May 28, 1940. W. P. MASON 2,202,391

PIEZOELECTRIC APPARATUS

Filed Nov. 3, 1938

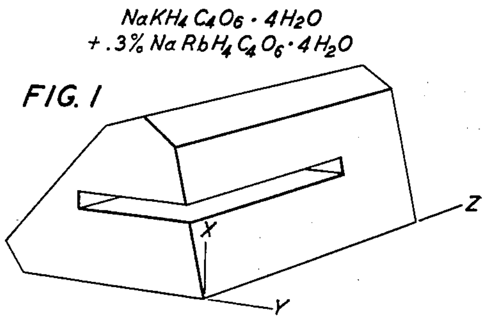


FIG. 1

FIG. 2

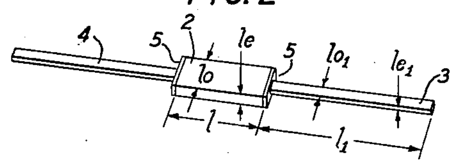


FIG. 3

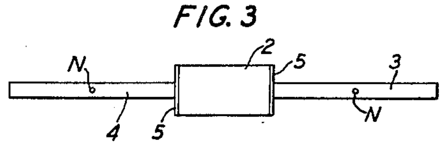


FIG. 4

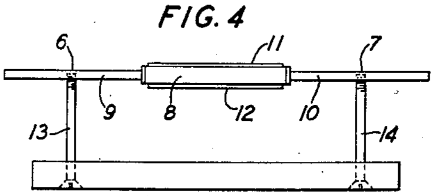


FIG. 5

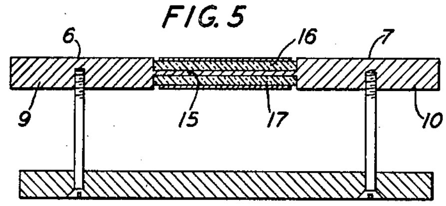


FIG. 6

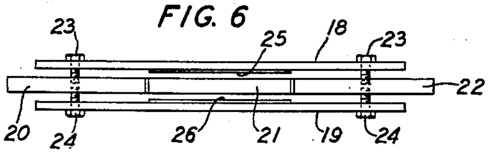
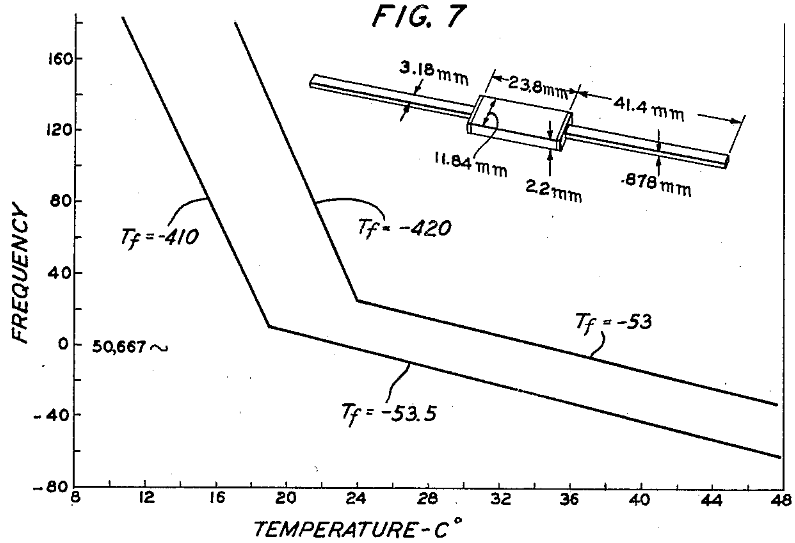


FIG. 7



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PIEZOELECTRIC APPARATUS

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Application November 3, 1938, Serial No. 238,530

12 Claims. (Cl. 171—327)

This invention relates to piezoelectric crystal apparatus and particularly to piezoelectric elements of the sodium potassium tartrate type in which it is desirable to be able to stabilize the resonance frequency with variations in temperature.

An object of the invention is to reduce the temperature coefficient of piezoelectric elements consisting of Rochelle salt.

A principal object of the invention is to provide a piezoelectric element of the Rochelle salt type that may have a substantially zero temperature coefficient of frequency.

Another object of the invention is to provide a piezoelectric element of the sodium potassium tartrate type that may be employed as a reactance of high stability during changing temperatures in frequency selective and control systems.

