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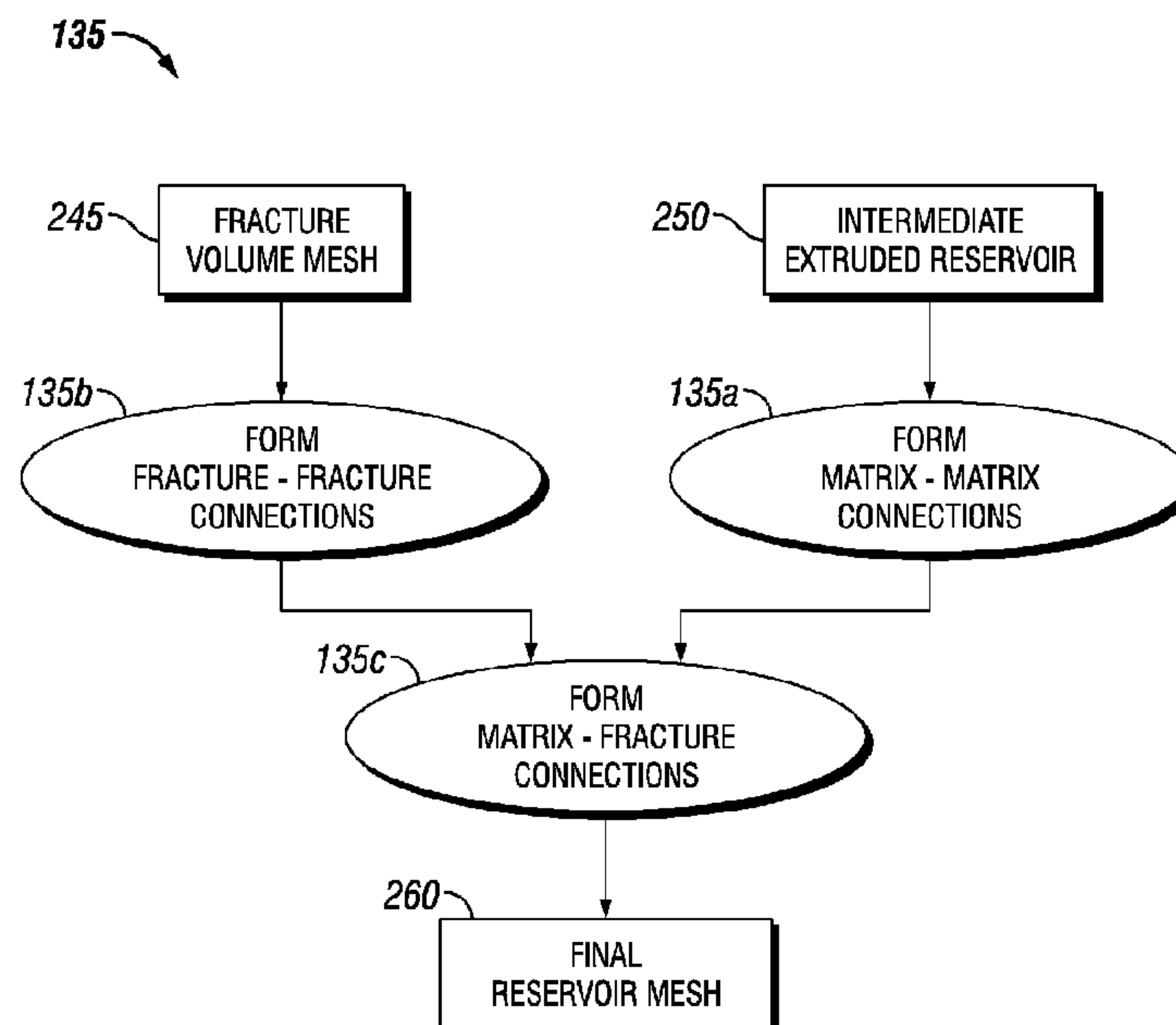


FIG. 1C

(57) **Abrégé/Abstract:**

The disclosed embodiments include a method, apparatus, and computer program product for generating multiple correlated meshes around complex and discrete fractures for the purpose of reservoir simulation. In an initial two-stage process, a 2.5-dimensional mesher algorithm, may be used in conjunction with an extended anisotropic geometry-adaptive refinement algorithm to produce, from a structured grid Earth Model, a refined reservoir model. Fractures may be modeled as a volumetric mesh that is independent of the refined reservoir model. Thereafter, the two independent, overlapping meshes are combined by forming matrix-matrix, fracture-fracture, and matrix-fracture connections to form a single model to allow rapid simulation of an extremely complex fracture network with sufficiently accurate results.

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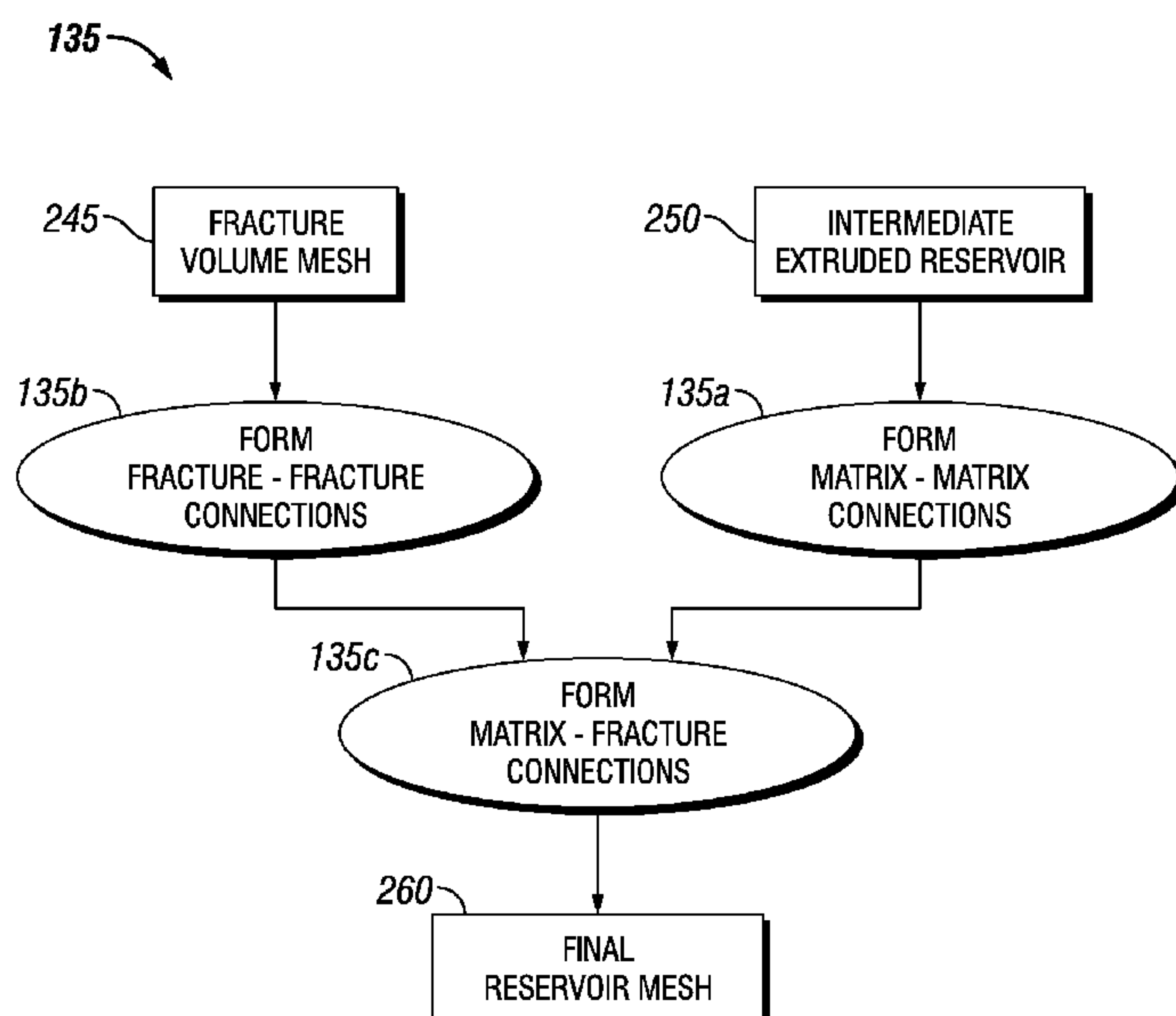


FIG. 1C

(57) Abstract: The disclosed embodiments include a method, apparatus, and computer program product for generating multiple correlated meshes around complex and discrete fractures for the purpose of reservoir simulation. In an initial two-stage process, a 2.5-dimensional mesher algorithm, may be used in conjunction with an extended anisotropic geometry-adaptive refinement algorithm to produce, from a structured grid Earth Model, a refined reservoir model. Fractures may be modeled as a volumetric mesh that is independent of the refined reservoir model. Thereafter, the two independent, overlapping meshes are combined by forming matrix-matrix, fracture-fracture, and matrix-fracture connections to form a single model to allow rapid simulation of an extremely complex fracture network with sufficiently accurate results.

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SIMULATING FRACTURED RESERVOIRS USING MULTIPLE MESHES

TECHNICAL FIELD

The present disclosure generally relates to a system and method for generating a grid that can
5 be used to construct a simulation model of a subsurface reservoir, and more particularly, to a
system and method configured for modeling geological fractures.

BACKGROUND

In the oil and gas industry, reservoir modeling involves the construction of a computer model
of a petroleum reservoir for the purposes of improving estimation of reserves and making
10 decisions regarding the development of the field. For example, geological models may be
created to provide a static description of the reservoir prior to production. Reservoir
simulation models may also be used to simulate the flow of fluids within the reservoir over its
production lifetime.

One challenge with reservoir simulation models is the modeling of fractures within a
15 reservoir, which requires a thorough understanding of matrix flow characteristics, fracture
network connectivity and fracture-matrix interaction. Fractures can be described as open
cracks or voids within the formation and can either be naturally occurring or artificially
generated from a wellbore. The correct modeling of the fractures is important as the
properties of fractures such as spatial distribution, aperture, length, height, conductivity, and
20 connectivity significantly affect the flow of reservoir fluids to the well bore.

Mesh generation techniques are used in reservoir modeling. Two traditional mesh generation
techniques for three-dimensional (3D) reservoir simulation are structured-based meshing and
extrusion based meshing. In structured techniques, hexahedra are connected in a logical 3D
i-j-k space with each interior mesh node being adjacent to 8 hexahedra. Extensions to
25 structured techniques include local grid refinement where local regions of an original grid are
replaced with finer grids. This can become time-consuming, computationally expensive, and
prohibitively burdensome when dealing with general reservoir geometries, such as arbitrary
3D fracture surfaces. Because of the inherent 2.5 dimensional (2.5D) nature of existing
extrusion techniques, similar limitations apply to these techniques. Alternative, fully
30 unstructured meshing techniques exist, including tetrahedralization and polyhedral meshing

schemes. The increased complexity of these techniques often leads to lower robustness as compared to structured techniques, especially, in the presence of imperfect geometry input.

Accordingly, simulation of reservoirs with large fracture systems, of arbitrary geometry and orientation, is difficult, with tradeoffs required between sufficient accuracy and reasonable computational time. As an example, simulation of a shale reservoir may generate a natural fracture network consisting of tens of thousands of fractures defined geometrically by hundreds of thousands of triangles. Such complex systems are not easily resolved using a 2.5-dimensional reservoir meshing system, and when resolved in three dimensions, often produce prohibitively large simulation models which take significant time to simulate.

10 BRIEF DESCRIPTION OF THE DRAWINGS

Embodiments are described in detail hereinafter with reference to the accompanying figures, in which:

Figures 1A–1C are high-level flow charts outlining a multiple mesh method for modeling a fractured hydrocarbon reservoir using a multiple mesh system according to one or more
15 embodiments;

Figure 2 perspective of the top surface of a structured or extruded mesh definition of an exemplary reservoir provided as a structured Earth Model, according to one or more
embodiments;

Figure 3 is a perspective view of an exemplary hydraulic fracture model defined using
20 surfaces, according to an embodiment;

Figure 4 is a perspective view of an exemplary natural fracture model defined using surfaces, according to an embodiment;

Figure 5A is a plan view of an exemplary initial structured reservoir model referenced in the flow chart of Figure 1A;

25 Figure 5B is an elevation cross-sectional view of the reservoir model of Figure 5A taken along lines 5B–5B of Figure 5A;

Figure 6A is a plan view of the reservoir model of Figure 5A, shown with a portion of the mesh removed for subsequent insertion of a 2.5 dimension unstructured fracture mesh;

Figure 6B is an elevation cross-sectional view of reservoir model of Figure 6A taken along lines 6B–6B of Figure 6A;

Figure 7A is a plan view of the reservoir model of Figure 6A, shown with an inserted 2.5 dimension unstructured fracture mesh in the top layer;

- 5 Figure 7B is an elevation cross-sectional view of the reservoir model of Figure 7A taken along lines 7B–7B of Figure 7A;

Figure 7C is the elevation cross-sectional view of the reservoir model of Figure 7B, shown with the unstructured fracture mesh extruded downwardly;

- 10 Figure 8 is a flowchart of a process for 2.5-dimension modeling of certain fractures according to Figure 1A and certain illustrative examples of the present disclosure;

Figure 9 depicts examples of refinement configurations for a single hexahedron cell within a three-dimensional U - V - W space, according to certain illustrative aspects of the present disclosure, which may be used in the flow chart of Figure 1A;

- 15 Figures 10 and 11 depict representative triangular prism cells showing a logical two-dimensional U - W coordinate system and a number scheme for edges of the prisms, according to certain illustrative aspects of the present disclosure, which may be used in the flow chart of Figure 1A;

- 20 Figure 12 depicts refinement configurations for a single prism cell using a single U -direction paradigm within the logical two-dimensional U - W coordinate system, according to certain illustrative examples of the present disclosure, which may be used in the flow chart of Figure 1A;

Figure 13 illustrates refinement configurations in four directions using a multiple U -direction paradigm within the logical two-dimensional U - W coordinate system, according to certain examples of the present disclosure, which may be used in the flow chart of Figure 1A;

- 25 Figure 14 illustrates a representative non-triangular prism (e.g., a general prismatic element) showing the logical two-dimensional coordinate system, according to certain features of the present disclosure, which may be used in the flow chart of Figure 1A;

Figure 15 illustrates possible refinement of a general prismatic element by subdividing the element into hexahedra, according to certain illustrative examples of the present disclosure, which may be used in the flow chart of Figure 1A;

5 Figures 16 and 17 are flowcharts illustrating a process for extended anisotropic, geometry-adaptive cell refinement, according to Figure 1A and certain illustrative examples of the present disclosure;

Figure 18 is a 2D schematic illustration of a scheme for generating reservoir-fracture mesh connections according to Figure 1C and certain illustrative examples of the present disclosure;

10 Figures 19A–19C illustrate an exemplary natural fracture mesh that has been processed in accordance with the method of Figures 1A–1C according to an embodiment; and

Figure 20 is a functional block diagram of an exemplary computer system according to an embodiment in which the method of Figures 1A–1C may be implemented.

DETAILED DESCRIPTION

15 The present disclosure may repeat reference numerals and/or letters in the various examples. This repetition is for the purpose of simplicity and clarity and does not in itself dictate a relationship between the various embodiments and/or configurations discussed. Further, spatially relative terms, such as “beneath,” “below,” “lower,” “above,” “upper,” “uphole,” “downhole,” “upstream,” “downstream,” and the like, may be used herein for ease of
20 description to describe one element or feature’s relationship to another element(s) or feature(s) as illustrated in the figures. The spatially relative terms are intended to encompass different orientations of the apparatus in use or operation in addition to the orientation depicted in the figures.

The present disclosure details a method and system to simulate reservoirs with large fracture
25 systems, of arbitrary geometry and orientation, with sufficient accuracy and in reasonable time using commercial simulator systems by selectively resolving data into significantly smaller simulation models. Various reservoir and fracture models are combined to form a single reservoir model which can be readily simulated with lower computational demand yet which has sufficient resolution to capture the flow patterns near fractures. Accordingly, the

disclosed embodiments provide a system, method, and computer program product for generating hybrid computational meshes around complex and discrete fractures for the purpose of reservoir simulation.

Figures 1A–1C together form a three-part high-level flow chart outlining a multiple mesh
5 method 100 for modeling a fractured hydrocarbon reservoir according to one or more
embodiments. Figure 1A outlines a first portion of method 100 to create a volumetric mesh
of the fractured reservoir that appropriately resolves desired fractures. More particularly, the
first portion of method 100 depicted in Figure 1A discloses an initial two-stage process
according to an embodiment in which a 2.5-dimensional (2.5D) mesher algorithm, such as a
10 stadium mesher, is used in conjunction with an extended anisotropic geometry-adaptive
refinement (AGAR) algorithm to produce, from a structured grid Earth Model, a refined
reservoir model. This combination may maintain computational speed and orientation
control advantages of a 2.5D mesher where desired, while also invoking an extended AGAR
algorithm to resolve general 3D fracture networks. A network of fractures can be represented
15 as geometry and fracture properties. The set of fractures may be divided into two groups:
Those to be resolved in the 2.5-dimensional mesh and those to be resolved only in the final,
three-dimensional mesh. The 2.5D-permitting fractures include fractures represented by
vertical planes or substantially vertical planes with respect to the horizon. The other fractures
can include fractures represented by horizontal planes or substantially horizontal planes. A
20 2.5-dimensional mesh is first created by using the 2.5 stadium mesher to resolve the 2.5D-
permitting fractures in a three-dimensional (3D) space. An extended AGAR algorithm then
anisotropically resolves the other fractures to produce a refined reservoir model. Refinement,
as used herein, refers to the splitting of cells in the 3D space that ultimately results in a higher
mesh resolution model. The extended AGAR algorithm may be applied directly to topologies
25 including hexahedral, triangular prisms and other general prismatic elements to provide
appropriate cell grading in the near-fracture region. Figure 1B outlines a second portion of
method 100 to model the fractures as a volumetric mesh that is independent of the reservoir
mesh. Finally, as illustrated in Figure 1C, the two independent, overlapping meshes are
combined to form a single model to allow rapid simulation of an extremely complex fracture
30 network with sufficiently accurate results.

However, depending on the type of input and the desired simulation, certain illustrated steps
of method 100 may be omitted. Moreover, in some alternative implementations, the steps

outlined in Figures 1A–1C may occur out of the order depicted in the figures. For example, two blocks shown in succession may, in fact, be executed substantially concurrently, or the blocks may sometimes be executed in the reverse order, depending upon the functionality involved.

5 The flowchart of Figures 1A–1C illustrate an architecture, functionality, and operation of possible implementations of systems, methods and computer program products according to various embodiments of the present disclosure. In one or more embodiments, each block and/or combinations of blocks of the illustrated flowchart may be implemented by special purpose hardware-based systems that perform the specified functions or acts, or combinations
10 of special purpose hardware and computer instructions.

Referring to Figure 1A, a reservoir model 200 initially provides subsurface stratigraphic detail of a reservoir, including, for example, geo science interpretation, petroleum system modeling, geochemical analysis, stratigraphic gridding, facies, net cell volume, and petrophysical property modeling. Earth modeling platforms include platforms such as, for
15 example, DecisionSpace®, which is commercially offered through Halliburton Energy Services Inc. of Houston Texas. A variety of other earth modeling platforms, however, may also or instead be utilized with the present disclosure. Moreover, reservoir model 200 may enable robust data retrieval and integration of historical and real-time reservoir related data that spans across all aspects of the well planning, construction and completion processes such
20 as, for example, drilling, cementing, wireline logging, well testing and stimulation. Such data may include, for example, logging data, well trajectories, petrophysical rock property data, mechanical rock property data, surface data, fault data, data from surrounding wells, data inferred from geostatistics, etc. A database (not illustrated) may store this information. An example database platform is, for example, the INSITE® software suite, commercially
25 offered through Halliburton Energy Services Inc. of Houston Texas. A variety of other database platforms, software platforms, and associated systems can be used to retrieve, store and integrate the well related data.

Reservoir simulation users may have reservoir description input in various forms. For example, users may begin with a structured Earth Model or a definition that can easily be
30 made into a structured Earth Model. Initial reservoir models often contain only vertical mesh columns. However, structured reservoir mesh 200 may be obtained by any suitable method and provided in any suitable form. Figure 2 graphically illustrates a structured or extruded

mesh definition of a reservoir provided as a structured Earth Model (EM). The shading of the meshes may indicate hydrocarbon content, density, porosity, combinations thereof, or the like.

Also depicted in Figure 1A, initial fractures model 220 provides the geometric definition of the fracture network, along with the petrophysical and mechanical property distributions for the fractures. Example fracture system modules may be implemented using, for example, the Fracpro® or Gohfer® platforms. Other platforms and software systems, however, can be used to implement fracture system model 220.

Fracture model 220 may be separated at step 105 into a first set of fractures 223, typically hydraulic fractures, that are well suited for 2.5 dimensional (2.5D) modeling and a second set of fractures 227, typically natural fractures, that are less suitable for 2.5D modeling. The two sets of fractures are not necessarily mutually exclusive. That is, some subset of fractures can be in both sets of fractures. Graphical illustrations of an exemplary hydraulic fracture model and a typical natural fracture model are shown in Figures 3 and 4, respectively, which may be defined as surfaces. The shading of the surfaces may represent the size or fluid resistance of fractures, for example.

The reservoir and fracture models of Figures 2–4 collectively comprise the initial data to form the final fracture network model of multiple mesh method 100. Referring to Figure 1A, structured reservoir model 200 may be modified so as to replace part of the existing structured reservoir mesh with one or more fracture models, as indicated in step 110. Recall that at step 105, a subset 223 of the fractures 220 (particularly hydraulic fractures) to be resolved in the structured reservoir model 200 via 2.5D meshing is determined. Typically, subset 223 is a simple selection of fractures that are connected to the well bore via high conductivity paths. Subset 223 may be integrated with structured reservoir model 200 at step 110 using a 2.5D meshing algorithm, such as a stadium mesher, to resolve planar, vertical fractures—an oft used geometry to represent hydraulic fractures—to create an unstructured extruded reservoir mesh 230.

For instance, Figure 5A is a plan view of an exemplary initial structured reservoir model 200. Figure 5B is an elevation cross section of initial structured reservoir model 200 of Figure 5A taken along lines 5B–5B. A region 202 associated with a single hydraulic fracture is identified. Figures 6A and 6B are plan and cross-sectional elevation views, respectively, of

structured reservoir model 200 shown in an intermediate stage of step 110 with fracture region 202 removed. Although a rectangular region 202 was identified and removed for insertion of fractures 220, any appropriate polygonal shape may be used as conditions dictate.

5 Figures 7A and 7B are plan and cross-sectional elevation views, respectively, of an unstructured reservoir model 230 with an unstructured 2D fracture mesh 204 originally projected onto the top surface of an otherwise structured mesh. A two-dimensional (2D) meshing scheme may be used to add resolution near fracture subset 223 to create fracture mesh 204. As shown in the cross-sectional elevation view of Figure 7C, fracture mesh 204 is then projected to the next lower mesh surface, and the process is repeated until region 202
10 has been filled with the new elements. Note that in the section view of Figure 7C the 2.5D fracture mesh 204 appears structured. The original structured reservoir model mesh 200 may be modified to remove section 202 to be replaced with the unstructured fracture mesh 204, and the layers of original structured reservoir model mesh 200 may be used to guide the extrusion of fracture mesh 204. The properties of original structured reservoir model 200 are
15 mapped onto the new topology of unstructured reservoir model mesh 230.

