272485 \cdot 10^{-5} h^4 + 1.454732 \cdot 10^{-8} h^6 + 5.59156 \cdot 10^{-11} h^8$$

h inches	$t - t_0$ inches	h inches	$t - t_0$ inches	h inches	$t - t_0$ inches
0	0	6.5	-.11344	10.0	-.17789
0.5	-.00081	7.0	-.12681	10.2	-.17776
1.0	-.00325	7.5	-.13956	10.4	-.17691
1.5	-.00726	8.0	-.15131	10.6	-.17529
2.0	-.01281	8.2	-.15563	10.8	-.17286
2.5	-.01983	8.4	-.15967	11.0	-.16953
3.0	-.02824	8.6	-.16341	11.2	-.16526
3.5	-.03791	8.8	-.16682	11.4	-.15994
4.0	-.04873	9.0	-.16985	11.6	-.15353
4.5	-.06052	9.2	-.17247	11.8	-.14593
5.0	-.07312	9.4	-.17462	12.0	-.13707
5.5	-.08631	9.6	-.17628	12.2	-.12684
6.0	-.09984	9.8	-.17738	12.4	-.11518

The remaining pairs of values of dt/dh and h are used to check the correctness of the calculation and to determine the accuracy with which

the curve represented by the power series fits the ideal curve. The maximum deviation in slope which has been found in any one case for the 22.5" plate is 34 seconds of arc, corresponding to a circle of diffusion with a diameter equal to 0.0004 of that of the picture.

If a constant thickness t_0 has been used in tracing the rays through the system, a certain error will result; a better approximation can be obtained by deriving the thickness at the points of incidence of the several rays from the formula

$$t = t_0 + ah^2 + bh^4 + ch^6 + dh^8$$

with the values for b , c and d just determined and then recalculating the coefficients for the new, slightly altered heights of incidence h . Normally, this is not necessary: for the 22.5" plate finally adopted the circle of diffusion resulting from carrying the approximation only to the first stage was found to have a diameter of less 0.0003 of that of the picture. For plates with much steeper curves there may be an advantage in going to the second approximation, but even here this can be avoided if the thickness variation is estimated from the profiles of plates which have already been calculated.

The shape of the 22.5" correcting plate finally adopted is found to be given by

$$t = 0.75 - 3.25284 \cdot 10^{-3} h^2 + 1.272485 \cdot 10^{-5} h^4 + 1.454732 \cdot 10^{-8} h^6 + 5.59156 \cdot 10^{-11} h^8 \text{ inches}$$

It has a minimum thickness of 0.57207 inch at $h = 10.0639$ inch and curls up slightly at the edge, having there (at $h = 11.25$ inch) a thickness of 0.58598 inch.

It has previously been stated that the best radius of curvature r for the tube face has been selected to make the image fall in focus at all points on a flat projection screen and that the focal length f of the correcting plate at its center is selected to give good marginal definition of the projected image as well as good definition at its center. The process by which these values r and f are selected will now be described more fully.

The procedure, when calculations are first begun on a new system, is, to select values for r and f that look like reasonable values. The system will then be calculated using these values and the correcting plate curve is obtained. (This procedure has been described in detail, the description being, of course, for a final calculation with the finally selected values of r and f .)

The next step is to trace a plurality of off-axis rays through the system to find out how much they deviate horizontally and vertically from a principal ray leaving the same off-axis point on the object. The principal ray is represented by a straight line drawn from the said off-axis point through the center of curvature of the spherical mirror.

Seven off-axis rays are traced in the particular example being given, five of them being in a vertical meridional plane (a meridional plane being one in which lies the optical axis of the system), and two of them (which are referred to as skew rays) being at an angle with respect to the meridional plane and in a horizontal plane passing through the off-axis point. The types of off-axis rays just referred to are illustrated in Fig. 7 where a plane surface is represented at P to aid in illustrating the above-mentioned vertical and horizontal planes.