A body of piezoelectric material has various resonance frequencies depending upon the mode of vibration, the dimensions of the material and its density. Since the dimensions and the density vary with temperature it is to be expected that the mechanical resonance frequency will likewise vary. However, when a body of piezoelectric material is associated with electrodes and is subjected to an electric field to excite it into vibration at or near one of the natural resonance frequencies of the body the electromechanical system involves an electrical driving circuit coupled electromechanically to the mechanical vibrating system. Because of their coupling the electrical and mechanical systems are not independent of each other and, accordingly, the effective electrical reactance of the coupled system as presented at the electrodes of the piezoelectric element experiences variations in consequence of the changes in the mechanical resonance characteristics of the piezoelectric body with changing temperature. In addition to this effect upon the reactance, there is, in the case of sodium potassium tartrate, a still more important effect occasioned by the variations in dielectric capacitance, in the piezoelectric constant and in the electro-mechanical coupling especially in the temperature region adjacent the Curie point which lies at approximately 23.6° C. At that point, these electric properties of the sodium potassium tartrate piezoelectric crystal vary in an extraordinary manner. The result is that both the series resonance frequency of the piezoelectric element and its anti-resonant frequency depart widely from the mechanical resonance frequency of the piezoelectric body. The change of frequency with temperature, or the temperature coefficient of frequency, as it is called, of sodium potassium tartrate is very high in the temperature region about the Curie point and amounts to about 500 parts per million per degree centigrade. This requires that piezoelectric elements of such material of the types hitherto generally used as reactances in systems for frequency selection or control must be placed in an environment of regulated temperature wherever stable electrical characteristics are essential.

The temperature coefficient of frequency of sodium potassium tartrate experiences a transition at the Curie point temperature of approximately 23.6° C. Below that temperature the coefficient is approximately −1200; above that temperature it is approximately −500. It follows that any system or set of conditions for compensating for the temperature coefficient on one side of the Curie point will not suffice on the other side. Since room temperatures frequently shift from one side of the Curie point to the other it is desirable to so modify the piezoelectric behavior of sodium potassium tartrate that the Curie point will fall below ordinary room temperatures in order that in the room temperature range no sudden shift of magnitude of the temperature coefficient of frequency will be experienced.

In accordance with one feature of the invention the Curie point of sodium potassium tartrate is shifted so as to fall outside of the ordinary room temperature range by introducing a certain percentage of an isomorphous substance as, for example, .3 per cent of sodium rubidium tartrate which reduces the Curie point of the resulting crystal to about +7° C.

In accordance with another feature of this invention, the high negative temperature coefficient of sodium potassium tartrate and isomorphous piezoelectric elements is compensated by constructing a composite vibrating or resonating device which includes a tartrate piezoelectric portion and a non-piezoelectric portion mechanically associated with it to vibrate therewith as an integral unit, the non-piezoelectric portion having a temperature coefficient of frequency of opposite sign or positive and of such magnitude as to give the combined unit a substantially zero coefficient.

A further feature of the invention relates to a composite bar operating at a triple frequency harmonic and having the junction points of the piezoelectric and non-piezoelectric portions at antinodes of motion to minimize the tendency of the vibration to cause the different portions of the bar to pull apart.

In the accompanying drawing:

Fig. 1 illustrates an isomorphous sodium potassium tartrate crystal containing a small percentage of sodium rubidium tartrate;

Fig. 2 illustrates in perspective a combined tartrate and metallic alloy piezoelectric vibrator having a zero temperature coefficient;

Fig. 3 shows a side view of the device of Fig. 2;

Fig. 4 illustrates a modified form of the resonator of Figs. 2 and 3 designed to operate at a triple frequency harmonic mode;

Fig. 5 illustrates an alternative design of the structure of Fig. 4;

Fig. 6 shows a modification in which the electrodes are adjustably spaced from the piezoelectric portion of the composite vibrator; and

Fig. 7 shows contrasting graphs of the variation in frequency with temperature of a composite device constructed with a piezoelectric element of sodium potassium tartrate and a similar device employing an isomorphous material containing sodium rubidium tartrate.

Fig. 1 indicates the manner of producing an X cut from an isomorphous crystal of the sodium potassium tartrate type. The Curie point of Rochelle salt (sodium tartrate) occurs at about +23.6° C. whereas the Curie point of the rubidium isomorphous material mixture of Fig. 1 occurs at about +7° C. It may be remarked that the temperature coefficient of frequency for longitudinal vibration of an X cut bar of Rochelle salt undergoes a marked discontinuity at the Curie point. Above that temperature the coefficient is approximately −500. Below that temperature it is about −1200. With the rubidium salt mixture of Fig. 1 the temperature coefficient of frequency is relatively constant throughout the entire temperature range above +7° C. It follows that the rubidium salt permits a single compensation to suffice throughout the most useful temperature range; moreover, the compensation required is only about half that required for ordinary Rochelle salt below its Curie point which is the range usually encountered in practical use of piezoelectric apparatus.

