(54) Title: SYSTEMS WITH INCREASED INFORMATION RATES USING EMBEDDED SAMPLE MODULATION AND PREDISTORTION EQUALIZATION

(57) Abstract

A system is described which achieves an increased rate of transmission by transmitting multiple symbols in one symbol time. The information in the overlapping symbols can be recovered if the symbol used for transmission is chosen so that a subset of the samples representing the symbol has an inverse. The construction of suitable symbols is illustrated. Effective operation of the system depends on equalization which will recover the baseband signal shape. Methods of equalization are depicted which are suitable for use with this system including equalization realized by predistorting the signal at the transmitter so that it arrives at the receiver undistorted. The invention is applied to baseband and passband systems and the application of predistortion equalization to the fading passband channel is described.
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Systems with increased information rates using
Embedded Sample Modulation and Predistortion Equalization

This invention relates to increasing the number of bits which can be transmitted in one cycle of bandwidth. Bandwidth efficient modulation methods have been studied in detail. These studies compared the number of bits of information which can be transmitted in one carrier cycle. The measure used in these comparisons is bits/hertz. Quadrature amplitude modulation, referred to as QAM, is often ranked as the most efficient modulation method. The invention described here transmits significantly more bits/hertz than QAM.

Modems are very often compared as to their information transmitting ability measured in bits/second. In the transmission of data over voice grade telephone lines the highest speed modems, measured in bits/second, achieve their high speed by using modulation methods known to have a high measure of bits/hertz. The invention described here can be used to increase the rate at which data can be transmitted over voice grade lines.

The frequency spectrum available in the large cities for cellular systems is limited. This results in congestion at rush hours in the existing analog systems. These analog systems are being supplemented by digital systems in order to increase the number of calls the system can support during busy hours. Using the modulation method described here the number of subscribers which can be accommodated could be further increased in the digital systems presently being deployed.

Accordingly, the invention describes a novel modulation and demodulation method which is capable of transmitting and receiving significantly more bits/hertz than any of the
conventional modulation methods.

Bandwidth efficient, reliable communication systems, in addition to using efficient modulation methods, use equalization techniques. To maximize the number of bits/hertz the modulation apparatus described here should be coupled with an equalization technique which carefully preserves the signal shape. Techniques which are suitable are described. In some cases these are novel in their own right and can be applied when conventional modulation apparatus is used. Accordingly, the invention describes equalization techniques which can be used in conjunction with the invention, some of these equalization techniques having novelty in their own right.

Voice grade telephone lines are one type of baseband channel. The application of the invention to baseband channels is described. Radio channels are often referred to as passband channels, as opposed to baseband channels. Passband channels use a carrier and some means of moving the baseband signal to the carrier frequency, followed by recovery of the baseband signal at the receiver. The problems involved in applying the modulation method to passband channels and providing equalization are somewhat different from those encountered in applying the invention to baseband channels. Accordingly, the invention shows how the modulation method can be combined with equalization techniques to achieve an increased information rate when using passband channels.

Communication systems use a sequence of symbols to transmit the data. Each symbol occupies a time slot, called a symbol time, and the symbol in each symbol time is modified in amplitude or phase in accordance with the incoming information. In many systems a Nyquist pulse is used as a basic symbol. The motivation for the use of this pulse comes from the lack of interference with adjacent pulses because of the way in which the zero crossings are distributed. Equalization involves removing the interference between symbols introduced by the channel. The amount of information which is carried by the system is determined by the maximum symbol rate which can be sent without the symbols overlapping, as well as the number of different symbols which can be used in a symbol time slot. Partial response signalling is a step in the direction of transmitting more information by putting the symbols closer together and allowing some controlled intersymbol interference which is removed at the receiver. In the modulation method described here multiple overlapping symbols are transmitted in a symbol time with interference extending over a number of symbols. This increases the number of bits transmitted in a symbol time without significantly increasing the bandwidth, thereby increasing the number of bits/hertz. In order to recover the information from the overlapping symbols the shape used in place of a Nyquist pulse is a band limited function, having a set of samples called embedded samples, whose Z transform has an inverse. An embedded sample inverse filter is the filter which when convolved with the embedded samples gives an impulse. The demodulator locates the embedded samples in the continuous wave which arrives at the
receiver then passes them through the inverse filter to recover the transmitted information from the overlapping symbols. This method of modulation is referred to as embedded sample modulation.

Equalization techniques are used to increase the rate at which information can be transmitted by removing the interference between adjacent symbols. These techniques usually involve filtering at the receiver. The novel equalization method described here involves predistorting the waveform at the transmitter so the waveform arriving at the receiver is undistorted.

In one embodiment of the invention a waveform is stored in memory which is referred to as a composing function. Modulation is accomplished by transmitting overlapping scaled versions of the composing function, the scaling being done in accordance with the information to be transmitted. It is easy to show this modulating method can be used to generate most forms of baseband signals required, including for example, the baseband I and Q channels required for Digital Cellular.¹

Application of the invention to Digital Cellular involves sending a test pattern on the I and Q channels. From the response two composing functions are generated which when used at the transmitter to generate predistorted I and Q baseband signals, causes undistorted I and Q baseband signals to arrive at the receiver.

It is shown that if the composing functions required at the receiver are continuously updated then the fading which is normally present when the receiver moves is eliminated. Diversity reception normally used to eliminate fades is not required. This equalization method is called predistortion equalization.

Predistortion equalization by itself can significantly increase the information rate transmitted. When used in combination with embedded sample modulation the information rate can be further increased. Accordingly, the invention described here includes a description of predistortion equalization as an invention, embedded sample modulation as an invention, as well as the combination.

The invention can be further described with reference to the following drawings.

DESCRIPTION OF THE DRAWINGS.

FIG. 1 : System Block Diagram.
FIG. 2 : Construction of a Composing Function with Embedded Samples for the Baseband Channel.
FIG. 3: Embedded Sample Equalizer: Inverse Filter for the Baseband Channel.

FIG. 4: Information to be Transmitted on a Simple Waveform.

FIG. 5: Composing Function Generation of a Simple Waveform and Reception of the Simple Waveform Using an Inverse Filter.

FIG. 6: Embedded Sample Location by Interpolation.

FIG. 7: Transmission and Reception of a Random Wave Using a Composing Function and an Inverse Filter.

FIG. 8: Power Spectrum of the Random Wave Required for Spectral Estimates.

FIG. 9: Channel Response Measurement: Construction of a Baseband Carrier Equalizer for a Baseband System.

FIG. 10: Equalization of a Baseband Signal by a Baseband Carrier Equalizer.

FIG. 11: Predistortion Equalization of the Baseband Channel Using a Dipulse Composing Function.

FIG. 12: Modulator for the I and Q Channel of a Passband System.

FIG. 13: Signal Constellation Used in Digital Cellular.

FIG. 14: Composing Function Generation from the Frequency Specification in Digital Cellular.

FIG. 15: Encoding of Simple I and Q Channel Baseband Waves for Digital Cellular.

FIG. 16: Construction of a Composing Function with Embedded Samples and an Inverse Filter for a Passband System.

FIG. 17: Transmission and Reception of a Random Wave in a Passband System Using a Composing Function and an Inverse Filter.

FIG. 18: Equalization of the Baseband Signals by a Baseband Carrier Equalizer.

FIG. 19: Random Wave Power Spectral Estimates Required in Truncating the Equalizing Filters Used at the Receiver.

FIG. 20: Dual Predistortion Composing Functions Used at the Transmitter. Receiver Response to the Predistortion Composing Functions.

FIG. 21: Dual Composing Function Predistortion Equalization for the Passband Channel.

FIG. 22: Channel Test Signal Distortion Caused by Transmitter Motion.

FIG. 23: Representative I and Q Signals Used to Demonstrate Test Signal Extraction.

FIG. 24: Power Spectral Estimates Required to Estimate Denominator Truncation.

FIG. 25: Recovered Test Signal Response.

DESCRIPTION OF THE INVENTION.

The invention is shown in block diagram form in FIG. 1. The modulator is comprised of a multilevel generator 12, a baseband modulator 16 and a composing function 14. The multilevel generator 12 converts the input data 10 to a multilevel signal. The baseband modulator 16 generates scaled versions of composing function 14 in accordance with the input from the multilevel generator 12. In one embodiment scaling is done by multiplication of the composing function by the input level. When predistortion
equalization is used the composing function is usually changed, therefore it is
convenient to store the composing function in ram memory, otherwise it can be stored in
rom. The baseband modulator 16 sums the time shifted and scaled composing functions to
generate the output baseband waveform 18. In baseband systems the baseband waveform 18
is then transmitted over the baseband channel, for example a voice grade telephone line, and
arrives at the demodulator as input baseband waveform 19. In passband systems two
baseband signals are used, referred to as the I and Q signals. These output baseband
waveforms 18 are shifted up to the carrier frequency and at the receiver shifted down to
become baseband waveforms 19. In passband systems with predistortion equalization
dual composing functions 14 are used.

The composing function 14, in a conventional modulator might be a Nyquist pulse or a
raised cosine pulse. One method of deriving the composing function from a raised cosine
pulse is discussed in connection with FIG.2. The composing function is shown to contain a
set of embedded samples whose Z transform has an inverse. When the embedded samples
associated with a scaled value of the composing function are passed through this inverse a
single pulse is generated at the output whose amplitude is dependent on the scaling factor
applied to the composing function. It follows from the superposition property of linear
systems that if two scaled composing functions are shifted in time relative to one another
and added and the embedded samples in the sum passed through the inverse filter, the output
will be two pulses separated in time whose relative amplitudes will depend on the original
scaling.

The demodulator in FIG.1 is comprised of the A/D converter 23, baseband carrier
equalizer 20, embedded sample locator 24, embedded sample equalizer 22 and the inverse
filter 26 which delivers the recovered input data 28. The baseband carrier equalizer is
required in many passband and baseband systems, however, in some cases it may be
possible to operate without this equalizer if the waveform distortion introduced by the
channel is very small. If all the equalization is done by predistortion then no baseband
carrier equalizer is required. The embedded sample locator 24 must locate the subset of
samples which are the embedded samples. One way of accomplishing this requires a signal
25 which is fed back from the embedded sample locator to change the sample timing. This
method and others are described in the operations section. The inverse filter 26, operates
on the embedded samples producing a string of pulses whose amplitudes are related to the
multilevel signals generated by the multilevel generator 12.