By means of this off-axis ray tracing, which

is done by means of the trigonometric formulas previously given and by means of the skew ray formulas given hereinafter, there is found the horizontal and vertical distances separating the point on the projection screen at which the off-axis ray falls and the point at which the principal ray falls.

After these deviation distances have been found for a system including the first calculated correcting plate (having the first selected value of f) and including a tube with the tube face having the first selected value of r , a different value for r is selected and the seven off-axis rays are again traced through the system using the same correcting plate. The curves of Fig. 8 may now be plotted by drawing a straight line through each pair of deviation points obtained with the two values of r . It will be noted that an additional pair of skew ray curves are plotted for skew rays on the opposite side of the vertical plane and symmetrical with respect to the two skew rays traced through the system. From these curves, the value of the tube face radius r which gives the minimum spread for a given correcting plate of focal length f may be determined. As indicated by the vertical arrows in Fig. 8, this minimum spread for the example plotted is obtained if the tube face radius r has a value of about 16.7 inches. It will be noted that this point of "minimum spread" is not necessarily the point either of minimum horizontal deviation or of minimum vertical deviation, but, instead, is a point where the deviation is the least, considering the two deviations together.

The procedure for tracing the five off-axis rays in a meridional plane up to the plane surface of the correcting plate is the same as previously described for rays leaving the center of the object at a substantial angle, the same trigonometric formulas being used. The ray tracing from the curved surface to the projection screen is done as follows:

Referring to Fig. 6 and to the angles indicated thereon for a ray incident to the curved surface of the plate and refracted thereby, the ray strikes the curved surface at a distance h from the axis which is found by the equation

$$h = h_1 + t_1 \tan \theta$$

where h_1 is the distance from the axis to the point at which the ray strikes the flat surface of the plate, and where t_1 is the thickness of the plate at this point and is also almost exactly the thickness of the plate at the very slightly different and as yet unknown distance h .

The value of h thus found is substituted in the equation

$$dt/dh = 2ah + 4bh^3 + 6ch^5 + 8ah^7$$

to obtain the value of $dt/dh = -\tan \phi$

Then the angle ϕ' may be calculated since

$$i = \phi - \theta$$

$$\sin i' = n_2 \sin i$$

where n_2 is the index of refraction of plate, and

$$\phi' = \phi - i'$$

Finally the height of incidence H on the projection screen is

$$H = h + [S' + (t_0 - t_1)] \tan \phi'$$

$$\Delta y = H - H_0$$

where H_0 is the height of incidence of the principal ray on the projection screen.

In tracing the two skew rays it is convenient

to employ Cartesian coordinates as shown in Fig. 7.

A particular ray passing through some point (x_1, y_1, z_1) will be represented by

$$\begin{aligned} y - y_1 &= a(x - x_1) \\ z - z_1 &= c(x - x_1) \end{aligned}$$

It will intersect a spherical surface of radius r with its vertex a distance s along the axis of symmetry (the x -axis) from the origin 0 in the point (x_0, y_0, z_0) given by

$$\begin{aligned} x_0 &= \{r + s - (ay_1 + cz_1) + (a^2 + c^2)x_1 - [(r + s - (ay_1 + cz_1) + (a^2 + c^2)x_1)^2 - (1 + a^2 + c^2)(r + s)^2 - r^2 + y_1^2 + z_1^2 - 2x_1(ay_1 + cz_1) + x_1^2(a^2 + c^2)]^{1/2}\} \\ &\quad \div (1 + a^2 + c^2); \end{aligned}$$

$$y_0 = y_1 + a(x_0 - x_1); \quad z_0 = z_1 + c(x_0 - x_1)$$

If the refracted or reflected ray is described by

$$\begin{aligned} y - y_0 &= a'(x - x_0) \\ z - z_0 &= c'(x - x_0) \end{aligned}$$

a' and c' are determined as follows:

Refraction at a spherical surface of radius r :

$$a' = \frac{ar^2 - y_0\{[(n'/n)^2 - 1](1 + a^2 + c^2)r^2 + (r + s - x_0 - ay_0 - cz_0)^2\}^{1/2} - (r + s - x_0 - ay_0 - cz_0)}{r^2 + (r + s - x_0)\{[(n'/n)^2 - 1](1 + a^2 + c^2)r^2 + (r + s - x_0 - ay_0 - cz_0)^2\}^{1/2} - (r + s - x_0 - ay_0 - cz_0)}$$

$$c' = \frac{cr^2 - z_0\{[(n'/n)^2 - 1](1 + a^2 + c^2)r^2 + (r + s - x_0 - ay_0 - cz_0)^2\}^{1/2} - (r + s - x_0 - ay_0 - cz_0)}{r^2 + (r + s - x_0)\{[(n'/n)^2 - 1](1 + a^2 + c^2)r^2 + (r + s - x_0 - ay_0 - cz_0)^2\}^{1/2} - (r + s - x_0 - ay_0 - cz_0)}$$

Refraction at a plane:

$$a' = \frac{a}{[(n'/n)^2 - 1](1 + a^2 + c^2) + 1}^{1/2}$$

$$c' = \frac{c}{[(n'/n)^2 - 1](1 + a^2 + c^2) + 1}^{1/2}$$

Reflection at a spherical surface of radius r :

$$a' = \frac{ar^2 + 2y_0(r + s - x_0 - ay_0 - cz_0)}{r^2 - 2(r + s - x_0)(r + s - x_0 - ay_0 - cz_0)}$$

$$c' = \frac{cr^2 + 2z_0(r + s - x_0 - ay_0 - cz_0)}{r^2 - 2(r + s - x_0)(r + s - x_0 - ay_0 - cz_0)}$$

Refraction at an aspheric surface of revolution $x = f(h)$ where $h = (y^2 + z^2)^{1/2}$:

$$a' = \frac{sh_o^2(1 + f'^2) - y_0f' \cdot M}{h_o^2(1 + f'^2) + h_o \cdot M}$$

$$c' = \frac{zh_o^2(1 + f'^2) - z_0f' \cdot M}{h_o^2(1 + f'^2) + h_o \cdot M}$$

where

$$M = [(n'/n)^2 - 1](1 + a^2 + c^2)(1 + f'^2)h_o^2 + (h_o - ay_0f' - cz_0f')^2]^{1/2} - (h_o - ay_0f' - cz_0f')$$

$$h_o = (y_o^2 + z_o^2)^{1/2}, \text{ and } f' = df/dh \text{ at } h = h_o$$

It will be apparent that, by means of the foregoing equations, a skew ray may be traced from point to point through the system to determine the point at which it falls on the projection screen. It is then a simple matter to calculate the distance horizontally and the distance vertically that this point is spaced from a corresponding principal ray.

Having traced the off-axis rays through the system for one correction plate of focal length f and for two values of tube face radii r whereby a set of curves such as those in Fig. 8 is obtained, the next step is to select another value of focal length f for another correcting plate and repeat the procedure; again tracing through a number of off-axis rays. Thus, there is obtained another group of curves similar to those in Fig. 8, but for a different f value.

For this second f value, the tube face radius r that will give minimum spread is again selected.

This procedure is repeated for several values

of f to obtain sufficient points to plot a curve of minimum spread vs. $1/f$, as shown in Fig. 9. From this curve, there is selected the focal length f that gives the least spread, this being the value that is used in the final calculation of the system.

There is also plotted a curve of optimum tube face radius r vs. $1/f$ as shown in Fig. 10 from which is selected the best value of r for the value of f just chosen from the curve of Fig. 9.

It will be seen that, by the above-described consideration of off-axis rays, the optimum values have been chosen for the tube face radius r and for the focal length f of the center of the correcting plate, whereby the system will project an image of excellent quality at the margin as well as at the center of the image.