Fig. 2 shows a longitudinally vibrating piezoelectric device comprising a central piezoelectric portion 2 preferably consisting of sodium potassium tartrate including about .3 per cent of sodium rubidium tartrate and end portions 3 and 4 of metal. Portions 3 and 4 may consist of an alloy as, for example, the so-called stoic metal which contains about 37 per cent nickel and the remainder iron. Portions 3 and 4 are of smaller cross-section than portion 2 except at their junction planes 5 where the metal is provided with a thin web coextensive in area with the piezoelectric material to facilitate making a strong glued joint. The metal portions 3 and 4 are glued or otherwise affixed to piezoelectric portion 2 to cause the ensemble to vibrate longitudinally as a unit when properly excited.

The design of a vibrating device such as is illustrated in Fig. 2 is based upon the fundamental principles developed in the articles entitled "An Electromechanical Representation of a Piezoelectric Crystal" appearing at pages 1252–1263 of the Proceedings of the Institute of Radio Engineers, vol. 23, October 1935, and "A Dynamic Measurement of the Elastic, Electric and Piezoelectric Constants of Rochelle Salt" published in the Physical Review, vol. 55, pages 775–789, April 15, 1939.

The resonant frequency for longitudinal vibration of a complex vibrator of the type illustrated in Fig. 2 is determined by the following equation which appears as equation (73) of the article, to which reference has been previously made, appearing at page 787 of the Physical Review published April 15, 1939.

$$\tan\frac{\omega l}{2v}\tan\frac{\omega l_1}{v_1}=\frac{l_e l_0 \rho v}{l_{e_1} l_{0_1}\rho_1 v_1} \tag{1}$$

In the foregoing equation ω represents the angular frequency $(2\pi f)$, l the length of piezoelectric portion 2, l_1 the length of each of the portions 3 and 4, v the propagation velocity of motion in a longitudinal direction in portion 2, v_1 the corresponding velocity in portions 3 and 4, ρ and ρ_1 the respective densities of the piezoelectric material and the metallic alloy and l_e, l_0 l_{e_1} and l_{0_1} the respective thicknesses and widths of portions 2 and 3 as indicated in Fig. 2. It will be understood that portions 3 and 4 are equal in dimensions and that the webs 5 are made as thin as possible and may be omitted if desired. In any case their mass is relatively small and may be neglected in the calculations which follow in which the length l_1 may be taken as the entire length of the metallic portion including the web 5 if such a web be employed.

As the temperature to which the system is exposed varies, changes will take place in each of the quantities of Equation 1. If, however, the quantities are so related that with change in temperature t the quantity ω which represents frequency remains constant or, in other words, the derivative of frequency with respect to temperature is zero the system possesses what is known as a zero temperature coefficient of frequency. To ascertain the conditions for this relationship we may differentiate Equation 1 with respect to t

$$\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}\left[\frac{v_1\omega\frac{dl_1}{dt}+v_1 l_1\frac{d\omega}{dt}-\omega l_1\frac{dv_1}{dt}}{v_1^2}\right]+\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}\left[\frac{2v\omega\frac{dl}{dt}+2vl\frac{d\omega}{dt}-2\omega l\frac{dv}{dt}}{(2v)^2}\right]=$$

$$\frac{l_{e_1}l_{0_1}\rho_1 v_1\left(l_e l_0\rho\frac{dv}{dt}+l_e l_0 v\frac{d\rho}{dt}+l_e\rho v\frac{dl_0}{dt}+l_0\rho v\frac{dl_e}{dt}\right)}{(l_{e_1}l_{0_1}\rho_1 v_1)^2}$$

$$-l_e l_0\rho v\frac{\left(l_{e_1}l_{0_1}\rho_1\frac{dv_1}{dt}+l_{e_1}l_{0_1}v_1\frac{d\rho_1}{dt}+l_{e_1}\rho_1 v_1\frac{dl_{0_1}}{dt}+l_{0_1}\rho_1 v_1\frac{dl_{e_1}}{dt}\right)}{(l_{e_1}l_{0_1}\rho_1 v_1)^2} \tag{2}$$

hence collecting terms

$$\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}\frac{\omega l_1}{v_1}\left[\frac{\frac{dl_1}{dt}}{l_1}+\frac{\frac{d\omega}{dt}}{\omega}-\frac{\frac{dv_1}{dt}}{v_1}\right]+\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}\frac{\omega l}{2v}\left[\frac{\frac{dl}{dt}}{l}+\frac{\frac{d\omega}{dt}}{\omega}-\frac{\frac{dv}{dt}}{v}\right]=$$

$$\frac{l_e l_0\rho v}{l_{e_1}l_{0_1}\rho_1 v_1}\left[\frac{\frac{dv}{dt}}{v}+\frac{\frac{d\rho}{dt}}{\rho}+\frac{\frac{dl_0}{dt}}{l_0}+\frac{\frac{dl_e}{dt}}{l_e}-\frac{\frac{dv_1}{dt}}{v_1}-\frac{\frac{d\rho_1}{dt}}{\rho_1}-\frac{\frac{dl_{0_1}}{dt}}{l_{0_1}}-\frac{\frac{dl_{e_1}}{dt}}{l_{e_1}}\right] \tag{3}$$