The equalizer 20 for passband systems described herein operates on the received I and Q
baseband waveforms to remove the distortion and recover the transmitted I and Q baseband
signals. The filters required for equalization are determined from the response of the
channel to specified test signals. The analysis to determine the equalizer 20 is extended to
determine the predistortion composing functions required when predistortion
equalization is done at the transmitter. Indeed the analysis shows that the equalization can be put at either end or distributed between the transmitter and receiver. A baseband carrier equalizer which does all the equalization is called a full baseband carrier equalizer. Similarly a predistortion equalizer which does all the equalization is called a full predistortion equalizer.

The equalization errors remaining after carrier equalization and embedded sample location can be cleaned up in the embedded sample equalizer 22. When there are residual errors the inverse filter output may have a prepulse and postpulse around the main pulse. A specific method of equalization which suppresses these side pulses is described in the operation section and is called embedded sample equalization. This equalizer can be combined by convolution with the inverse filter so that the side pulses are suppressed. In a system where all the equalization is done by the embedded sample equalizer it is referred to as full embedded sample equalization.

OPERATION OF THE BASEBAND INVENTION.

The composing function 14, of FIG.1 is derived from the raised cosine 30 of FIG.2. The raised cosine, has a Z transform defined as \( H(z) \). The zeros of \( H(z) \) are shown in the Z plane 34 of FIG.2. Some of the zeros of \( H(z) \), are on the unit circle 35, such as the zero 36. Whenever there are zeros of \( H(z) \) on the unit circle the function \( 1/H(z) \), called the inverse of \( H(z) \) cannot be determined. Therefore to create a function \( H(z) \) with an inverse each zero on the unit circle is replaced by a zero inside the unit circle distance \( r, 39 \) from the origin and another zero outside the unit circle distance \( 1/r \) from the origin. This construction is shown in FIG.2. Here the pair of zeros 37 and 38 replace the zero 36 of FIG.2. The Z transform of the desired composing function is deduced by forming a polynomial in Z, in factored form, using all the zeros shown as 41 and multiplying these factors together to obtain the Z transform polynomial of the desired composing function. Composing function 40, shown in FIG.2 is the result. The Z transform of this composing function will not have any zeros on the unit circle, and so this composing function will have an inverse. A subset of samples, from the composing function 40 is shown as 42. This subset is comprised of every second sample from the composing function 40. These samples 42 are termed embedded samples. Experience has shown that if the zeros of the composing function 40 are all off the unit circle, then the zeros of a Z transform \( HE(z) \) of the embedded samples 42 will be further off the unit circle. The zeros of \( HE(z) \) are shown as 44 in FIG.2.

Since the function \( HE(z) \) has all its zeros off the unit circle it has an inverse. The power series expansion of \( 1/HE(z) \) is the desired inverse. The power series will converge if there are no zeros on the unit circle. The rate of convergence is determined by the distance \( r, 39 \) of FIG.2. The closer \( r \) is to the unit circle the greater will be the number of terms.
required in the inverse. A truncated power series represents an FIR filter and is the inverse filter if when convolved with $HE(z)$ an impulse is generated.

One method of generating the inverse is described in the literature. The method used here is a modification of this reference and is shown in Fig. 3.

The method involves finding a first estimate of the inverse and passing embedded samples 42, of Fig. 2 through this inverse and examining how close the result is to an impulse. The process is repeated until an impulse is obtained. Fig. 3 shows the inverse estimate 52 at the end of the first iteration 56. The convolution of the embedded samples 42, and the inverse 52 is shown as $R(z)$ 54, in Fig. 3. The residual error 58 is measured by the number and size of the impulses surrounding the main pulse. The third iteration 66 and the fifth iteration 76 show decreasing residual errors 68 and 78 respectively. The inverse 72 is the desired result.

The operations performed to obtain the first estimate 52 of the inverse 72 are as follows. The peak and the samples to the right of the peak of the embedded samples 42 of Fig. 2 form a decreasing function. Call the $Z$ transform of these samples $MF(z)$. The zeros of such a function tend to be inside the unit circle in which case the inverse of $MF(z)$ can be determined by long division and truncation. The samples to the left of the peak and including the peak form an increasing function. Use these samples to form a $Z$ transform $MR(z)$. Such a function tends to have all its zeros outside the unit circle. The inverse of $MR(z)$ can be determined by time reversal, which moves all the zeros inside the unit circle, long division, truncation, and time reversal. The first inverse estimate 52 was the convolution of the inverse of $MF(z)$ and the inverse of $MR(z)$. $R(z)$ 54 is the result of passing the embedded samples 42 through the inverse 52. As long as the input function has no zeros on the unit circle it has been found by experience that this process will tend to reduce the size of the samples surrounding the main pulse. This is evident if $R(z)$ 54 is compared with the embedded samples 42. For the second iteration the process was repeated with $R(z)$ 54 as the input. The inverse of $MR(z)$ and $MF(z)$ were convolved with the inverse 52 to obtain a new inverse estimate. The embedded samples 42 were then passed through this inverse to generate a new $R(z)$. The third iteration used this new $R(z)$ as the input, and so forth. In summary the inverse 72 when convolved with the embedded samples 42, generated the impulse 74.

A simple example of the system shown in Fig. 1 is shown in Fig. 4 and Fig. 5. Input binary bits 79 associated with the word Hah! were generated (Fig. 4). This binary string was converted to the octal string 81. This octal string 81 represents the multilevel samples with levels from 0 through 7. Next a prepulse was added to the information portion of the wave consisting of pulse of amplitude 7 followed by 3 zeros, a pulse of amplitude 7 followed...
by two zeros. This signal with prepulse is shown as 82 in FIG.4.

Although it is not an essential part of the invention it is advantageous to use positive and negative composing functions as this will tend to concentrate the power spectrum. The composing function sign is shown as 83 in FIG.4. It is clear that the first three composing functions will have positive signs and after that the function will change sign with period 10. Since a composing function will be sent out separated by the embedded sample time, which is this case is twice the sample separation in the composing function, it follows that the generated baseband signal will have a fundamental frequency whose period is 20 sample times. In summary the baseband wave was generated by sending out scaled values of the composing function 40, scaled according to 82 in FIG.4 and with signs generated in accordance with 83. A new scaled composing function was generated every two sample times and the baseband modulator 16 of FIG.1 added the scaled and time shifted composing functions to generate the output wave.

The modulated wave 87 is shown in FIG.5. At the receiver the embedded samples 89, in this case every second sample, have been extracted and are shown in FIG.5. These samples have been applied to the inverse filter 72 and the demodulator output 86 is shown in FIG.5. The allowed levels 88 are comprised of 7 positive levels, zero and seven negative levels. The prepulse inverse 84 establishes an amplitude reference at the receiver. It is clear that an output is generated every second sample time as expected, and the output levels, with associated sign, reproduce the encoded data 82, shown in FIG.4.

The normal period of the modulated wave 87, excluding the positive and negative prepulses will be 20 sample times as discussed above. This period can also be estimated from FIG.5. During this period 10 information bearing impulses arrive each carrying 3 bits (8 levels). Therefore in one cycle of the modulated wave 87, 30 bits are transmitted thereby realizing a rate of 30 bits/hertz. QAM is regarded as the most bandwidth efficient classical modulation method. If 8 levels are allowed QAM will transmit 6 bits in one cycle, requiring 5 cycles to send the same information that is sent here in one cycle.

The embedded sample locator 24 of FIG.1 must locate the embedded samples. One method involves preceding the information bearing portion of the wave with a bipolar pulse whose zero crossing defines a reference time position, and locating the scaled composing functions and therefore the embedded samples at a known time position from this reference. The input baseband waveform 19, in FIG.1 will usually be a continuous wave and after sampling a bipolar prepulse 90 might appear as shown in FIG.6. If the continuous wave had been sampled at the zero crossing the sampled bipolar pulse would have the samples shown as 92 in FIG.6. It is necessary to determine the time shift required to recover samples of this type. This time shift must then be applied to the information bearing portion of the wave to establish the value of the embedded samples. Some form of interpolation is
required. Since the discrete Fourier transform (DFT) was used for signal processing an interpolation technique based on the DFT was chosen as it minimized the signal processing required.

The DFT and IDFT are given by

\[ X_p(k) = \sum_{n=0}^{N-1} x_p(n)e^{-j(2\pi/N)nk} \]

\[ x_p(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_p(k)e^{j(2\pi/N)kn} \]  

(1)

In the usage here \( x_p(n) \) is a real periodic sequence and \( X_p(k) \) is a complex pair for every frequency \( k \).

Interpolation is accomplished by relating the DFT to the standard Fourier transform. The exponential term in the DFT above has a corresponding term in the direct Fourier transform of the form

\[ e^{-j\omega t} \]  

Whenever the Fourier transform is applied to samples from a continuous function, time and frequency are quantized. In this case relation (2) above becomes

\[ e^{-j2\pi f \Delta t} \]  

(3)

where \( \Delta f \) and \( \Delta t \) are real time and frequency increments. The exponential form above becomes the same as the exponential form in the DFT if

\[ \Delta f \Delta t = 1/N \]  

(4)

In the application to voice grade telephone lines which follows \( \Delta f \) is taken as 100 cycles and \( \Delta t \) is taken as 50 microseconds. This gives a value for \( N \) of 200.

To perform interpolation the DFT of the samples from the continuous wave was determined. It is easy to show this amounts to finding the direct Fourier transform with the values of \( \Delta f \) and \( \Delta t \) given above. If the IDFT is now taken the original samples will be recovered. Consider finding the IDFT with \( n \) taking on an offset from the integer values of 1, 2, etc.

It is easy to show that this amounts to taking the inverse Fourier transform at a time point offset from the existing samples. For example if \( n \) in the IDFT takes on a value of 1.40 then the IDFT will provide an interpolated value of the samples at a point 40 percent of the way between the first and second sample.