Although any appropriate 2.5D meshing algorithm may be used for step 110, in one or more embodiments, a stadia mesher, which is operable to quickly generate unstructured grids using structured elements around complex geometries, may be used as follows: Fractures subset 223 may be discretized in a two-dimensional plane by a collection of line segments. The
20 collection of line segments represents the intersection between the two-dimensional plane and the three-dimensional geological fractures. Each fracture in subset 223 may be represented by a collection of straight line segments to approximate a curvature of the fracture.

For each fracture line segment in the two-dimensional plane, a set of stadia is generated at a specified radii from the respective fracture line segment. Thereafter, closed loops are
25 generated around all of the line segments of a fracture. The process of generating the closed loop around line segments of the fracture may include computing an intersection of all stadia sides for each specified radius for each line segment of the fracture and discarding the contained segments for each straight line segment in each fracture line segment that are wholly contained by stadia of other line segments in the fracture line segment.

30 Following the creation of closed loops, shape elements may be generated within the closed loops of the straight line segment. For example, parametrical segments may be generated

along a length and radius of each straight line segment. Where possible, quadrilateral elements are formed within the structured region, and polygons are formed within the remaining regions of the closed loops.

Once the shape elements are generated, a constrained mesh is generated around the closed
5 loops of the set of fracture line segments, filling the remainder of the two-dimensional plane. In one or more embodiments, a Delaunay triangulation algorithm is employed to generate the constrained mesh around the closed loops of the set of fracture line segments. Thus, the two-dimensional plane now consists entirely of cell elements of the fracture line segments and the constrained mesh. From here, the process can extrude each of the cells in the two-
10 dimensional plane to a third dimension for creating one or more layers of three-dimensional cells.

The cells within a closed loop of a fracture line segment represent a three-dimensional fracture, whereas the cells within the mesh represent the rock layers encompassing the fractures. Thus, the process can assign reservoir properties such as, but not limited to,
15 porosity and permeability, to each of the three-dimensional cells for modeling the fluid flow of the reservoir. A 2.5D stadia meshing algorithm is discussed in greater detail hereinafter.

Figure 8 is a flow chart outlining a process 2500 for modeling 3D fractures using a 2.5D stadium mesher according to one or more embodiments. The process 2500 begins at block 2502 by receiving a set of fractures 223 (Figure 1A) with geometry that has been discretized
20 in a 2D plane by a collection of line segments. In an alternative, the process 2500 may begin by performing the discretization of a set of fractures 223 in a 2D plane by a collection of line segments. The collection of line segments can represent the intersection between the 2D plane and the 3D geological fractures. As shown in block 2503, each fracture can be represented by a collection of straight line segments to approximate a curvature of the
25 fracture.

At block 2504, for each fracture line segment in the 2D plane, process 2500 performs block 2506 to generate a set of stadia at a specified radii from the respective fracture line segment. Then, at block 2508, process 2500 then generates closed loops around all of the line segments of a fracture. In certain embodiments, block 2508 includes a process of generating the closed
30 loops around line segments of the fracture. This can include computing an intersection of all stadia sides for each specified radius for each line segment of the fracture, as shown at block

2509 in Figure 8, and discarding the contained segments for each straight line segment in each fracture line segment that are wholly contained by stadia of other line segments in the fracture line segment, as shown at block 2511.

After completing block 2508 (and optional blocks 2509 and 2511 in certain embodiments),
5 the process 2500 proceeds to block 2510. At block 2510, shape elements may be generated within the closed loops of the straight line segment. For example, in one embodiment, the process 2500 generates parametrical segments along a length and radius of each straight line segment, as shown in block 2513.

Process 2500 then forms quadrilateral elements where possible within the structured region at
10 block 2515, and forms polygons within the remaining regions of the closed loops at block 2517. After generating the shape elements, process 2500 can generate a constrained mesh around the closed loops of the set of fracture line segments, filling the remainder of the 2D plane at block 2512. In one embodiment, a Delaunay triangulation algorithm can be utilized to generate the constrained mesh around the closed loops of the set of fracture line segments.
15 Thus, the 2D plane now consists entirely of cell elements of the fracture line segments and the constrained mesh.

At this point, process 2500 can extrude each of the cells in the 2D plane to a third dimension for creating one or more layers of 3D cells. The cells within a closed loop of a fracture line segment can represent a 3D fracture, whereas the cells within the mesh can represent rock
20 layers encompassing the fractures. Thus, the process 2500 can assign reservoir properties such as, but not limited to, porosity and permeability, to each of the 3D cells for modeling the fluid flow of the reservoir, as shown at block 2516.

Referring back to Figure 1A, once unstructured model mesh 230 has been created to capture the pressure field near the first subset of fractures 223, at step 115, an extended anisotropic
25 geometry-adaptive refinement (AGAR) mesher algorithm may be used to capture the flow near the remaining second subset of fractures 227. That is, an unstructured AGAR algorithm may be used to refine the unstructured mesh of model 230 to resolve the remaining fractures 227, thereby creating refined reservoir model mesh 235.

In one or more embodiments, refinement can be coarse for all the remaining fractures 227.
30 Often, a significant portion of the fractures of subset 227 will contribute less significantly to the flow of the reservoir system and produce less severe pressure gradients. When this is

expected to be the case, these areas do not require as much mesh resolution as those fractures
 223 handled primarily with the 2.5D meshing scheme. Accordingly, a reduction in element
 count may be obtained as compared to a reservoir-wide use of the unstructured AGAR
 algorithm. However, the unstructured AGAR algorithm may be tuned to provide higher
 5 resolution near particular fractures that are expected to contribute more significantly to the
 flow of the reservoir.

An extended AGAR mesher algorithm suitable for use in step 115 according to one or more
 embodiments is now described. In general, the AGAR mesher algorithms refines cell edges
 up to a number n times and does not split any edge (*i.e.*, splitting the cell) that will produce an
 10 edge shorter than a *TargetSize*. As defined herein, *TargetSize* is the desired mesh size, or
 edge length, for resolving the fracture width. In certain embodiments, the *TargetSize* value is
 provided to the system based upon a desired level of accuracy and the desired time to
 solution. It may determine whether a given cell should be refined and in what direction
 refinements should occur. In some embodiments, there may be two types of rules for
 15 determining whether a cell should be refined: 1) gradation rules (rules that ensure slow
 transitions in element size); and 2) intersection rules (rules that ensure the fractures are
 adequately represented).

To summarize the AGAR implementation of step 115, consider the U -direction (one direction
 of three cardinal directions in a 3D space). Note that U is a direction in a topological sense,
 20 not in a Euclidean sense. The U -direction for each cell is independent of the U -direction for a
 neighboring cell. Further, within a cell, one U edge might point in a slightly different
 direction than another U edge. As will be described in more detail below, considering the U -
 direction, the extended AGAR algorithm will analyze all cells within the model and refine
 those near-fracture cells in the U -direction if all U edges are longer than $C \times \text{TargetSize}$ and
 25 at least one of the following five rules are met: 1) a scaled U edge intersects a fracture; 2) for
 any opposing pair of edges in the U -direction, exactly one of the two scaled edges intersect a
 fracture; 3) a U edge has two or more ‘hanging’ nodes; 4) for any opposing pair of edges in
 the U -direction, exactly one of the two edges has two or more hanging nodes; or 5) the
 unscaled cell intersect a fracture but no scaled edge of the cell intersects a fracture. Referring
 30 to $C \times \text{TargetSize}$, for an implementation-specific or user-supplied scalar variable, C , step
 115 (Figure 1A) will not split any edges (thus, cells) that are shorter than $C \times \text{TargetSize}$. In

this illustrative method, C is selected as $\sqrt{2}$, but could be other variables such as, for example, $4/3$, or any finite value. This process is repeated for each direction in the 3D space.

As used herein, the term ‘hanging node’ is generally used to refer to a node created during the refinement of a neighboring cell, which is not required to maintain the underlying geometry of the cell of interest. For example, the underlying geometry of may be a hexahedron (such as shown in the examples of Figure 9), a prism (such as shown in the examples of Figures 10 – 13), or other extruded element types. Consider a cell H , when a neighboring cell is refined, an extra node is added the hexahedron-shaped cell H . This node is ‘hanging’ with respect to cell H . In yet other examples, the edge length requirement may be omitted for rules 3 and 4 if strict adherence to the one hanging node guideline is adopted.

Figure 9 illustrates a set 300 of possible refinement configurations 306, 312, and 316 for a single hexahedron cell 304 within a three-dimensional U - V - W space 302. In particular, Figure 9 depicts how, for hexahedra, some embodiments can utilize a local, logical U - V - W space 302, and each cell is analyzed and refined in some subset of these three dimensions, U , V , and W , as shown in sub-steps 308, 310, and 314 of Figure 9. Once the desired refinement directions have been determined for a cell, extended AGAR step 115 (Figure 1A) performs the refinement (i.e., splitting of edges) for that cell. Figure 9 illustrates eight possible refinement configurations for a single cell within a 3D U - V - W space 302. Extended AGAR step 115 (Figure 1A) may iteratively performs refinement by first determining whether the near-fracture cells should be refined in the U , V , or W dimensions/directions (sub-steps, 308, 310, and 314 shown in Figure 9), and then, if so, spitting the edges in the directions as shown to create cell refinements 306, 312, and/or 316. The eight possible configurations of cell refinements 304, 306, 312, and 314 range from refining in no refinement directions (hexahedron 304) to refining in all three refinement directions (the right-most refinement 316). Since these cells will be those cells positioned in the near-fracture region, the mesh resolution of the resulting images will be greatly improved.

Figure 10 shows a representative triangular prism cell 404 and a logical two-dimensional U - W coordinate system 402 and a number scheme for edges of the prism 404. Certain embodiments include two alternatives for extending the above-noted refinement concept to prisms. One is the single U -direction option shown in Figure 10. The single U -direction encompasses all of the triangular face edges of prism 404. According to this definition of the directions, refinement, when it occurs on the triangular faces of prism 404, is always isotropic

with respect to the face. In the example of Figure 10, using the single U -direction option 400, each of edges 1 through 6 is considered to be in the U -direction. Refinement in U would refine all of these edges and refinement can be done in any subset of $\{U, W\}$.

Another option is referred to herein as the multiple U -directions option, illustrated in Figure 11. Figure 11 shows a triangular prism cell 504 and a logical two-dimensional U - W coordinate system 502 and a number scheme for edges of the prism 504. Using the multiple U -directions option 500 of Figure 11, refinement would be done in U -direction pairs. Thus, refinement could be done in a subset of the $\{U_1U_2, U_1U_3, U_2U_3, W\}$ directions 502 shown in Figure 11. Generally, in the rules applied by certain embodiments, not all 16 permutations of the four directions (e.g., W, U_1, U_2 , and U_3) are performed during the AGAR refinement step 115 (Figure 1A). In the example of Figure 11, the multiple U directions are considered in pairs. For example, refinement in U_1U_2 would refine edges 1, 2, 4, 5 of prism 504.

Figure 12 provides an AGAR refinement example of the single U -direction paradigm 600. Following the general notation and the extended AGAR algorithm discussed above, and using the canonical triangular prism numbering in Figure 11, the edges of the cell, C_i , are labeled as $\{e_{ij}\}$ for $j = 1 \dots 9$. Consider now an abstract Boolean function on the edges, $\text{Test}(\{e_{ij}\})$. For example, the cell can be refined in: U if any edge from the set $\{e_{i,1}, e_{i,2}, e_{i,3}, e_{i,4}, e_{i,5}, e_{i,6}\}$ satisfies $\text{Test}(\{e_{ij}\})$ (see sub-step 606 of Figure 12), and W if any edge from the set $\{e_{i,7}, e_{i,8}, e_{i,9}\}$ satisfies $\text{Test}(\{e_{ij}\})$ (see sub-step 610 of Figure 12).

The above condition is referred to as the primary condition for gradation. Opposing conditions are now defined. For the quadrilaterals in the prism 604, the rule is the same as it is for hexahedra (e.g., as discussed above with reference to Figure 9). If, given two opposing edges on face, exactly one of those edges satisfies $\text{Test}(\{e_{ij}\})$, then the direction that divides those two edges will also be refined. For example, edges $\{e_{i,4}, e_{i,1}\}$ are opposing edges on a quadrilateral face. If edge $\{e_{i,4}\}$ satisfies $\text{Test}(\{e_{ij}\})$ but edge $\{e_{i,1}\}$ does not, then this condition would be satisfied, and direction W would be added the refinement set. Similarly, if edge $\{e_{i,7}\}$ satisfies $\text{Test}(\{e_{ij}\})$ but edge $\{e_{i,8}\}$ does not, direction W would be added the refinement set. For the triangular face, if exactly one satisfies $\text{Test}(\{e_{ij}\})$, then U is added to the refinement set. For example, if edge $\{e_{i,4}\}$ satisfies $\text{Test}(\{e_{ij}\})$ but edges $\{e_{i,5}, e_{i,6}\}$ do not, direction W would be added to the refinement set.

For the extended AGAR algorithm's gradation rules, $\text{Test}(\{e_{i,j}\})$ is defined as true if and only if $\{e_{i,j}\}$ has been refined at least twice. For the extended AGAR algorithm's intersection rules, $\text{Test}(\{e_{i,j}\})$ is defined as true if the scaled edge $\{e_{i,j}\}$ intersects a fracture of interest. Figure 12 depicts the possible combinations of refinements 608 and 612 using the single U -direction paradigm 600.

Figure 13 shows how, in an alternative to the single U -direction discussed above with reference to Figure 12, a multiple U -directions paradigm 700. In particular, Figure 13 shows refinements 708, 712, 716, 720, and 722 in the four logical dimensions/directions 702 for a multiple U directions paradigm 700 of a prism 704. Using the multiple U -directions paradigm 700, the cell will be refined in: U_1U_2 and U_1U_3 if any edge from the set $\{e_{i,1}, e_{i,4}\}$ satisfies $\text{Test}(\{e_{i,j}\})$ (see sub-steps 706 and 710 of Figure 13); U_1U_2 and U_2U_3 if any edge from the set $\{e_{i,2}, e_{i,5}\}$ satisfies $\text{Test}(\{e_{i,j}\})$ (see sub-steps 706 and 714 of Figure 13); U_1U_3 and U_2U_3 if any edge from the set $\{e_{i,3}, e_{i,6}\}$ satisfies $\text{Test}(\{e_{i,j}\})$ (see sub-steps 710 and 714 of Figure 13); and W if any edge from the set $\{e_{i,7}, e_{i,8}, e_{i,9}\}$ satisfies $\text{Test}(\{e_{i,j}\})$ (see sub-step 718 of Figure 13).

In addition to these primary conditions, opposing conditions exist. For quadrilaterals, the two U paradigms are very similar. If, given two opposing edges on a face, exactly one of those edges satisfies $\text{Test}(\{e_{i,j}\})$, then the directions that divide those two edges will also be refined. If edge $\{e_{i,7}\}$ satisfies $\text{Test}(\{e_{i,j}\})$ but edge $\{e_{i,8}\}$ does not, direction W would be added to the refinement set. In one example, for the triangular face rule, if exactly one edge satisfies $\text{Test}(\{e_{i,j}\})$, then U_1U_2 and U_1U_3 can be added to the refinement set. In this paradigm, no opposing condition is added for the triangular faces. However, alternative rules can be determined and tested as part of paradigms beyond the paradigms 600 and 700 shown in Figures 13 and 13, respectively.

Figure 14 illustrates a representative non-triangular prism 1004 (e.g., a general prismatic element) showing the logical two-dimensional coordinate system 1002. Figure 15 illustrates possible refinement of a general prismatic element 1104 by subdividing the element into hexahedra. As described above, the extended AGAR algorithm step 115 (Figure 1A) can be used to implement methods that extend the hexahedra-based AGAR scheme to also include triangular prisms, such as those shown in Figures 10–13. For extruded triangle and quadrilateral meshes, that combination is sufficient. Assuming a consistent extruded topology, the paradigms discussed above with reference to Figures 10–13 will result in a

valid mesh even if some of the elements are degenerate. Figures 14 and 15 show how certain embodiments can directly extend the single U -direction paradigm to non-triangular prisms, utilizing a mid-point subdivision algorithm. For non-triangular prisms, such as non-triangular prism 1004 of Figure 14, the anisotropy may be maintained in the U - W space 1002.

As can be seen in refinement 1104 of Figure 14, using this refinement algorithm, the non-hexahedral prismatic elements 1004 are subdivided into hexahedra. Since hexahedra have three logical dimensions, compared to the two shown in Figures 14 and 15 for the non-hexahedral prismatic elements, the opportunity for anisotropic refinement is increased after the refinement of the original element.

To generalize further, a mid-point subdivision algorithm may be applied for all convex polyhedral cells, including non-extruded cells. Note that this would change the refinement algorithm for triangular prisms from that discussed above with reference to Figures 10–13. However, for the non-extruded cells, a single dimensional U space may be used by dropping the anisotropic refinements for these general cells. In cases where the majority of cells are extruded (hexahedral cells or other prismatic cells), the anisotropy for those cells will still provide a significant advantage over fully isotropic refinement. Further, anisotropic refinement on a regular cell neighboring an irregular cell does not violate the conformal requirement on the mesh. The irregular neighbor cell's polygonal face is simply divided.

In view of the foregoing detailed description, one illustrative process for step 115 of Figure 1A will now be described with reference to the flow charts of Figures 16 and 17. Referring to Figure 16, process 2100 begins at block 2102. In one example, block 2102 includes data from unstructured extruded reservoir model 230. At block 2104, a cell in unstructured extruded reservoir model 230 may be analyzed to determine whether the cell in a mesh should be anisotropically refined (i.e., refined in any particular direction in 3D space).

If it is determined that the cell being analyzed does not require refinement, control is passed to block 2109. At block 2109, it is determined if there are other cells to be analyzed. If there are no other cells to be analyzed, control is passed to block 2110. Otherwise, if it is determined that there is another cell remaining to be analyzed, control is passed back to block 2104 where that cell will then be analyzed to determine if refinement may be necessary.

If, at block 2104, it is determined that the cell requires refinement, the algorithm moves onto block 2108 where that cell is refined. As previously described, gradation and intersection rules may be employed to determine whether a cell should be refined. Once the cell is

refined in all directions determined to be necessary, the algorithm proceeds onto block 2109 where a determination is made as to whether there are other cells to be analyzed. The process repeats until all cells have been analyzed. Upon completion of analysis of all cells, at block 2110, the fracture network within the reservoir model is resolved using the refined cells. The resulting refined reservoir model will possess a higher level of mesh resolution in those areas surrounding the fractures (e.g., near-fracture areas and the fractures), and lower mesh resolution in those non-near-fracture areas.

Figure 17 is a flow chart of an illustrative refining process 2200 used in block 2104 of Figure 16. The location of fractures within the fracture system is known based upon the fracture data 220 (Figure 1A). Using this data, along with the gradation and intersection rules previously described, those cells in the near-fracture regions of the model that require refinement may be located. For those cells requiring refinement, all edges (and, thus, the cells) may be split in half in each refinement direction. The newly created cells will then be analyzed again in the next iteration, thereby ultimately increasing the mesh resolution. The resulting high resolution mesh provides more accurate numerical results in the near-fracture/fracture regions. The $C \times TargetSize$ (e.g., where $C = \sqrt{2}$) may be determined as previously described.

At block 2104(i), it may be determined whether all edges in the U - V - W directions for the cell (each direction is analyzed separately) are longer than $C \times TargetSize$. If none of the edges are longer than $C \times TargetSize$, no refinement is required, and control is passed to block 2109. If, however, one or more edges are longer than $C \times TargetSize$, the five rules described above can then be applied. At this point, at blocks 2104(ii)-(vi), it may be determined if one or more of the following are met: ii) a scaled directional edge intersects a fracture; iii) for any opposing pair of edges in the 3D direction, exactly one of the two scaled edges intersect a fracture; iv) a directional edge has two or more ‘hanging’ nodes; v) for any opposing pair of edges in the 3D direction, exactly one of the two edges has two or more hanging nodes; and vi) the un-scaled cell intersect a fracture but no scaled edge of the cell intersects a fracture.

At block 2104(ii), it may be determined if the scaled version of the cell has an edge in the analyzed direction that intersects a fracture. At block 2104(iii), an opposing pair of edges in a given direction may be analyzed to determine if only one of the scaled edges intersects a fracture. At block 2104(iv), it may be determined if an edge in the analyzed direction has

two or more hanging nodes. In Figure 17, element 2209 is provided as an illustration of this principle. Element 2209 is described below.

As shown in Figure 17, element 2209 includes two adjacent cell faces 2210 and 2220 (shown in 2D for simplicity) having two nodes 2212 and 2214 positioned between them. In the
5 example of element 2209, the first refinement would have split cell 2210 in half using node 2212. The second refinement would have split one of the new cells in half using node 2214. Therefore, nodes 2212 and 2214 are hanging with respect to cell 2220 because, although cell 2210 has been refined (in a previous iteration), cell 2220 has not been refined. Since cell 2220 has at least two hanging nodes, it will also be refined in subsequent iterations.

10 At block 2104(v) of process 2200, opposing edge pairs in the analyzed direction may be reviewed to determine whether only one of the edges has two or more hanging nodes. At block 2104(vi), it may be determined whether an un-scaled cell intersects a fracture but no scaled edge of the cell intersects a fracture.

If the determination is affirmative to any of the five rules described above with reference to
15 blocks 2104(i-v), process 2200 passes control to block 2108 where that cell is refined. Otherwise, if the determination is negative with respect to all of the five rules, control is passed to block 2109. This process will continue iteratively until each cell in the model has been analyzed, as discussed above with respect to Figure 16.

Referring now to Figure 1B, in addition to creating refined reservoir mesh 235, method 100
20 includes the creation of a mesh for the fracture system. The fracture mesh may be represented as a volumetric mesh or a surface mesh with a thickness (aperture) property. In some instances, the fracture data may be initially provided as a discretized surface. If this surface is discretized with sufficient resolution for simulation, it may be used directly, in conjunction with a provided aperture value. In other cases, the surfaces will need to be
25 meshed or refined to achieve the desired resolution.

Accordingly, an appropriate fracture surface mesh 240 may be constructed from fracture data at step 120. For fractures 220 already defined as a surface mesh, that mesh may be used as-is, or refinement or coarsening algorithms may be employed at step 120 to create the desired resolution. At step 125, each element of fracture surface fracture mesh 240 may then be
30 thickened by a fracture aperture value to form fracture volume mesh 245.

Although typically the volume of the fracture network in fracture mesh 245 will not be a significant fraction of any reservoir cell in refined reservoir mesh 235, at this point how much fracture volume is located inside any given reservoir cell may be determined. If desired, the reservoir properties of refined reservoir mesh 235 can be modified at step 130 to account for the volume that is present in fracture volume mesh 245. Because the refined reservoir and fracture meshes 235, 245 will overlap, as discussed hereafter, the pore volume in refined reservoir mesh 235 can be reduced to account for the volume now represented by the overlapping fracture volume mesh 245. In other words, the volumes of the matrix blocks surrounding the fractures in refined reservoir mesh 235 may be modified at step 130 to maintain the correct pore volume by removing pore volume from matrix control volumes that connect with fractures, thereby resulting in intermediate reservoir mesh 250. The amount of pore volume removed depends on the number and size of fractures to which the matrix control volume is connected.