Since the derivative of any function with respect to temperature divided by the function is the temperature coefficient of that function we may write

$$\frac{\frac{dl_1}{dt}}{l_1}=T_{l_1},\quad \frac{\frac{d\omega}{dt}}{\omega}=T\omega,\ \text{etc.}$$

where T_{l_1}, T_ω etc., are respective temperature coefficients of l_1, ω, etc. Hence Equation 3 may be rewritten

$$\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}\frac{\omega l_1}{v_1}[T_{l_1}+T_\omega-T_{v_1}]+\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}\frac{\omega l}{2v}[T_l+T_\omega-T_v]=$$

$$\frac{l_e l_0\rho v}{l_{e_1}l_{0_1}\rho_1 v_1}[T_v+T_\rho+T_{l_0}+T_{l_e}-T_{v_1}-T_{\rho_1}-T_{l_{0_1}}-T_{l_{e_1}}] \quad (4)$$

The mass M of piezoelectric portion 2 is

$$M=ll_e l_0\rho \quad (5)$$

which remains constant with changes in temperature. Hence

$$\frac{dM}{dt}=0=ll_e l_0\frac{d\rho}{dt}+ll_e\rho\frac{dl_0}{dt}+ll_0\rho\frac{dl_e}{dt}+l_e l_0\rho\frac{dl}{dt} \quad (6)$$

Dividing by $ll_e l_0\rho$

$$T_\rho+T_{l_0}+T_{l_e}+T_l=0 \quad (7)$$

In similar fashion it may be shown that

$$T_{\rho_1}+T_{l_{0_1}}+T_{l_{e_1}}+T_{l_1}=0 \quad (8)$$

Substituting the values shown by Equations 7 and 8 in Equation 4

$$\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}\frac{\omega l_1}{v_1}[T_{l_1}+T_\omega-T_{v_1}]+\frac{\tan\frac{\omega l_1}{v}}{\cos^2\frac{\omega l}{2v}}\frac{\omega l}{2v}[T_l+T_\omega-T_v]=$$

$$\frac{l_e l_0\rho v}{l_{e_1}l_{0_1}\rho_1 v_1}[T_v-T_l-T_{v_1}+T_{l_1}] \quad (9)$$

Solving for T_ω

$$T_\omega=\frac{\frac{l_e l_0\rho v}{l_{e_1}l_{0_1}\rho_1 v_1}[T_v-T_l-T_{v_1}+T_{l_1}]+\frac{\omega l_1}{v_1}\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}[T_{v_1}-T_{l_1}]}{\frac{\omega l}{2v}\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}+\frac{\omega l_1}{v_1}\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}}$$

$$+\frac{\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}\frac{\omega l}{2v}[T_v-T_l]}{\frac{\omega l}{2v}\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}+\frac{\omega l_1}{v_1}\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}} \quad (10)$$

The frequency f_0 of a longitudinal vibration in a bar of piezoelectric material of length l and wave propagation velocity v is

$$f_0=\frac{v}{2l} \quad (11)$$

$$\frac{df_0}{dt}=\frac{2l\frac{dv}{dt}-v\frac{d(2l)}{dt}}{4l^2} \quad (12)$$

$$\frac{\frac{df_0}{dt}}{\frac{v}{2l}}=\frac{\frac{dv}{dt}}{v}-\frac{\frac{dl}{dt}}{l} \quad (13)$$

$$T_{f_0}=T_v-T_l \quad (14)$$

Similarly if T_{f_1} be the frequency of longitudinal vibration of a bar of material having a length l_1 and propagation velocity v_1

$$T_{f_1}=T_{v_1}-T_{l_1} \quad (15)$$

Substituting from (14) and (15) into (10)

$$T_\omega=\frac{\frac{l_e l_0\rho v}{l_{e_1}l_{0_1}\rho_1 v_1}[T_{f_0}-T_{f_1}]+\frac{\omega l_1}{v_1}\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}[T_{f_1}]+\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}\frac{\omega l}{2v}[T_{f_0}]}{\frac{\omega l}{2v}\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}+\frac{\omega l_1}{v_1}\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}} \quad (16)$$