The application of this interpolation method is illustrated in FIG.6. The smallest sample of waveform 90 of FIG.6 was located. The DFT of the waveform, made up of the smallest sample and 15 samples on either side, was determined. Next the offset in the sample point was
estimated which would make the sample 98, following the smallest sample 96, the same size as the sample 94. This is a simple linear interpolation as shown in Fig.6. The IDFT was then used to determine the value of the samples on either side of the smallest sample using the estimated offset. The process was repeated until the samples on either side of the smallest sample were equal. In this case the process involved 6 estimates -0.532, -0.336 -0.432, -0.384, -0.408, -0.396. These values converge on the the shift of +0.40 used to generate the samples 90 from samples 92. The IDFT with an offset of -0.396 was then evaluated for all the samples and the result is shown as 99 in Fig.6. This is a very good approximation to the samples 92.

To evaluate the embedded samples in the following information portion of the wave the DFT of a wave segment is first determined then the IDFT determined using the established offset value. This method of locating the embedded samples is referred to as offset evaluation and waveform interpolation.

The offset can be used in another way to establish the embedded samples. Knowledge of the offset can be used to change the sampling time of the input A/D converter 23 of Fig.1, using feedback loop 25. In this case no interpolation of the information bearing portion of the wave is required. This method of establishing the embedded samples will be referred to as offset determination and sample point adjustment.

In cases where a prepulse is not available the offset can be determined by other methods. Eye patterns are a common method of sample point adjustment. In Fig.5 the end point 85 of the output sample will tend to lie substantially on the desired output level if sampling is done when an embedded sample is present. If the sampling point is moved away from the embedded samples the end points will move away from the desired levels and the eye is said to close. It follows that a systematic adjustment of the sampling time followed by examination of the eye pattern and continued adjustment until the eye is open will locate the embedded samples. This method of establishing the offset requires a string of data and some of the data will be lost while the sampling time is adjusted to open the eye. A variant of this approach uses a fixed segment of the signal stored in memory. The interpolation technique described above can be used to establish sets of samples. Each of these sets will have a fixed offset from the given samples. The sets are passed through the inverse filter. The offset associated with the set which creates the largest eye opening is the offset to be used. These approaches which use the eye pattern to locate the embedded samples will be referred to as embedded sample location by eye pattern adjustment.

RECTIFIED SHEET (RULE 91)
EQUALIZATION OF THE BASEBAND CHANNEL.

The inverse filter 26 of FIG. 1 operates on the set of embedded samples to recover the encoded information. It follows that the equalizer must remove any signal distortion introduced in the embedded samples by the transmission medium. The equalization method described here acts to remove distortion on the total baseband wave thereby removing distortion of the embedded samples.

The baseband carrier equalizer 20 of FIG. 1 is described using a voice grade channel as the transmission medium. Although described using a specific medium the method is applicable to any baseband channel.

The composing function 14 of FIG. 1 is shown in sample data form as 14 in FIG. 7. The inverse filter 26 of FIG. 1 is shown in sample data form as 26. In finding the inverse filter from the embedded samples the distance, r, 39 in FIG. 2, was assigned the value 0.70. FIG. 7 shows a random wave 102, generated by the baseband modulator 16 of FIG. 1 by scaling and addition. The modulation values 100 were used in generating the the random wave 102. Embedded samples 104 from the random wave are shown in FIG. 7. FIG. 7 shows the recovered samples 106 generated by passing the embedded samples through the inverse filter 26. The recovered samples 106 clearly recover the modulation values 100. The number of bits/hertz is 30 as discussed previously. This can be confirmed by observing that 150 bits are transmitted in the 5 cycles of the random wave 102.

The fundamental frequency of the random wave 102 was chosen as 1,000 hertz. Since there are 20 samples in one cycle, the time between samples Δt is 50 microseconds. Δf was chosen as 100 cycles therefore from equation (4) N=1/Δf*Δt=200. Using this value of N the DFT of the random wave 102 was determined using the first of equations (1). The power spectrum 108 associated with this DFT is shown in FIG. 8. The power spectrum is formed by taking the square root of the sum of the squares of each complex term in the DFT.

From FIG. 8 it is estimated that an equalizer would likely reproduce the random wave 102 if equalized from 100 cycles, 110 to 4Khz, 114. This is a first estimate and can be confirmed after testing on the random wave.

Consider representing the random wave 102 by the Z transform X(z). Represent the voice grade channel by H(z) and the output response when X(z) is applied to H(z) by Y(z). Then
\[ Y(z) = H(z)X(z) \]  
(6)

\[ X(z) = Y(z)/(1/H(z)) \]  
(7)

\[ H(z) = Y(z)/X(z) \]  
(8)

Equation (7) is a classic relationship which states that given the distorted signal \( Y(z) \) the input \( X(z) \) can be determined by multiplying the distorted signal by \( 1/H(z) \). Since every set of samples represented by a Z-transform has a DFT, the inverse of \( H(z) \) represented by \( 1/H(z) \) is determined by finding the DFT of the samples, then for each frequency multiplying the value of the DFT by its complex conjugate. In most cases once calculation is initiated in the DFT domain it is usually convenient to multiply and add DFT's to evaluate relationships expressed in Z-transform form.

Before inverting \( H(z) \) consider measuring \( H(z) \) using equation (8). The operations to be performed are then; apply a signal \( X(z) \), measure the response \( Y(z) \), invert \( X(z) \), multiply by \( Y(z) \). FIG.9 shows these operations in the DFT domain. Here the scale 123 is 100 hertz/div. The applied signal \( X(z) \) is shown as 116. The power spectrum of the DFT of this signal is shown as 118. The power spectrum of the DFT of \( Y(z) \) is shown as 120. The DFT of \( Y(z) \) was determined by multiplying the DFT of \( X(z) \) by the DFT of a known line whose frequency and phase were known in 100 hertz increments from 100 hertz to 4KHz. The inverse of the DFT of \( X(z) \) was then multiplied by the DFT of \( Y(z) \) to obtain the DFT of the channel whose power spectrum is shown as 122. Finally the DFT of \( 1/H(z) \) was found by inverting the DFT of the channel. The power spectrum associated with the DFT of the inverse is shown as 124. In practice the DFT of \( Y(z) \) is determined by digitizing the response \( Y(z) \) and finding the DFT of this response. This response will usually be an extended damped ringing signal.

The applied signal 116 has a period of 10 samples therefore a fundamental frequency of 2000 hertz. This is confirmed in FIG.9 where the power spectrum of the DFT has a peak in the neighborhood of 2000 hertz. The applied signal has a significant frequency content at all the frequencies in the range of interest from 100 cycles to 4KHz. This is important if inversion is to be accomplished easily.

FIG.10 shows the performance of the equalizer whose power spectrum is shown as 124.

The standard test wave 102 of FIG.7 is reproduced as 130 in FIG.10. Waveform 130 is the signal applied to the channel whose DFT has the power spectrum 122 of FIG.9. The signal received at the output of the channel is shown as 132 in FIG.10. Waveform 134 is the output after applying the equalizer whose DFT has the power spectrum 124 of FIG.9 to the received signal 132. A comparison of 130 and 134 indicates the equalizer has removed substantially all the distortion introduced by the channel. Equalizers of this kind
are usually employed at the receiver. Consider next achieving equalization at the transmitter by predistorting the composing function so the signal arriving at the receiver is undistorted.

The composing function $14$ of FIG. 7 is to be predistorted so as to arrive at the receiver in undistorted form. Represent this composing function by $X(z)$ and consider the equality

$$X(z) = (X(z)/(1/H(z)))/H(z)$$

This can be interpreted to say that the signal $X(z)/(1/H(z))$ will arrive at the receiver undistorted when passed through the channel $H(z)$. The DFT of $1/H(z)$ is known. Its power spectrum is shown as 124 in FIG. 9. The DFT of the composing function $14$ of FIG. 7 can be found, however these DFT's cannot be multiplied together as the DFT of $1/H(z)$ is not defined at zero frequency and the DFT of the composing function $14$ of FIG. 7 has a DC component. The solution to this problem is shown in FIG. 11.

The composing function $140$ is used in place of composing function $14$. Composing function $140$ is made up of two superimposed functions of the same shape as $14$, separated by 10 sample intervals with the second pulse inverted. When waves were generated using $14$ a sign reversal was introduced every 10 sample intervals. It follows that the composing function $140$ when used in a first time slot will introduce into a following slot the proper shape of signal for that slot; the proper sign but not likely the proper amplitude. When composing function $140$ is used to generate the signal in the following slot, the signal previously introduced is considered, and the scaling value modified so the amplitude required in the following slot is generated. This procedure is repeated for all the required time slots. The generated wave usually has a non-information-bearing tail. Waveform $142$ was generated in this manner. With the exception of the non-information-bearing tail it is the same as waveform $102$ of FIG. 7.

The composing function $144$ of FIG. 11 was obtained by multiplying the DFT of $140$ by the DFT of the inverse of $H(z)$ whose power spectrum is shown as 124 in FIG. 9 and taking the IDFT. This multiplication is now possible and meaningful as $144$ only has significant spectral components in the region where the inverse DFT is defined. Waveform $146$ was generated using composing function $144$, with scaling and addition according to the modulation shown as 100 in FIG. 7. Finally the signal $146$ was passed through the channel whose spectrum is shown as 122 of FIG. 9. The waveform which arrives at the receiver is shown as $148$ in FIG. 11. Comparison with waveform $142$ which is the undistorted signal shows that the arriving waveform is substantially undistorted.

Although the waveform $148$ arrives substantially undistorted this example illustrates the possibility of time domain aliasing. The waveform $144$ is 200 samples long and $N=200$. Furthermore waveform $144$ does not show any region where the value is effectively zero.
In practice, aliasing can be avoided by the choice of $\Delta f$ used in analyzing the time response when the channel characteristic is being measured. From equation (4) it is clear that if $\Delta f$ is made smaller then $N$ will be larger, providing more samples in the IDFT for the time response to decay. This option was ruled out in this example as the voice channel data was available at fixed frequency increments of 100 hertz.