Fracture mesh 245 and intermediate reservoir mesh 250 define two discrete volumetric meshes that overlap in space. Referring now to Figure 1C, according to one or more embodiments, method 100 includes combining fracture volume mesh 245 with intermediate reservoir mesh 250 to form a final reservoir model 260. At step 135, three distinct sets of connections between fracture volume mesh 245 and intermediate reservoir mesh 250 are formed: Matrix-matrix connections, fracture-fracture connections, and matrix-fracture connections.

A first set connections, matrix-matrix connections, may be formed at step 135a between adjacent cell pairs in intermediate reservoir mesh 250. Such connections may be calculated in a standard way using two-point flux approximations (TPFA), multi-point flux approximations (MPFA), or other approximations. The TPFA scheme uses the pressure potential at the centers of two adjacent cells for computing the flow between said cells. The MPFA scheme additionally uses the pressure potential from all the neighboring cells of two adjacent cells to compute the flow between cell pairs. With orthogonal grids (the grid may be orthogonal relative to the permeability field—often referred to as "k-orthogonal"), the TPFA scheme may be sufficiently accurate. However for non-orthogonal grids, as is often the case with unstructured grids, the TPFA scheme becomes less accurate, and the MPFA scheme may be preferred to maintain accuracy.

A second set of connections, fracture-fracture connections, may be formed at step 135b between adjacent cell pairs in fracture volume mesh 245. In one or more embodiments, a control volume finite-difference discretization technique using a two-point flux approximation may be used. For both single- and multi-phase flow, the material balance for each control volume requires the knowledge of neighboring control volumes—a connectivity list—and the flow rate associated with each connection, which may be determined using the transmissibility for a fractured porous medium, as described below.

For any control-volume shape and problem dimension, the flow rate may be given as:

$$Q_{12} = T_{12} \lambda (p_2 - p_1), \quad (\text{Eq. 1})$$

where p is pressure, Q_{12} is the flow rate from cell 1 to cell 2, T_{12} is the geometric part of the transmissibility, and λ represents the fluid mobility using upstream information. In the case of multiphase flow, different flow rates, pressures, and mobilities are applicable for each phase. The mobility component of the transmissibility, which is different for each phase, may be computed in the usual way.

The geometric component of the transmissibility, T_{12} , which is the same for each phase may be expressed as:

$$T_{12} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} \text{ with } \alpha_i = \frac{A_i k_i}{D_i} n_i \cdot f_i, \quad (\text{Eq. 2})$$

where, A_i is the area of the interface between two control volumes (using information from CV_i , where CV_i designates the i th control volume), k_i is the permeability of CV_i , D_i is the distance between the centroid of the interface and the centroid of CV_i , n_i is the unit vector normal to the interface inside CV_i , and f_i is the unit vector along the direction of the line joining the control volume centroid to the centroid of the interface. All of the geometrical information needed to compute T_{12} is defined in the grid domain. The above transmissibility calculation may be used directly for either 2D or 3D problems. In a 2D configuration, the interface is a segment, while in a 3D configuration the interface is a polygon.

The numerical connection between two fractures may be accomplished through use of an intermediate control volume (CV_0). The purpose of this intermediate control volume is to allow for flow redirection and thickness variation between the two fractures. However, in

one or more embodiments, it may be advantageous not to introduce CV_0 , and its associated unknowns, into the numerical model, as an intermediate control volume may introduce numerical problems because of the small size of CV_0 relative to the control volumes of the two cells, CV_1 and CV_2 . The transmissibility between CV_1 and CV_2 may be expressed, while
 5 implicitly accounting for the intermediate control volume CV_0 , as follows:

$$T_{12} = \frac{T_{10}T_{02}}{T_{10} + T_{02}} \quad (\text{Eq. 3})$$

where T_{10} is the transmissibility between CV_1 and CV_0 and T_{02} is the transmissibility between CV_0 and CV_2 . Accordingly, T_{12} is simply a harmonic average of T_{10} and T_{02} , which is appropriate for cells in series.

10 The definition of T_{12} requires knowledge of T_{10} and T_{02} , which necessitates a geometrical definition of the intermediate control volume. In order to avoid introducing this geometrical definition, which may be complex in some configurations, the following simplification may be possible. Because the size of the intermediate control volume is generally small compared to the adjacent control volumes, and the intermediate control volume is typically
 15 characterized by a similar permeability to that of the surrounding fracture control volumes, it may be assumed that:

$$\left. \begin{array}{l} D_1 \gg D_0 \\ k_1 \approx k_0 \end{array} \right\} \Rightarrow \alpha_1 \ll \alpha_0 \Rightarrow T_{10} \approx \alpha_1 \quad (\text{Eq. 4})$$

Similarly, it may be assumed that $T_{02} \approx \alpha_2$. Therefore, T_{12} may be approximated by:

$$T_{12} \approx \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} \text{ with } \alpha_i = \frac{A_i k_i}{D_i}, \quad (\text{Eq. 5})$$

20 where A_i is the fracture aperture, and the other variables are as defined above. In this configuration, $n_i \cdot f_i = 1$.

The above 1D technique may be extended for establishing fracture-fracture connections between two coplanar 2D objects intersecting in 3D space. An intermediate control volume may be introduced to connect the 2D objects to allow expression of the transmissibility
 25 between two control volumes with different thicknesses and orientations. Moreover, because the intermediate control volume has a relatively small size and nearly the same permeability

as the surrounding control volumes the simplifications introduced above apply. Accordingly, the definition of the transmissibility from Equation 2 above may be used, noting that n_i , f_i , and CV_i are coplanar. The variable A_i is the area of the interface between CV_i and CV_0 , which is $l \times e_i$, where l is the length of the interface calculated in the grid domain, and the variable e_i is the thickness of CV_i .

With respect to establishing a connection between three 1D objects, an intermediate control volume CV_0 may be introduced for flow redirection and thickness adjustment between the different branches. The approximation of Equation 4 above may be applied to simplify the transmissibility between CV_0 and the surrounding control volumes ($T_{10} = \alpha_1$, $T_{20} = \alpha_2$, and $T_{30} = \alpha_3$). But, because of the three-way connectivity, it is not possible to remove the intermediate control volume by simply using a harmonic mean, as for the case of 1D/1D and 2D/2D connections. Rather, the connection may be modeled by analogy between flow through porous media and conductance through a wye network of resistors. A wye-delta transformation borrowed from electrical engineering may be employed to eliminate an intermediate three-way control volume CV_0 and allow the harmonic mean simplification provided by Equation 4. Thus, the transmissibility between CV_i and CV_j , with $i, j = 1, 2, 3$ (without using the intersection control volume), may be expressed as:

$$\begin{aligned} T_{12} &= \frac{T_{10}T_{20}}{T_{10} + T_{20} + T_{30}} \approx \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2 + \alpha_3} \\ T_{13} &= \frac{T_{10}T_{30}}{T_{10} + T_{20} + T_{30}} \approx \frac{\alpha_1\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} \\ T_{23} &= \frac{T_{20}T_{30}}{T_{10} + T_{20} + T_{30}} \approx \frac{\alpha_2\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3}. \end{aligned} \quad (\text{Eq. 6})$$

For an intersection with n connections, Equation 6 can be generalized as

$$T_{ij} \approx \frac{\alpha_i\alpha_j}{\sum_{k=1}^n \alpha_k}. \quad (\text{Eq. 7})$$

The above transformations maybe applied using only information contained in the connectivity list and may be applied to 3D problems to eliminate all intersection control volumes. It should be noted that the above transformations are for cells not forming the

boundary elements of the reservoir volume. Additionally, connections should be established for nearby but disconnected fractures accounting for the fracture rock that would be involved in such flow. Connections should be created between fractures that are close relative to the grid size in the reservoir of that region to ensure that small gaps in the fracture network do not force the flow to be routed through large reservoir blocks.

Finally, according to step 135c, a third set of connections, matrix-fracture connections, may be formed between cells of fracture volume mesh 245 and cells of intermediate reservoir mesh 250 to create a final model 260 for simulation. Fracture mesh 245 and intermediate reservoir mesh 250 define two discrete volumetric meshes that overlap in space. Intersections (*i.e.*, geographically intersecting faces) of reservoir mesh cells with fracture mesh cells are first determined.

In one or more embodiments, a standard two-point flux approximation for computing transmissibility t_i across a face for some cell, C_i , may be computed as follows:

$$t_i = \frac{A_i k_i (c_i \cdot f_i)}{d_i}, \quad (\text{Eq. 8})$$

where A_i is the area of the face, k_i is the permeability of the cell, c_i is the direction between the cell and face centroid, f_i is the normal for the face, and d_i is the distance between the cell and face centroids. The harmonic average may then be taken to determine the transmissibility between the two cells. For C_{mat} , a reservoir (matrix) mesh cell, and C_{frac} , a fracture mesh cell, the transmissibility between the two faces may be given as:

$$T_{mat,frac} = \frac{t_{mat} t_{frac}}{t_{mat} + t_{frac}}. \quad (\text{Eq. 9})$$

In one or more embodiments, Equation 9 may be used directly to generate matrix-fracture connections. However, in one or more embodiments, matrix-fracture connections may be more accurately generated as follows:

The fracture cell, C_{frac} , was originally a face that was then thickened in step 125 (Figure 1B) to have an aperture width Ap ; that original face is referred to as F_{orig} . For small values of aperture Ap , the primary flow will be from the two faces extruded from F_{orig} , but only a portion of the original face may be in C_{mat} . Thus, we let F_{inter} be the intersection of C_{mat} and

F_{orig} . The fracture area A_{frac} can then be assumed to be twice the area of F_{inter} (i.e., $A_{frac} = 2 \| F_{inter} \|$). It follows, then, that:

$$t_{frac} = \frac{4 \cdot \| F_{inter} \| \cdot k_{frac}}{Ap}. \quad (\text{Eq. 10})$$

To avoid small variations in the fractures position relative to the reservoir cell centroid having great impact on the flow between the two cell, it may be preferable to not simply compute the centroid differences between the reservoir (matrix) cell centroid and the face centroid, geometrically. Letting $\tilde{d}_{mat} = \frac{d_{mat}}{c_{mat} \cdot f_{mat}}$ denote a representative distance between the matrix cell and the fracture faces through which the flow is modeled, only an appropriate value for $\tilde{d}_{mat}$ is needed to solve for t_{mat} , and thus $T_{mat,frac}$:

$$t_{mat} = \frac{2 \cdot \| F_{inter} \| \cdot k_{mat}}{\tilde{d}_{mat}} \quad (\text{Eq. 11})$$

In one or more embodiments, an initial value for $\tilde{d}_{mat}$ may be given by the following conceptual model:

$$\tilde{d}_{mat} = \frac{\| l_{seg} \|}{4} \quad (\text{Eq. 12})$$

where l is a line passing through the centroid of C_{mat} along the direction of the face normal, and the line segment l_{seg} is determined by:

$$l_{seg} = C_{mat} \cap l. \quad (\text{Eq. 13})$$

However, computational experiments and comparisons may also be used to determine the appropriate equation, correlation, or refinement for $\tilde{d}_{mat}$.

Figure 18 is a 2D schematic of the foregoing interpretation of $\tilde{d}_{mat}$. The representative distance from the reservoir (matrix) cell to the fracture faces is computed assuming the fracture was at the centroid and flow needed to traverse a quarter of the cell going in either direction.

Figures 19A–19C illustrate an exemplary natural fracture mesh that has been processed in accordance with method 100 (Figures 1A–1C) described above, except that no 2.5D meshing of step 110 was performed in this specific example. The model was simulated using commercially available Nexus® Reservoir Simulation Software. Figure 19A is a perspective view of a natural fracture network, Figure 19B is a perspective view of the processed mesh in accordance with method 100, and Figure 19C is a horizontal cross section of Figure 19B. Shading of Figures 19B and 19C is indicative of the pressure field. In one or more embodiments, the fracture network may be trimmed (not illustrated) to avoid fractures extending outside of the reservoir being modeled.

Examples of embodiments have been described in terms of apparatuses, systems, services, and methods. However, certain aspects of the disclosed embodiments may be embodied in software that is executed using one or more processing units/components. Program aspects of the technology may be considered as "products" or "articles of manufacture" typically in the form of executable code and/or associated data that is carried on or embodied in a type of machine readable medium. This, to implement the various features and functions described above, some or all elements of the systems and methods may be, in one or more embodiments, implemented using elements of the computer system 3000 of Figure 19. Computer system 3000 is operable to manipulate or transform data represented as physical electronic or magnetic quantities within memories, registers, or other information storage devices, transmission devices, or display devices of the computing platform. Computer system 3000 may include hardware, software, firmware, non-transitory computer readable media having logic or instructions stored thereon, or a combination thereof. Any suitable programming, scripting, or other type of language or combinations of languages may be used to implement method 100 in software to be executed by computer system 3000. Throughout this specification, disclosure using terms such as “processing,” “computing,” “calculating,” “determining,” “generating,” “identifying,” or the like may refer to actions or processes of computing computer system 3000. Computer system 3000, as disclosed herein, is not limited to any particular hardware architecture or configuration.

In particular, computer hardware, software, or any combination of such may embody certain modules and components used to implement method 100 illustrated by Figures 1A–1C discussed above. Although operations may be described as a sequential process, some of the operations may in fact be performed in parallel, concurrently, and/or in a distributed

environment, and with program code stored locally or remotely for access by single or multi-processor machines. In addition, in some embodiments the order of operations may be rearranged without departing from the spirit of the disclosed subject matter. If programmable logic is used, such logic may execute on a commercially available processing platform or a special purpose device. Embodiments of the disclosed subject matter may be implemented by various computer system configurations, including multi-core multiprocessor systems, minicomputers, mainframe computers, computers linked or clustered with distributed functions, as well as pervasive or miniature computers that may be embedded into virtually any device.

Referring now to Figure 19, computer system 300 may include a processor device 3004. Processor device 3004 may be a special purpose or a general purpose processor device and may include digital circuitry such as microprocessors, microcontrollers, field-programmable gate arrays, digital-to-analog converters, analog-to-digital converters, buffers, latches, memory, drivers, multiplexors, universal asynchronous receivers/transmitters, and the like. Discrete electronic components may be combined in one or more application-specific integrated circuits (ASICs) as appropriate. Processor device 3004 may be a single processor in a multi-core/multiprocessor system, such system operating alone, or in a cluster of computing devices operating in a cluster or server farm. Processor device 3004 may be connected to a communication infrastructure 3006, for example, a bus, message queue, network, or multi-core message-passing scheme.

Computer system 3000 may also include a main memory 3008 and a secondary memory 3010. Main memory 3008 may include volatile memory, for example, random access memory (RAM), that temporarily stores software instructions and data for execution and or manipulation by processor device 3004. Secondary memory 3010 may include, for example, non-volatile memory such as Flash memory or electrically erasable programmable read-only memory (EEPROM), a hard disk drive 3012, and/or removable storage 3014 for storing persistent software code and data. Software code and data may be transferred between secondary memory 3010 and main memory 3008 as necessary. Removable storage drive 3014 may include magnetic or optical storage media, Flash memory, or the like. Removable storage 3014 may include a non-transitory computer software and/or data.

Computer system 3000 may also include a communications interface 3024. Communications interface 3024 allows software and data to be transferred between computer system 3000 and

one or more external devices or communications networks (not illustrated). For example, communications interface 3024 may enable computer system 3000 receive user input (e.g., from a keyboard and mouse) and output information to one or more devices such as, but not limited to, printers, external data storage devices, and audio speakers. A computer display 5 3030 of computer system 3000 may be implemented as a conventional or touch sensitive display (i.e., a touch screen). The computer display 3030 may connect to communications infrastructure via display interface 3002 to display received electronic content. For example, the computer display 3030 can be used to display input models and refined fractures. Display interface 3002 may include instructions or hardware (e.g., a graphics card or chip) for 10 providing enhanced graphics, touchscreen, and/or multi-touch functionalities associated with computer display 3030.

Communications interface 3024 may include a network interface card and/or a wireless transceiver, for example. Software and data transferred via communications interface 3024 may be in the form of signals, which may be electronic, electromagnetic, optical, or other 15 signals capable of being received by communications interface 3024. These signals may be provided to communications interface 3024 via a communications path 3026. Communications path 3026 carries signals and may be implemented using wire or cable, fiber optics, a phone line, a cellular phone link, a radio-frequency (RF) link or other communications channels. The communications network may be any type of network 20 including a combination of one or more of the following networks: A wide area network, a local area network, one or more private networks, the Internet, a telephone network such as the public switched telephone network (PSTN), one or more cellular networks, and wireless data networks. The communications network may include a plurality of network nodes such as routers, network access points/gateways, switches, domain name service (DNS) servers, 25 proxy servers, and other network nodes for assisting in routing of data/communications between devices.

In one or more embodiments, computer system 3000 may interact with one or more database servers (not illustrated) for performing method 100 (Figures 1A–1C). Computer system 3000 may query a database for geological information for assigning reservoir properties to cells for 30 performing a simulation. For instance, computer system 3000 may query a database for well log information for determining fracture orientation or density for enabling modeling of the fractures in accordance with the disclosed embodiments. Further, in certain embodiments,

computer system 3000 may act as a server system for one or more client devices or a peer system for peer to peer communications or parallel processing with one or more devices.

In summary, a method, system, and computer readable storage medium with executable instructions to model three-dimensional geological fractures within a reservoir have been
5 described. Embodiments of the method to model three-dimensional geological fractures within a reservoir may generally include: Generating a volumetric matrix mesh of the reservoir; generating a volumetric fracture mesh of the geological fractures; establishing matrix-matrix connections within the matrix mesh; establishing fracture-fracture connections within the fracture mesh; and establishing matrix-fracture connections between the matrix
10 and fracture volume meshes. Embodiments of the system to model three-dimensional geological fractures within a reservoir may generally have: A processor; and a memory having instructions stored thereon, that, if executed by the processor, cause the processor to perform operations comprising, generating a volumetric matrix mesh of the reservoir, generating a volumetric fracture mesh of the geological fractures, establishing matrix-matrix
15 connections within the matrix mesh, establishing fracture-fracture connections within the fracture mesh, and establishing matrix-fracture connections between the matrix and fracture volume meshes. Embodiments of the computer readable storage medium with executable instructions to model three-dimensional geological fractures within a reservoir may generally have instructions including: Generating a volumetric matrix mesh of the reservoir;
20 generating a volumetric fracture mesh of the geological fractures; establishing matrix-matrix connections within the matrix mesh; establishing fracture-fracture connections within the fracture mesh; and establishing matrix-fracture connections between the matrix and fracture volume meshes.

Any of the foregoing embodiments may include any one of the following elements or
25 characteristics, alone or in combination with each other: The matrix mesh is orthogonal; the establishing matrix-matrix connections further comprises discretizing using two-point flux approximations of pressure potential between adjacent cells in the matrix mesh; the matrix mesh includes non-orthogonal cells; the establishing matrix-matrix connections further comprises discretizing using multi-point flux approximations of pressure potential of all
30 neighboring cells of adjacent cells in the matrix mesh; approximating a transmissibility between adjacent cells in the fracture mesh; determining a flow rate between the adjacent cells using the transmissibility; discretizing using a two-point or multi-point flux

approximations of the flow rate; determining geographical intersecting faces between cells of the matrix mesh and cells of the fracture mesh; for each intersecting face, calculating a matrix transmissibility value at the face of the matrix mesh, calculating a fracture transmissibility value at the face of the fracture mesh, and calculating a transmissibility across the face
 5 between the matrix mesh and the fracture mesh as a harmonic average of the matrix and fracture transmissibility values; calculating the matrix transmissibility value using a two-point flux approximation; calculating the fracture transmissibility value using a two-point flux approximation; the generating a volumetric fracture mesh includes extruding an original face of each cell within a surface fracture mesh by a corresponding aperture width; the
 10 establishing matrix-fracture connections further comprises, for each face, calculating the fracture and matrix transmissibility values as functions of the intersection of the corresponding original face of the surface fracture mesh with the face of the matrix mesh; for each face, calculating the fracture and matrix transmissibility values as functions of the intersection of the corresponding original face of the surface fracture mesh with the face of
 15 the matrix mesh; determining a representative distance between the centroid of a cell in the matrix mesh and the intersecting face of the cell; calculating the matrix transmissibility value for the face of the cell as a function of the representative distance; receiving a reservoir specification; identifying, based on the reservoir specification, a set of fractures including 2.5-dimensional-permitting fractures and other fractures; generating an unstructured reservoir
 20 model including an extrusion mesh which models the 2.5-dimensional-permitting fractures in three-dimensions; anisotropically refining one or more cells in the unstructured reservoir model corresponding to the other fractures; resolving a fracture network within the unstructured reservoir model using the refined cells; generating a refined reservoir model using the fracture network; the 2.5-dimensional-permitting fractures have geometry that has
 25 been discretized in a two-dimensional plane by a collection of line segments; for each line segment associated with each fracture in the 2.5D-permitting fractures, generating a set of stadia at a specified radii from the line segment, generating closed loops around the line segment, and generating shape elements within the closed loops of the line segment; generating the mesh as a constrained mesh around closed loops of the 2.5D-permitting
 30 fractures to fill in a remainder space of the two-dimensional plane; identifying a direction within the three dimensions in which the cells should be refined; splitting an edge of the cells, the edge being in the direction within the three dimensions; and the method to model three-dimensional geological fractures within the reservoir is performed by a computer system.

The Abstract of the disclosure is solely for providing the a way by which to determine quickly from a cursory reading the nature and gist of technical disclosure, and it represents solely one or more embodiments.

5 While various embodiments have been illustrated in detail, the disclosure is not limited to the embodiments shown. Modifications and adaptations of the above embodiments may occur to those skilled in the art. Such modifications and adaptations are in the spirit and scope of the disclosure.

WHAT IS CLAIMED

1. A method to model three-dimensional geological fractures within a reservoir, comprising:

generating a volumetric matrix mesh of said reservoir;

5 generating a volumetric fracture mesh of said geological fractures;

establishing matrix-matrix connections within said matrix mesh;

establishing fracture-fracture connections within said fracture mesh; and

establishing matrix-fracture connections between said matrix and fracture volume meshes.

10

2. The method of claim 1 wherein:

said matrix mesh is orthogonal; and

said establishing matrix-matrix connections further comprises discretizing using two-point flux approximations of pressure potential between adjacent cells in said matrix mesh.

15

3. The method of claim 1 wherein:

said matrix mesh includes non-orthogonal cells; and

said establishing matrix-matrix connections further comprises discretizing using multi-point flux approximations of pressure potential of all neighboring cells of adjacent cells in said matrix mesh.

20

4. The method of claim 1 wherein said establishing fracture-fracture connections further comprises:

approximating a transmissibility between adjacent cells in said fracture mesh;

25 determining a flow rate between said adjacent cells using said transmissibility; and

discretizing using a two-point or multi-point flux approximations of said flow rate.

5. The method of claim 1 wherein said establishing matrix-fracture connections further comprises:

30 determining geographical intersecting faces between cells of said matrix mesh and cells of said fracture mesh; and

for each intersecting face, calculating a matrix transmissibility value at the face of said matrix mesh, calculating a fracture transmissibility value at the face of said fracture

mesh, and calculating a transmissibility across the face between said matrix mesh and said fracture mesh as a harmonic average of said matrix and fracture transmissibility values.

6. The method of claim 5 wherein said establishing matrix-fracture connections further
5 comprises:

calculating said matrix transmissibility value using a two-point flux approximation;
and

calculating said fracture transmissibility value using a two-point flux approximation.