or

$$T_\omega=\frac{T_{f_0}\left[\frac{\omega l}{2v}\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}+\frac{l_e l_0\rho v}{l_{e_1}l_{0_1}\rho_1 v_1}\right]+T_{f_1}\left[\frac{\omega l_1}{v_1}\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}-\frac{l_e l_0\rho v}{l_{e_1}l_{0_1}\rho_1 v_1}\right]}{\frac{\omega l}{2v}\frac{\tan\frac{\omega l_1}{2v}}{\cos^2\frac{\omega l}{2v}}+\frac{\omega l_1}{v_1}\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}} \quad (17)$$

T_ω becomes zero if

$$T_{f_0}\left[\frac{\omega l}{2v}\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}+\frac{l_e l_0\rho v}{l_{e_1}l_{0_1}\rho_1 v_1}\right]=T_{f_1}\left[\frac{l_e l_0\rho v}{l_{e_1}l_{0_1}\rho_1 v_1}-\frac{\omega l_1}{v_1}\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}\right] \quad (18)$$

Substituting the value of

$$\frac{l_e l_0\rho v}{l_{e_1}l_{0_1}\rho_1 v_1}$$

from Equation 1,

$$T_{f_0}\left[\frac{\omega l}{2v}\frac{\tan\frac{\omega l_1}{v_1}}{\cos^2\frac{\omega l}{2v}}+\tan\frac{\omega l}{2v}\tan\frac{\omega l_1}{v_1}\right]=$$

$$T_{f_1}\left[\tan\frac{\omega l}{2v}\tan\frac{\omega l_1}{v_1}-\frac{\omega l_1}{v_1}\frac{\tan\frac{\omega l}{2v}}{\cos^2\frac{\omega l_1}{v_1}}\right] \quad (19)$$

Writing

$$\frac{\omega l}{2v}=x$$

$$\frac{\omega l_1}{v_1}=y$$

$$T_{f_0}\left(x\frac{\tan y}{\cos^2 x}+\tan x\tan y\right)=$$

$$T_{f_1}\left(\tan x\tan y-y\frac{\tan x}{\cos^2 y}\right) \quad (20)$$

Dividing by $\tan x \tan y$

$$T_{f_0}\left(\frac{x}{\tan x\cos^2 x}+1\right)=T_{f_1}\left(1-\frac{y}{\tan y\cos^2 y}\right) \quad (21)$$

$$T_{f_0}\left(\frac{x}{\sin x\cos x}+1\right)=T_{f_1}\left(1-\frac{y}{\sin y\cos y}\right) \quad (22)$$

$$T_{f_0}\left(\frac{2x}{\sin 2x}+1\right)=T_{f_1}\left(1-\frac{2y}{\sin 2y}\right) \quad (23)$$

$$T_{f_0}\left[\frac{\frac{\omega l}{v}}{\sin\frac{\omega l}{v}}+1\right]=T_{f_1}\left[1-\frac{\frac{2\omega l_1}{v_1}}{\sin\frac{2\omega l_1}{v_1}}\right] \quad (24)$$

Equation 24 is the criterion for zero temperature coefficient of frequency of the composite bar for longitudinal vibration. The equation is independent of the relative cross-sections of portions 2 and 3. However, since the impedances to longitudinal vibration are different in portions 2 and 3 there will occur in addition to the fundamental longitudinal vibrations, reflections at the junctions of portions 2 and 3 with portion 4. Such reflections may be eliminated if the impedances be matched. The characteristic impedance Z_0 of the piezoelectric portion 2 is

$$Z_0 = l_e l_0 \rho v \tag{25}$$

and the characteristic impedance Z_1 of the non-piezoelectric member 3 is

$$Z_1 = l_{e_1} l_{0_1} \rho_1 v_1 \tag{26}$$

For the matched impedance condition

$$Z_0 = Z_1 \tag{27}$$

or

$$l_e l_0 \rho v = l_{e_1} l_{0_1} \rho_1 v_1 \tag{28}$$

Assuming as before that portion 2 consists of a bar cut from a crystal of a mixture of sodium rubidium tartrate and sodium potassium tartrate having an X cut, $v=4200$ meters per second and $\rho=1.79$. For portions 3 and 4 assumed to be of stoic metal, that is, 37 per cent nickel and the remainder iron, the magnitudes are as given in Table VIII by Professor Pierce at page 27 of the Proceedings of American Academy of Arts and Sciences, vol. 63, No. 1, April, 1928,

$$v_1 = 4150 \text{ (approx.)}$$

$$\rho_1 = 8.02$$

$$\frac{l_e l_0}{l_{e_1} l_1} = \frac{(8.02)(4150)}{(1.79)(4200)} = 4.42+ \tag{29}$$