Fig. 10 demonstrates baseband carrier equalization at the receiver using the DFT whose power spectrum is 124 of Fig. 9. Fig. 11 demonstrates how to perform predistortion equalization, the composing function being determined from this same DFT. Splitting the equalization between the transmitter and receiver is accomplished by expressing each vector component of this DFT as the product of two vectors. For example consider splitting the equalization equally between the transmitter and receiver. If this DFT has a term represented by $r$ and $\theta$ at a particular frequency then the component filters will both be the same with component $\sqrt{r}$ and $\theta/2$. The predistortion composing function at the transmitter will be generated as described previously but with this new filter.

Baseband carrier equalization as described above is supplemented by the embedded sample equalizer 22 of Fig. 1. The random wave 142 of Fig. 11 has at the start of the wave a prepulse of the type 140. The embedded sample equalizer operates on the distortion of this prepulse after passage through the inverse filter. Fig. 7 pulse 105 shows the shape of this prepulse after passage through the inverse filter. When residual equalization is required there will be small samples before and after the pulse 105. Since the pulse 105 is made up of the sum of two identical waves it is easy to split it into the sum of a positive pulse and a negative pulse. This positive pulse will appear much the same as 64 in Fig. 3 showing the residual error 68. The process described in connection with finding the inverse filter can be applied to find a filter which will reduce this residual error. This filter can be convolved with the inverse filter. This equalization can be regarded as a fine tuning of the inverse filter. In summary, the procedure described in Fig. 3 to find the inverse filter is the procedure required to implement the embedded sample equalizer 22.

In the example of Fig. 7 a 1000 hertz baseband wave was used with 8 levels and a rate of 30 kilobits/sec was achieved. The standards for Voice Band Modems use various forms of Trellis coding to permit reliable transmission of more levels. If 32 levels are transmitted then one can expect to receive 16 levels reliably on some voice grade lines. If Trellis coding is applied in the same way here the rate would be 40 kilobits/sec. If the baseband carrier is increased to 1600 hertz then the rate would be 64 kilobits/sec.
OPERATION OF THE PASSBAND INVENTION.

The use of the invention in Digital Cellular Systems is described. It will be clear the invention can be applied as described to any passband system which uses a linear modulation method, i.e. the frequency shifts up to the carrier and down from the carrier to baseband are accomplished by multiplication and filtering.

Fig. 1 has been used to describe the baseband invention. The same diagram is used to describe the passband invention. In the passband case two output baseband waveforms referred to as I and Q are generated by the baseband modulator of Fig. 1. At the receiver the two baseband waveforms are received. In the passband case the problems of embedded sample location and equalization have solutions analogous to the solutions in the baseband case.

Fig. 12 shows a standard modulator used to generate a radio signal \( x(t) \). This standard modulator is used to generate the modulation for Digital Cellular, referred to as \( \pi/4 \) shifted, differentially encoded, quadrature phase shift keying (DQPSK).

In the implementation used here the baseband signal \( I(t) \) is generated by adding scaled and time separated versions of a composing function related to \( g(t) \), the scaling being determined by the value of \( I_k \). This implements the transmit filter. \( Q(t) \) is generated in a similar manner, the scaling being determined by \( Q_k \). It will be shown that predistortion equalization, analogous to that described for baseband, can be accomplished by making the transmit filter, different from the transmit filter. In this invention this is accomplished in practice by making the composing function associated with the transmit filter different from the composing function associated with the transmit filter. In a standard Digital Cellular system the same composing function is used in both channels and will be referred to as \( G(z) \). Consider first the use of the modulator of Fig. 12 to generate a standard Digital Cellular signal, and the recovery of the encoded data at the receiver.

The transmitter shown in Fig. 12 accepts bits \( 202 \). These bits are encoded two at a time and determine a value for \( I_k, Q_k \). The transmitter filter then generates an output \( I(t) \) given by
\[
I(t) = \sqrt{2} I_k g(t)
\]
and similarly for the Q channel. \( I(t) \) and \( Q(t) \) are then shifted in frequency to the carrier frequency \( \omega_c \), by multiplication, and added to generate the real time signal \( x(t) \). This \( x(t) \) arriving at the receiving antenna is shifted down to baseband by multiplication and filtering to recover \( I(t) \) and \( Q(t) \).
Digital data sequences are encoded as shown below:

\[
I_k = I_{k-1} \cos[\Delta \Phi(x_k, y_k)] - Q_{k-1} \sin[\Delta \Phi(x_k, y_k)]
\]

\[
Q_k = I_{k-1} \sin[\Delta \Phi(x_k, y_k)] - Q_{k-1} \cos[\Delta \Phi(x_k, y_k)]
\]

\(x_k\) and \(y_k\) represent the input data and the \(I_k\) and \(Q_k\) represent the peak amplitude of \(I(t)\) and \(Q(1)\) respectively.

\(x_k\) and \(y_k\) are related to \(\Delta \Phi\) by the following encoding/decoding table:

<table>
<thead>
<tr>
<th>(x_k)</th>
<th>(y_k)</th>
<th>(\Delta \Phi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>(-\frac{3\pi}{4})</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>(\frac{3\pi}{4})</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>(\frac{\pi}{4})</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>(-\frac{\pi}{4})</td>
</tr>
</tbody>
</table>

From the above it is clear each incoming pair \(x_k, y_k\) is associated with a unique phase change. It is also clear that the next value of \(I_k\) and \(Q_k\) depends on the values of \(I_{k-1}, Q_{k-1}\).

The channel coding is summarized in a different form in FIG.13. Here transitions can take place from any of the + states 218 to any of the x states 216 and vice versa.

The amplitude of \(I_k\) and \(Q_k\) for any state is given by the coordinate axes. For example in state 0, 220, \(I_k = 0, Q_k = 1\), and in state 1, 222, \(I_k = -0.707, Q_k = 0.707\). Suppose at an initial time the system is in state 0 and one symbol time later the system is in state 1, then the phase change is \(\pi/4\) and from the encoding/decoding table it is clear a 0, 0 has been transmitted.

It follows that for demodulation the peak value of \(I(t)\) and \(Q(t)\), \((I_k, Q_k)\) are determined thus providing the entry point into FIG.13. \(I_{k+1}\) and \(Q_{k+1}\), the peaks at the subsequent symbol time, are then determined to establish the next state. The angle between the two states serves as an entry point into the encoding/decoding table to determine the transmitted data.

In summary demodulation involves determining an angle associated with state 0 and a second angle associated with state 1, followed by a table lookup to determine the transmitted data. The vectors in FIG.13 form a signal constellation, and demodulation involves finding the angle between the vectors.

In the Digital Cellular standard, the filter \(g(t), 207\) in FIG.12 is specified in the frequency domain. In the implementation used here this frequency specification must be transformed into an equivalent time domain function. The frequency specification from the
Digital Cellular specification IS-54 is:

\[
\begin{align*}
H(f) &= 1 \text{... for } 0 \leq f \leq (1-\alpha)/2T \\
H(f) &= -\sqrt{0.5(1-\sin((\pi/2\alpha)(2fT - 1)))} \text{... for } (1-\alpha)/2T \leq f \leq (1+\alpha)/2T \\
H(f) &= 0 \text{... for } f > (1+\alpha)/2T
\end{align*}
\]

Here T is the symbol period and \( \alpha = 0.35 \) is the roll-off factor. The symbol rate in Digital Cellular is 24.3 kilosymbols/second.

This frequency response 224 is plotted in FIG.14. The impulse response \( G(z) \), 226, associated with this frequency response is shown in FIG.14. \( G(z) \) is derived from the frequency response by taking the inverse Fourier transform. In practice the baseband signal \( I(t) \), 208 in FIG.12 is generated by adding scaled values of \( G(z) \) scaled according to \( I_k \), 204. \( Q(t) \), 210 is generated in a similar manner. Scaled values of \( G(z) \) can be stored in RAM or ROM memory. RAM memory is used when the baseband signal is to be predistorted to achieve predistortion equalization as will be discussed in the equalization section.

In practice, in Digital Cellular, successive symbols are transmitted so that the peak of one corresponds to the zero crossing of the preceding symbol. It is clear from FIG.14 that the peak 225 and zero crossing 227 of \( G(Z) \) are separated by eight sample intervals. Therefore, in the absence of transmission distortion, symbols separated by eight sample intervals will not have any intersymbol interference, i.e. samples taken at the peak will not have any contribution from adjacent symbols. In the invention described here the symbols are placed closer together than eight sample intervals thereby achieving a higher bit rate, as will be described.

In summary a standard Digital Cellular system can be implemented using the system shown in FIG.1. The composing function for the \( I \) and \( Q \) baseband channels is \( G(z) \), 226 of FIG.14. Values of \( G(z) \) are scaled according to \( I_k \) and \( Q_k \), the scaled values are separated by eight sample intervals and the scaled values are added to generate the \( I \) and \( Q \) waves. At the receiver the samples associated with the peak of \( G(z) \) are located, then the decoding operation can proceed as described in connection with FIG.13.

Generation of a typical modulated wave as used in Digital Cellular is shown in FIG.15. Here "Hehi!" is encoded in DQPSK format. First the bits \( X_k, Y_k \) to be transmitted were determined from the ASCII characters. Then a sequence of \( I_k \) and \( Q_k \) values was determined from FIG.13. These values of \( I_k \) and \( Q_k \) were used to scale \( G(z) \) and the scaled values added to generate the iWave 228 shown in FIG.15. The QuWave, 230, is shown in FIG.15.
Consider the IWave and QWave of FIG. 15 to be the waves received at the receiver after modulation and demodulation. Assume the waves have been sampled at the peaks of the scaled composing functions. One sample of this sample set is shown as 234 in FIG. 15. If the largest sample is taken to have a magnitude of 1.0 then the smaller non zero samples will be found to have a magnitude of 0.707. These samples can be used to establish values of \( l_k, q_k \) and \( l_k+1, q_k+1 \) and using FIG. 13 the modulation can be recovered.

In the passband invention described here the composing function \( G(z) \) is first modified as shown in FIG. 16. The procedure is analogous to that used in the baseband case as described in connection with FIG. 2.