10 7. The method of claim 5 wherein:

said generating a volumetric fracture mesh includes extruding an original face of each
cell within a surface fracture mesh by a corresponding aperture width; and

said establishing matrix-fracture connections further comprises, for each face,
calculating said fracture and matrix transmissibility values as functions of the intersection of
15 the corresponding original face of said surface fracture mesh with the face of said matrix
mesh.

8. The method of claim 7 wherein said establishing matrix-fracture connections further
comprises:

20 for each face, calculating said fracture and matrix transmissibility values as functions
of the intersection of the corresponding original face of said surface fracture mesh with the
face of said matrix mesh.

9. The method of claim 8 wherein said establishing matrix-fracture connections further
25 comprises:

determining a representative distance between the centroid of a cell in said matrix
mesh and the intersecting face of said cell; and

calculating said matrix transmissibility value for said face of said cell as a function of
said representative distance.

30 10. The method of claim 1 wherein said generating said volumetric matrix mesh further
comprises:

receiving a reservoir specification;

identifying, based on the reservoir specification, a set of fractures including 2.5-dimensional-permitting fractures and other fractures;

generating an unstructured reservoir model including an extrusion mesh which models the 2.5-dimensional-permitting fractures in three-dimensions;

5 anisotropically refining one or more cells in the unstructured reservoir model corresponding to the other fractures;

resolving a fracture network within the unstructured reservoir model using the refined cells; and

generating a refined reservoir model using the fracture network.

10

11. The method of claim 1 wherein:

the 2.5-dimensional-permitting fractures have geometry that has been discretized in a two-dimensional plane by a collection of line segments; and

said generating said volumetric matrix mesh further comprises,

15

for each line segment associated with each fracture in the 2.5D-permitting fractures, generating a set of stadia at a specified radii from the line segment, generating closed loops around the line segment, and generating shape elements within the closed loops of the line segment, and

generating the mesh as a constrained mesh around closed loops of the 2.5D-permitting fractures to fill in a remainder space of the two-dimensional plane.

20

12. The method of claim 10 wherein said anisotropically refining cells further comprises:

identifying a direction within the three dimensions in which the cells should be refined; and

25

splitting an edge of the cells, the edge being in the direction within the three dimensions.

13. The method of claim 1 wherein:

the method to model three-dimensional geological fractures within the reservoir is performed by a computer system.

30

14. A computer readable storage medium having executable instructions stored thereon, that, if executed by a computing device, cause the computing device to perform operations, the instructions comprising:

generating a volumetric matrix mesh of said reservoir;
 generating a volumetric fracture mesh of said geological fractures;
 establishing matrix-matrix connections within said matrix mesh;
 establishing fracture-fracture connections within said fracture mesh; and
 5 establishing matrix-fracture connections between said matrix and fracture volume
 meshes.

15. The computer readable storage medium of claim 14 wherein:

said matrix mesh is orthogonal; and

10 said instructions further comprise discretizing using two-point flux approximations of
 pressure potential between adjacent cells in said matrix mesh to establish said matrix-matrix
 connections.

16. The computer readable storage medium of claim 14 wherein:

15 said matrix mesh includes non-orthogonal cells; and

said instructions further comprise discretizing using multi-point flux approximations
 of pressure potential of all neighboring cells of adjacent cells in said matrix mesh to establish
 said matrix-matrix connections.

20 17. The computer readable storage medium of claim 14 wherein, to establish said
 fracture-fracture connections, said instructions further comprise:

approximating a transmissibility between adjacent cells in said fracture mesh;

determining a flow rate between said adjacent cells using said transmissibility; and

discretizing using a two-point or multi-point flux approximations of said flow rate.

25

18. The computer readable storage medium of claim 14 wherein, to establish said matrix-
 fracture connections, said instructions further comprise:

determining geographical intersecting faces between cells of said matrix mesh and
 cells of said fracture mesh; and

30 for each intersecting face, calculating a matrix transmissibility value at the face of
 said matrix mesh, calculating a fracture transmissibility value at the face of said fracture
 mesh, and calculating a transmissibility across the face between said matrix mesh and said
 fracture mesh as a harmonic average of said matrix and fracture transmissibility values.

19. The computer readable storage medium of claim 18 wherein, to establish said matrix-fracture connections, said instructions further comprise:

calculating said matrix transmissibility value using a two-point flux approximation;
and

5 calculating said fracture transmissibility value using a two-point flux approximation.

20. The computer readable storage medium of claim 18 wherein:

to generate a volumetric fracture mesh, said instructions further include extruding an original face of each cell within a surface fracture mesh by a corresponding aperture width;
10 and

to establish said matrix-fracture connections, said instructions further includes, for each face, calculating said fracture and matrix transmissibility values as functions of the intersection of the corresponding original face of said surface fracture mesh with the face of said matrix mesh.

15

21. The computer readable storage medium of claim 20 wherein to establish said matrix-fracture connections, said instructions further comprise:

for each face, calculating said fracture and matrix transmissibility values as functions of the intersection of the corresponding original face of said surface fracture mesh with the
20 face of said matrix mesh.

22. The computer readable storage medium of claim 21 wherein to establish said matrix-fracture connections, said instructions further comprise:

determining a representative distance between the centroid of a cell in said matrix
25 mesh and the intersecting face of said cell; and

calculating said matrix transmissibility value for said face of said cell as a function of said representative distance.

23. The computer readable storage medium of claim 14 wherein to generate said
30 volumetric matrix mesh, said instructions further comprise:

receiving a reservoir specification;

identifying, based on the reservoir specification, a set of fractures including 2.5-dimensional-permitting fractures and other fractures;

generating an unstructured reservoir model including an extrusion mesh which models the 2.5-dimensional-permitting fractures in three-dimensions;

anisotropically refining one or more cells in the unstructured reservoir model corresponding to the other fractures;

5 resolving a fracture network within the unstructured reservoir model using the refined cells; and

generating a refined reservoir model using the fracture network.

24. The computer readable storage medium of claim 14 wherein:

10 the 2.5-dimensional-permitting fractures have geometry that has been discretized in a two-dimensional plane by a collection of line segments; and

to generate said volumetric matrix mesh, said instructions further comprise,

15 for each line segment associated with each fracture in the 2.5D-permitting fractures, generating a set of stadia at a specified radii from the line segment, generating closed loops around the line segment, and generating shape elements within the closed loops of the line segment, and

generating the mesh as a constrained mesh around closed loops of the 2.5D-permitting fractures to fill in a remainder space of the two-dimensional plane.

20 25. The computer readable storage medium of claim 23 wherein to anisotropically refine cells, said instructions further comprise:

identifying a direction within the three dimensions in which the cells should be refined; and

25 splitting an edge of the cells, the edge being in the direction within the three dimensions.

26. A system to model three-dimensional geological fractures within a reservoir, comprising:

a processor; and

30 a memory having instructions stored thereon, that, if executed by the processor, cause the processor to perform operations comprising,

generating a volumetric matrix mesh of said reservoir,

generating a volumetric fracture mesh of said geological fractures,

establishing matrix-matrix connections within said matrix mesh,

establishing fracture-fracture connections within said fracture mesh, and
establishing matrix-fracture connections between said matrix and fracture
volume meshes.

5 27. The system of claim 26 wherein:

said matrix mesh is orthogonal; and

said instructions further comprise discretizing using two-point flux approximations of
pressure potential between adjacent cells in said matrix mesh to establish said matrix-matrix
connections.

10

28. The system of claim 26 wherein:

said matrix mesh includes non-orthogonal cells; and

said instructions further comprise discretizing using multi-point flux approximations
of pressure potential of all neighboring cells of adjacent cells in said matrix mesh to establish
15 said matrix-matrix connections.

29. The system of claim 26 wherein, to establish said fracture-fracture connections, said
instructions further comprise:

approximating a transmissibility between adjacent cells in said fracture mesh;

20

determining a flow rate between said adjacent cells using said transmissibility; and

discretizing using a two-point or multi-point flux approximations of said flow rate.

30. The system of claim 26 wherein, to establish said matrix-fracture connections, said
instructions further comprise:

25

determining geographical intersecting faces between cells of said matrix mesh and
cells of said fracture mesh; and

for each intersecting face, calculating a matrix transmissibility value at the face of
said matrix mesh, calculating a fracture transmissibility value at the face of said fracture
mesh, and calculating a transmissibility across the face between said matrix mesh and said
30 fracture mesh as a harmonic average of said matrix and fracture transmissibility values.

31. The system of claim 30 wherein, to establish said matrix-fracture connections, said
instructions further comprise:

calculating said matrix transmissibility value using a two-point flux approximation;
and

calculating said fracture transmissibility value using a two-point flux approximation.

5 32. The system of claim 30 wherein:

to generate a volumetric fracture mesh, said instructions further include extruding an original face of each cell within a surface fracture mesh by a corresponding aperture width;
and

10 to establish said matrix-fracture connections, said instructions further includes, for each face, calculating said fracture and matrix transmissibility values as functions of the intersection of the corresponding original face of said surface fracture mesh with the face of said matrix mesh.

15 33. The system of claim 32 wherein to establish said matrix-fracture connections, said instructions further comprise:

for each face, calculating said fracture and matrix transmissibility values as functions of the intersection of the corresponding original face of said surface fracture mesh with the face of said matrix mesh.

20 34. The system of claim 33 wherein to establish said matrix-fracture connections, said instructions further comprise:

determining a representative distance between the centroid of a cell in said matrix mesh and the intersecting face of said cell; and

25 calculating said matrix transmissibility value for said face of said cell as a function of said representative distance.

35. The system of claim 26 wherein to generate said volumetric matrix mesh, said instructions further comprise:

receiving a reservoir specification;

30 identifying, based on the reservoir specification, a set of fractures including 2.5-dimensional-permitting fractures and other fractures;

generating an unstructured reservoir model including an extrusion mesh which models the 2.5-dimensional-permitting fractures in three-dimensions;

anisotropically refining one or more cells in the unstructured reservoir model corresponding to the other fractures;

resolving a fracture network within the unstructured reservoir model using the refined cells; and

5 generating a refined reservoir model using the fracture network.

36. The system of claim 26 wherein:

the 2.5-dimensional-permitting fractures have geometry that has been discretized in a two-dimensional plane by a collection of line segments; and

10 to generate said volumetric matrix mesh, said instructions further comprise,

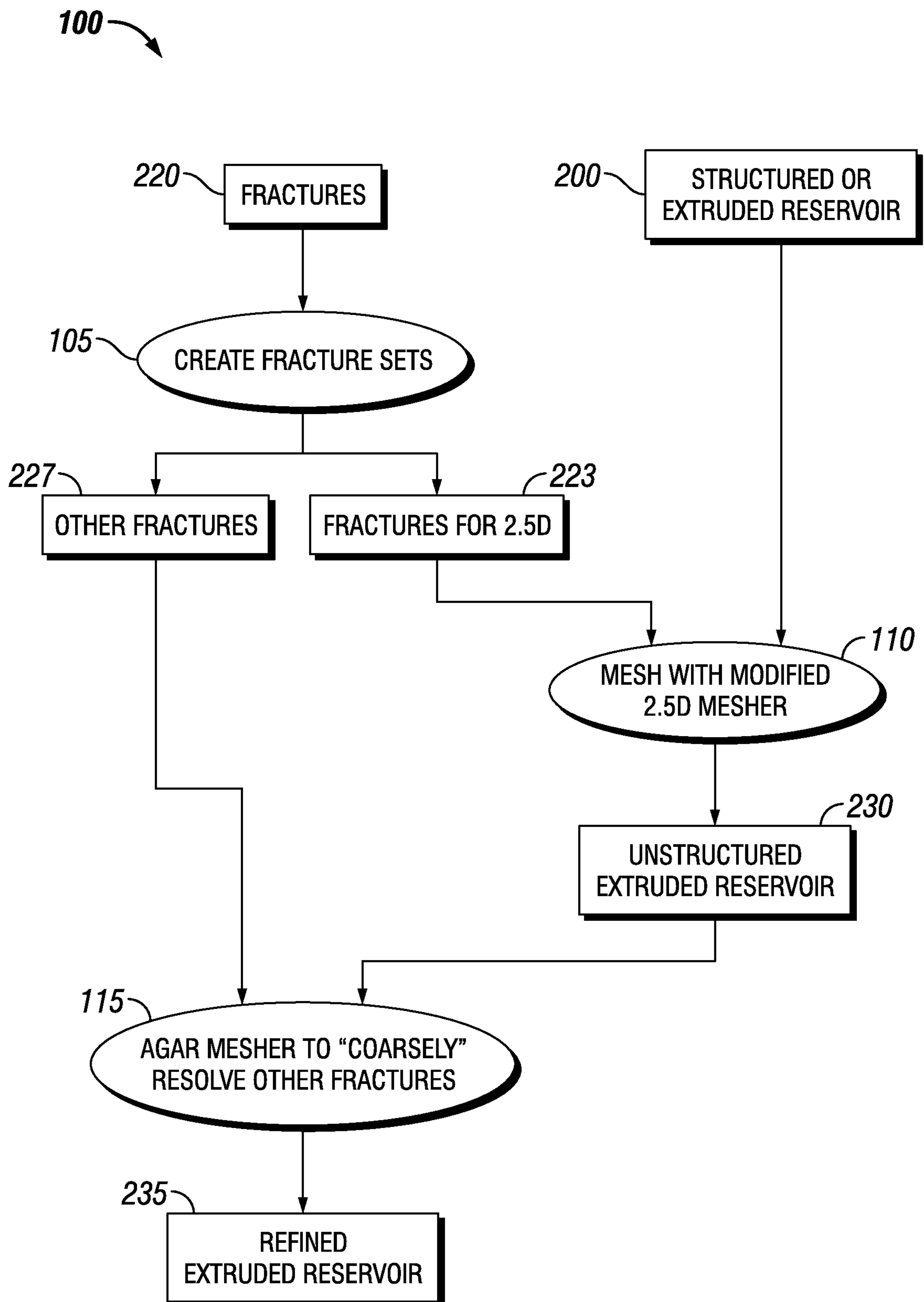
for each line segment associated with each fracture in the 2.5D-permitting fractures, generating a set of stadia at a specified radii from the line segment, generating closed loops around the line segment, and generating shape elements within the closed loops of the line segment, and

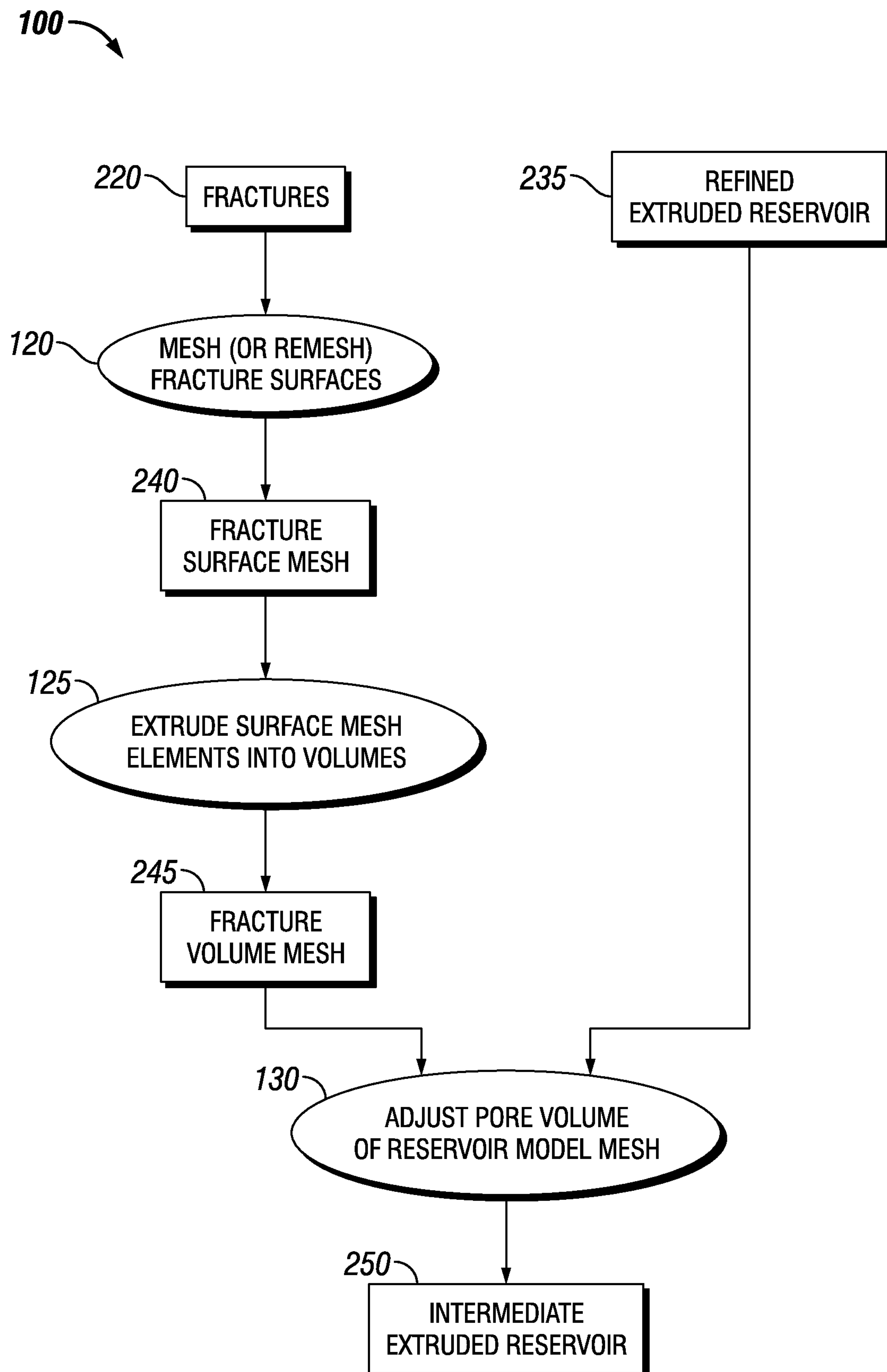
15 generating the mesh as a constrained mesh around closed loops of the 2.5D-permitting fractures to fill in a remainder space of the two-dimensional plane.

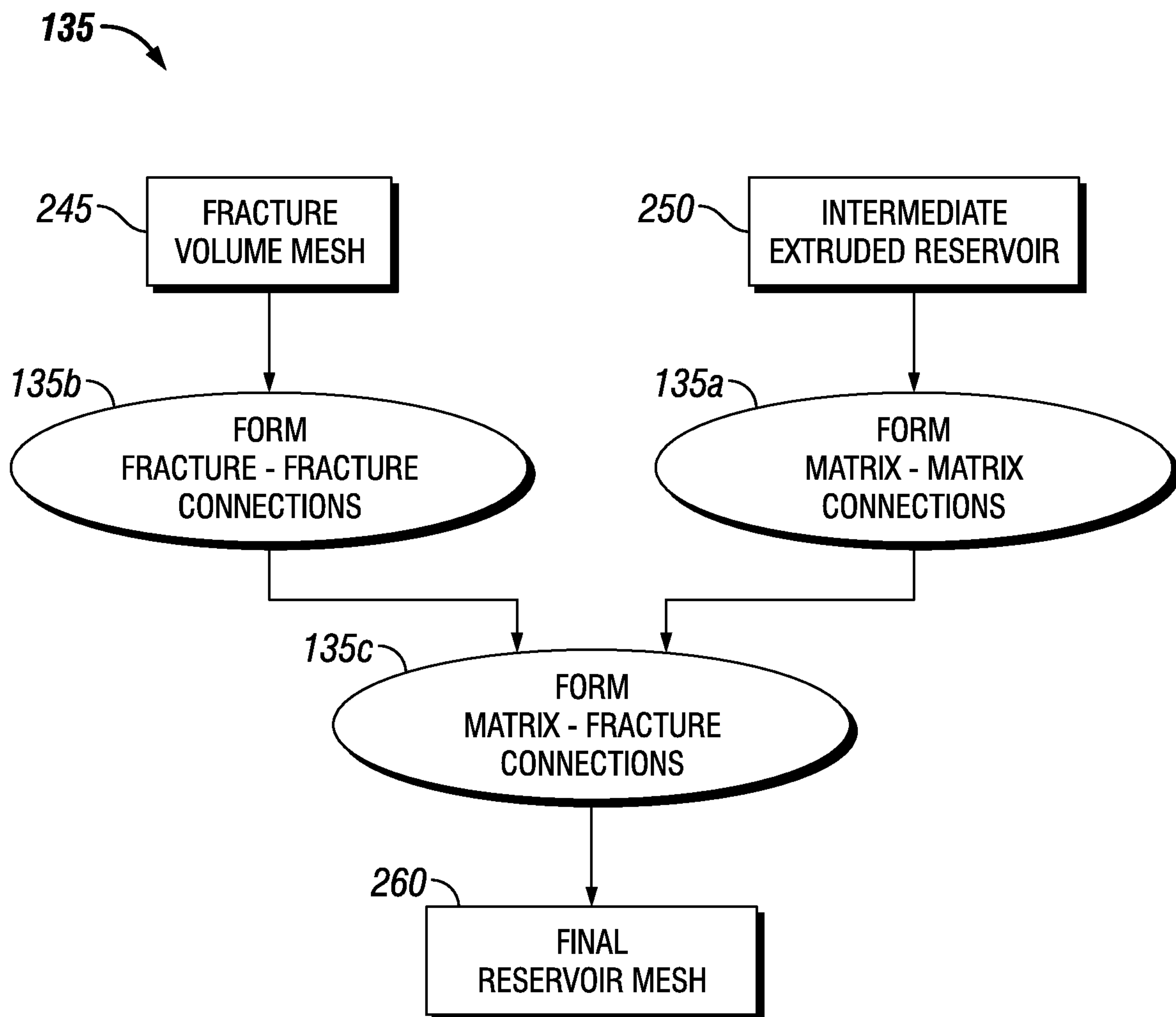
37. The system of claim 35 wherein to anisotropically refine cells, said instructions further comprise:

20 identifying a direction within the three dimensions in which the cells should be refined; and

splitting an edge of the cells, the edge being in the direction within the three dimensions.