It may be noted that the system which has been described is well corrected for chromatic aberration, this type of aberration being small as compared with the geometrical aberrations considered in the foregoing calculations.

With respect to the shape or curve of the cor-

recting plate when it is figured in accordance with my invention as compared with its shape when figured without taking into account the thickness of the tube face, it has been pointed out with reference to Fig. 2 that the correcting plate in the improved system is relatively less concave at the edge. This difference in the corrected and uncorrected curve of the correcting plate may be expressed mathematically as explained below. The correcting plate curve without tube face correction may be defined as follows:

$$t = t_0 + a_0h^2 + b_0h^4 + c_0h^6 + d_0h^8$$

Considering the tube face as a flat plate of thickness s :

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$$\sin \theta = \frac{1}{n} \sin \theta'$$

$$s' = \frac{s \tan \theta}{\tan \theta'} = \frac{s}{\tan \theta'} \frac{\sin \theta'}{n \sqrt{1 - \frac{\sin^2 \theta'}{n^2}}} = \frac{s \cos \theta'}{\sqrt{n^2 - \sin^2 \theta'}}$$

50

where the distance s' and the angles θ and θ' are those shown in Fig. 6 at the tube face and where n is the index of refraction of the tube face.

Let Δt = difference in thickness of a correcting plate which is not corrected for the thickness of the tube face and of a correcting plate which is so corrected.

The ray leaving the center point of the object at the angle θ will, continued backward after refraction at the tube face, intersect the virtual image plane at a distance

$$\left(\frac{s}{n} - s'\right) \tan \theta'$$

65

from the axis. If, as with the correcting plate uncorrected for the thickness of the tube face, the virtual object plane is imaged sharply on the projection screen, the actual ray leaving the object at the angle θ will strike the projection screen a distance

$$m \left(\frac{s}{n} - s'\right) \tan \theta'$$

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from the axis, m being the magnification. Thus,

to correct the plate for the tube face thickness its slope at the height of incidence of the ray considered must be changed by an amount

$$\Delta \frac{dt}{dh}$$

such that the angle of refraction at the correcting plate is increased by an amount

$$\frac{m}{S'} \left(\frac{s}{n} - s' \right) \tan \theta'$$

i. e. just enough to cause the ray considered to strike the projection screen at the axis.

$$t = t_0 + a_0 h^2 + \left[b_0 - \frac{2s}{n(n_2-1)R^2} \left(1 - \frac{1}{n^2} \right) \right] h^4 + \left[c_0 - \frac{4s}{n(n_2-1)R^2} \left(1 - \frac{1}{n^2} \right) \right] h^6 + \left[d_0 - \frac{10s}{n(n_2-1)R^2} \left(1 - \frac{1}{n^2} \right) \right] h^8$$

As, in order to produce an angular deflection $\Delta \theta'$ the slope of the refracting surface has to be changed by an angle

$$\Delta \phi = \Delta \frac{dt}{dh}$$

such that

$$\Delta \phi = \frac{\Delta \theta'}{n_2 - 1}$$

the needed change in the slope of the correcting plate is

$$\Delta \frac{dt}{dh} \approx \frac{m}{S'(n_2-1)} \left(\frac{s}{n} - s' \right) \tan \theta'$$

where S' is the throw distance and n_2 is the index of refraction of the correcting plate.

As

$$\frac{S'}{m} \approx \frac{R}{2}$$

$$\Delta \frac{dt}{dh} = \frac{-2s}{(n_2-1)R^2} \left(\frac{1}{n} - \frac{\cos \theta'}{\sqrt{n^2 - \sin^2 \theta'}} \right) \tan \theta'$$

where R is the radius of curvature of the spherical mirror. Again, to a first approximation

$$\frac{h}{F} = \sin \theta$$

where F is the focal length of the system

$$\Delta \frac{dt}{dh} = \frac{-2s}{(n_2-1)R^2} \left(\frac{1}{n} - \frac{\sqrt{1 - \frac{h^2}{F^2}}}{\sqrt{n^2 - \frac{h^2}{F^2}}} \right) \frac{h}{F}$$

$$\Delta \frac{dt}{dh} = -\frac{h}{F} \frac{2s}{(n_2-1)R^2} \left(\frac{1}{\sqrt{1 - \frac{h^2}{F^2}}} - \frac{1}{\sqrt{n^2 - \frac{h^2}{F^2}}} \right)$$