Accordingly the ratio of cross-sectional areas for matched impedances is indicated by Equation 29. It is, however, unnecessary to match impedances to obtain the zero temperature coefficient condition. If instead of the relationship of Equation 28 we were to select a higher ratio of

$$\frac{l_e l_0 \rho v}{l_{e_1} l_{0_1} \rho_1 v_1}$$

that is, by making the cross-sectional area of the metal portions relatively smaller the conditions for zero temperature coefficient could be met as indicated by Equation 24 and the electromechanical coupling coefficient could be materially raised as may sometimes be desirable.

For the matched impedance case the lengths of the materials in the composite case can be calculated by substituting Equation 28 in Equation 1.

$$\tan\frac{\omega l}{2v}\tan\frac{\omega l_1}{v_1} = 1 \tag{30}$$

$$\tan\frac{\omega l}{2v} = \frac{1}{\tan\frac{\omega l_1}{v_1}} = \cot\frac{\omega l_1}{v_1} \tag{31}$$

$$\frac{\omega l}{2v} + \frac{\omega l_1}{v_1} = \frac{\pi}{2} \tag{32}$$

$$\frac{\omega l}{v} + \frac{2\omega l_1}{v_1} = \pi \tag{33}$$

Substituting the value of

$$\frac{2\omega l_1}{v_1}$$

from Equation 33 in Equation 24

$$T_{f_0}\left[\frac{\frac{\omega l}{v}}{\sin\frac{\omega l}{v}} + 1\right] = T_{f_1}\left[1 - \frac{\pi - \frac{\omega l}{v}}{\sin\frac{\omega l}{v}}\right] \tag{34}$$

From Equation 34 we may calculate the relative lengths of the piezoelectric portion 2 and the metallic portions 3 and 4. Taking the values $T_{f_0} = -500$, $T_{f_1} = +230$

$$-500\left[\frac{\frac{\omega l}{v}}{\sin\frac{\omega l}{v}} + 1\right] = 230\left[1 - \frac{\pi - \frac{\omega l}{v}}{\sin\frac{\omega l}{v}}\right] \tag{35}$$

which reduces to

$$\frac{\omega l}{v} + \sin\frac{\omega l}{v} = .99 \tag{36}$$

The solution of Equation 36 may be obtained by reference to trigonometric tables from which it is found that

$$\frac{\omega l}{v} = 29° = .506$$

$$\frac{2\omega l_1}{v_1} = \pi - \frac{\omega l}{v} = 2.636 \tag{37}$$

The actual length of the parts of the composite bar may be determined by reference to Equation 37 since it is apparent therefrom that

$$l = \frac{.506 v}{\omega} = \frac{.506(4200)}{2\pi f} = \frac{95.7}{f}$$

and

$$l_1 = \frac{1.318 v_1 l}{.506 v} = \frac{246}{f} \text{ or } 2.57 l. \tag{38}$$

The actual dimensions of the composite bar vibrator for any particular frequency may, therefore, be quite simply computed. As has been explained, a somewhat different result will be obtained if a different relation than that of Equation 28 is selected. However, the basic method of design remains the same although the actual calculations will be somewhat more involved.

Fig. 3 shows how the vibrator of Fig. 2 may be constructed so as to be supported by small wires passing horizontally through holes N at nodal points. Electrodes may be electroplated on the piezoelectric portion 2 or otherwise associated with it in any desired manner.

Where a higher vibration frequency and a lower impedance are desirable resort may be had to a triple frequency mode of vibration. In this case Equation 24 is still valid as the criterion for zero temperature coefficient of frequency as it is perfectly general in character for longitudinal vibration of a composite bar. For matched impedance we may take the condition of Equation 28. Bearing in mind that we are now dealing with a frequency ω' which is three times as high as that involved in the design of the structure of Fig. 1,

$$\omega' = 3\omega \tag{39}$$

Equation 32 becomes for the triple frequency

$$\frac{\omega' l}{2v} + \frac{\omega' l_1}{v_1} = \frac{3\pi}{2} \text{ or } \frac{\omega' l}{v} + \frac{2\omega' l_1}{v_1} = 3\pi \tag{40}$$

From Equation 24

$$\frac{T_{f_0}}{T_{f_1}} = R = \frac{1 - \frac{\frac{2\omega' l_1}{v_1}}{\sin\frac{2\omega' l_1}{v_1}}}{1 + \frac{\frac{\omega' l}{v}}{\sin\frac{\omega' l}{v}}} \tag{41}$$