1/21**FIG. 1A**

2/21**FIG. 1B**

3/21**FIG. 1C**

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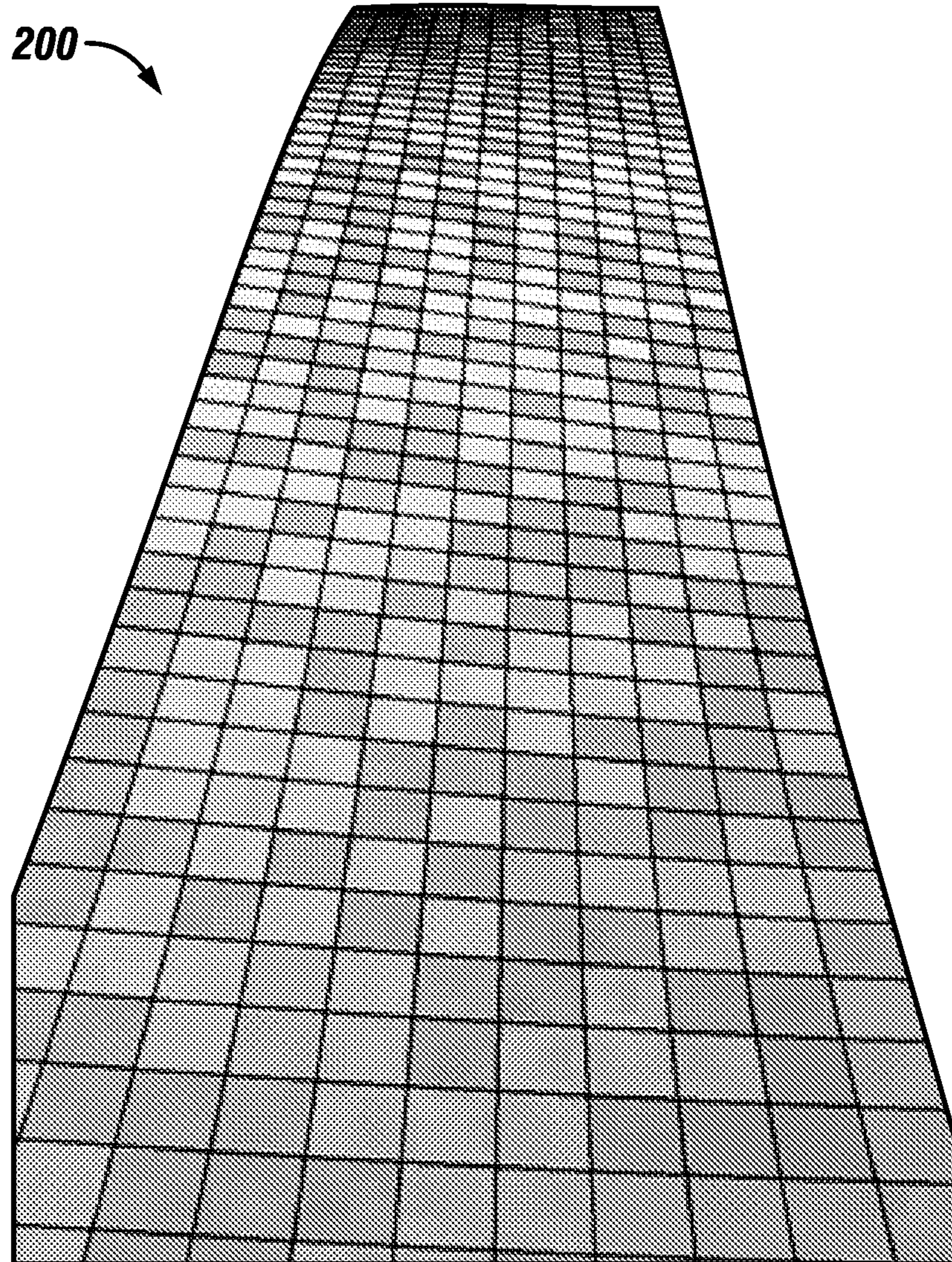


FIG. 2

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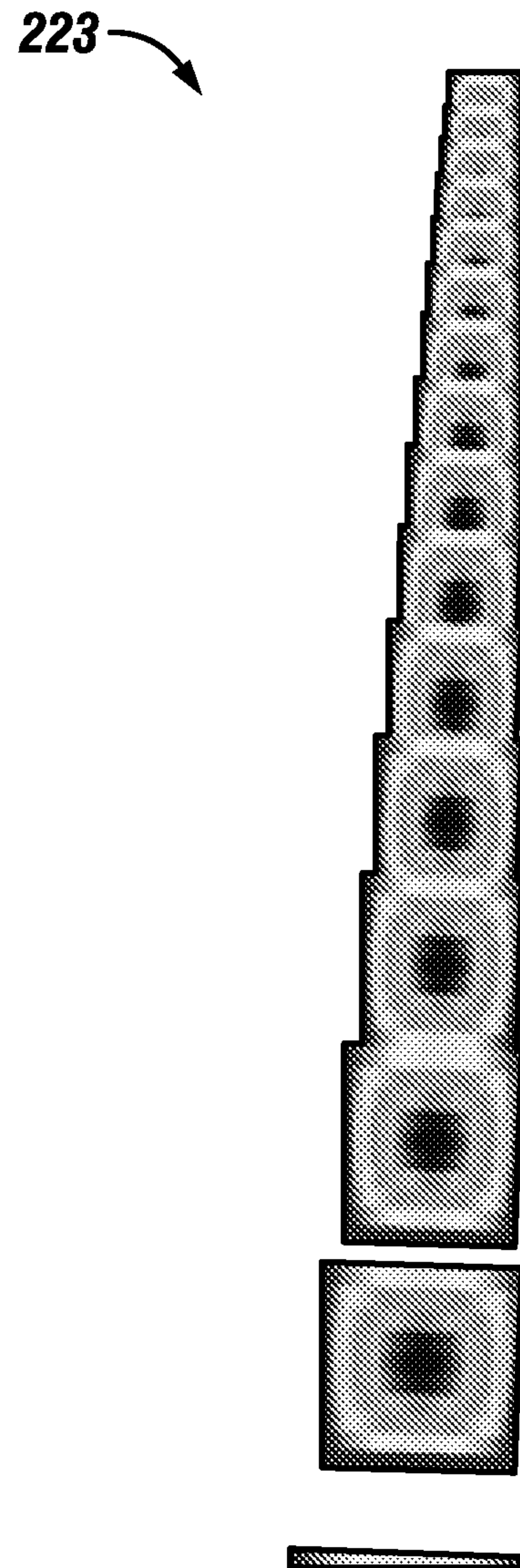


FIG. 3

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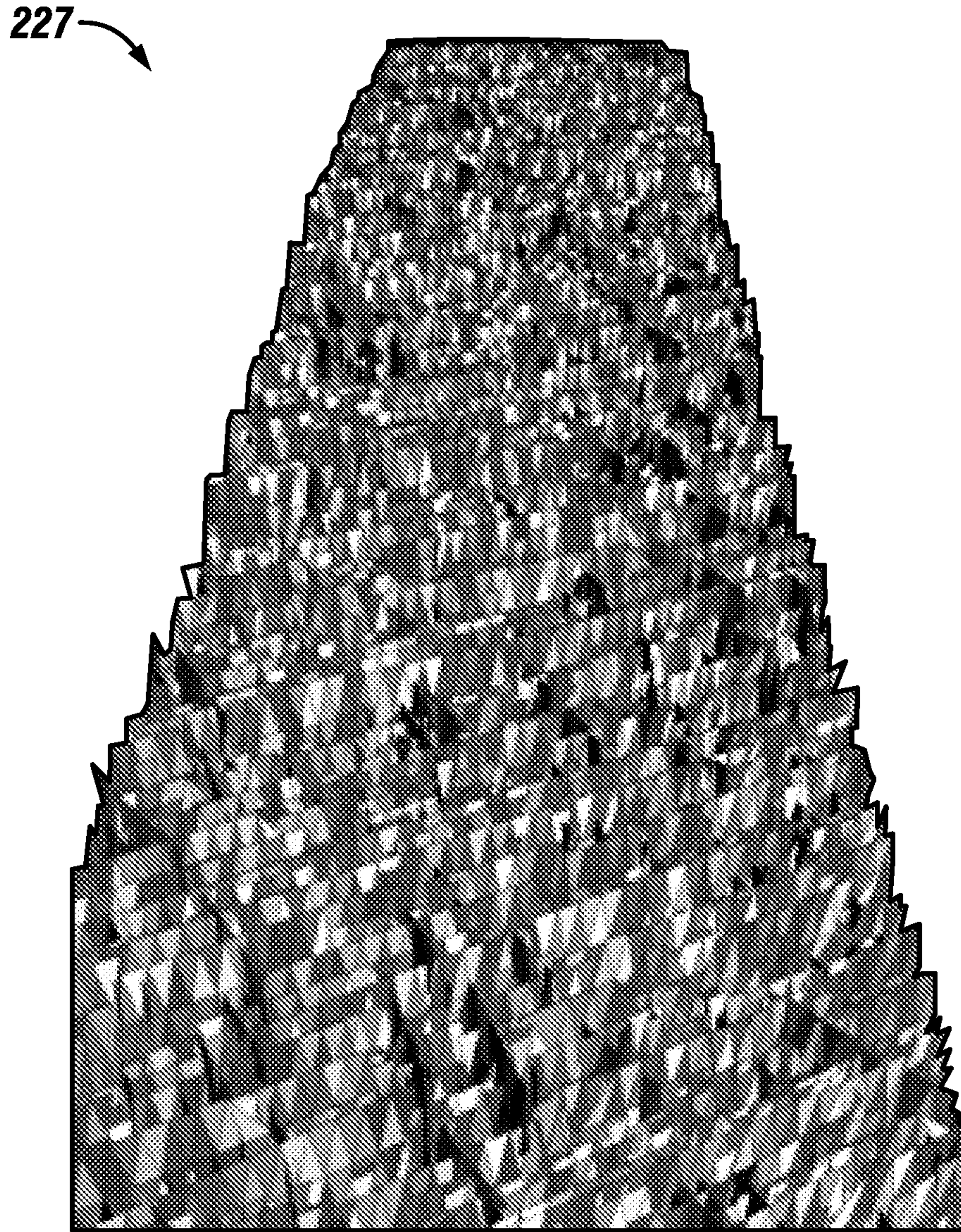
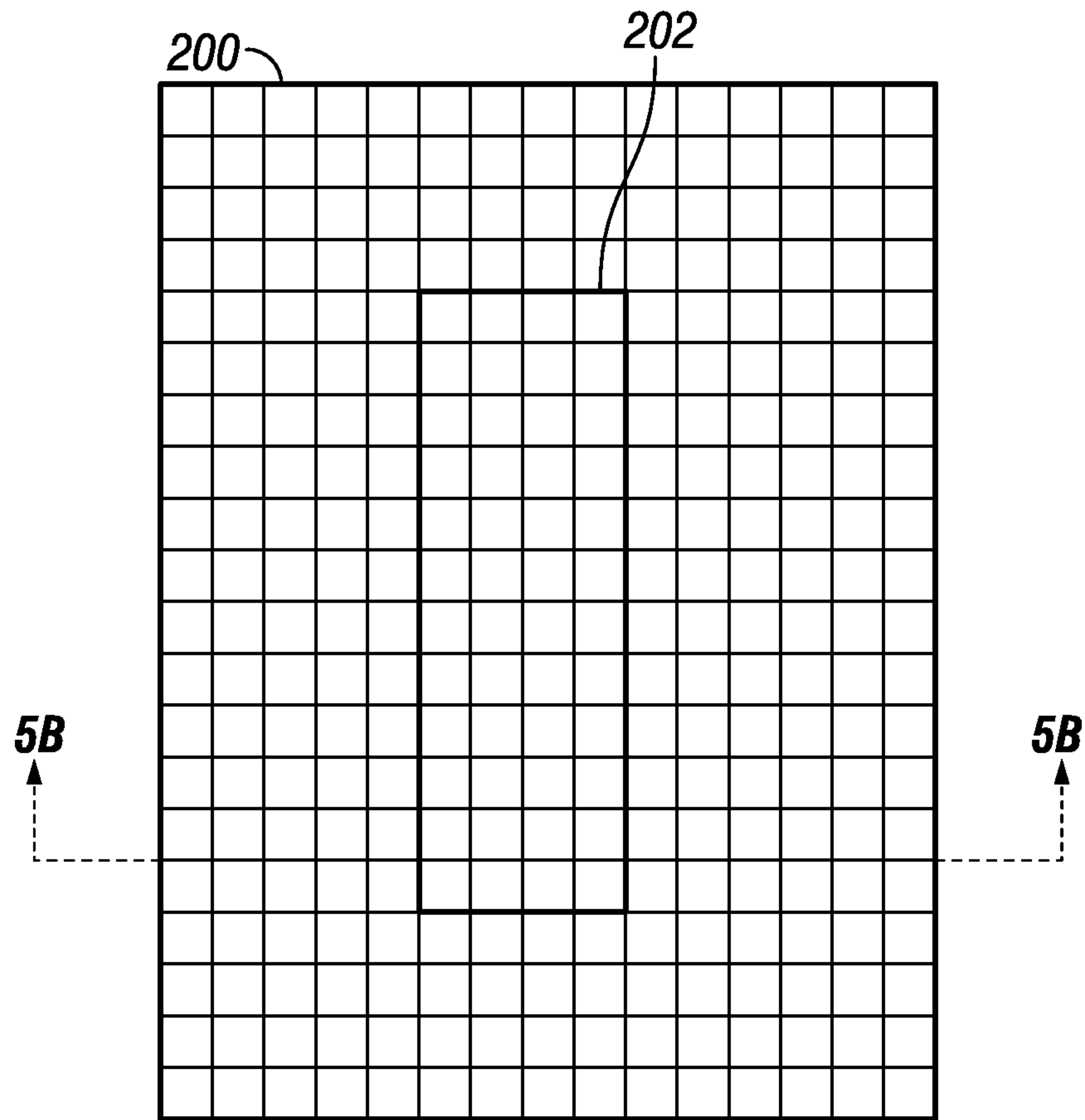
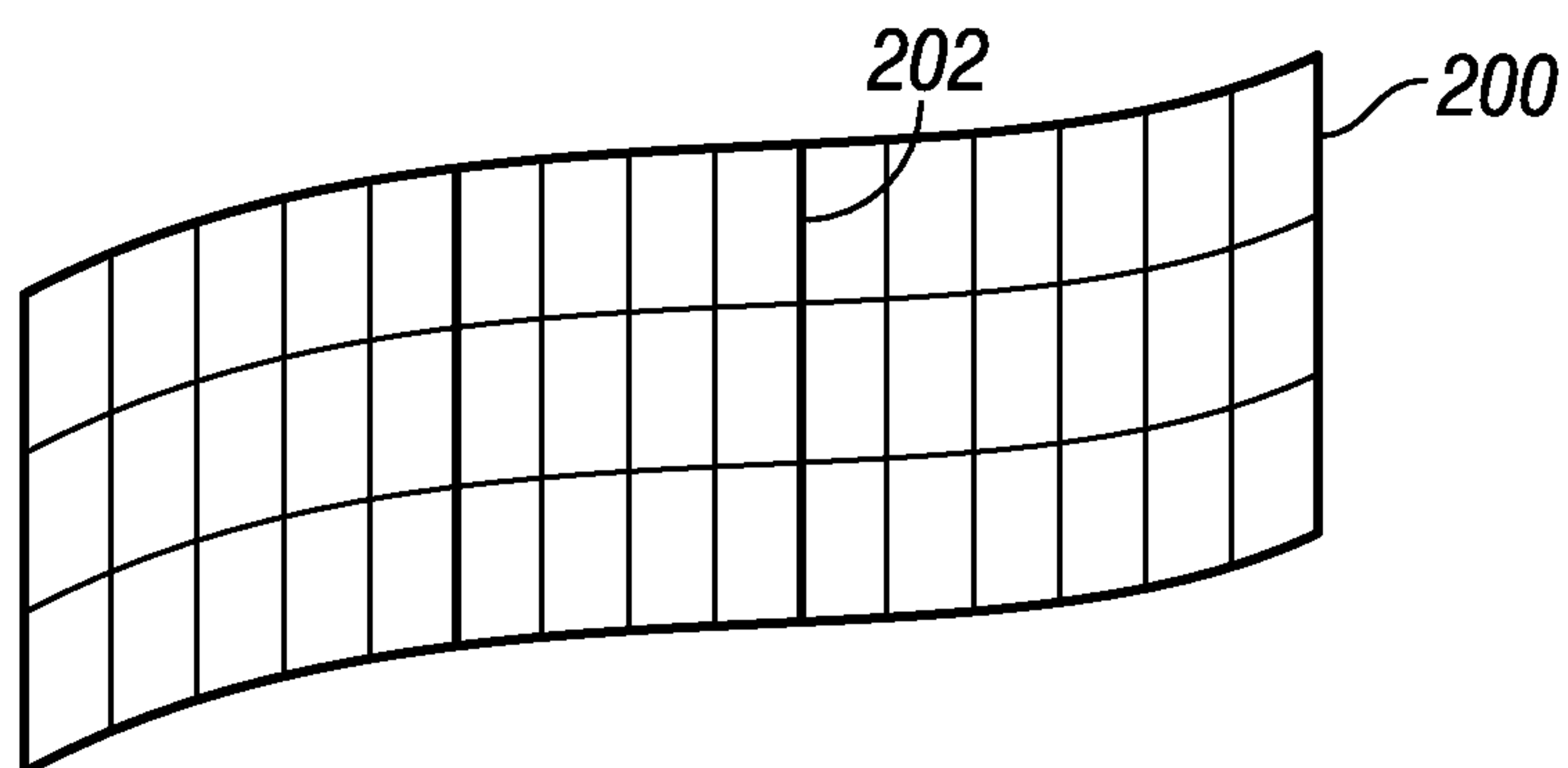
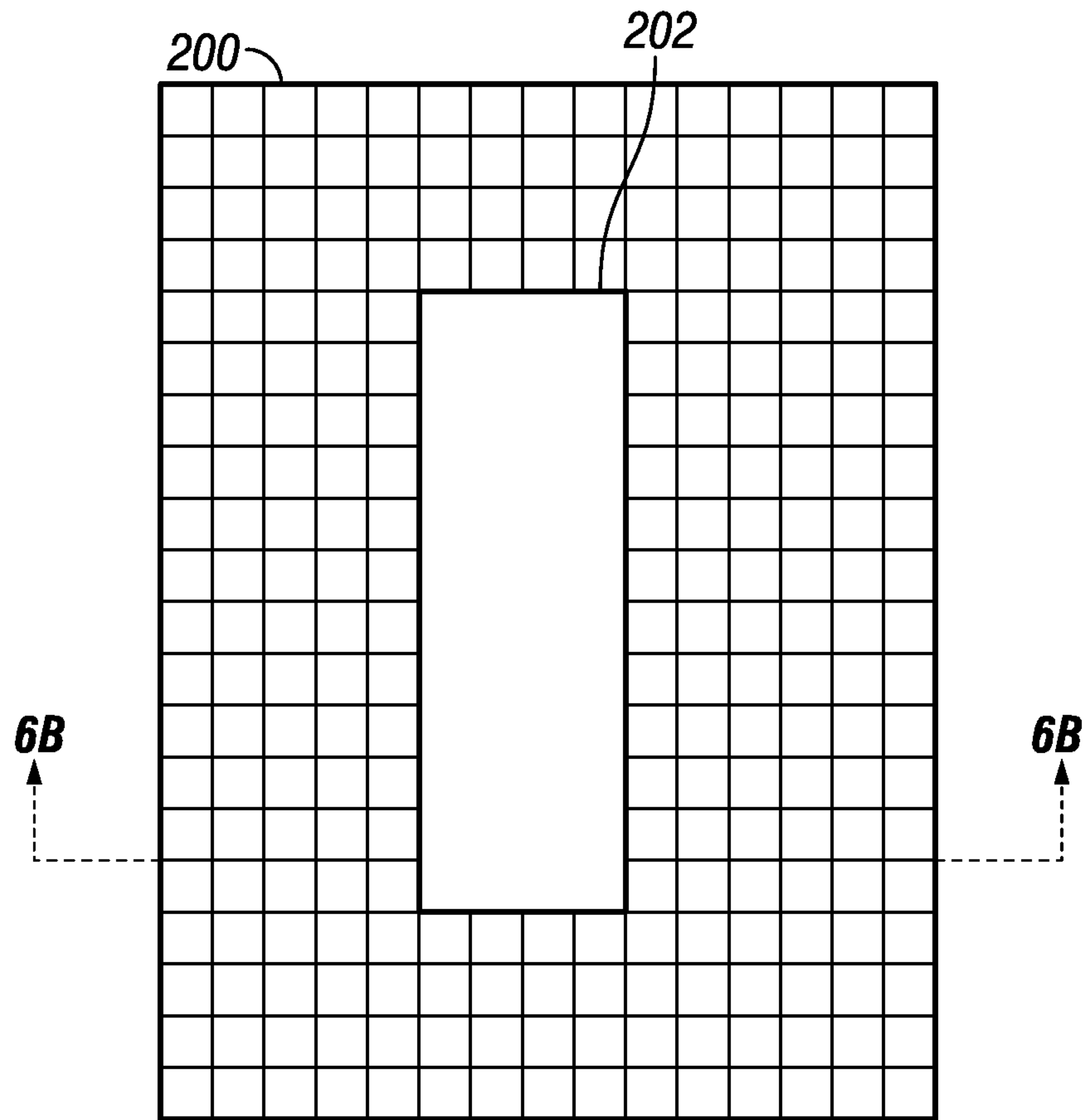
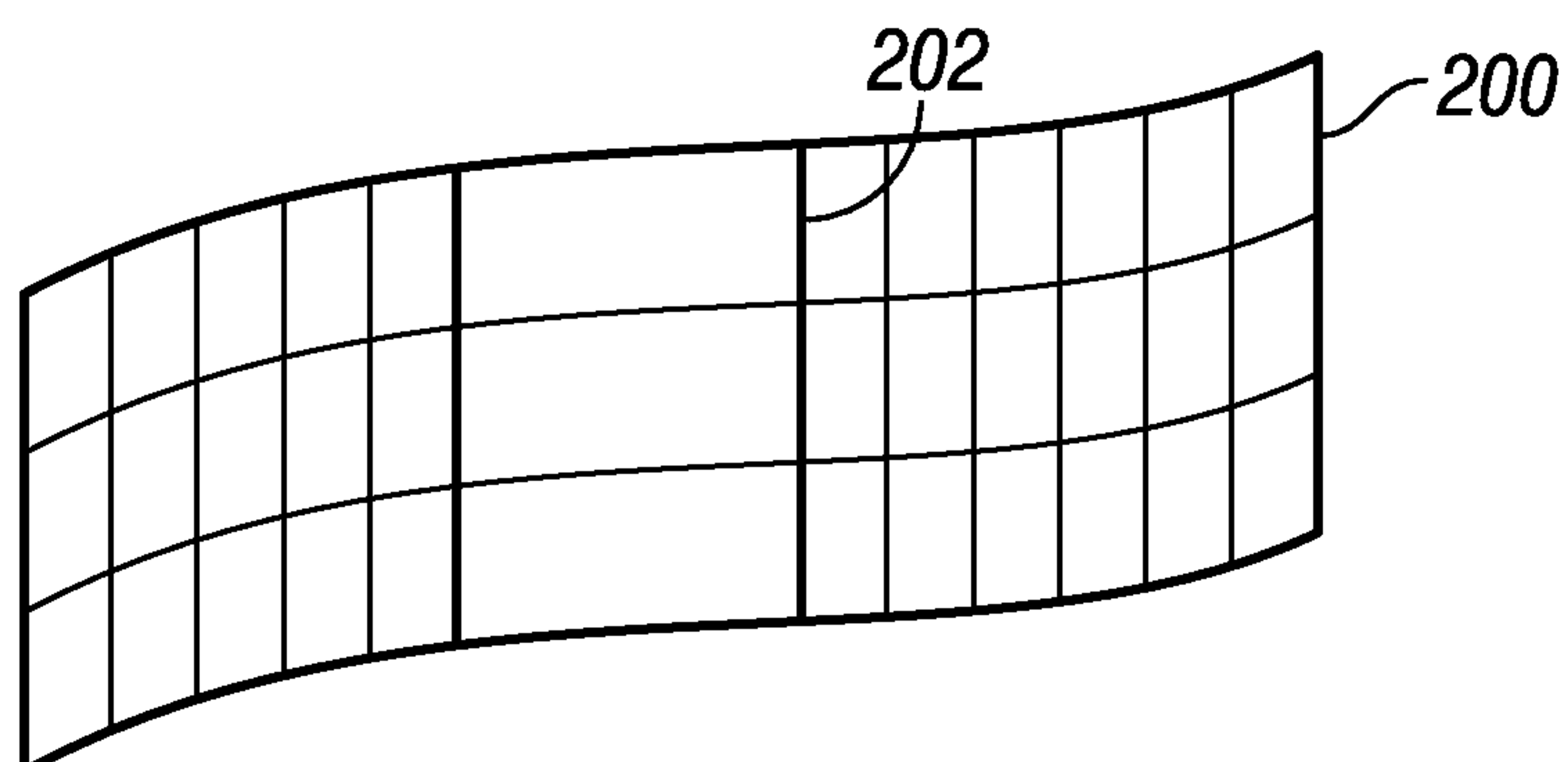
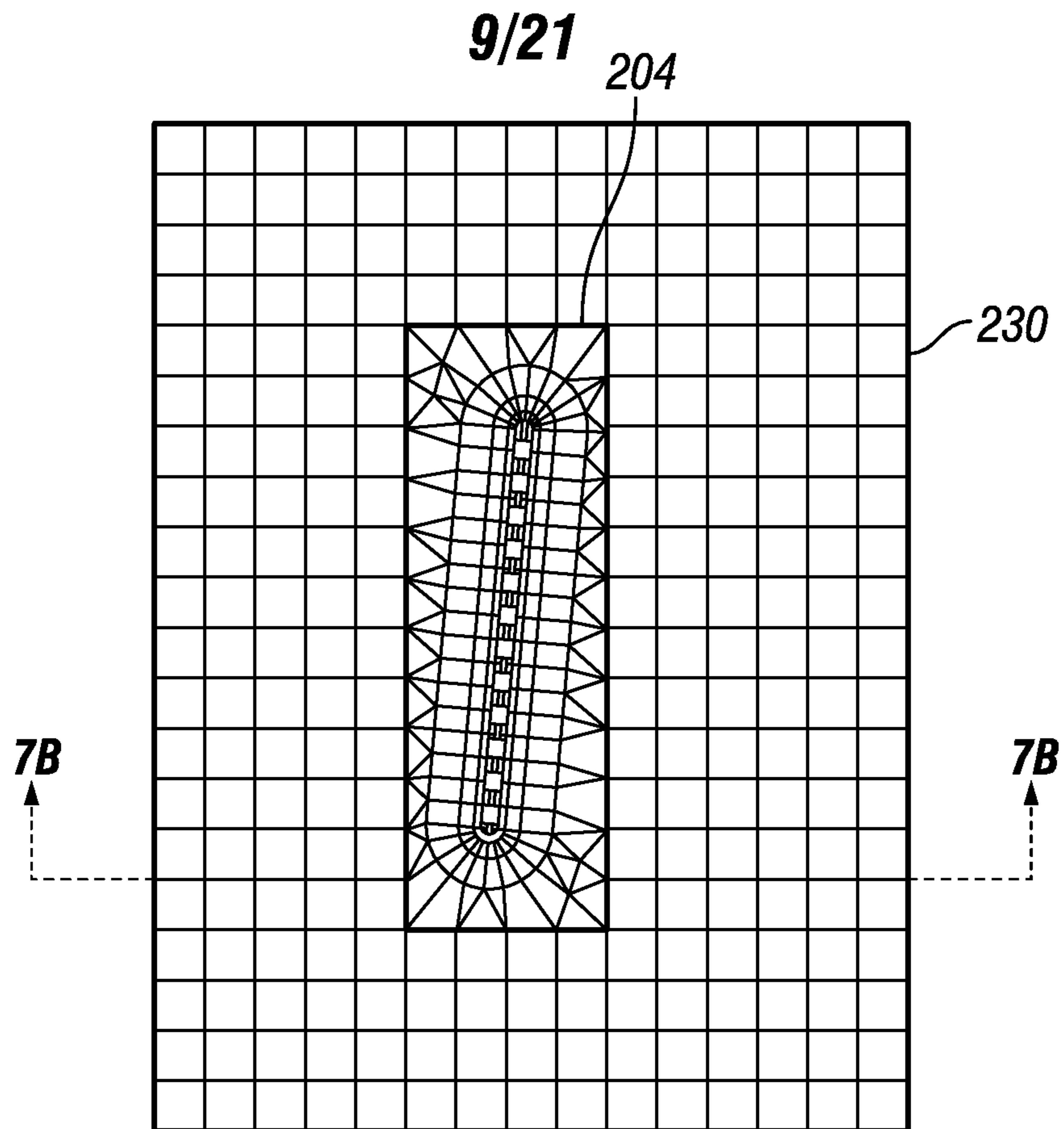
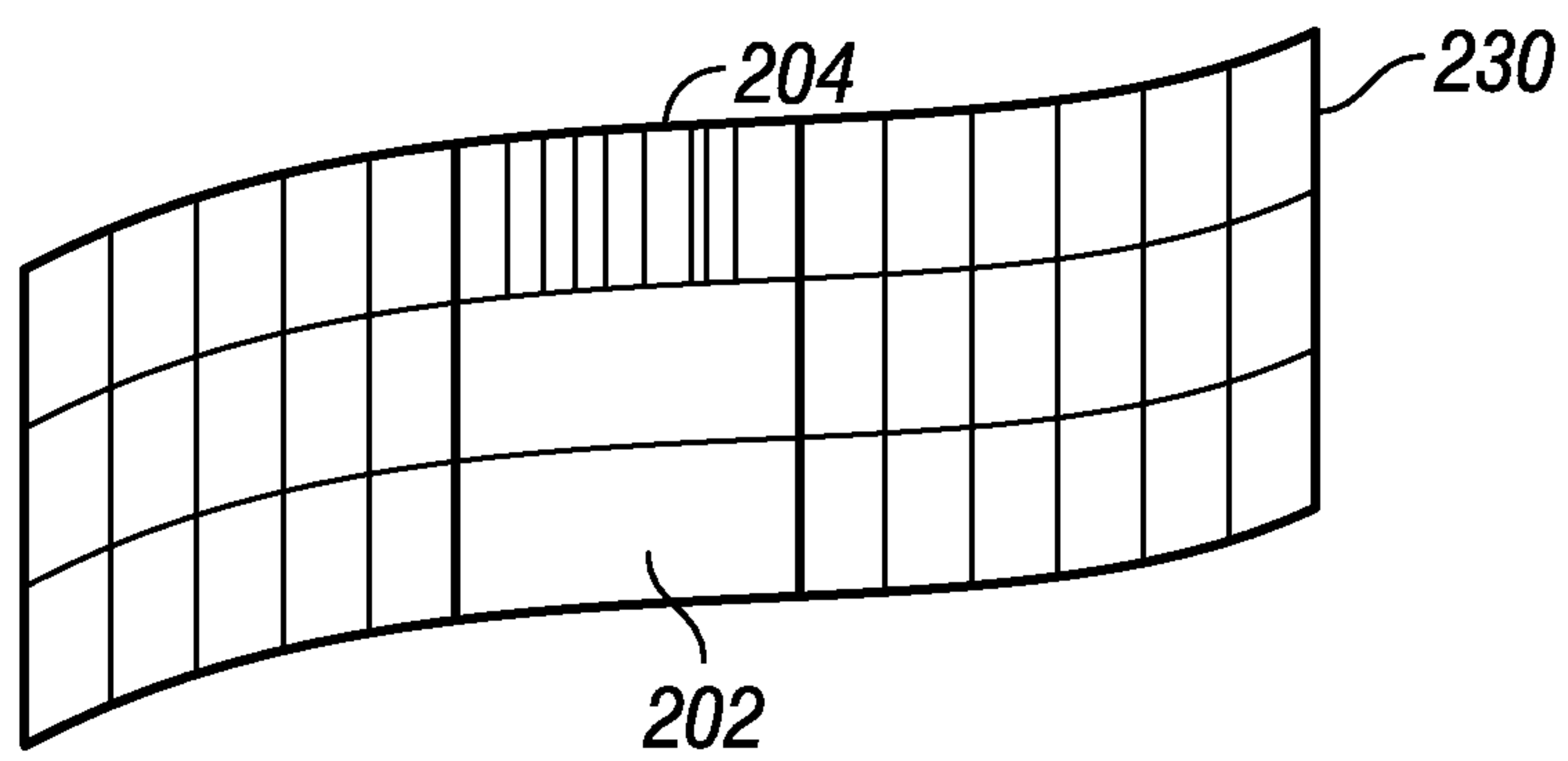
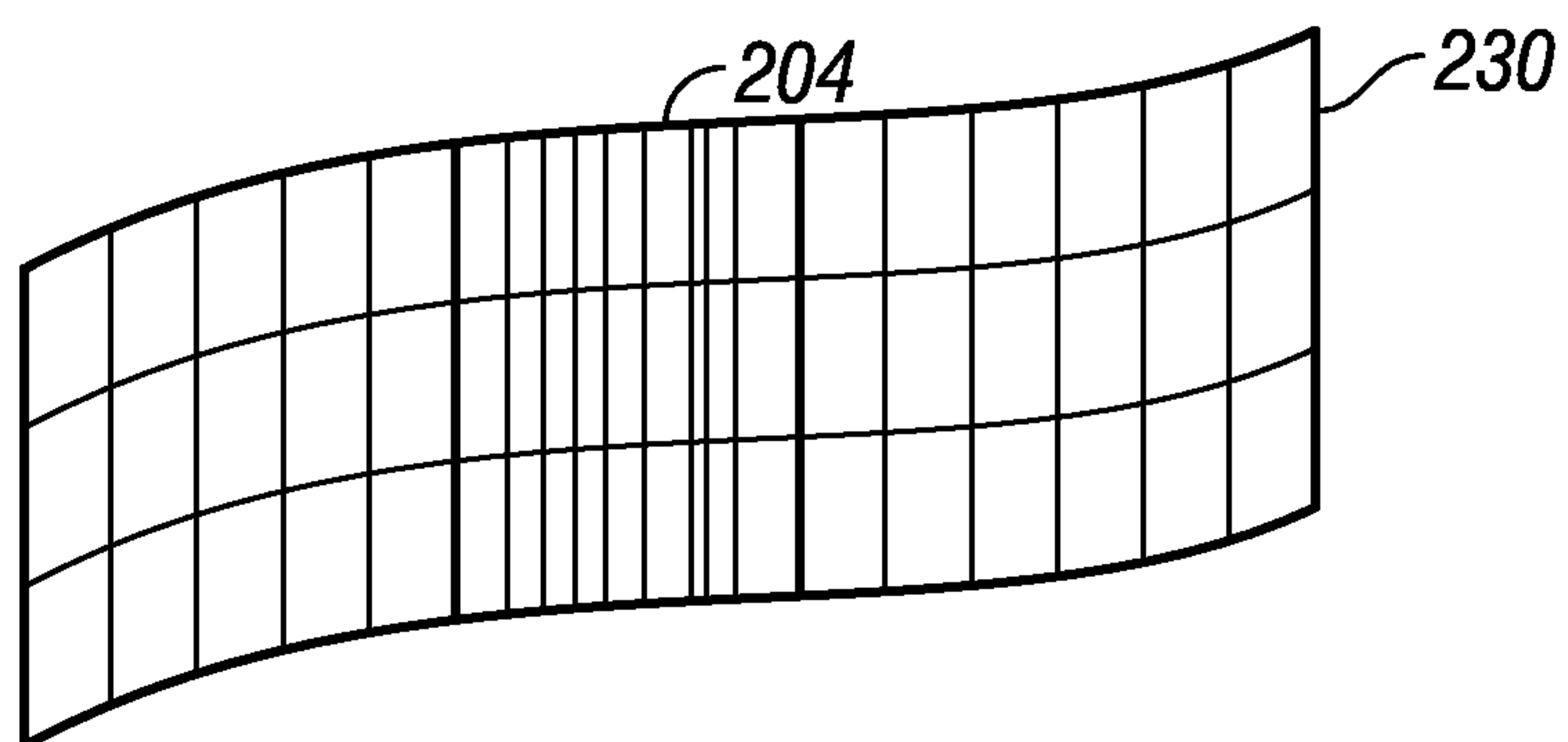
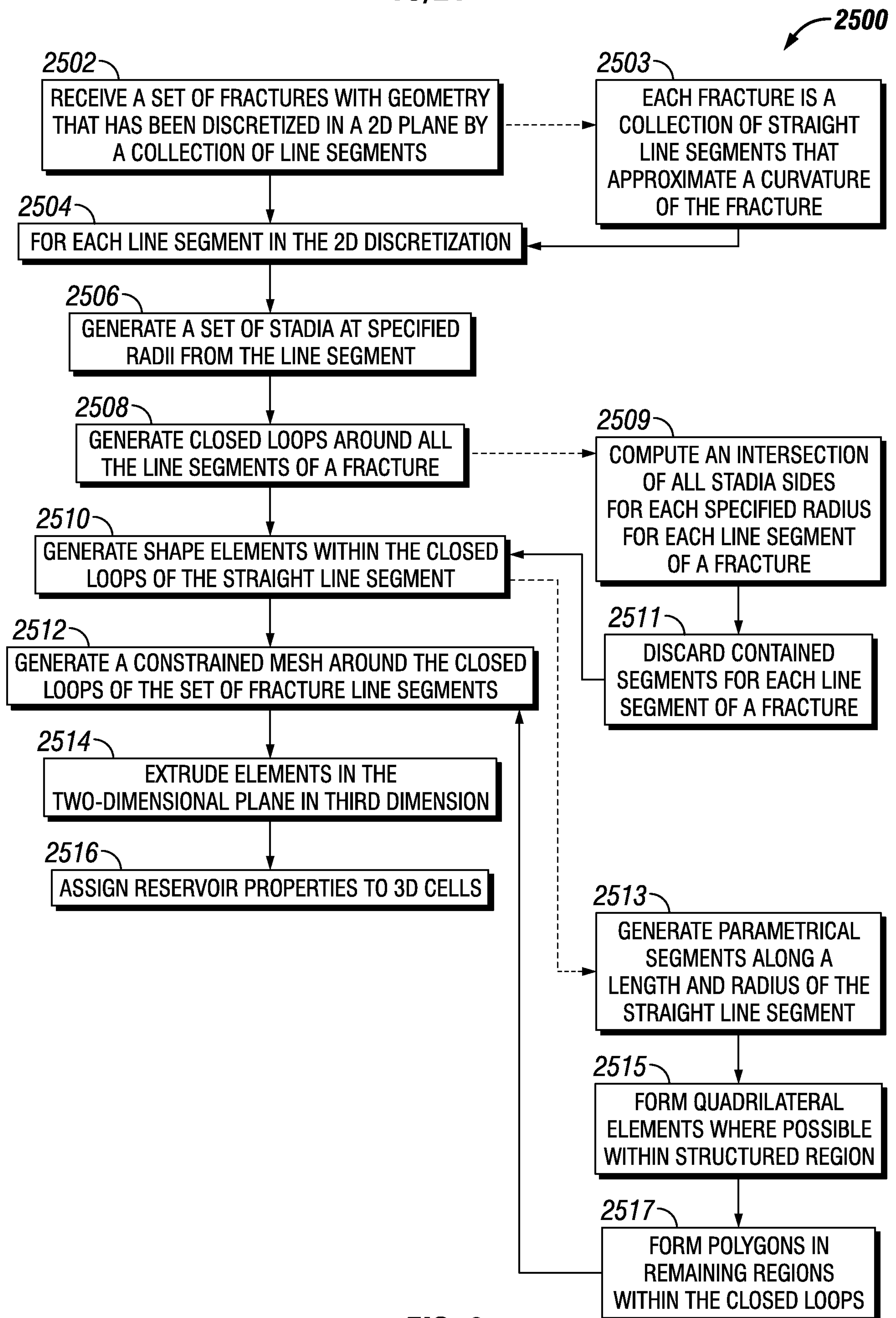