As

$$F \approx \frac{R}{2} \Delta \frac{dt}{dh} = \frac{-4s}{n(n_2-1)R^2} \left[\left[1 + \frac{1}{2} \frac{h^2}{F^2} + \frac{3}{8} \frac{h^4}{F^4} + \frac{5}{16} \frac{h^6}{F^6} + \frac{35}{128} \frac{h^8}{F^8} \right] - \left[1 + \frac{1}{2} \frac{h^2}{n^2 F^2} + \frac{3}{8} \frac{h^4}{n^2 F^4} + \frac{5}{16} \frac{h^6}{n^2 F^6} + \frac{35}{128} \frac{h^8}{n^2 F^8} \right] \right]$$

$$\Delta t = \frac{2s}{n(n_2-1)R^2} \left(1 - \frac{1}{n^2} \right) h^2 - \frac{4s}{n(n_2-1)R^2} \left(1 - \frac{1}{n^2} \right) h^4 - \frac{10s}{n(n_2-1)R^2} \left(1 - \frac{1}{n^2} \right) h^6$$

We have now found how the coefficient b , c and d in the equation for the correcting plate spherical mirror, a correcting plate positioned at least approximately at the center of curvature of

curve differ for this curve when corrected for the tube face thickness and when not so corrected. The values for b , c and d are smaller for the correcting plate of the present invention than for a plate uncorrected for tube face thickness by the above-indicated quantities Δb , Δc and Δd , respectively. Thus, the correcting plate curve with correction for the tube face thickness may be defined as follows:

$$t = t_0 + a_0 h^2 + (b_0 - \Delta b) h^4 + (c_0 - \Delta c) h^6 + (d_0 - \Delta d) h^8$$

or

The numerical values for Δb , Δc and Δd are given below for the particular projection unit described in this application.

for $s = 0.1875$; $n = 1.474$; $(n_2 - 1) = 0.532$; $R = 32.4$,

$$\Delta b = -\frac{0.375 \cdot 0.543}{1.474 \cdot 0.532 (32.4)^4} = -2.60 \cdot 10^{-7}$$

$$\Delta c = -\frac{0.75 \cdot 0.791}{1.474 \cdot 0.532 (32.4)^6} = -6.53 \cdot 10^{-10}$$

$$\Delta d = -\frac{1.875 \cdot 0.905}{1.474 \cdot 0.532 (32.4)^8} = -1.778 \cdot 10^{-12}$$

	Corrected	Uncorrected
$b =$	$1.2725 \cdot 10^{-5}$	$(1.2935 \cdot 10^{-5})$
$c =$	$1.4547 \cdot 10^{-8}$	$(1.5200 \cdot 10^{-8})$
$d =$	$5.5916 \cdot 10^{-11}$	$(5.7694 \cdot 10^{-11})$

I claim as my invention:

1. A projection system comprising a concave spherical mirror, a correcting plate positioned at least approximately at the center of curvature of said mirror, and a projection tube having an object surface therein which is positioned at a conjugate focus of the system and having a transparent refractive end or tube face of a certain thickness, said correcting plate being figured to correct both for the spherical aberration of the mirror and for the error introduced by said thickness of the tube face.

2. A projection system comprising a concave spherical mirror, a correcting plate positioned at least approximately at the center of curvature of said mirror, and a cathode ray tube having a luminescent screen therein which is positioned at a conjugate focus of the system and having a transparent refractive end or tube face of a certain thickness, said correcting plate being figured to correct both for the spherical aberration of the mirror and for the spherical aberration introduced by said thickness of the tube face.