The composing function 236 in FIG. 16 has a Z transform \( G(z) \). The zeros of this Z transform are shown as 237 in FIG. 16. The zeros moved off the unit circle are shown in as 239 in FIG. 16. Each zero is replaced by a pair of zeros, the zero inside the unit circle being distance \( r, 241 \) from the origin and the zero outside the unit circle being distance \( 1/r \) from the origin. The zeros 239 in FIG. 16 are used to form a polynomial in factored form. The factors are multiplied together to form a Z transform whose values are shown as 238. This composing function 238 is used as the composing function for the I and Q channels of the system in FIG. 1. A set of embedded samples 240 chosen from the composing function 238, is shown in FIG. 16. The inverse of these embedded samples is shown as 242 in FIG. 16. This inverse is used in the received I and Q channels of FIG. 1 as the inverse filter 26.

A number of options exist in constructing the new composing function 238 of FIG. 16. As long as the zeros are moved off the unit circle an inverse can be found and the system shown in FIG. 1 will operate. They need not be moved distance \( r \) and \( 1/r \) as described above. If each zero on the unit circle is replaced by a pair as described above then from filter theory it is known that the composing function 238 will be symmetric about the midpoint. In general the number of terms in the inverse 242 will decrease as \( r, 241 \) decreases. The value of \( r \) used here is 0.70.

The spacing between embedded samples 240 is another parameter. In this case every third sample from the composing function 238 was chosen. This implies that in the system of FIG. 1 a scaled composing function will be transmitted every third sample interval. Since the standard Digital Cellular system transmits a scaled composing function every eighth sample interval this implies an increased symbol rate of 8/3 and a corresponding increase in bit rate. If every second sample from composing function 238 is chosen as an embedded sample the bit rate will increase by 8/2. In this case the number of samples in the inverse 242 will increase, and more precision will be required in locating the embedded samples at the receiver.

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Finally the starting composing function 236 can be altered. The samples making up the composing function can be interpolated and a new set chosen. The zeros of the Z tranform of this new set of samples can be determined to establish 237 in FIG.16 and the process repeated. This approach can be useful in modifying the number of significant samples in the inverse filter 242:

FIG. 17 shows the typical waveforms present in one embodiment of the invention. The lWave 250 is generated by the system of FIG.1 when the composing function 238 of FIG.16 is used as the composing function 14 of FIG.1. In this case scaled composing functions are transmitted every three sample intervals, therefore generating embedded samples separated by three sample intervals. QWave 252 is the corresponding Q wave. The modulating I_k and Q_k signals used in the information portion of the wave were generated by random bit generation followed by encoding using the previously described encoding/decoding table. The information portion of the wave was preceded by a prepulse. The values of I_k and Q_k used in this and subsequent examples are:

\[
I_k = 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, -0.707, -1.0, -0.707, 0.0, -0.707,
0.0, 0.707, -1.0, 0.707, 0.0, 0.707, -1.0
\]

\[
Q_k = 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, -0.707, 0.0, -0.707, -1.0, 0.707,
-1.0, -0.707, 0.0, 0.707, 1.0, -0.707, 0.0
\]

(11)

lWave 250 and the QWave 252 are modulated and demodulated and arrive as input baseband waveforms 19 at the receiver shown in FIG.1. Assume the embedded samples in the arriving I and Q baseband signals have been located. Embedded samples 254 arrive in the lWave and embedded samples 256 arrive in the QWave. These embedded samples are passed through the inverse filter 26 of FIG.1. The inverse filter used is 242 of FIG.16. The output samples 258 are the I channel output and the output samples 260 are the Q channel output. If the magnitude of the largest signal is taken as 1.0, then the magnitude of the smaller non-zero sample is found to be of the order of 0.707. The received values of I_k and Q_k as shown in 258 and 260 respectively are found to correspond to the transmitted values shown in relationships (11).

In summary in this embodiment of the invention an increase in the bit rate of Standard Digital Cellular has been achieved by transmitting overlapping composing functions, the composing function having embedded samples. The inverse of these embedded samples is the inverse filter. Decoding is accomplished by locating the embedded samples in the received I and Q baseband signals and passing the embedded samples through the inverse filter.

It is clear the passband invention is not limited to the above embodiment. Consider for example using two baseband waves of the type for baseband systems, one for I and one for Q.
If the composing function 238 of FIG.16 is used and the system is limited to 4 levels, i.e.
two bits/symbol, then this system will transmit the same number of bits/second as the
modified Digital Cellular system, however the benefits of spectral shaping by polarity
inversion can be employed to minimize spectral spreading. It is clear the invention is not
restricted to a particular kind of modulation, i.e. DQPSK or amplitude modulation.

Baseband carrier equalization, predistortion equalization, embedded sample equalization,
have a parallel in the passband case. These equalization methods are very important
for the transmission of waves with embedded samples. In addition they can be applied to
passband channels which do not use embedded sample transmission. For example they
can be applied in a novel manner to the existing Digital Cellular system to significantly
improve the performance. Accordingly, the following development is done without regard
to the modulation method. The interaction of multiple waves arriving at the receiver is
analyzed and novel methods proposed to remove the intersymbol interference introduced
by the channel.

EQUALIZATION OF THE PASSBAND CHANNEL.

The analysis below develops the following results for the fading passband channel:

- a method of removing the distortion on the I and Q baseband waves by filtering the
  I and Q waves arriving at the receiver. This result is illustrated in FIG.18 and
  FIG.19. This is the baseband carrier equalizer 20 of FIG.1.
- a method of predistorting the I and Q waves at the transmitter so that the I and Q
  baseband waves arriving at the receiver are undistorted. This result is illustrated in
  FIG.20 and FIG.21
- a method of removing the distortion introduced by the fading channel when the
  receiver is in motion. This result is illustrated in FIG.22 and FIG.23.

These analytical results are achieved in the following steps. An analytical expression for
the standard modulator shown in FIG.12 is introduced. The process of up and down
conversion is analyzed when one wave arrives at the receiver. This requires the
introduction of known trigonometric expressions. Next two waves arriving at the receiver
are considered. The distortion introduced can easily be analyzed using additional known
trigonometric expressions. These results are extended to describe the real physical
situation where multiple waves arrive. At this point in the analysis it is clear that the end
to end baseband channel is a linear system which can be described by Z transforms. The
channel description in Z transform form is manipulated to obtain the results described
above.
The modulator shown in Fig. 12 is a standard method of generating I and Q waves. It has a complex representation and a representation which displays I and Q channels as real time functions. The description from Lee/Messerschmitt\textsuperscript{2} is

\[ x(t) = \sqrt{2} \text{Re}\left( e^{j\omega t} \sum_{k=-\infty}^{\infty} a_k g(t - kT) \right) \]

where \( a_k \) is complex. The real part of the above is

\[ x(t) = \sqrt{2} \cos(\omega t) \sum_{k=-\infty}^{\infty} I_k g(t - kT) \]
\[ - \sqrt{2} \sin(\omega t) \sum_{k=-\infty}^{\infty} Q_k g(t - kT) \]

(12)

(13)

\( x(t) \) is the waveform generated by the system of Fig. 12. Consider the demodulation of this wave when arriving at the receiver. To demodulate the I channel multiply (13) by \( \sqrt{2}\cos(\omega c t) \) and use the relation

\[ 2\cos^2(\omega c t) = 1 + \cos(2\omega c t) \]

Filter out the term \( \cos(2\omega c t) \) using a low pass filter, then from the first product term in (13) above obtain

\[ \sum_{k=-\infty}^{\infty} I_k g(t - kT) \]

which is the original modulation. Multiplication of the second term in (13) by \( \sqrt{2}\cos(\omega c t) \) gives a term of the form \( \cos(\omega c t) \sin(\omega c t) \), and since

\[ 2\sin(\omega c t) \cos(\omega c t) = \sin(2\omega c t) \]

this product term will be removed by the low pass filter. To recover the Q channel multiply equation (13) by \( \sqrt{2}\sin(\omega c t) \). Use the relation

\[ 2\sin^2(\omega c t) = 1 - \cos(2\omega c t) \]

Filter with a low pass filter to obtain from the second term

\[ \sum_{k=-\infty}^{\infty} Q_k g(t - kT) \]

Multiplication of the first term in (13) by \( \sqrt{2}\sin(\omega c t) \) generates a double frequency term which is removed by filtering. It is clear from the above that terms involving \( \sin^2(\omega c t) \) or \( \cos^2(\omega c t) \) contribute to the demodulated wave, whereas terms of the form \( \cos(\omega c t) \sin(\omega c t) \) are filtered out. This is used throughout the following analysis. Consider the interaction of a primary wave given by (13) above and a secondary wave.
delayed by \( \Delta t \). The delayed wave will have the same form as (13) above with \( t \) replaced by \( t + \Delta t \). Let the primary wave have amplitude \( b_0 \) and the delayed wave have amplitude \( b_1 \) then the \( \cos(\omega_c t) \) terms of the sum wave is given by

\[
\sqrt{2} b_0 \cos(\omega_c t) \sum_{k=\infty}^{\infty} l_k g(t - kT) + \sqrt{2} b_1 \cos(\omega_c t + \Delta \tau) \sum_{k=\infty}^{\infty} l_k g(t + \Delta \tau - kT)
\]

Since,

\[
\cos(\omega_c t + \omega_c \Delta \tau) = \cos \omega_c t \cos \omega_c \Delta \tau - \sin \omega_c t \sin \omega_c \Delta \tau
\]

expression (14) can be written

\[
\sqrt{2} \cos(\omega_c t) \left[ \sum_{k=\infty}^{\infty} b_0 l_k g(t - kT) + b_1 \cos(\omega_c \Delta \tau) \sum_{k=\infty}^{\infty} l_k g(t + \Delta \tau - kT) \right] - \sqrt{2} \sin(\omega_c t) b_1 \sin(\omega_c \Delta \tau) \sum_{k=\infty}^{\infty} l_k g(t + \Delta \tau - kT)
\]

Similarly the sum of the \( \sin(\omega_c t) \) terms of the primary and delayed waves can be written

\[
- \sqrt{2} b_0 \sin(\omega_c t) \sum_{k=\infty}^{\infty} Q_k g(t - kT) - \sqrt{2} b_1 \sin(\omega_c t + \omega_c \Delta \tau) \sum_{k=\infty}^{\infty} Q_k g(t + \Delta \tau - kT)
\]