Substituting Equation 40 in Equation 41,

$$R=\frac{1-\dfrac{3\pi-\dfrac{\omega' l}{v}}{\sin\dfrac{\omega' l}{v}}}{1+\dfrac{\dfrac{\omega' l}{v}}{\sin\dfrac{\omega' l}{v}}} \tag{42}$$

$$R\left[\frac{\sin\dfrac{\omega' l}{v}+\dfrac{\omega' l}{v}}{\sin\dfrac{\omega' l}{v}}\right]=\frac{\sin\dfrac{\omega' l}{v}-3\pi+\dfrac{\omega' l}{v}}{\sin\dfrac{\omega' l}{v}} \tag{43}$$

$$R\sin\frac{\omega' l}{v}=R\frac{\omega' l}{v}=\sin\frac{\omega' l}{v}-3\pi+\frac{\omega' l}{v}$$

$$\sin\frac{\omega' l}{v}(R-1)=\frac{\omega' l}{v}(1-R)-3\pi \tag{44}$$

$$\sin\frac{\omega' l}{v}=-\frac{\omega' l}{v}-\frac{3\pi}{R-1}$$

$$R \doteq -2 \text{ in this case}$$

Hence Equation 44 is satisfied by

$$\frac{\omega' l}{v}=\pi$$

and from Equation 40

$$\frac{\omega' l_1}{v_1}=\frac{3\pi}{2}-\frac{\omega' l}{2v}=\frac{3\pi}{2}-\frac{\pi}{2}=\pi$$

and

$$l_1=l$$

This partition into thirds is a peculiarly advantageous expedient since the dividing line comes at a loop of the motion and hence at a node of tensile stress so that there is no vibrating force tending to pull the constituent bars apart at their terminal junctions. Nodes of motion occur at central points in each constituent bar. It is, therefore, expedient to support the composite bar at nodal points **6** and **7** shown, for example, in Figs. 4, 5 and 6.

Fig. 4 illustrates a vibrating system in which a piezoelectric element bar **8** is glued at its ends to equal length bars **9** and **10**. The bar **8** may consist of the rubidium sodium tartrate material which has been described and bars **9** and **10** of the 37 per cent nickel-iron alloy. The piezoelectric element is provided with platings or coatings **11** and **12** of electrically conducting material to which the charging electromotive force may be applied. The coatings are carefully spaced from metallic portions **9** and **10**. The composite bar **8**, **9**, **10**, is supported at nodal points of motion **6** and **7** by means of supports **13**, **14** which may, if desired, be screws extending upwardly from a base member of insulating material.

Fig. 5 illustrates another method of accomplishing the same result as in Fig. 4. In this structure, a small web **15** of metal is left connecting members **9** and **10**. The two piezoelectric crystal bars **16** and **17** are glued to the web. **15**. These bars may be used as a divided plate crystal as, e. g., in such selective circuits as those of U. S. Patents 2,081,405, May 25, 1937, and 2,094,044 of September 28, 1937.

Fig. 6 shows a top view of a modified apparatus resembling that of Figs. 2 and 3 but in which it is desired to drive the piezoelectric element through an air-gap electrode arrangement. In this device horizontally mounted supporting members **18** and **19** are provided at opposite sides of the horizontally mounted composite vibrator **20**, **21**, **22**. Adjusting screws **23**, **24** serve to hold the composite bar at its nodal points, and also to enable the members **18** and **19** to be spaced closer to or farther from the composite bar. The members **18** and **19** are supported in any desirable manner as by flexible vertical stems to permit their separation to be thus varied. The conductive coatings or platings **25**, **26** which are to serve as electrodes are located on the inner sides of members **18** and **19** closely adjacent the piezoelectric portion **21**. The mechanical design of the composite vibrating bar may be in accordance with that of the structure of Figs. 2 and 3 which has been described in detail.

Fig. 7 illustrates graphs of the frequency characteristic of a composite vibrating bar with the dimensions indicated in the figure as plotted from measurements of its performance. The frequency deviation with temperature above and below a standard frequency of 50,667 cycles is indicated by the ordinates and the temperatures are abscissae. The upper curve shows the performance of a sodium potassium tartrate—37 per cent iron-nickel combination. The lower curve shows the performance of the same metallic end pieces glued to a piezoelectric element of the sodium potassium tartrate type containing about .1 per cent of sodium rubidium tartrate cut perpendicular to the y or b axis of the crystal with its length and an angle of 45 degrees from the x or a axis of the crystal. It will be apparent that the mixture improves the performance of the device by extending the low range temperature coefficient portion to approximately 19° C. as against approximately 24° C. for the sodium potassium tartrate. The result shows that the over-all temperature coefficient has also been reduced from a magnitude of approximately 500 parts per million per degree centigrade for the composite device for sodium potassium tartrate (Rochelle salt) to about .1 of that magnitude. Obviously, this temperature coefficient may be made as small as is desired by the method of this invention.