FIG. 4

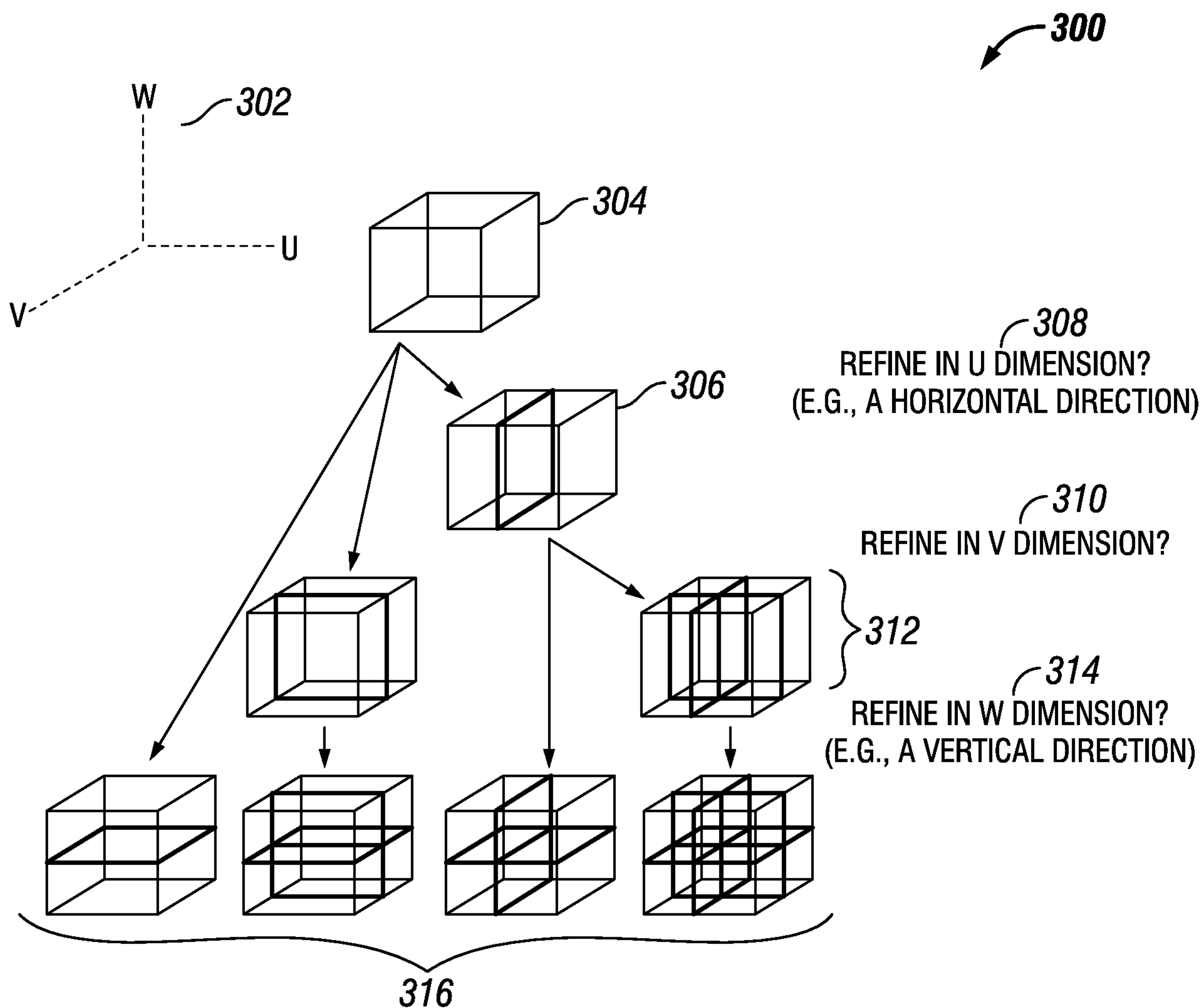
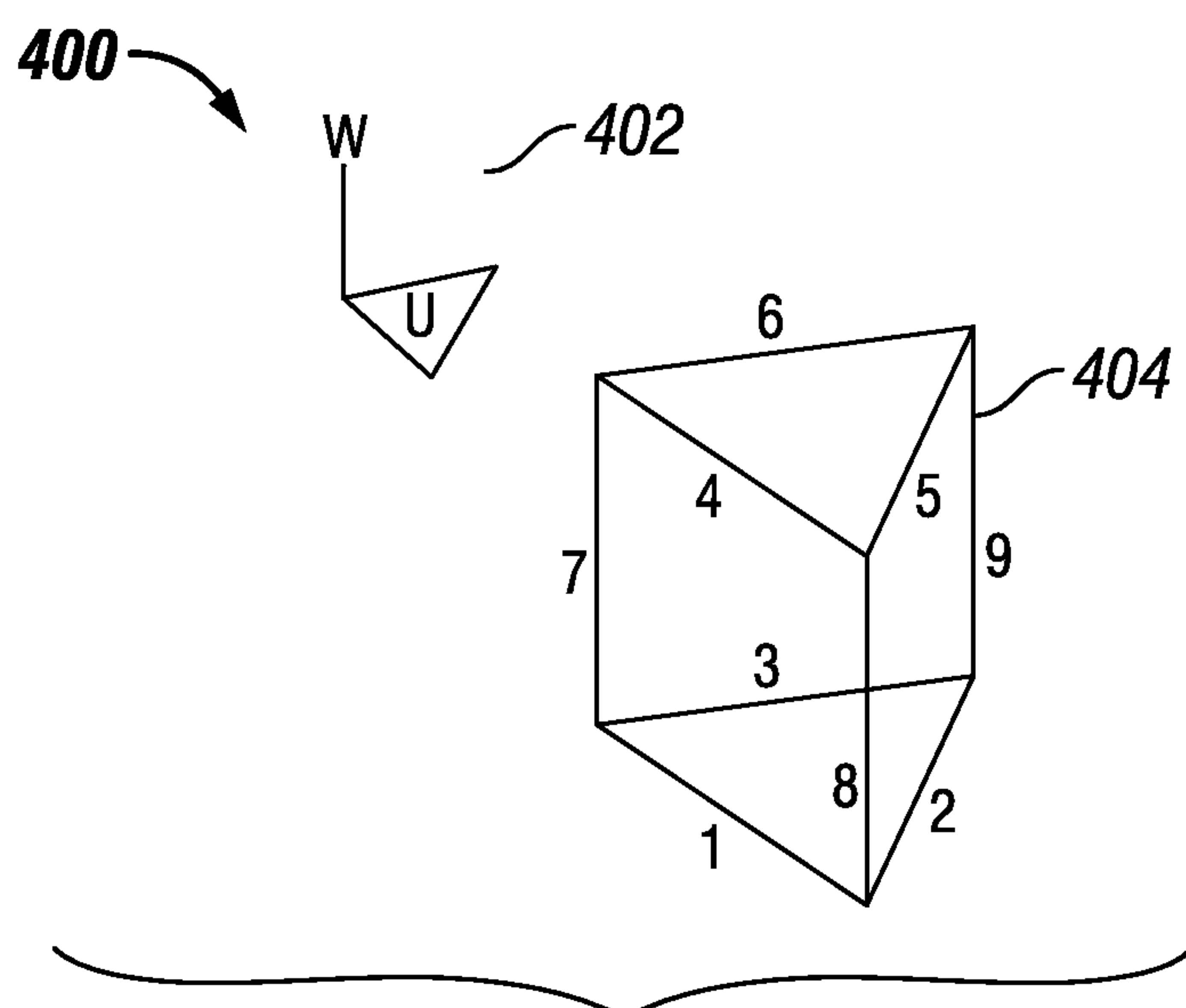
7/21**FIG. 5A****FIG. 5B**