Since,

\[
\sin(\omega_c t + \omega_c \Delta \tau) = \sin \omega_c t \cos \omega_c \Delta \tau + \cos \omega_c t \sin \omega_c \Delta \tau
\]

expression (16) above can be written

\[
- \sqrt{2} \sin(\omega_c t) \left[ b_0 Q_0 g(t - kT) + b_1 \cos(\omega_c \Delta \tau) \sum_{k=\infty}^{\infty} Q_k g(t + \Delta \tau - kT) \right] - \sqrt{2} \cos(\omega_c t) b_1 \sin(\omega_c \Delta \tau) \sum_{k=\infty}^{\infty} Q_k g(t + \Delta \tau - kT)
\]

To recover the I channel multiply (15) and (17) by \( \sqrt{2} \cos(\omega_c t) \) and filter to obtain

\[
\sum_{k=\infty}^{\infty} b_0 l_k g(t - kT) + b_1 \cos(\omega_c \Delta \tau) \sum_{k=\infty}^{\infty} l_k g(t + \Delta \tau - kT)
\]

\[
- b_1 \sin(\omega_c \Delta \tau) \sum_{k=\infty}^{\infty} Q_k g(t + \Delta \tau - kT)
\]
To recover the Q channel multiply (15) and (17) by \( \sqrt{2} \sin(\omega_c t) \) and filter to obtain

\[
-\sum_{k=-\infty}^{\infty} b_0 q_k g(t - kT) - b_1 \cos(\omega_c \Delta t) \sum_{k=-\infty}^{\infty} q_k g(t + \Delta t - kT) \\
- b_1 \sin(\omega_c \Delta t) \sum_{k=-\infty}^{\infty} l_k g(t + \Delta t - kT)
\]

(19)

Consider the recovered baseband signal in the I channel given by (18) above. The first term is the undistorted primary wave with scale factor \( b_0 \). The second term is a copy of the primary wave delayed by \( \Delta t \) and scaled by \( b_1 \cos(\omega_c \Delta t) \). The third term is a cross modulation term whose value depends on the modulation on the Q channel as well as the scale factor \( b_1 \sin(\omega_c \Delta t) \). The terms \( \sin(\omega_c \Delta t) \) and \( \cos(\omega_c \Delta t) \) can be evaluated at a particular point in space if the carrier frequency is known and the differential delay between the primary wave and delayed wave is known. If the receiver moves to a point where \( \Delta t \) takes on a new value then the scale factors change. At a fixed point in space the baseband channel associated with the I signal can be represented by a linear system with two inputs, the I baseband wave and the Q baseband wave, designated IWave and QWave respectively. This system scales the IWave by \( b_0 \), and also scales it by \( b_1 \cos(\omega_c \Delta t) \), and the QWave is scaled by \( b_1 \sin(\omega_c \Delta t) \). The output of this linear system is the sum of the three terms and is designated IFade.

Since the IWave and QWave are themselves the sum of scaled values of \( G(z) \) it follows from the superposition property of linear systems that the baseband wave received at the receiver will be equalized by the same filter which removes the distortion in \( G(z) \). It is also clear that if we move to a different point in space so the scaling factors change, the linear system will change and the equalization required will change.

This two ray model with a primary and delayed wave will be used in the examples which follow as it has been found to demonstrate all the features which are present in the multiple ray model whose development follows.

Consider a multiplicity of waves arriving and choose one as a reference. Suppose it arrives at the receiver with amplitude \( b_0 \). If there are \( m \) arriving waves in addition to the reference wave, expression (15) can be modified to give the \( \cos(\omega_c t) \) terms of the sum wave;
\[
-2\sqrt{2} \cos(\omega_c t) \left( \sum_{k=\infty}^{\infty} /k b_0 g(t - kT) + \sum_{n=1}^{m} \sum_{k=\infty}^{\infty} b_n \cos(\omega_c \Delta_n r) /k g(t + \Delta_n r - kT) \right) \\
- \sqrt{2} \sin(\omega_c t) \left( \sum_{n=1}^{m} b_n \sin(\omega_c \Delta_n r) /k g(t + \Delta_n r - kT) \right)
\]

(20)

Similarly expression (17) can be modified to give the sum of the \( \sin(\omega_c t) \) terms of the sum wave

\[
-\sqrt{2} \sin(\omega_c t) \left( \sum_{k=\infty}^{\infty} b_0 g(t - kT) - \sum_{n=1}^{m} \sum_{k=\infty}^{\infty} b_n \cos(\omega_c \Delta_n r) /k g(t + \Delta_n r - kT) \right) \\
- \sqrt{2} \cos(\omega_c t) \left( \sum_{n=1}^{m} b_n \sin(\omega_c \Delta_n r) /k g(t + \Delta_n r - kT) \right)
\]

(21)

To recover the I channel multiply (20) and (21) above by \( \sqrt{2} \cos(\omega_c t) \) and filter to obtain

\[
\sum_{k=\infty}^{\infty} /k b_0 g(t - kT) + \sum_{n=1}^{m} \sum_{k=\infty}^{\infty} b_n \cos(\omega_c \Delta_n r) /k g(t + \Delta_n r - kT) \\
- \sum_{n=1}^{m} \sum_{k=\infty}^{\infty} b_n \sin(\omega_c \Delta_n r) /k g(t + \Delta_n r - kT)
\]

(22)

To recover the Q channel multiply (20) and (21) above by \( \sqrt{2} \sin(\omega_c t) \) and filter to obtain

\[
- \sum_{k=\infty}^{\infty} b_0 g(t - kT) - \sum_{n=1}^{m} \sum_{k=\infty}^{\infty} b_n \cos(\omega_c \Delta_n r) /k g(t + \Delta_n r - kT) \\
- \sum_{n=1}^{m} \sum_{k=\infty}^{\infty} b_n \sin(\omega_c \Delta_n r) /k g(t + \Delta_n r - kT)
\]

(23)

Consider transmitting a unit pulse on the I channel \((b_0 = 1)\) and a zero pulse on the Q channel \((b_0 = 0)\). This involves sending a signal \(1 b_0 g(z)\) on the I channel and zero signal on the Q channel. Consider sampling the I channel and the Q channel. Represent the sampled version of the first term in (22) above by \(b_0 g(z)\) and the second term by \(c(z)\), and the third term by \(c(z)\). If the sampled I channel response is given by \(R(z)\), then it follows from (22),
\[ b_0 G(z) + E(z) = R(z) \]

It also follows from expression (23) that the response on the Q channel is \( C(z) \).

Next transmit an arbitrary amplitude \( I_0 \) on the I channel and an arbitrary amplitude \( Q_0 \) on the Q channel. Represent the I channel response by \( I(z) \) and the Q channel response by \( Q(z) \) then the following equations apply:

\[
I_0 b_0 G(z) + I_0 (R(z) - b_0 G(z)) + Q_0 C(z) = I(z) \\
- Q_0 b_0 G(z) - Q_0 (R(z) - b_0 G(z)) + I_0 C(z) = Q(z)
\]

(24)

Solving the second in (24) for \( Q_0 \) and substituting into the first gives

\[
I_0 = \frac{I(z)}{(R(z)/I(z) + C(z)Q(z))/(R^2(z) + C^2(z))}
\]

(25)

Equation (25) describes how to obtain \( I_0 \) given the I and Q channel responses.

An equalized channel would have the response \( I_0 Q_0(z) \) to an transmitted pulse \( I_0 \). It follows that the equalizer required to equalize the I channel is

\[
I_0 G(z) = \frac{[G(z)R(z)/(R^2(z) + C^2(z))]/(z)}{[G(z)C(z)/(R^2(z) + C^2(z))]Q(z)} +
\]

(26)

In (26) the coefficients of \( I(z) \) and \( Q(z) \) are determined by exciting the system with a unit pulse on the I channel and zero on the Q channel and recording the response on the I channel as \( R(z) \) and recording the response on the Q channel as \( C(z) \), followed by the operations indicated above.

Similarly the Q channel can be equalized by

\[
Q_0 G(z) = \frac{[G(z)C(z)/(R^2(z) + C^2(z))]/(z)}{[G(z)R(z)/(R^2(z) + C^2(z))]Q(z)} -
\]

(27)

Equations (26) and (27) can be written

\[
I_0 G(z) = F_1(z)/I(z) + F_2(z)Q(z)
\]

(28)

\[
Q_0 G(z) = F_2(z)/I(z) - F_1(z)Q(z)
\]

(29)
where:

\[
F_1(z) = G(z)R(z)/(R^2(z) + C^2(z))
\]
\[
F_2(z) = G(z)C(z)/(R^2(z) + C^2(z))
\]

(30)

Equations (28) and (29) show how transmitted pulses I(t)G(z) and Q(t)G(z) can be recovered from the response I(z) on the I channel and Q(z) on the Q channel. It follows from the superposition property of linear systems, since the transmitted baseband I and Q channels are scaled versions of G(z), that the received waves will be equalized by using the filtering operations specified in equations (28) and (29). Designate the transmitted baseband wave in the I channel by IWave and the transmitted baseband wave in the Q channel by QWave. Designate the received baseband I channel wave by IFade and the Q channel received wave by QFade. It follows that if I(z) and Q(z) in equation (28) are IFade and QFade then the output will be IWave. Similarly if I(z) and Q(z) in equation (29) are IFade and QFade then the output will be QWave.

FIG.18 illustrates equalization of IFade and QFade. First the waveforms labelled IWave,270 and QWave,272 were generated using the system of FIG1. The composing function, 238 of FIG.16 was used as composing function 14 in FIG.1. The I_k and Q_k values were those in expression (11). The waves 270 and 272 are then the same as the waves 250 and 252 of FIG.17.

The waves IFade,274 and QFade,276 were then generated using the two ray model.

Equation (18) was used to generate IFade and equation (19) was used to generate QFade. The primary wave was taken to have amplitude 1.0. The delayed wave had amplitude 0.75 with a delay of 40 microseconds corresponding to the maximum delay specified in 1S-54. Since the symbol rate is 24.3 kilosymbols per second, the symbol time is of the order of 41 microseconds, so the delay is of the order of one symbol time. The carrier phase shift shown in equation (18) as \( \omega_c \Delta t \) was taken as \( \pi/4 \) radians.