3. A projection system comprising a concave

said mirror, and a cathode ray tube having a luminescent screen therein which is positioned at a conjugate focus of the system and having a transparent refractive end or tube face of a certain thickness, said correcting plate being figured to correct for the spherical aberration of the mirror and for the additional error introduced by the refraction caused by the end of said tube whereby said plate is relatively less concave on the edge than a correcting plate figured to correct only for the spherical aberration of the mirror.

4. A projection system comprising a concave spherical mirror, a correcting plate positioned at least approximately at the center of curvature of said mirror, and a projection tube having an object surface therein which is positioned at a conjugate focus of the system and having a transparent refractive end or tube face of a certain thickness, said correcting plate being figured to correct both for the spherical aberration of the mirror and for the error introduced by said thickness of the tube face, the correcting plate curve being substantially defined by the equation

$$t = t_0 + a_0 h^2 + \left[b_0 - \frac{2s}{n(n_2-1)R^2} \left(1 - \frac{1}{n^2} \right) \right] h^4 + \left[c_0 - \frac{4s}{n(n_2-1)R^2} \left(1 - \frac{1}{n^2} \right) \right] h^6 + \left[d_0 - \frac{10s}{n(n_2-1)R^2} \left(1 - \frac{1}{n^2} \right) \right] h^8$$

where t is the thickness of the correcting plate at the distance h , h is the radial distance measured from the center of the correcting plate, t_0 is the thickness of the correcting plate at its center, a_0 , b_0 , c_0 and d_0 are coefficients from the equation for the correcting plate curve when it is uncorrected for the tube face thickness, s is the thickness of the tube face, n is the index of refraction of the tube face, n_2 is the index of refraction of the correcting plate, and R is the radius of curvature of the spherical mirror.

5. An image projection device comprising a cathode ray tube having a bidimensional image area on which there may be produced a light image, said tube having a transparent refractive end or tube face of substantial thickness through which the light rays from said light image pass, and an optical system including a concave reflecting surface of revolution having its concave surface area positioned to receive said light rays from the image, and an aspherical zone plate positioned external to the path of the light projected from the light image to the reflector and positioned to receive the reflected light from the reflector, said zone plate having such curvature as to correct both for the spherical aberration introduced by the reflector and for the spherical aberration introduced by said thickness

of the tube face, whereby the optical system including the reflector and the zone plate is adapted to form a projected enlarged image of the said light image upon a viewing area located at a finite distance from the zone plate.

6. An image projection device comprising a cathode ray tube having a bidimensional image area on which there may be produced a light image, said tube having a transparent refractive end or tube face of substantial thickness through which the light rays from said light image pass, and an optical system including a substantially spherical mirror having its concave surface area positioned to receive said light rays from the image, and an aspherical zone plate positioned external to the path of the light projected from the light image to the reflector and located at or near the center of curvature of said mirror to receive the reflected light from the reflector, said zone plate having such curvature as to correct both for the spherical aberration introduced by the reflector and for the spherical aberration introduced by said thickness of the tube face,

whereby the optical system including the reflector and the zone plate is adapted to form a projected enlarged image of the said light image upon a viewing area located at a finite distance from the zone plate.

7. An image projection device comprising a cathode ray tube having a bidimensional image area on which there may be produced a light image, said tube having a transparent refractive end or tube face of substantial thickness through which the light rays from said light image pass, and an optical system including a substantially spherical mirror having its concave surface area positioned to receive said light rays from the image, and an aspherical zone plate positioned external to the path of the light projected from the light image to the reflector and located at or near the center of curvature of said mirror to receive the reflected light from the reflector, said zone plate having such curvature as to correct both for the spherical aberration introduced by the reflector and for the spherical aberration introduced by said thickness of the tube face, said plate curvature also being such as to reduce the said spherical aberration to a minimum in a viewing area located at a finite distance from the said zone plate.

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