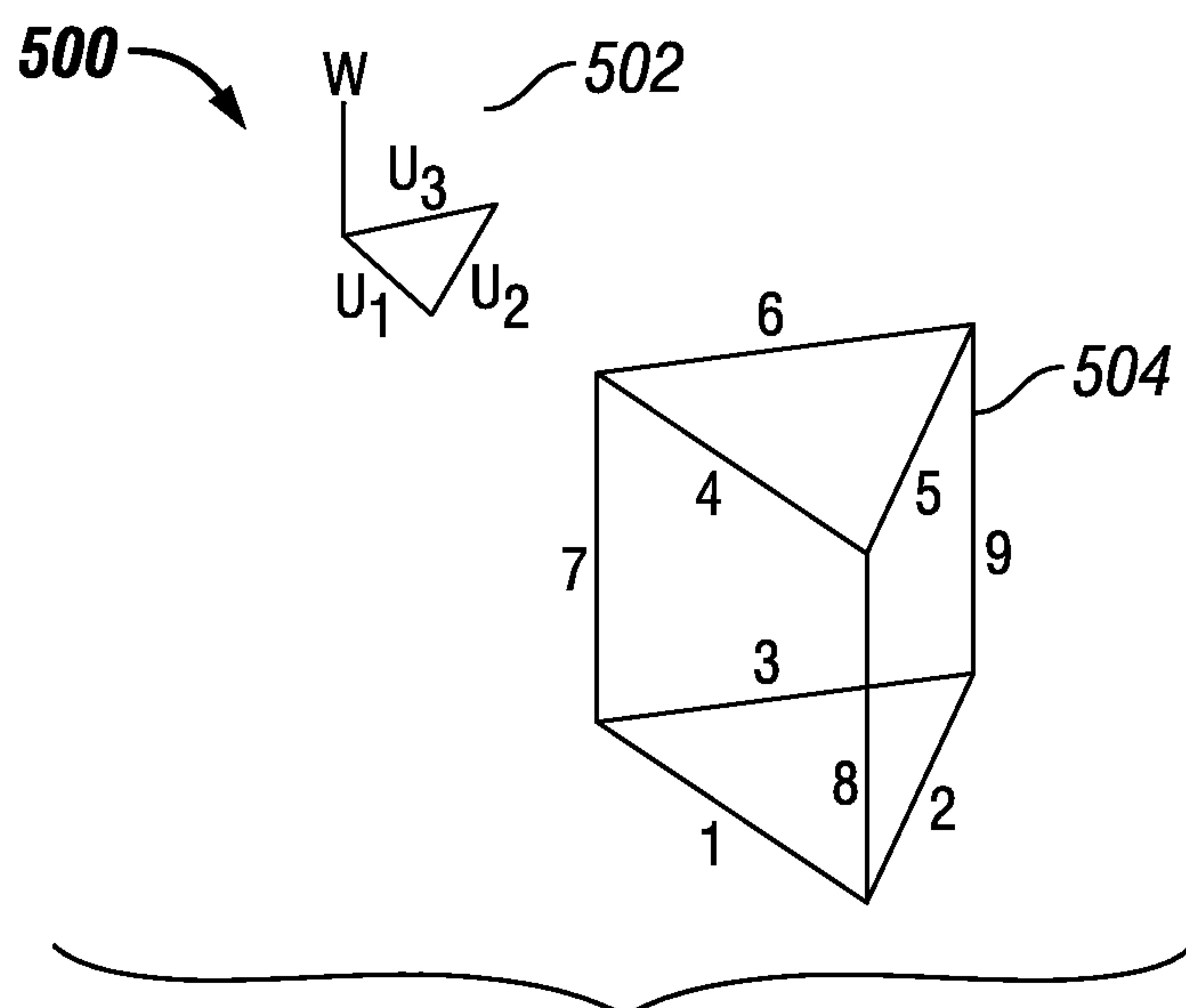
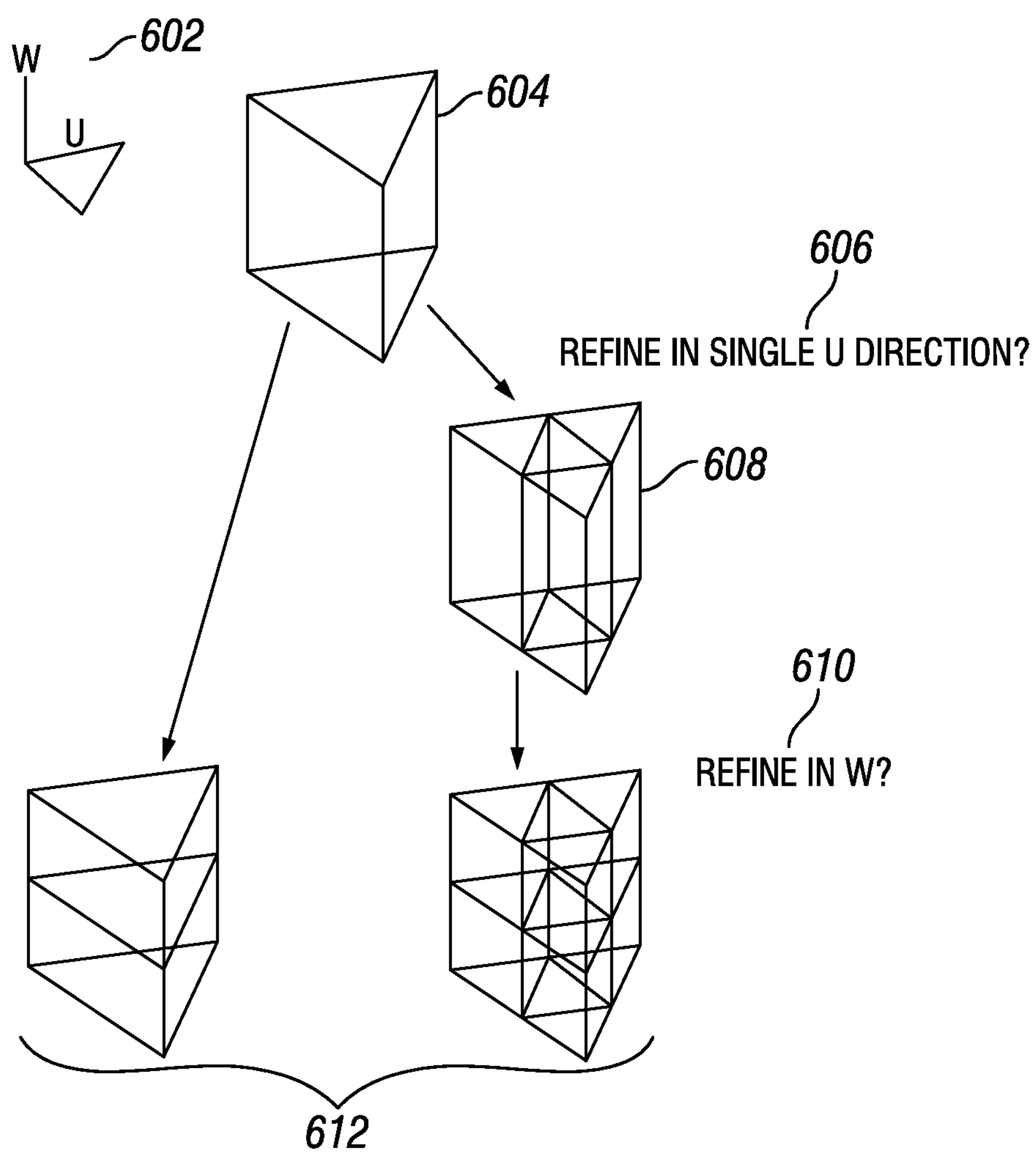
8/21**FIG. 6A****FIG. 6B**

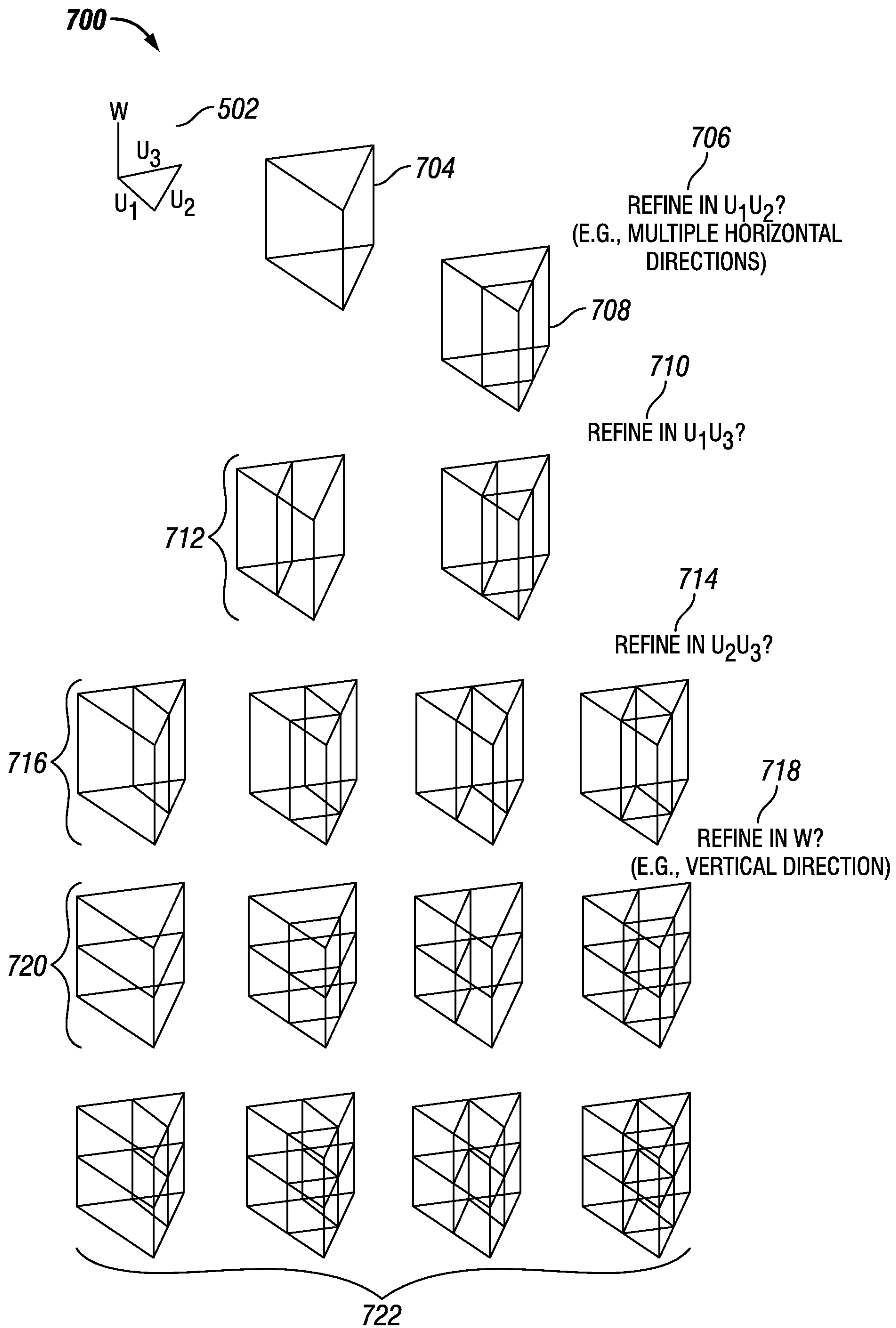
**FIG. 7A****FIG. 7B****FIG. 7C**

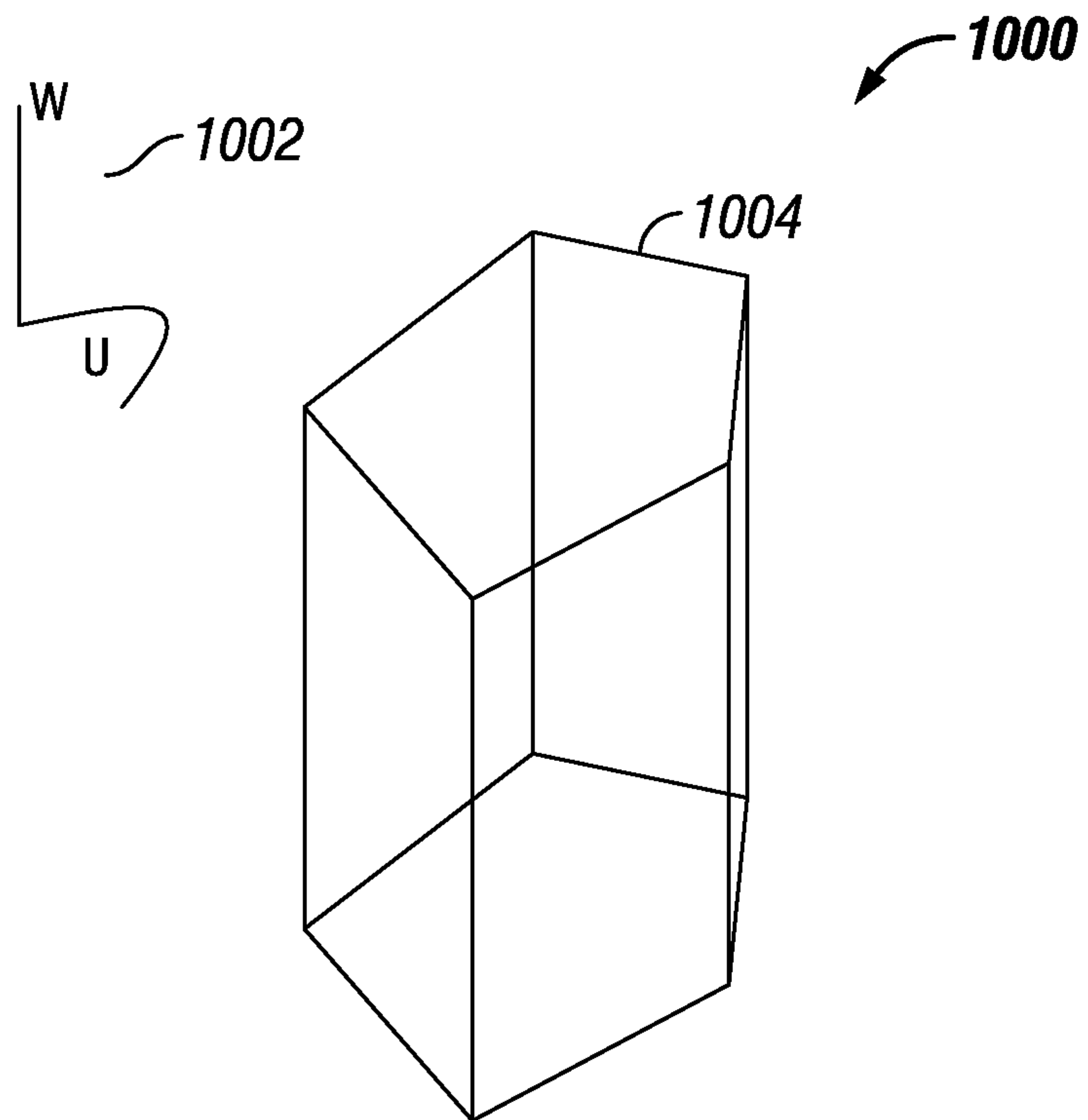
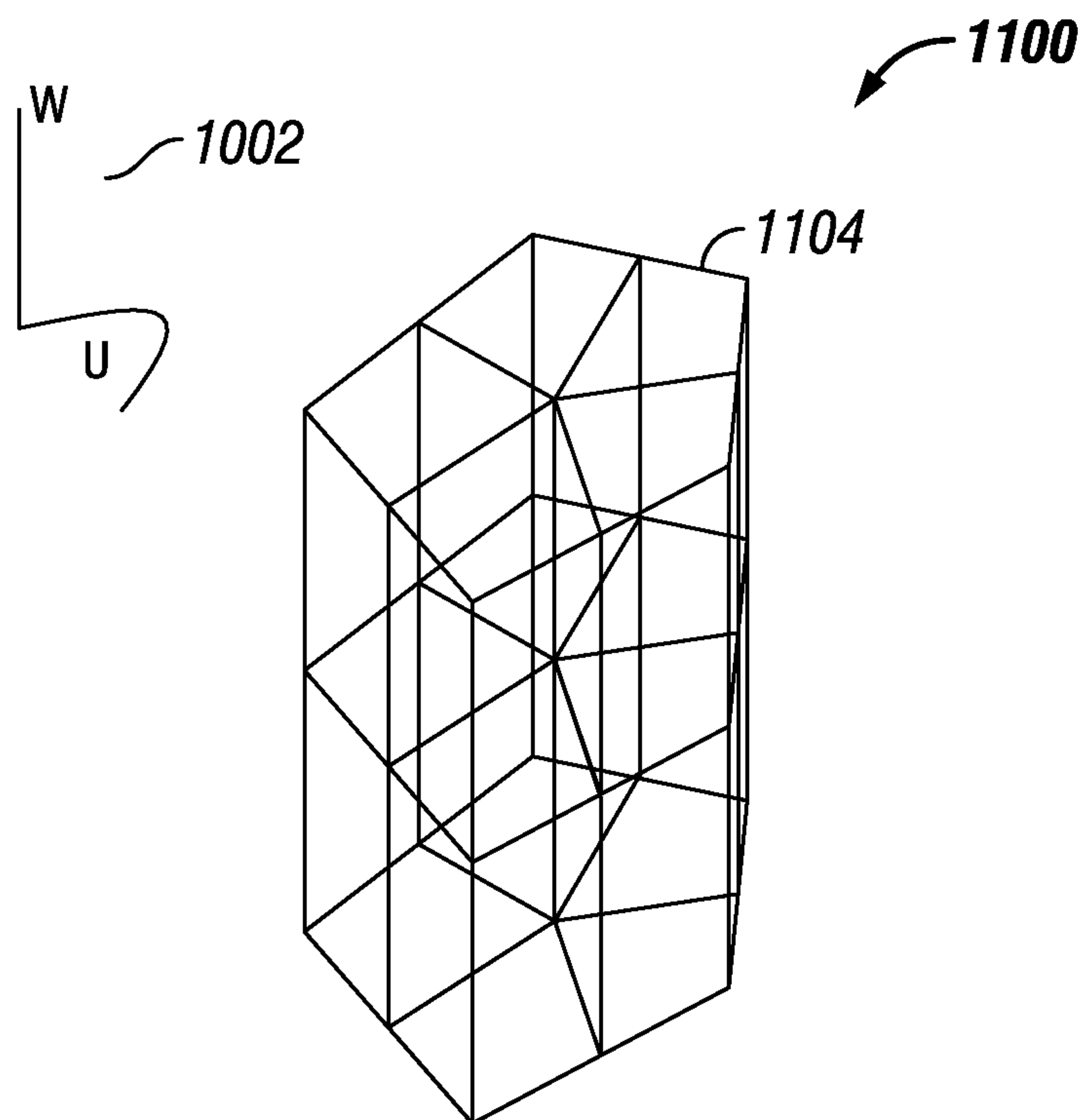
10/21**FIG. 8**

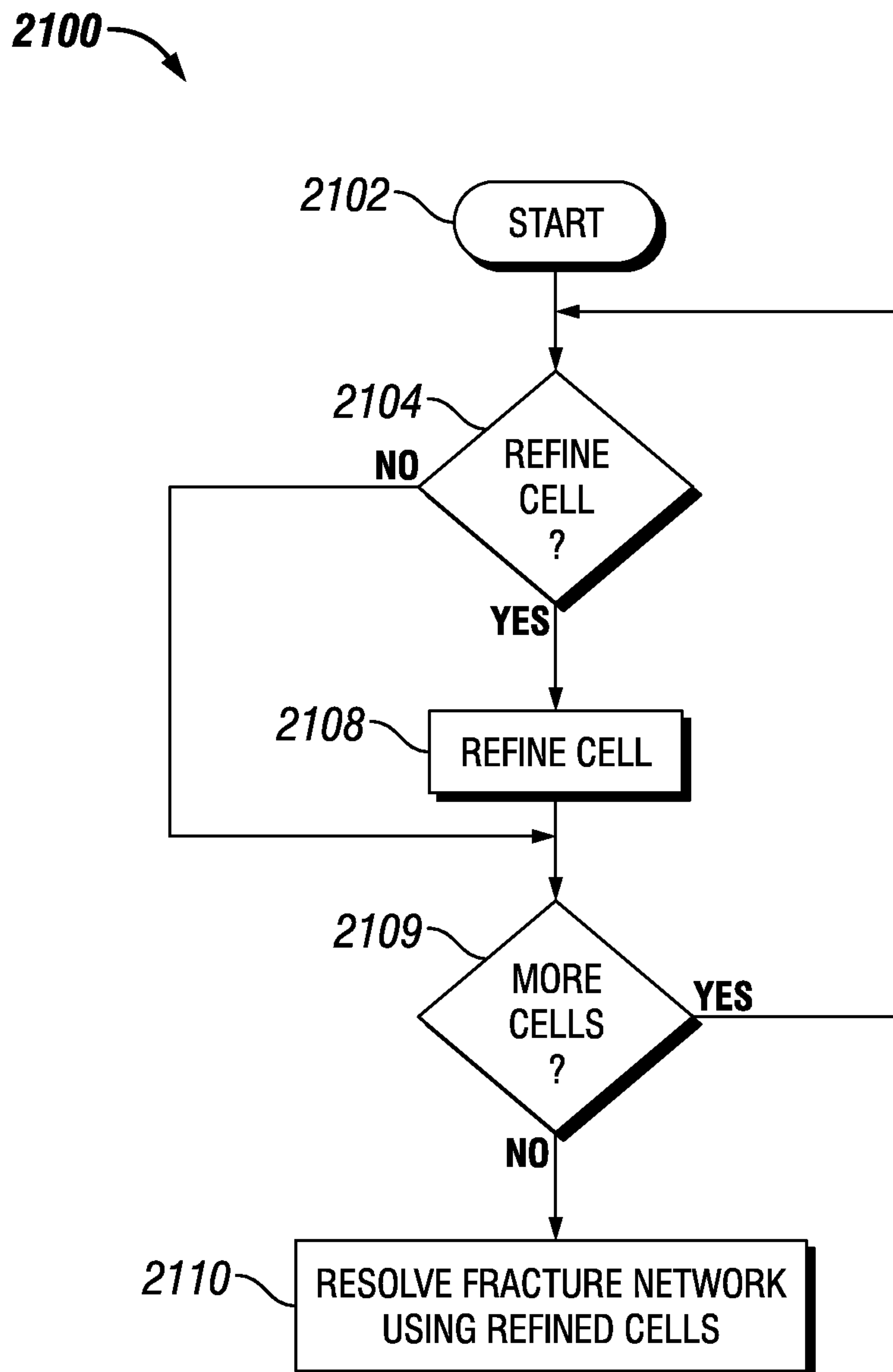
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**FIG. 9****FIG. 10**