Generation of IWave_Eq.278 and QWave_Eq.280 requires manipulation of equations (30). Consider the denominator. This involves convolving the sample sequence \( R(z) \) with itself by multiplication of Z transforms and adding to the convolution of the sequence \( C(z) \) with itself. The same sample sequence will be obtained if we take the DFT, equation (1), of the sequence represented by \( R(z) \) multiply it by itself, add it to the DFT of \( C(z) \), multiplied by itself, followed by the IDFT. It turns out to be convenient to stay in the frequency domain in order to perform inversion.

Consider finding the DFT of I(z) as given in equation (30). The prepulse at the start of the IFade,274 in FIG.18 is \( R(z) \) and the prepulse at the start of the QFade,276 is \( C(z) \).
Consider the DFT of the denominator. This involves finding the DFT of the sample sequence represented by \( R(z) \), squaring it and adding to the square of the DFT of \( C(z) \). At this point the number of frequency terms to be taken in the denominator is not clear. Taking a number of terms equal to the number of significant frequency terms in the numerator is a good first estimate. In this example 40 frequency terms were taken. Next invert the denominator by multiplication of each term by its complex conjugate. The DFT of the samples in \( F_1(z) \), hereafter called \( F_1 \), is then found by multiplication of the inverted frequency function by the DFT of \( G(z) \) and the DFT of \( R(z) \) as indicated in equation (30). \( F_2 \) can be calculated in the same manner.

To find the DFT of the equalized waveform it follows from equation (28) that multiplication of the DFT’s of \( |\text{Fade}_1|, |\text{Fade}_2|, F_1, F_2 \) are required. Designate the power spectrum of \( F_1 \) by \( PS\_F_1 \). The power spectrum of \( F_1, F_2, |\text{Fade}_1|, |\text{Fade}_2| \) are shown in Fig.18 as 282, 284, 286, 288 respectively. It is clear that in performing the transform multiplications required the DFT of \( |\text{Fade}_1| \) and \( |\text{Fade}_2| \) will act to truncate the DFT of \( F_1 \) and \( F_2 \). The IDFT of the DFT from equation (28) is shown in Fig.18 as IWave\_Eq, 278. Similarly the wave output from equation (29) is shown as QWave\_Eq. Comparison of the equalized waves 278 and 280 with the transmitted waves 270 and 272 shows negligible distortion.

In summary, equalization at the receiver requires sending prepulses to provide information about \( R(z) \) and \( C(z) \). Frequency domain filters \( F_1 \) and \( F_2 \) can then be calculated from equations (30). The I and Q transmitted waves can then be recovered from the I and Q received waves by filtering according to equations (28) and (29).

In the example shown here \( |\text{Fade}_1| \) and \( |\text{Fade}_2| \) were short signals. If \( |\text{Fade}_1| \) and \( |\text{Fade}_2| \) are long signals to be equalized then they may be segmented as described in the Overlap-Save method in Rabiner and Gold 3 and \( F_1 \) and \( F_2 \) applied as described. The frequency spectrum of a typical segment should be determined to establish the number of frequency terms required in \( F_1 \) and \( F_2 \).

Consider predistorting the transmitted signals so that the transmission medium acts as an equalizer and the signals arrive at the receiver undistorted.

The response to signals \( I_{0}G(z) \), \( Q_{0}G(z) \) is given by (24):

\[
I_{0}R(z) + Q_{0}C(z) = I(z)
- Q_{0}R(z) + I_{0}C(z) = Q(z)
\]

(31)

It is convenient to view \( I_{0} \) and \( Q_{0} \) as impulses exciting \( G(z) \). Consider inserting an additional filter ahead of \( G(z) \) in the I channel and similarly in the Q channel. Let these filters be represented by \( I_{\text{in}}(z) \) and \( Q_{\text{in}}(z) \). Consider seeking a value for these filters such RECTIFIED SHEET (RULE 91)
that the signals arriving in the I and Q channel are undistorted. The following equations then apply:

\[
\begin{align*}
I_{0}^{o}(z)R(z) + Q_{0}Q_{o}(z)C(z) &= I_{0}G(z) \\
- Q_{0}O_{o}(z)R(z) + I_{0}O_{o}(z)C(z) &= Q_{0}G(z)
\end{align*}
\]

(32)

Solving the above for \(I_{0}I_{0}(z)\) and \(Q_{0}Q_{0}(z)\) gives

\[
\begin{align*}
I_{0}^{o}(z) &= (R(z)G(z)/(R^{2}(z) + C^{2}(z)))I_{0} + (C(z)G(z)/(R^{2}(z) + C^{2}(z)))Q_{0} \\
Q_{0}O_{o}(z) &= (C(z)G(z)/(R^{2}(z) + C^{2}(z)))I_{0} - (R(z)G(z)/(R^{2}(z) + C^{2}(z)))Q_{0}
\end{align*}
\]

(33)

Identifying \(F_{1}(z)\) and \(F_{2}(z)\) the above can be written

\[
\begin{align*}
I_{0}^{o}(z) &= F_{1}(z)I_{0} + F_{2}(z)Q_{0} \\
Q_{0}O_{o}(z) &= F_{2}(z)I_{0} - F_{1}(z)Q_{0}
\end{align*}
\]

(34)

The above equations (34) give the output of the filters preceding \(G(z)\). The signals to send in the I and Q channels are then

\[
\begin{align*}
P_{1}(z)I_{0} + P_{2}(z)Q_{0} \\
P_{2}(z)I_{0} - P_{1}(z)Q_{0}
\end{align*}
\]

(35) (36)

where

\[
\begin{align*}
P_{1}(z) &= F_{1}(z)G(z) \\
P_{2}(z) &= F_{2}(z)G(z)
\end{align*}
\]

(37)

The DFT of \(F_{1}(z)\) and \(F_{2}(z)\) are available, therefore the DFT of \(P_{1}(z)\) and \(P_{2}(z)\) can be determined by multiplication by the DFT of \(G(z)\). The IDFT can then be applied to find \(P_{1}(z)\) and \(P_{2}(z)\). \(P_{1}(z),300\) and \(P_{2}(z),302\) are shown in FIG 20.

As in the baseband case the equalization can be split between the transmitter and receiver.

Some reflection shows that if the DFT of \(F_{1}(z)\) and \(F_{2}(z)\) are split into the product of two DFT, in the manner described for baseband systems, then the equalization will be split between predistortion at the transmitter and filtering at the receiver.

It follows from equations (35) and (36) that if \(I_{0}=1\) and \(Q_{0}=0\), then sending \(P_{1}(z)\) in the I channel and \(P_{2}(z)\) in the Q channel, the received signals after transmission over the fading channel should be \(G(z)\) and 0. The signals which arrive are shown as \(I\text{Arriv},304\) and \(Q\text{Arrive},306\) in FIG 20 and indeed are as expected.

\textbf{RECTIFIED SHEET (RULE 91)}
It further follows from equations (35) and (36) that if the information to be transmitted is encoded using $P_1(z)$ and $P_2(z)$ as composing functions then the I and Q channels should arrive at the receiver undistorted. $P_1(z)$ and $P_2(z)$ of Fig.20 were used as composing functions. Expression (35) was used to generate the I channel signal and expression (36) was used to generate the Q channel signal, where the modulation was as described in expression (11). Fig.21 shows these signals.

Undistorted signals IWave 308 and QWave 310 are shown for reference. The predistorted signals are shown as IWPredis 312 and QWPredis 314. The signals which arrive after passing through the fading channel model are shown as IArriv 316 and QArriv 318. Comparison of the arriving signals with the reference signals 308 and 310 shows negligible distortion as expected.

In existing Digital Cellular systems vehicle motion can cause the channel response to change significantly between the beginning and end of a packet. Consider the application of the receiver equalization method described herein to this environment. At a fixed point in space $R(z)$ and $C(z)$ are fixed, they can be measured and the receiver equalizer can be determined. At a different point in space these parameters will be different and a new equalizer will be required. The change in $R(z)$ and $C(z)$ can be estimated from equation (18) and (19). Previously in this model the primary wave was taken to have a value of 1.0, the delayed wave an amplitude of 0.75 and the delay of the secondary wave was taken as 40 microseconds corresponding to the maximum delay specified in IS-54. The carrier phase shift $\omega_0 \Delta t$ was taken as $\pi/4$. With motion the carrier phase shift can cause a significant change in $R(z)$ and $C(z)$. Consider the change in the parameter $\omega_0 \Delta t$ from the start of the reception of a packet to the end of the packet, maintaining the same values for the other parameters.

The symbol rate in Digital Cellular is 24.3 kilosymbols/sec and there are 162 symbols in a packet. During the arrival time of a packet a car travelling at 120km/hr will move about 0.22 meters, which takes the EM wave moving at the speed of light about 0.733ms to traverse. If it is assumed that the car is moving directly up the primary wave and at right angles to the secondary wave, then the multipath change in delay during the reception of a packet is 0.733ms. At a digital cellular frequency of 825MHz, the change in $\omega_0 \Delta t$ is approximately 1.20 $\pi$. The initial value of $\omega_0 \Delta t$ used previously was 0.25 $\pi$. Using the same initial value it follows that the values of $\omega_0 \Delta t$ at the beginning, middle, and end of the packet are 0.25 $\pi$, 0.85 $\pi$, 1.45 $\pi$.

The test pattern, $\delta_0=1; \delta_0=0$, used to measure $R(z)$ and $C(z)$ was sent through the fading channel model for these three values of $\omega_0 \Delta t$. The composing function for Digital Cellular 238 of Fig.16 was used.
Fig. 22 shows the results. Since this response determines the equalizer required, it is clear from the nature of the change in $R(z) 310$ and $C(z) 312$ that the equalizer required at the beginning, middle, and end of the packet will be different. It should be noted that a worst case has been considered, ie a differential delay of 40 microseconds was used as a beginning point, which is at the extremity of the specification, and it was assumed the vehicle moved directly up the primary wave.

Equalization can be performed by breaking the packet up into smaller packets and sending a prepulse at the head of each of these smaller packets and using the prepulse to equalize the following symbols. Alternatively, the information in the packet can be used to estimate $R(z)$ and $C(z)$. In this case a prepulse at the beginning of the packet is used to find the equalizer, the first part of the packet is passed through the equalizer and the encoded information is recovered. The process is continued until the wave distortion is such that a new equalizer is required. The problem is then to find from the received wave, given the recovered data and the symbol position, the effective values of $R(z)$ and $C(z)$. This development follows.