What is claimed is:

1. A composite vibrator comprising a piezoelectric element, a non-piezoelectric loading means fixedly attached thereto whereby the device may vibrate as a unit, the temperature coefficients of frequency of the piezoelectric element and of the non-piezoelectric loading means having opposite signs and the density and dimensions of the loading means being so predesigned with respect to that of the piezoelectric element and with respect to the relative magnitudes of the temperature coefficients of frequency that in the region of a desired frequency of vibration the device is resonant at a substantially constant frequency irrespective of temperature variations.

2. In combination, a piezoelectric element consisting of sodium potassium tartrate, loading means fixedly secured thereto, the temperature coefficients of frequency of the sodium potassium tartrate and the loading means having opposite signs and the dimensions and masses of the tartrate and loading means being so predesigned that together the tartrate element and the loading means constitute a unit having a desired resonance frequency and a subsantially zero temperature coefficient of frequency.

3. A composite resonator comprising elements of Rochelle salt and of a metallic alloy fixedly secured together whereby they may vibrate as a mechanical unit the temperature coefficients of frequency of the elements having opposite signs and the temperature coefficients, densities and dimensions of the respective elements being so related numerically that the resonator vibrates at a desired frequency at a given temperature and exhibits a substantially zero temperature coefficient of frequency over a relatively wide temperature range in the region of the given temperature.

4. A piezoelectric device comprising a sodium potassium tartrate element and means for causing the device to vibrate at a desired resonance frequency with a substantially zero temperature coefficient of frequency consisting of loading elements fixedly secured to the tartrate element to vibrate therewith in unitary manner, the magnitudes of the density, dimensions and temperature coefficients of the elements being so predesigned as to preclude frequency variations from the desired resonance frequency over a relatively wide range of temperatures and the characteristic impedances of the tartrate element and of the loading elements for longitudinal vibration being matched to eliminate reflections whereby the composite structure tends to vibrate in longitudinal manner as a unitary structure having a single resonance frequency.

5. A bar adapted for longitudinal vibration at the triple frequency harmonic corresponding to its length comprising three longitudinally aligned portions of substantially equal length the middle portion consisting of piezoelectric material and the other portions of metallic material fixedly attached thereto, the portions being so proportioned in mass as to yield a zero temperature coefficient of frequency for the bar vibrating as a whole.

6. The combination of claim 5, characterized in this that means are provided for mounting the bar at nodal points for the triple frequency harmonic mode of vibration.

7. A piezoelectric resonator comprising an element cut from an isomorphous crystal of sodium potassium tartrate, the material of the crystal including approximately .3 per cent of sodium rubidium tartrate.

8. A composite vibrator comprising a bar-shaped member of an isomorphous crystal of sodium potassium tartrate including a small percentage of a substance which lowers the effective Curie point and similar bars of an alloy cemented at the ends of the bar of crystalline material to enable the composite bar to vibrate longitudinally as a unit the alloy having a temperature coefficient of frequency of opposite sign to that of the crystalline material and the lengths of the crystalline and alloy portions being such as to render the temperature coefficient of frequency of the composite device substantially equal to zero.

9. A device comprising three longitudinally aligned bars cemented together at their ends to constitute a composite longitudinal vibrator, the central bar comprising piezoelectric material and the other two bars consisting of metal, the relative lengths of the bars being so chosen that the loops of maximum motion occur at the junction points whereby the tensional stress at the joints is a minimum and means for supporting the device at two nodal points which are relatively widely spaced whereby the damping introduced by the supporting structure is rendered relatively small.

10. A composite vibrator comprising a pair of longitudinally aligned bars connected by an integral web member of relatively small cross-sectional area and a piezoelectric member extending between the bars and cemented thereto at an end of each whereby the longitudinally aligned piezoelectric member and bars are adapted to vibrate longitudinally as a unit.

11. A piezoelectric element of sodium potassium tartrate including a small percentage of sodium rubidium tartrate to reduce the Curie point of the element.

12. A piezoelectric element of the sodium potassium tartrate type including a small percentage of an isomorphous substance sufficient to shift the Curie point below the range of ordinary room temperatures and loading means fixedly secured thereto, the density and dimensions of the loading means being so designed and its temperature coefficient being so chosen that the combined structure vibrates at a desired resonance frequency with a substantially zero temperature coefficient of frequency.

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