12/21**600** →**FIG. 12**

13/21**FIG. 13**

14/21**FIG. 14****FIG. 15**

15/21**FIG. 16**

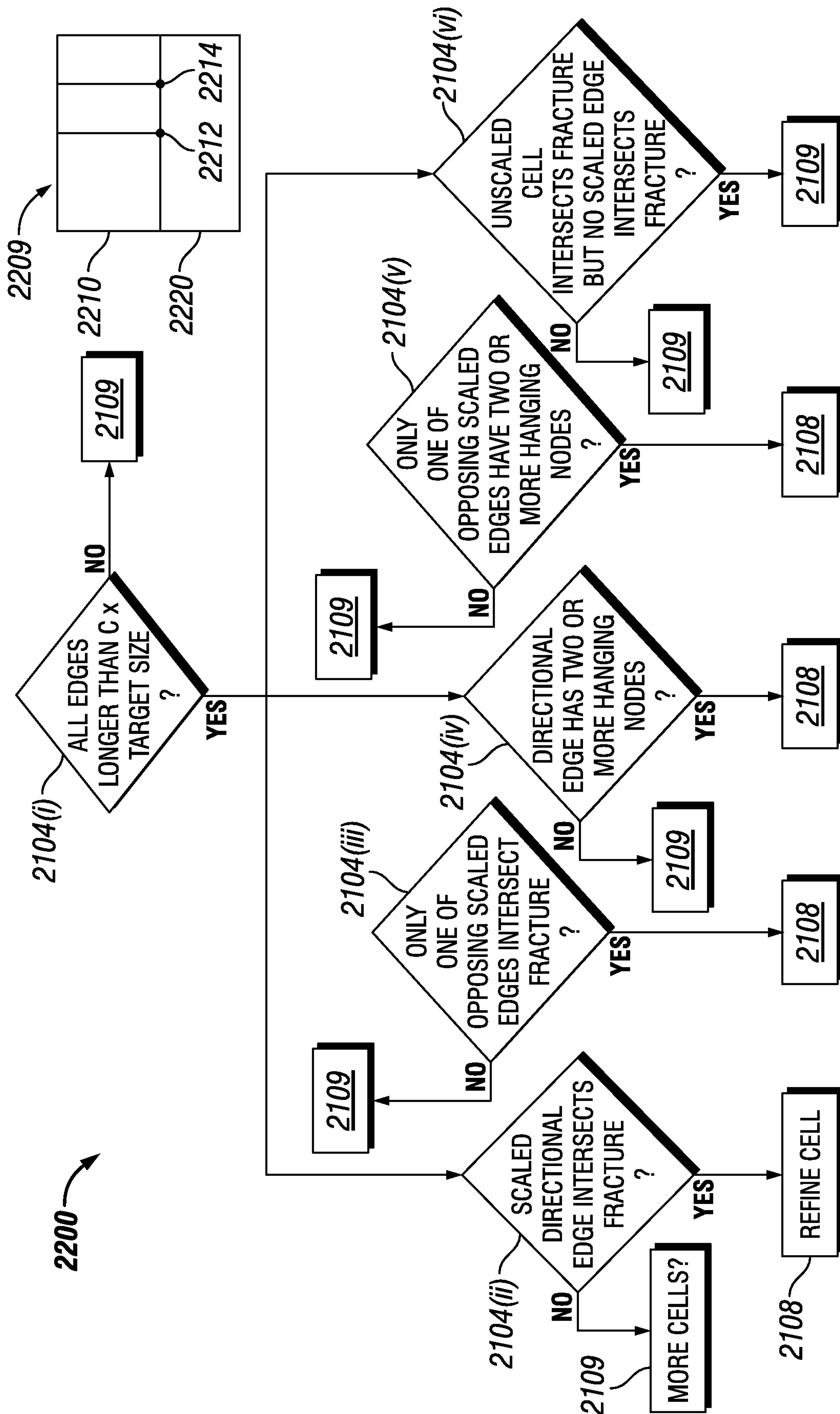


FIG. 17

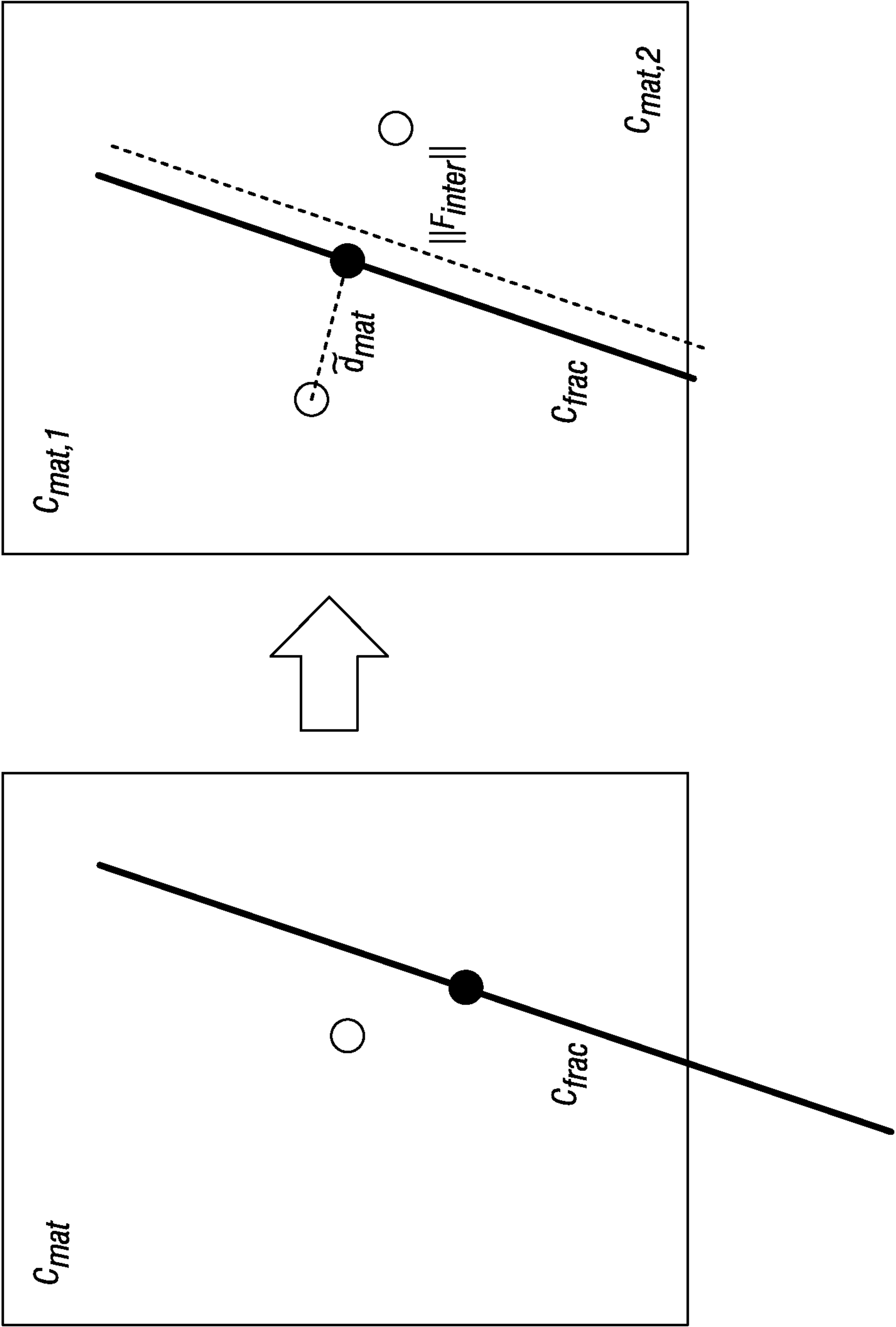


FIG. 18

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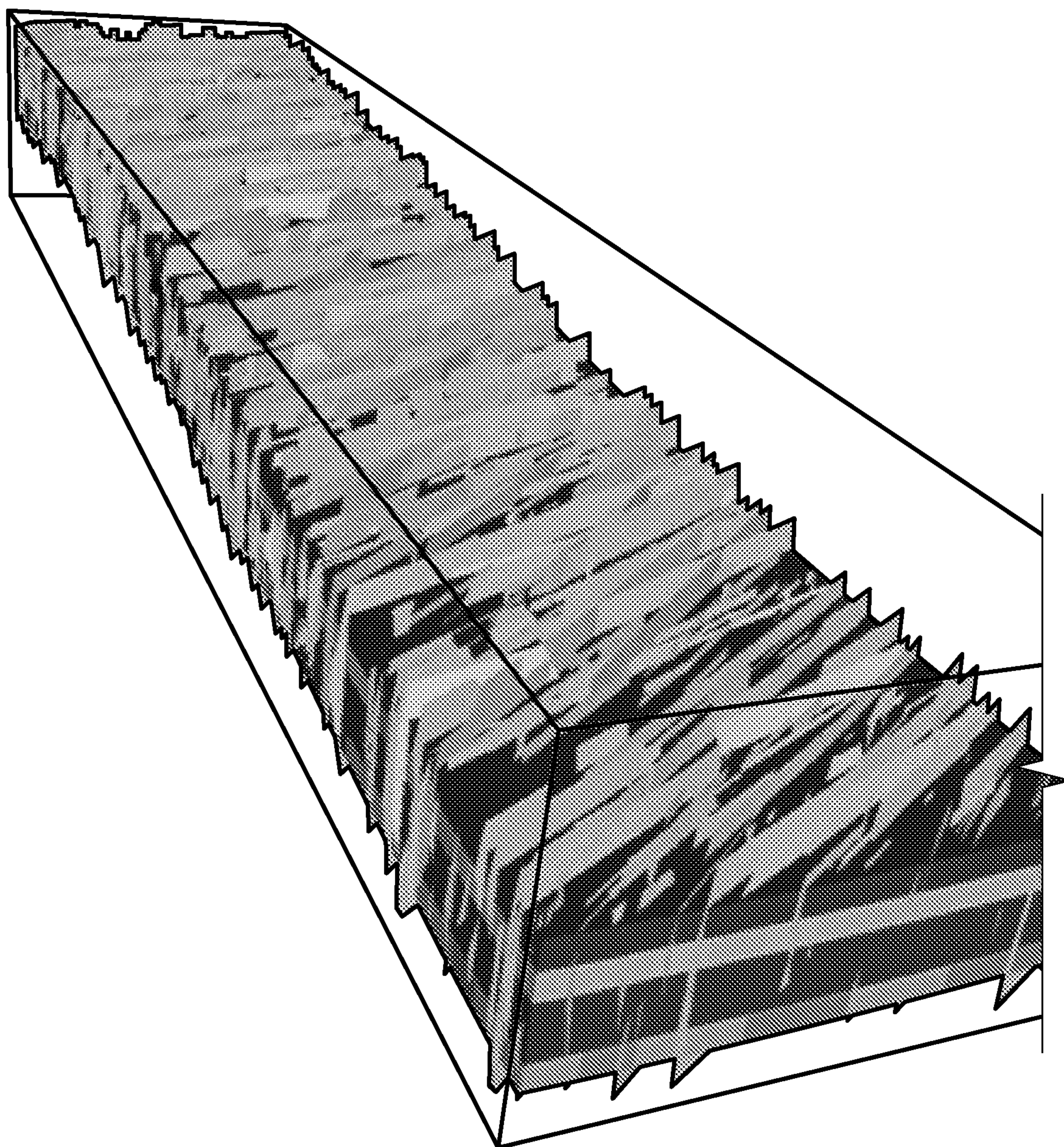


FIG. 19A

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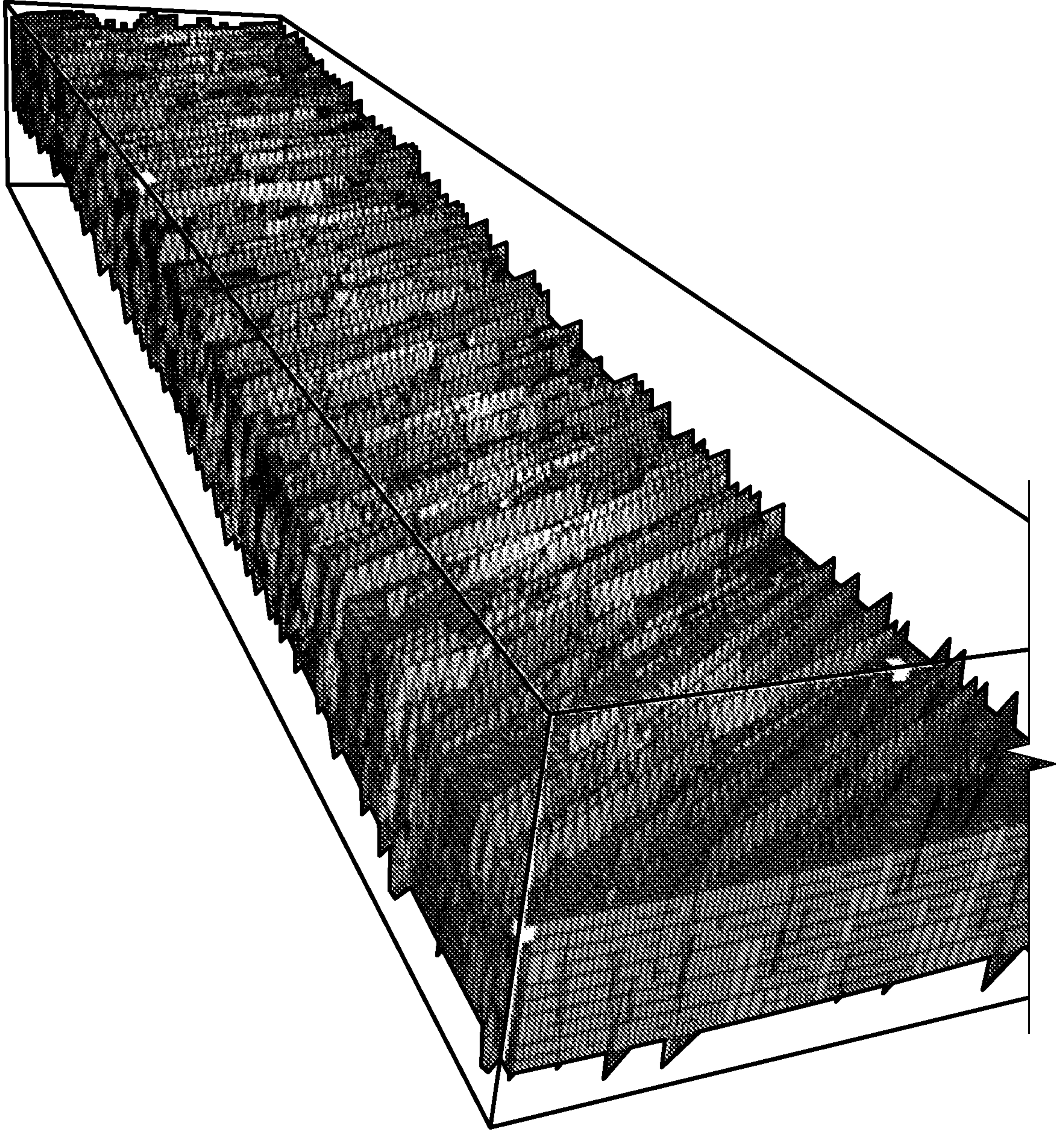


FIG. 19B

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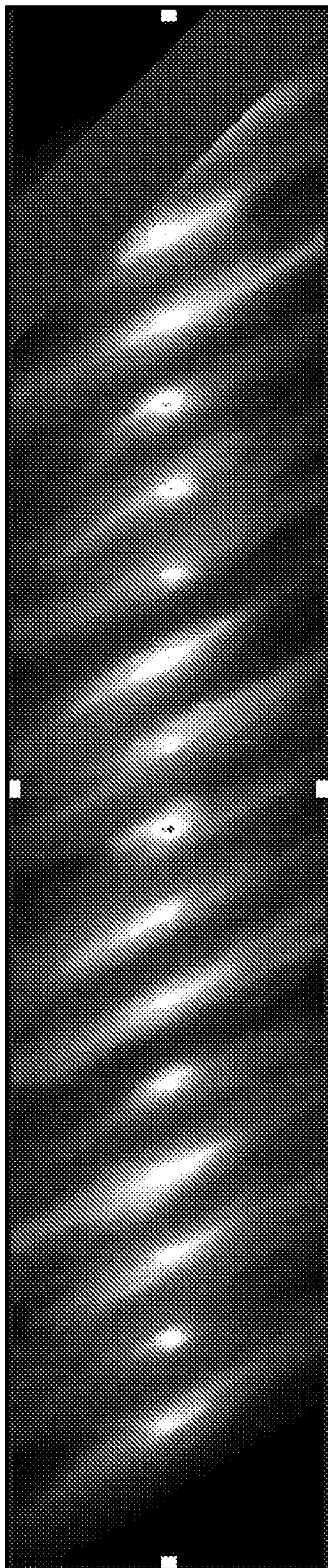
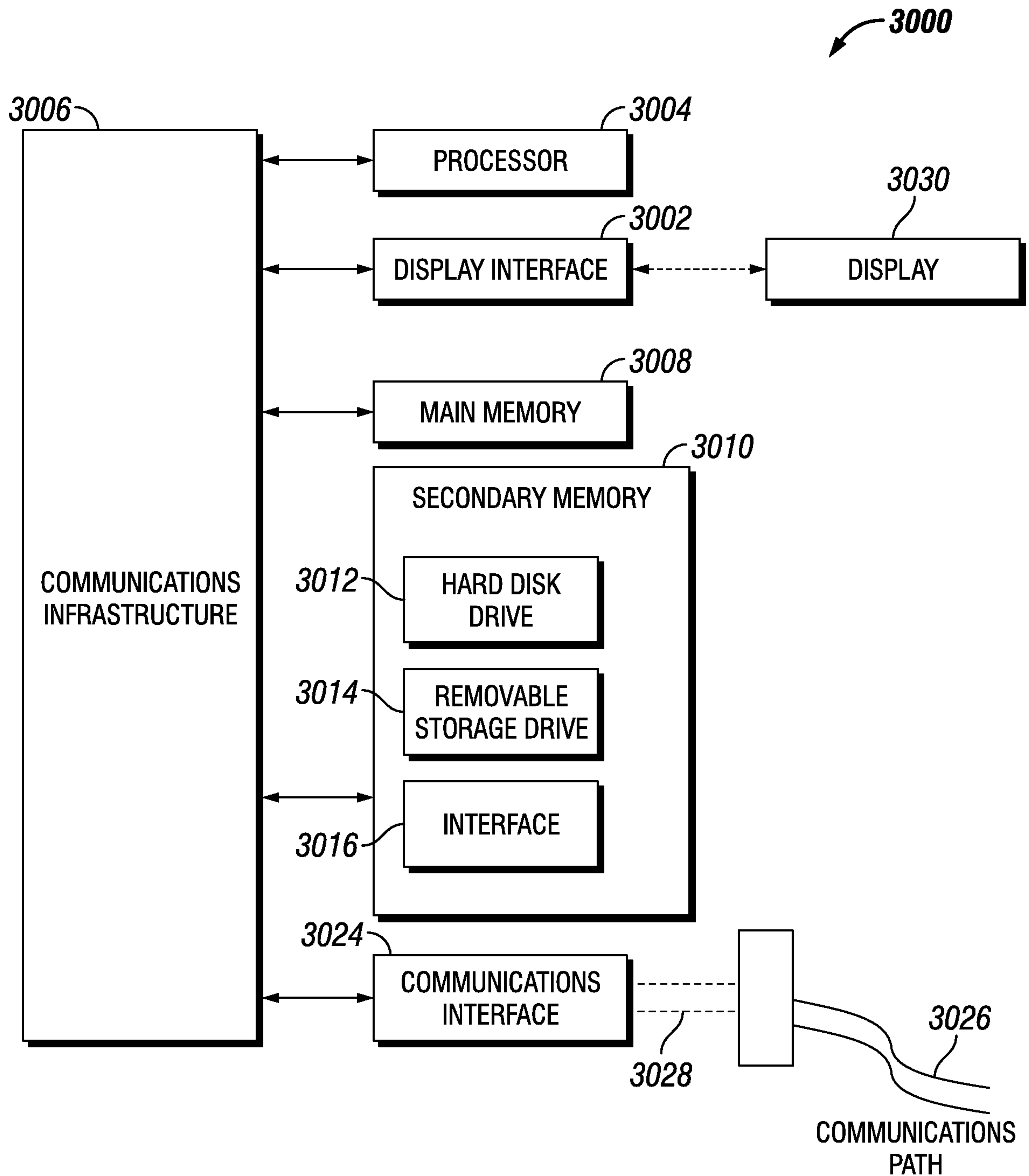


FIG. 19C

21/21**FIG. 20**

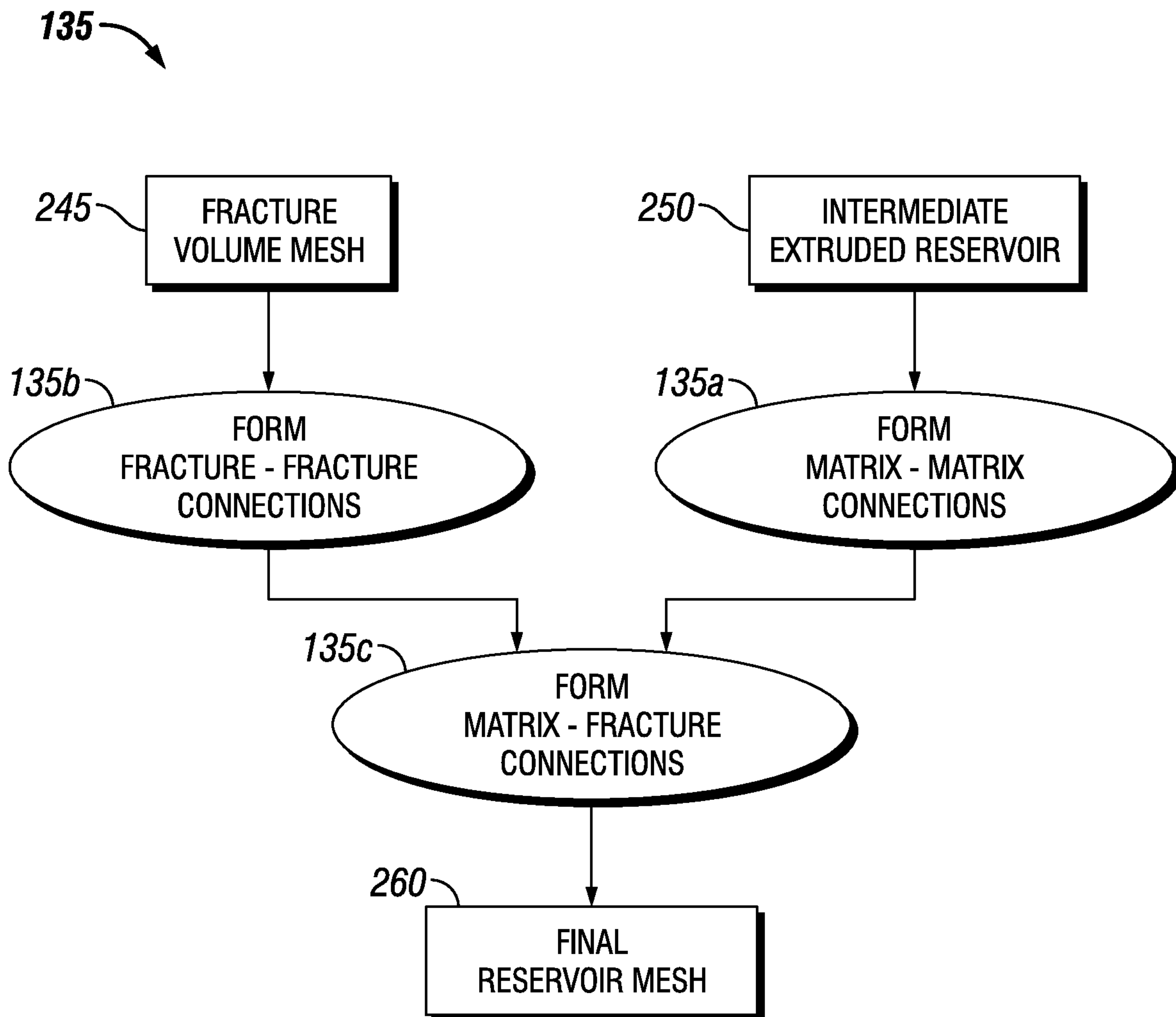


FIG. 1C