Consider signals $I_0$ and $Q_0$ to be applied followed by $I_1$ and $Q_1$ and preceded by $I_{-1}$ and $Q_{-1}$ separated by 8 time units. The channel response to each of the $I_i, Q_i$ pairs is given by the first of equations (24) and can be written:

$$I_0 z^{-6} R(z) + Q_0 z^{-6} C(z) = I_0(z)$$

$$I_0 R(z) + Q_0 C(z) = I_0(z)$$

$$I_0 z^{-6} R(z) + Q_0 z^{-6} C(z) = I_0(z)$$

(38)

Assume a packet of limited time duration with $m$ symbols preceding $I_0$ and $n$ symbols following $I_0$, in which case there are $n + m + 1$ equations of the above form. Recognize that the sum of the right sides of the above format is the total channel response $I(z)$. Adding the $n + m + 1$ equations of the above form for the $I$ channel and a similar set of equations for the $Q$ channel where $Q(z)$ is the response of the $Q$ channel then,

$$F_1(z) R(z) + F_2(z) C(z) = I(z)$$

$$- F_2(z) R(z) + F_1(z) C(z) = Q(z)$$

(39)

where

$$F_1(z) = (I_{-m} z^{-6m} + ... + I_{-n} z^{-6n})$$

$$F_2(z) = (Q_{-m} z^{-6m} + ... + Q_{-n} z^{-6n})$$

(40)

Solving (39) for $R(z)$ and $C(z)$ gives

$$R(z) = (I(z)F_1(z) - Q(z)F_2(z))/(F_1(z) + F_2(z))$$

$$C(z) = (Q(z)F_2(z) + I(z)F_1(z))/(F_1(z) + F_2(z))$$

(41)
Application of (41) is shown in FIG. 23, 24 and 25. Symbols are transmitted separated by 8 sample intervals. The start of the first symbol is shown as 314 in FIG 23. The encoded information was:

\[ \begin{align*}
1080 \hspace{0.5cm} & 1.0,0.0,0.0,0.0,0.0,0.0,0.0,1.0,-0.707,-1.0,-0.707,0.0,-0.707, \\
& 0.0,0.707,-1.0,0.707,0.0,0.707,-1.0 \\
Qk... & 0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,-0.707,0.0,-0.707,-1.0,0.707, \\
& -1.0,-0.707,0.0,0.707,1.0,-0.707,0.0 \\
\end{align*} \]

(42)

The signals IFade 317 and QFade 319 of FIG.23 were generated with the above data using the baseband modulator 16 of FIG.1 with the Digital Cellular composing function 238 of FIG.16, followed by passage through the fading channel model. The steps implied in equation (41) were then applied to these waves to recover R(z) and C(z).

The DFT was used to evaluate F1(z) and F2(z) defined in equation(40) and used in equation (41), using the input data in expression(42). The number of terms required in this evaluation is apparent from FIG.24. The power spectrum of F1, F2, IFade, QFade, are shown as 320, 322, 324, 326 respectively. It is apparent when terms of the form I(z)F(z), Q(z)F(z), which appear in the numerator of equations (41), are formed the DFT will be truncated, and these terms can be evaluated.

The recovered R(z) 328 and C(z) 330 are shown in FIG. 25. These are to be compared with R(z) 332 and C(z) 334 which are the pre pulses shown in FIG 23.

In this analysis the denominator in equations (41), in some cases, had terms in the frequency domain which were very small making inversion difficult. FIG 25 shows the power spectrum of the denominator of equations (41) after inversion as 336. The difference between the smallest and largest amplitude provides a measure of the how close the inversion process is to being unmanageable. As the number of symbols increases the difference between the largest and smallest amplitude decreases.

The equalization methods for baseband and passband systems are analogous but each has a different implementation. The embedded sample locator 22 in FIG.1 can be implemented in the same manner in both baseband and passband systems. FIG.6 and FIG.5 provide examples of possible implementations.

In standard Digital Cellular systems the specification as to the shape for the symbols to be transmitted is provided in the frequency domain. The filter 207 in FIG.12 is specified in the frequency domain. As a consequence of the fourier transform relation discussed in connection with these two different implementations the output waveforms will be substantially the same. The I and Q channel may then be viewed as the sum of scaled composing functions although a composing function is not in evidence.
As described above according to this invention a system for the transmission of information at rates substantially above those achieved by conventional modulation, containing a novel equalization apparatus achieved by predistorting the signal to accomplish the equalization.

What is claimed is:

1. A system to transmit information comprising:

   an inverse filter which when operating to filter a unique sample sequence produces a unit pulse output, said output being scaled in accordance with the scaling on said unique sample sequence and,

   modulating means which generates one or more sample streams which may be regarded as the sum of scaled basic sample sequences said scaling being determined by the information to be transmitted and,

   sample conditioning means which operates on said sample streams to generate conditioned sample streams which may be regarded as the sum of scaled values of said unique sample sequence, said conditioned sample streams when filtered by said inverse filter generating an output sample sequence whose values are related to the scaling applied when encoding said information permitting the recovery of said information.

2. The system according to claim 1 wherein said basic sample sequences contain said unique sample sequence as a subset, said sample streams from said modulating means defining continuous undistorted waveforms from which the defining samples may be recovered, said sample conditioning means comprising means for recovering said defining samples.

3. The system according to claim 2 wherein said means for recovering said defining samples comprising eye pattern adjustment.

4. The system according to claim 2 wherein said means for recovering said defining samples comprising offset evaluation and waveform interpolation.

5. The system according to claim 2 wherein said modulating means is connected to said sample conditioning means by a channel and wherein said sample streams from said modulator define distorted waveforms said distortion being predetermined so that after transmission through the channel the output is said undistorted waveforms, said predetermined distortion being determined by setting the values
of said basic sample sequences.

6. The system according to claim 5 wherein said system further comprises equalizing means said equalizing means operating on said distorted waveforms to generate said undistorted waveforms.

7. The system according to claim 6 wherein said modulator operates with one of said basic sample sequences and one of said output streams and said equalizing means comprising a baseband carrier equalizer for baseband systems.

8. The system according to claim 6 wherein said modulator operates with two basic sample sequences and two of said output streams said equalizing means comprising a baseband carrier equalizer for passband systems.

9. The system according to claim 7 or 8 further comprising an embedded sample equalizer.

10. An apparatus for equalization using controlled distortion of a modulator output comprising:

modulating means which generates one or more sample sequences which may be regarded as the sum of one or more basic sample sequences said scaling being determined by the information to be transmitted, said sample sequences defining a distorted waveform, said distortion being determined by the values of said basic sample sequences, said distorted waveform when passed through a channel arriving undistorted at the receiver, the values of said basic sample sequences being determined in a deterministic manner from the characteristics of said channel.

11. The apparatus according to claim 10 wherein said basic sequences are stored in memory and said sample streams are generated by adding scaled version of said basic sequences.

12. The apparatus according to claim 11 wherein said modulator comprises one basic sequence and one of said streams.

13. The apparatus according to claim 11 wherein said basic sequences comprising a dipulse.

14. The apparatus according to claim 11 wherein said modulator comprises two basic sequences and two of said streams.
15. The apparatus according to claim 14 further comprising a band pass carrier equalizer for baseband systems.

1.0 Electronic Industries Association, EAI/TIA Interim Standard, Cellular System, IS-54-A.
6.0 CCITT Recommendation V.32 bis, Data Communication Over the Telephone Network.
FIG. 1
FIG. 2
FIG. 3

(A) 50 - $1/H(z)$  52 - INVERSE
      56 - FIRST ITERATION:

(B) 60 - $1/H(z)$  62 - INVERSE
      66 - THIRD ITERATION:

(C) 70 - $1/H(z)$  72 - INVERSE
      76 - FIFTH ITERATION

54 - R(z)  58 - RESIDUAL ERROR

64 - R(z)  68 - RESIDUAL ERROR

74 - R(z)  78 - RESIDUAL ERROR
80-Hah!

79-Binary: 0 0 0 1 0 0 1 0 1 0 0 0 0 1 1 0 0 0 0 1 0 1 1 0 1 0 0 0 0 1 0 0
81-Octal: 0 4 5 0 3 0 2 6 4 1 0
82-With Prepulse: 7 0 0 0 7 0 0 0 4 5 0 3 0 2 6 4 1 0
83-Composing Func Sign: + + + - - - - - - + + + + - - - -

FIG. 4
FIG. 5

87-MODULATED WAVE

89-EMBEDDED SAMPLES

88-LEVELS

84-PREPULSE INVERSE

86-DEMODULATOR OUTPUT

85-END POINT

RECTIFIED SHEET (RULE 91)
FIG. 6
FIG. 7
FIG. 8

FIG. 9
FIG. 12

FIG. 13
FIG. 16
FIG. 17
FIG. 18
FIG. 19

FIG. 20
**INTERNATIONAL SEARCH REPORT**

**International application No.**
PCT/CA 93/00282

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**A. CLASSIFICATION OF SUBJECT MATTER**

H 04 L 27/00, H 04 B 3/14

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**B. FIELDS SEARCHED**

Minimum documentation searched (classification system followed by classification symbols)

H 04 L, H 04 B

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Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

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Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

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**C. DOCUMENTS CONSIDERED TO BE RELEVANT**

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<th>Category</th>
<th>Citation of document, with indication, where appropriate, of the relevant passages</th>
<th>Relevant to claim No.</th>
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**Date of the actual completion of the international search**

09 February 1994

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**Date of mailing of the international search report**

03.03.94

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**Authorized officer**

HAJOS e.h.
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**ANHANG**

zum internationalen Recherchebericht über die internationale Patentanmeldung Nr.

**ANNEX**


**ANNEXE**

au rapport de recherche international relatif à la demande de brevet international n°

PCT/CA 93/00282 SAE 77707

In diesem Anhang sind die Mitglieder der Patentfamilien der in obengenannten internationalen Rechercheberichte angeführten Patentdokumente angegeben. Diese Angaben dienen nur zur Unterrichtung und erfolgen ohne Gewähr.

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